College Algebra: Module 10 What You Need To Know

3-22-15

1 Synthetic Division; the Remainder and Factor Theorems (Section 4.2)

I will list some internet links because they can do a much better job explaining both of these methods than I can in here.

Internet Links:

- mathbff - Long Division
- patrickJMT - Long Division
- Khan Academy - Long Division

Remainder Theorem:

If a polynomial $p(x)$ is divided by $x - c$ using long division, the remainder is equal to $p(c)$

Factor Theorem:

For a polynomial $p(x)$

1. If c is a zero of $p(x)$, then $x - c$ is a factor of $p(x)$
2. If $x - c$ is a factor of $p(x)$, then c is a zero of $p(x)$ $\Big($i.e. $p(c) = 0\Big)$

2 The Zeroes of a Polynomial Function

Linear Factorization Theorem:

If $p(x)$ is a polynomial function of degree $n \geq 1$, then p has exactly n linear factors and can be written in the form:

$$p(x) = a(x - c_1) \cdot (x - c_2) \cdots (x - c_n)$$

Multiplicity of Zeroes - if $p(x)$ is a polynomial function and $(x - c)$ occurs as a factor of $p(x)$ exactly m times, then c is called a zero of *multiplicity* m

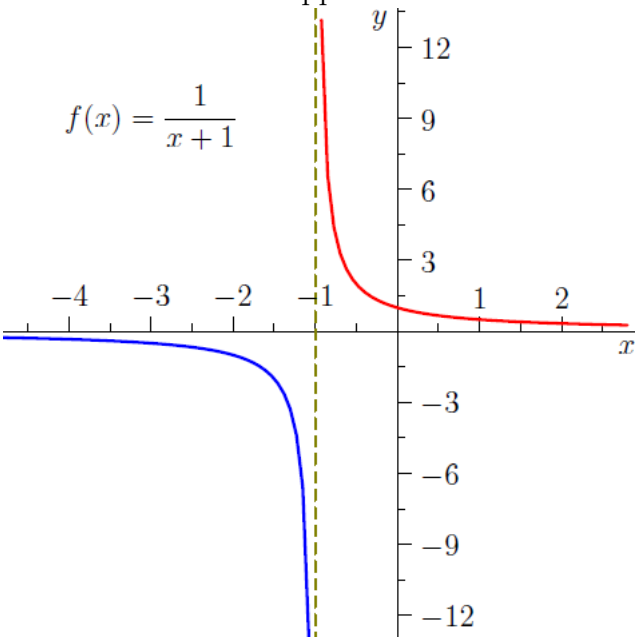
Conjugate Pairs Theorem:

If $p(x)$ is a polynomial with real coefficients, complex zeroes must occur in conjugate pairs. That means, **if $a + bi$ is a zero, then $a - bi$ will also be a zero**

3 Graphing Rational Functions (Section 4.5)

Vertical Asymptotes:

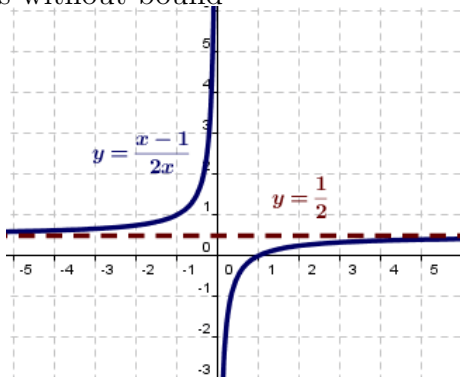
The line $x = a$ is a **vertical asymptote** for the graph of a function if $f(x)$ either increases or decreases without bound as x approaches from either the right or left.



Finding Vertical Asymptotes: Set the denominator equal to 0

Horizontal Asymptotes:

The line $y = a$ is a **horizontal asymptote** for the graph of a function if $f(x)$ approaches a as x increases or decreases without bound



Finding Horizontall Asymptotes: Compare the degree of the numerator and the degree of the denominator of a rational function.

1. Degree of Numerator $<$ Degree of Denominator $\implies$ $y = 0$ is the horizontal asymptote
2. Degree of Numerator $=$ Degree of Denominator $\implies$ horizontal asymptote is the ratio of the leading coefficients in the numerator and denominator

3. Degree of Numerator $>$ Degree of Denominator $\implies$ there is no horizontal asymptote

Steps to Graphing Rational Functions: $R(x) = \dfrac{N(x)}{D(x)}$

1. Find the y-intercept, $R(0)$, if it exists
2. Find the zeroes of $R(x)$ by setting the numerator $N(x) = 0$
3. Find the vertical asymptotes. In other words, find the zeroes of $D(x)$
4. Find the horizontal asymptote, if it exists
5. Determine if the graph will cross the horizontal asymptote
6. Compute any additional points needed to sketch the graph

4 Polynomial and Rational Inequalities (Section 4.6)

Steps to Solving a Polynomial Inequality:

1. Move every term to one side of the inequality
2. Find the zeroes of the resulting polynomial
3. Drawn a number line and plot the zeroes from step 2
4. Test points **inside** each interval to see if the function is positive or negative
5. See which interval(s) make the inequality true

(See handwritten notes on Polynomial Inequalities on Icon)

Steps to Solving a Rational Inequality:

1. Move every term to one side of the inequality
2. Put everything over a common denominator
3. Find the points where the function is undefined by setting the denominator equal to 0 and solving
4. Ignore the denominator and set only the numerator equal to 0 to find the points where the function is equal to 0
5. Draw a number line and plot the points where the function is undefined **and** where the function is 0
6. Test points **inside** each interval to see if the function is positive or negative
7. See which interval(s) make the inequality true

(See handwritten notes on Rational Inequalities on Icon)