College Algebra: Module 11 What You Need To Know

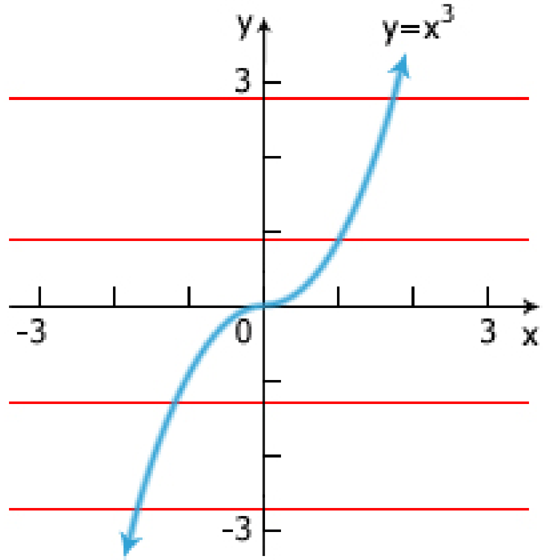
3-28-15

1 One-to-One and Inverse Functions (Section 5.1)

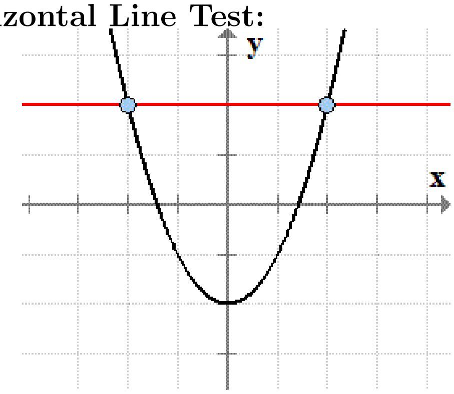
One-to-One - a function f is *one-to-one* if every element in the range corresponds to **only one** element of the domain

Horizontal Line Test: - a nice, easy, graphical test to determine if a function is one-to-one. It says that a function is **one-to-one** if every horizontal line intersects a graph **at most once**

Function That Passes the Horizontal Line Test:



Function That Fails Horizontal Line Test:

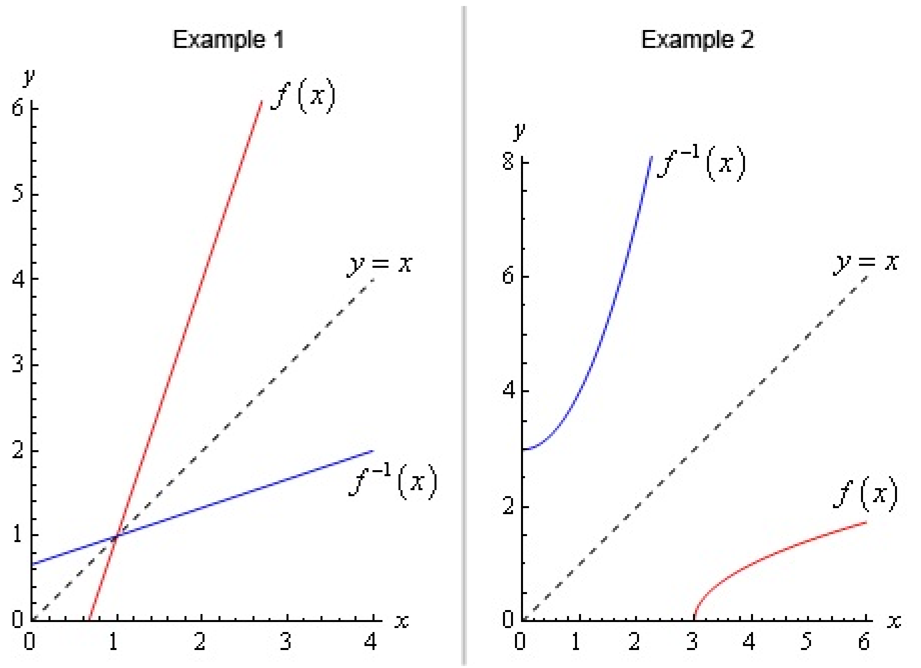


Inverse Function: - if a function f is one-to-one then the **inverse function** denoted by $f^{-1}(x)$ is the function that "undoes" the original function f so that

$$f^{-1}\Big(f(x)\Big) = x = f\Big(f^{-1}(x)\Big)$$

Properties of Inverse Function $f^{-1}(x)$ to its Original Function $f(x)$

- The domain of f^{-1} is the range of f
- The range of f^{-1} is the domain of f
- Graphically, the inverse function, $f^{-1}(x)$, is the graph of the original function, $f(x)$, reflected about the line $y = x$



Note: Notice how in Example 2 that the range of $f^{-1}(x)$ is the domain of $f(x)$

Finding the Inverse Function:

1. Set $y = f(x)$
2. Swap x and y
3. Solve for y in terms of x
4. The result gives the inverse function: Replace y with $f^{-1}(x)$

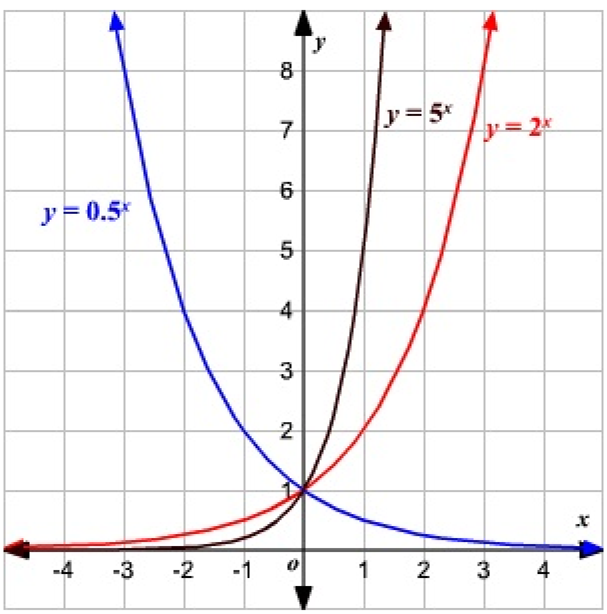
2 Exponential Functions (Section 5.2)

Exponential Properties

- $b^m \cdot b^n = b^{m+n}$
- $\dfrac{b^m}{b^n} = b^{m-n}$
- $(b^m)^n = b^{mn}$
- $(a \cdot b)^n = a^n \cdot b^n$
- $b^{-n} = \dfrac{1}{b^n}$
- $\left(\dfrac{b}{a}\right)^{-n} = \left(\dfrac{a}{b}\right)^n$

Exponential Function

$$f(x) = b^x \qquad\qquad b \neq 1 \text{ and } b > 0$$



We say b is the **base** of the exponential function.

There is a special kind of exponential function that we single out because of its significance and we call it the **Natural Exponential Function**. It is the function

$$f(x) = e^x$$

Note: e is simply a number; $e \approx 2.71828...$

Solving Exponential Equations with Common Bases

- $b^m = b^n \implies m = n$ (Equal bases imply equal exponents)