Rational Inequalities

Solving rational inequalities is very similar to how we solve polynomial inequalities except in addition to finding the zeros of the rational function we also find the points where it is undefined

Step-By-Step Procedure

Step 1 Move every term to one side of the inequality

Step 2 Put everything underneath a common denominator

Step 3 Find the points where the function is undefined by setting the denominator equal to 0 and solving

Step 4 Ignore the denominator and set the numerator equal to 0 to find the points where the function is 0

Step 5 Draw a number line and plot the points where the function is undefined and where the function is 0.

Test points inside each interval to see if the function is positive or negative

Step 6 See which interval(s) make the inequality true and write in interval notation.

$\boxed{\text{Example 1}}$ Solve $\dfrac{3t^2}{t+2} \geq 5t$

Step 1

$$\frac{3t^2}{t+2} - 5t \geq 0$$

Step 2

$\boxed{\text{LCD: } t+2}$

$$\frac{3t^2}{t+2} - 5t \cdot \frac{t+2}{t+2} \geq 0$$

$$\Rightarrow \frac{3t^2}{t+2} - \frac{5t(t+2)}{t+2} \geq 0$$

$$\Rightarrow \quad \frac{3t^2 - 5t(t+2)}{t+2} \geq 0$$

$$\Rightarrow \quad \frac{3t^2 - 5t^2 - 10t}{t+2} \geq 0$$

$$\Rightarrow \quad \frac{-2t^2 - 10t}{t+2} \geq 0$$

Step 3

$$t+2=0 \Rightarrow t=-2$$

$$\boxed{\text{Undefined at } t=-2}$$

Step 4

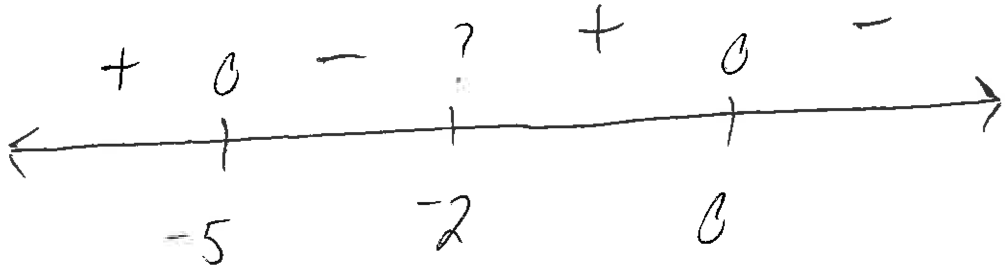
$$-2t^2-10t=0$$

$$\Rightarrow \quad -2t(t+5)=0$$

$$\Rightarrow \quad t=0 \text{ or } t=-5$$

So the function is zero at $t=-5, 0$

Step 5



$\underline{(-\infty, -5)}$

$\boxed{t=-6}$

$$\rightarrow \frac{-2(-6)(-6+5)}{-6+2} = \frac{+(-)}{-} = +$$

$\underline{(-5, -2)}$

$$t=-3 \Rightarrow \frac{-2(-3)(-3+5)}{-3+2} = \frac{+(+)}{-} = -$$

$\underline{(-2, 0)}$

$$t = -1 \Rightarrow \frac{-2(-1)(-1+5)}{-1+2} = \frac{+(+)}{+} = +$$

$\underline{(0, \infty)}$

$$t = 1 \Rightarrow \frac{-2(1)(1+5)}{1+2} = \frac{-(+)}{+} = -$$

Step 6

Inequality: $\frac{3t^2}{t+2} - 5t \geq 0$

$$\boxed{\text{Sol'n: } (-\infty, -5] \cup (2, 0]}$$

$\boxed{\text{Example 2}}$ $$\frac{2}{2x-3} - \frac{1}{x+1} \leq \frac{1}{2x^2-x-3}$$

Step 1

$$\frac{2}{2x-3} - \frac{1}{x+1} - \frac{1}{2x^2-x-3} \leq 0$$

Step 2

$$\frac{2}{2x-3} - \frac{1}{x+1} - \frac{1}{(2x-3)(x+1)} \leq 0$$

$$\Rightarrow \frac{2(x+1)}{(2x-3)(x+1)} - \frac{(2x-3)}{(2x-3)(x+1)} - \frac{1}{(2x-3)(x+1)} \leq 0$$

$$\Rightarrow \frac{2x+2 \;-2x+3\; -1}{(2x-3)(x+1)} \leq 0$$

$$\Rightarrow \quad \frac{4}{(2x-3)(x+1)} \le 0$$

Step 3

$$(2x-3)(x+1) = 0$$

$$\Rightarrow \quad x = -1 \quad \text{or} \quad x = +\tfrac{3}{2}$$

Undefined

$x = -1, \tfrac{3}{2}$

Step 4

$4 = 0$ NEVER

This function is NEVER 0

Step 5

$\underline{(-\infty, -1)}$

$$x = -2 \Rightarrow \frac{4}{(2(-2)-3)(-2+1)} = \frac{4}{(-)(-)} = +$$

$\underline{(-1, 3/2)}$

$$x = 0 \Rightarrow \frac{+}{(2(0)-3)(0+1)} = \frac{+}{(-)(+)} = -$$

$\underline{(3/2, \infty)}$

$$x = 2 \Rightarrow \frac{+}{(2(2)-3)(2+1)} = \frac{+}{(+)(+)} = +$$

Step 6

Inequality: $\dfrac{4}{(2x-3)(x+1)} \leq 0$

$\boxed{\text{Soln: } \left(-1, \tfrac{3}{2}\right)}$