College Algebra: Module 5 What You Need To Know

2-12-15

1 Complex Numbers (Section 1.4)

Complex Numbers - *Complex Numbers* are numbers that can be written in the form

$$a + bi$$

where a and b are real numbers and i is the imaginary unit

Imaginary Unit: $i = \sqrt{-1}$ and $i^2 = -1$

Property 1: $\sqrt{-k} = i\sqrt{k} \qquad (k > 0)$

Algebraic Rules:

1. **Addition/Subtraction:** $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$
2. **Multiplication:** $(a + bi) \cdot (c + di) = (ac - bd) + (bc + ad)i$
3. **Division:** $$\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di}$$
$$= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

Note: To divide two complex numbers you need to multiply the top and bottom by the **conjugate** of the bottom.

2 Solving Quadratic Equations (Section 1.5)

Quadratic Equation - a *quadratic equation* is an equation of the form

$$ax^2 + bx + c$$

where a, b, c are all real numbers and $a \neq 0$

Zero Product Property: $A \cdot B = 0 \implies A = 0$ or $B = 0$

Square Root Property of Equality: $x^2 = a \implies x = \pm\sqrt{a}$

Completing The Square: In order to solve equations of the following form

$$ax^2 + bx + c = 0$$

by *completing the square* we follow these steps:

1. Subtract the constant c from both sides
2. Divide both sides by a
3. Compute $\left(\frac{b}{2a}\right)^2$ and add the result to both sides
4. Factor left hand side and simplify right hand side
5. Solve using the square root property of equality

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant of the Quadratic Formula: Given a quadratic

$$ax^2 + bx + c$$

where $a \neq 0$ then we can determine the number and type of solutions to

$$ax^2 + bx + c = 0$$

by examining the discriminant of the quadratic formula which is given by

$$b^2 - 4ac$$

(i.e. the terms underneath the square root in the quadratic formula) The rules are given below:

1. $b^2 - 4ac > 0 \implies$ 2 real roots
2. $b^2 - 4ac = 0 \implies$ 1 real (repeated) root
3. $b^2 - 4ac < 0 \implies$ 2 nonreal roots

Problem 1: Write the quadratic equation whose roots are -2 and 6 and whose leading coefficient is 3

Problem 2: Write the quadratic equation whose roots are 1 and 6 and whose leading coefficient is 4

Problem 3: The length of a rectangle is 7 ft leass than three times the width. The area of the rectangle is 66 ft. Find the dimensions of the rectangle.

Solving a word problem using a quadratic equation with irrational roots

A ball is thrown from a height of 110 feet with an initial downward velocity of $6\,\text{ft/s}$. The ball's height h (in feet) after t seconds is given by the following.

$$h = 110 - 6t - 16t^2$$

How long after the ball is thrown does it hit the ground?

Round your answer(s) to the nearest hundredth.
(If there is more than one answer, use the "or" button.)