Polynomial Inequalities

In this handout I will provide a step-by-step guide for solving polynomial inequalities and do some examples.

Step-By-Step Procedure

Step 1 Move every term to one side of the inequality

Step 2 Find the zeros of the resulting polynomial

Step 3 Draw a number line and plot the zeros from step 2

Step 4 Test points <u>inside</u> each interval and see which interval(s) make the inequality true

$\boxed{\text{Example 1}}$ Solve for x:

$$2x^2 - 3 \le x$$

Step 1

$$2x^2 - x - 3 \le 0$$

Step 2

$$2x^2 - x - 3 = 0$$

$$\Rightarrow (2x-3)(x+1) = 0$$

$$\Rightarrow x = -1, \ 3/2$$

Zeros

$x = -1$ and $x = 3/2$

Step 3

Step 4

$x = -2 \Rightarrow (2(-2)-3)(-2+1) = (-)(-) = +$

$x = 0 \Rightarrow 2(0)^2 - 0 - 3 = -3 < 0$

$x = 2 \Rightarrow (2(2)-3)(2+1) = (+)(+) = +$

Inequality: $2x^2 - x - 3 \le 0$

Solution: $-1 \le x \le 3/2$ OR $[-1, 3/2]$

Example 2 Solve the inequality and write sol'n in interval notation:

$$x^2 - 4x < 6$$

Step 1

$$x^2 - 4x - 6 < 0$$

Step 2

$$x^2 - 4x - 6 = 0$$

Zeros

$$x = \frac{4 \pm \sqrt{16 - 4(1)(-6)}}{2}$$

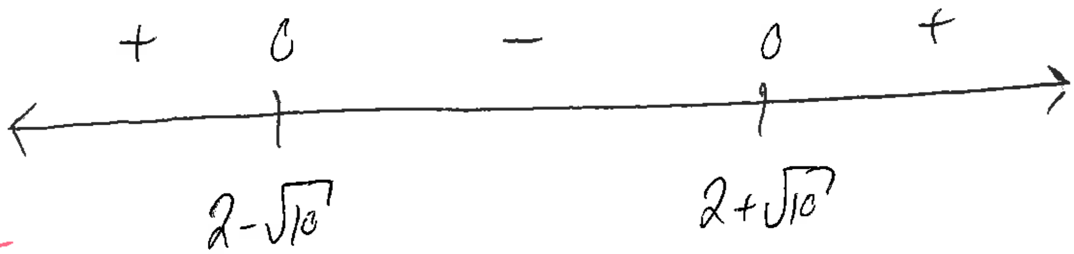
$$= \frac{4 \pm \sqrt{16 + 24}}{2}$$

$$= \frac{4 \pm \sqrt{40}}{2} = \frac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$$

Step 3

We know $\sqrt{9} < \sqrt{10} < \sqrt{16}$

$$\Rightarrow \quad 3 < \sqrt{10} < 4$$



Step 4

Interval	Test Value of X	Function Eval
$(-\infty, 2-\sqrt{10})$	$x = 2-4 = -2$	$(-2)^2 - 4(-2) - 6$ $= 4 + 8 - 6 > 0$
$(2-\sqrt{10}, 2+\sqrt{10})$	$x = 0$	$0^2 - 4(0) - 6 = -6 < 0$
$(2+\sqrt{10}, +\infty)$	$x = 2+4 = 6$	$6^2 - 4(6) - 6$ $= 36 - 24 - 6$ $= 36 - 30$ $= 6 > 0$

Step 4

The inequality was $x^2 - 4x - 6 < 0$

$$\boxed{\begin{array}{l} \underline{\text{Solution}}: \quad 2-\sqrt{10} < x < 2+\sqrt{10} \\ \qquad\qquad\qquad \underline{\text{or}} \\ \qquad\qquad (2-\sqrt{10},\ 2+\sqrt{10}) \end{array}}$$

Example 3 Solve $2x^2 - 3x > -x^3$

Step 1

$$x^3 + 2x^2 - 3x > 0$$

Step 2

$$x^3 + 2x^2 - 3x = 0$$

$$\Rightarrow \quad x(x^2+2x-3)=0$$

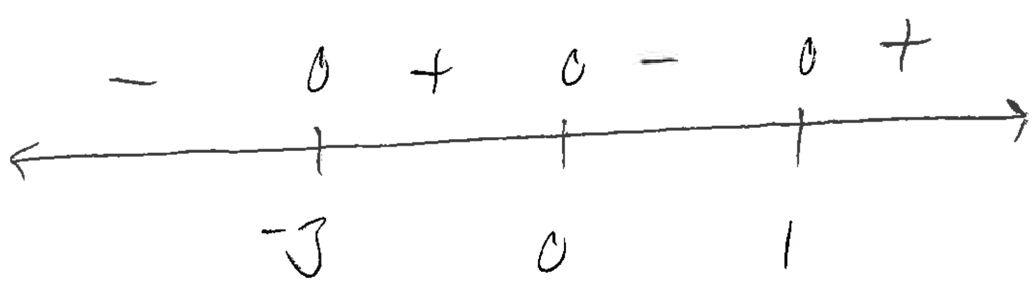
$$\Rightarrow \quad x(x+3)(x-1)=0$$

$$\Rightarrow x=0 \quad \text{or} \quad x=-3 \quad \text{or} \quad x=1$$

Zeros

$x=-3, 0, 1$

Step 3



Step 4

$$x=-4 \Rightarrow -4(-4+3)(-4-1) = -(-)(-) = -$$

$$x=-1 \Rightarrow -1(-1+3)(-1-1) = -(+)(-) = +$$

$$x=\tfrac{1}{2} \Rightarrow \tfrac{1}{2}\left(\tfrac{1}{2}+3\right)\left(\tfrac{1}{2}-1\right) = +(+)(-) = -$$

$x = 2 \Rightarrow 2(2+3)(2-1) = +(+)(+) = +$

$$\boxed{\begin{array}{l}\text{Solution} \\ \text{The inequality was } x^3 + 2x^2 - 3x > 0 \\ \text{So in interval notation we have} \\ (-3, 0) \cup (1, +\infty)\end{array}}$$