## Key:

## Green: Correct

Red: Problem has no solution if red in "Number" problem, otherwise this just means the given column didn't return a value or a "good" value

Yellow: Problem solvable is yellow in "Cond #" column, but not to machine precision. If in "Notes" column, means problem is solvable but not fast enough

## Black: Values are probably wrong, need to be recomputed

Blue: Function is unimplemented and/or values are uncomputed

| Num<br>ber | Continuous<br>? Smooth?<br>Where?  | Other Observations  | Preferred quadrature method                            | Integration Value  | Interval      | Cond #                      | Notes   |
|------------|--|---|--|--|---------------|-----------------------------|---|
| 1          | Yes, yes everywhere  | Should be very easy   | Clenshaw Curtis or Classical Gaussian/Gauss-Kronrod    | 1.718281828459045  | [0, 1]        | 1.5819767068693264<br>24385 | CC, p=270   |
| 2          | No, not at x=.3  | Constant on two separate intervals, derivative 0  | Adaptive Quadrature/Simpson's Rule                     | 0.70000000000000000  | [0, 1]        | 1.43                        | AS- tol=10^(-16)  |
| 3          | No, not at x=0   | N/A   | Adaptive Quadrature/Simpson's Rule                     | 0.3785300171241612   | [0, 1]        | 2.22                        | AS recurses infinitely. CC converges very slowly, accurate to about 11 digits with 100000 points. Not sure what to do on this one  QUADGK COMES CLOSEST                               |
| 4          | No, not at x= -1, this should be continuous on the interval given though | CONDITION NUMBER UNDERFLOWS IN DENOMINATOR -> ILL CONDITIONED PROBLEM   | Clenshaw Curtis or Classical<br>Gaussian/Gauss-Kronrod | -3.9062500000000000*10^-15   | [0, 10^(-12)] | BLOWS UP TO<br>INFINITY     | SEE IF YOU CAN BEAT<br>UNDERFLOW  |
| 5          | Yes, yes,<br>everywhere  | Spikes at x=.4  Possible cancellation errors?  Looking at graph should show why this one is continuous but not easy with gauss due to spike | Adaptive Quadrature/Simpson's Rule?                    | 0.1727205034243749<br>(PRETTY SURE THIS IS<br>WRONG)<br>0.1717416237343111 | [0, 1]        | 1.3 * 10^(-6)               | AS, tol = 10^(-10) should work in theory but doesn't.  Are we sure our integral value here is correct?? CC also seems to think the second digit is a 6 not a 7.  Yeah just checked on |

|    |   |  |   |   |          |  | wolfram and the value should<br>be closer to .163102. Weird<br>tho since both CC and AS<br>give values around .163494<br>QUAD and SIMPSON GIVE<br>EXACT SAME VALUE =><br>THAT IS CORRECT<br>(MAYBE??)   |
|----|---|--|---|---|----------|--|---|
| 6  | Yes, yes, everywhere                                | Should be easy   | Clenshaw Curtis or Classical<br>Gaussian/Gauss-Kronrod                            | 1.564396444069050   | [-1, 1]  | .32                                      | CC, p=270   |
| 7  | Yes, but with noise                                 | Noise, do a least squares  | Least Squares   | 1.692603132845149 (give or take .05 due to choice of epsilon) | [-1, 1]  | .32                                      | QUAD OR QUADGK<br>MIGHT BE CLOSE WHO<br>KNOWS   |
| 8  | Yes, yes, everywhere                                | Should be easy   | Clenshaw Curtis or Classical<br>Gaussian/Gauss-Kronrod                            | 1.000000000000000   | [0, 10]  | 6.67*10^(-107)                           | CC, p=270   |
| 9  | Yes, yes, everywhere                                | Possible cancellation errors in numerator Possible overflow errors in numerator and denominator. Should probably rewrite/simplify Simplifies to: $(E^x)^{-1/10} - (E^x)^{-9/10}/E^{-11}$ | Clenshaw Curtis or Classical<br>Gaussian/Gauss-Kronrod                            | 9.900536099712276   | [0, 100] | .00046                                   | CC, p=270, had to rewrite into (exp(x)).^(-1/10) - (exp(x)).^(9/10)./(exp(11*x)) first. Weird that we got this to machine precision, condition number may be off  |
| 10 | Yes, yes,<br>everywhere                             | Should be very easy  | Clenshaw Curtis or <mark>Classical</mark><br><mark>Gaussian</mark> /Gauss-Kronrod | 1.83333333333333  | [0, 1]   | 1.63636363636363636363636363636363636363 | CC, p=5000  |
| 11 | No, not<br>where e^x =<br>.5, 1.5, 2.5,<br>3.5 27.5 | Multiple discontinuities, Will be pretty tough Looking at graph helps a lot  | Adaptive Quadrature/Simpson's Rule  | 17.66446061011451   | [0, 3]   | 3.4??                                    | AS loses 13 digits, CC loses at least 10, looks like errorestimate in AS is giving some kind of 0 error like a cancellation error, AS does not appear to be working properly; continues running even when it gives errorestimate = 0.  If true value is 17.66 etc. then quadgk comes closest. Else, |

|    |  |   |  |                    |        |                    | if true value is 17.78 etc.then quad comes closest  |
|----|--|---|--|--------------------|--------|--------------------|---|
| 12 | Yes, yes, everywhere                               | Should be easy                                    | Clenshaw Curtis or Classical Gaussian/Gauss-Kronrod    | 0.400000000000000  | [0, 1] | 2.5000000000000000 | Works with felencurt for high n values, CC, p=2000  |
| 13 | No, not at x=0                                     | N/A   | Adaptive Quadrature/Simpson's Rule                     | 2.000000000000000  | [0, 1] | .5                 | AS appears not to converge (may not be properly written) Quadgk comes very close but loses 3 digits instead of 1                  |
| 14 | No, not continuous when x=3. Not smooth at 1 or 3. | N/A   | Adaptive Quadrature/Simpson's Rule                     | 7.50000000000000   | [0, 5] | 1.333              | AS is off by approximately 10^(-16). Is this good? What does this mean? That is, AS gives 7.4999999999999999999999999999999999999 |
| 15 | Yes, yes, everywhere                               | Should be easy<br>Should be similar to integral 6 | Clenshaw Curtis or Classical<br>Gaussian/Gauss-Kronrod | 0.8669729873399110 | [0, 1] | .57                | CC, p=270   |

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| Number | Continuous? Smooth?<br>Where? | Other Observations                    | Preferred quadrature method | Integration Value | Interval | Cond numb | Notes                               |
|--------|-------------------------------|---------------------------------------|-----------------------------|-------------------|----------|-----------|-------------------------------------|
| 16     | Not continuous at x=0         | Not continuous<br>Oscillates heavily. | Clenshaw Curtis             | 0.498986808693045 | [0, 1]   | 0         | CC, p=2000, QUADGK<br>IS VERY CLOSE |

|    |  | This is bad. Can't use either may need to combine and do a piecewise gauss  |   |                             |               |        | LOSES 2 DIGITS  |
|----|--|---|---|-----------------------------|---------------|--------|---|
| 17 | No, not at $x=-1$                        | N/A   | Adaptive<br>Quadrature/Simpson's<br>Rule                  | -1                          | [0, 1]        | 0      | AS appears not to converge (may not be properly written).   |
|    |  |   |   |                             |               |        | QUAD gets it for tol = $10^{(-16)}$   |
| 18 | Yes, yes, everywhere on its domain       | N/A   | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod |                             | [-1, 1]       |        |   |
| 19 | Yes, yes, everywhere on its domain       | N/A   | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod | -0.000200020002000200<br>0  | [-1, 1]       | 4999.5 | CC, p=5000, have to take real part for some weird reason  |
| 20 | Yes, yes, everywhere (it's a polynomial) | Computing the det as a polynomial is going to be hard   | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod |                             | [-1, 1]       |        |   |
| 21 | Discontinuous on x=0                     | N/A   | Adaptive<br>Quadrature/Simpson's<br>Rule                  | 0.3817732906760363          | [-1, 1]       | 1.41   | AS appears not to converge (may not be properly written)  |
|    |  |   |   |                             |               |        | EVERYTHING GIVES6366 SOMETHING IS THE VALUE WRONG??? ALL OF THEM ARE VERY VERY CLOSE, CC, QUAD, and QUADGK agree up to the last digit |
| 22 | Yes, yes, everywhere                     | Numerator is very small<br>compared to denominator<br>May have to integrate<br>first and then divide by<br>constant | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod | 1.273239544735163*10^-<br>9 | [-1, 1]       | 0      | CC, p=270   |
| 23 | Yes, yes                                 | Approaches 0 as x approaches infinity   | Adaptive<br>Quadrature/Simpson's                          | 1.000000000000000           | [0, infinity] |        | AS appears not to converge (may not be  |

|    |                               | (quickly) (for x greater<br>than 36.5 the function<br>underflows below<br>machine precision<br>Will need something<br>piecewise most likely | Rule??  |  |         |     | properly written) RECHECK MAYBE WE CAN FIGURE OUT HOW TO BEAT THE UNDERFLOW |
|----|-------------------------------|---|---|--|---------|-----|---|
| 24 | Discontinuous at x=0          | N/A   | Adaptive<br>Quadrature/Simpson's<br>Rule                  | .2375252341425132                        | [0,.5]  | 0   |   |
| 25 | Discontinuous at endpoints??? | N/A   | Adaptive<br>Quadrature/Simpson's<br>Rule                  |  | [0,1]   |     |   |
| 26 | Yes, yes, everywhere          | N/A   | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod | -0.5818757877653272                      | [0, pi] | 0   | CC, p=270   |
| 27 | Yes, yes, everywhere          | N/A   | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod | 2.925303491814363                        | [-1,1]  | .93 | CC, p=600   |
| 28 | Discontinuous at x=0          | INTEGRAL<br>APPARENTLY<br>DIVERGES  | Adaptive<br>Quadrature/Simpson's<br>Rule                  | -124.8993729727460<br>(doesn't converge) | [0,1]   |     |   |
| 29 | Yes, yes, everywhere          | Possible cancellation errors around x=0??   | Clenshaw Curtis or<br>Classical<br>Gaussian/Gauss-Kronrod | 1.047197551196598                        | [5,.5]  | .55 | CC, p=4600  |
| 30 | Discontinuous at x=0          | Also equals log(x)  | Adaptive<br>Quadrature/Simpson's<br>Rule                  | -1.000000000000000                       | [0,1]   | 0   | Quad gets it for tol<br>=10^(-16)   |

16. Sin[100 Pi x]/(Pi x)

17. Log[x]

18. Cos[10000 ArcCos[x]]

19. Cos[100 ArcCos[x]]

20. ???

21. Piecewise[ $\{\{\sin[x], -1 \le x \le 0\}, \{\cos[x], 0 \le x \le 1\}\}$ ]

22. Cos[Pi (x/2)]/10^9

23. E^(-x)

24. Log[2 x]/Log[x]

- 25. ???
- 26. Sin[5 Sin[x]]
- 27. E^x^2
- 28. Sin[x]/Log[x]
- 29. 1/Sqrt[1 x^2]
- 30. Log[2 x] Log[2]

SetPrecision[N[FullSimplify[Integrate[f9[x], {x, 0, 100}]]], 16]