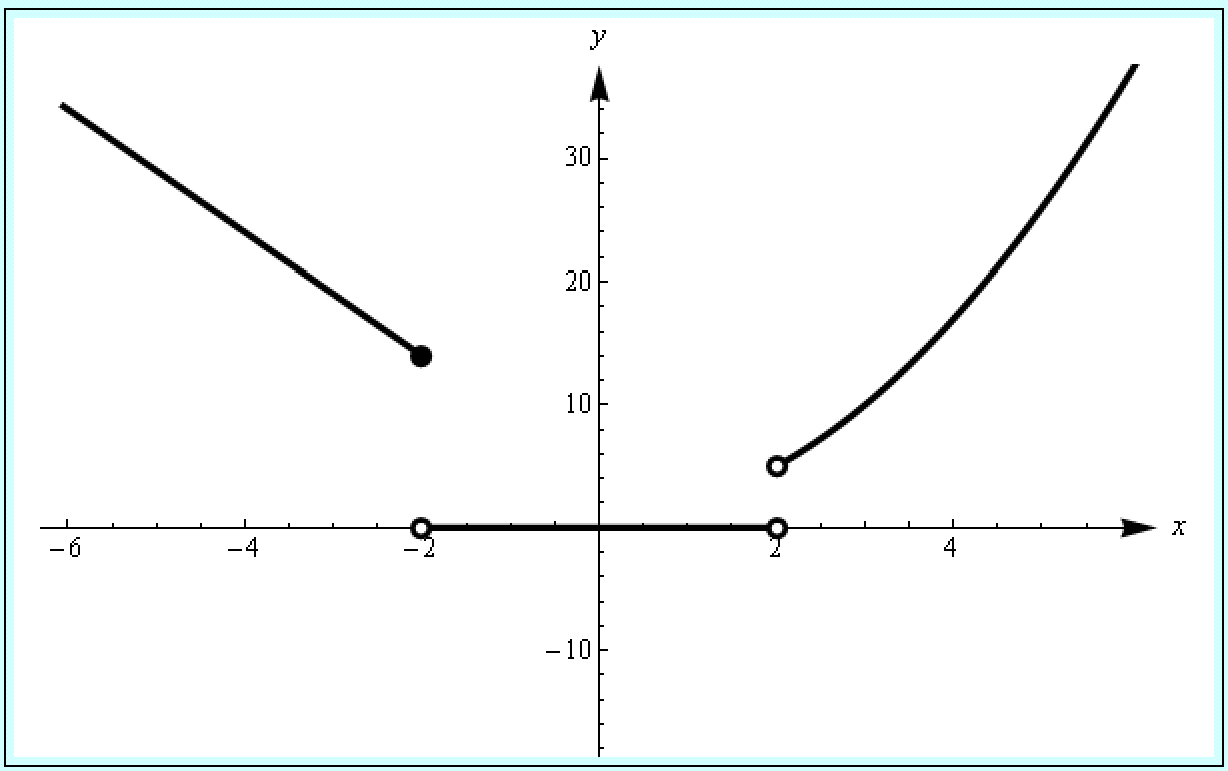
College Algebra: Module 9 What You Need To Know

3-22-15

1 Piecewise Defined Functions (Section 3.4)

Piecewise Function - a *piecewise function* is a function that is a combination of two or more sub-functions where each sub-function is defined over its own unique interval. Below is an example of a piecewise function:

$$f(x) = \begin{cases} 4 - 5x & \text{if} \quad x \leq -2 \\ 0 & \text{if} \quad -2 < x < 2 \\ x^2 + 1 & \text{if} \quad x > 2 \end{cases}$$



Evaluating Piecewise Functions

Consider the same piecewise function that was given on the first page of the notes:

$$f(x) = \begin{cases} 4-5x & x \leq -2 \\ 0 & -2 < x < 2 \\ x^2+1 & x > 2 \end{cases}$$

Let's try find the values $f(-4), f(-2), f(0), f(2),$ and $f(3)$.

$f(-4) = 4 - 5(-4) = 4 + 20 = 24$

$f(-2) = 4 - 5(-2) = 4 + 10 = 14$

$f(0) = 0$

$f(2)$ is undefined

$f(3) = 3^2 + 1 = 10$

2 The Algebra and Composition of Functions (Section 3.5)

Sum, Difference, Product, and Quotient of Functions:

Let $f(x)$ and $g(x)$ be any two functions. We can find the sum, difference, product, and quotient of them as given by the following rules:

1. **Sum:** $(f+g)(x) = f(x) + g(x)$
2. **Difference:** $(f-g)(x) = f(x) - g(x)$
3. **Product:** $(f \cdot g)(x) = f(x) \cdot g(x)$
4. **Quotient:** $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)}$

Function Composition:

$(f \circ g)(x) = f\Big(g(x)\Big)$

Example: Let $f(x) = 3x^2 - 2$ and $g(x) = \dfrac{x+2}{x-1}$

Then $f \circ g = 3\left(\dfrac{x+2}{x-1}\right)^2 - 2$

Note: BE CAREFUL when finding the domain of a composition of functions.

Example: Let $f(x) = \sqrt{x-12}$ and $g(x) = x^2 - 4$

Find

a) $f \circ g$

b) $g \circ f$

and state the domain of each

Answer: a) $f \circ g = \sqrt{(x^2 - 4) - 12}$

$$= \sqrt{x^2 - 16}$$

Domain: $x^2 - 16 \geq 0 \implies x \leq -4$ and $x \geq 4$

So the domain in interval notation is $(-\infty, -4] \cup [4, \infty)$

Answer: b) $g \circ f = \left(\sqrt{x - 12}\right)^2 - 4$

$$= x - 12 - 4$$

$$= x - 16$$

Domain: $x - 12 \geq 0 \implies x \geq 12 \implies [12, \infty)$

3 Quadratic Functions and Applications (Section 4.1)

Standard Form of a Quadratic Function

$$f(x) = ax^2 + bx + c$$

$$a > 0 \rightarrow \text{Upward Pointing}$$

$$a < 0 \rightarrow \text{Downward Pointing}$$

Vertex Form of a Quadratic Function:

$$f(x) = a(x - h)^2 + k$$

$$\text{Vertex: } (h, k)$$

$$a > 0 \rightarrow \text{Upward Pointing}$$

$$a < 0 \rightarrow \text{Downward Pointing}$$

Vertex of a Parabola:

$$\left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$$

Converting Between Standard Form and Vertex Form: COMPLETE THE SQUARE

Example: Write $2x^2 + 8x + 7$ in vertex form and identify the vertex

1. Group first two terms: $(2x^2 + 8x) + 7$
2. Factor out a from the first two terms: $2(x^2 + 4x) + 7$

3. Compute $\left(\frac{b}{2a}\right)^2$ and add and subtract this inside: $2(x^2+4x+4-4)+7$
4. Factor: $2\Big((x+2)^2-4\Big)+7$
5. Distribute a back inside: $2(x+2)^2-8+7$
6. Combine the constant terms: $2(x+2)^2-1$

This is now in vertex form. The vertex is $(-2,-1)$

Finding the X-Intercepts of a Quadratic - Set $y=0$ and solve for x either by factoring, completing the square, or using the quadratic formula

Finding the Maximum/Mininum of a Quadratic ax^2+bx+c

$a>0 \rightarrow$ parabola is upward pointing so there is no maximum, but there is a minimum. The minimum is the y-value of the vertex. If you can find the vertex, then you will find the minimum value.

$a<0 \rightarrow$ parabola is downward pointing so there is no minimum, but there is a maximum. The maximum is the y-value of the vertex. If you can find the vertex, then you will find the maximum value.

Problem 1: Find all entire functions f such that $\int\int|f(x+iy)|^2dxdy<\infty$.

Problem 2: If $\mathbb{D}$ is the open unit disk and $f:\mathbb{D}\rightarrow\overline{\mathbb{D}}$ is an analytic function then show that $\left|f^{(n)}(z)\right|\leq n!(1-|z|)^n$, for all $z\in\mathbb{D}$.