College Algebra: Module 8 What You Need To Know

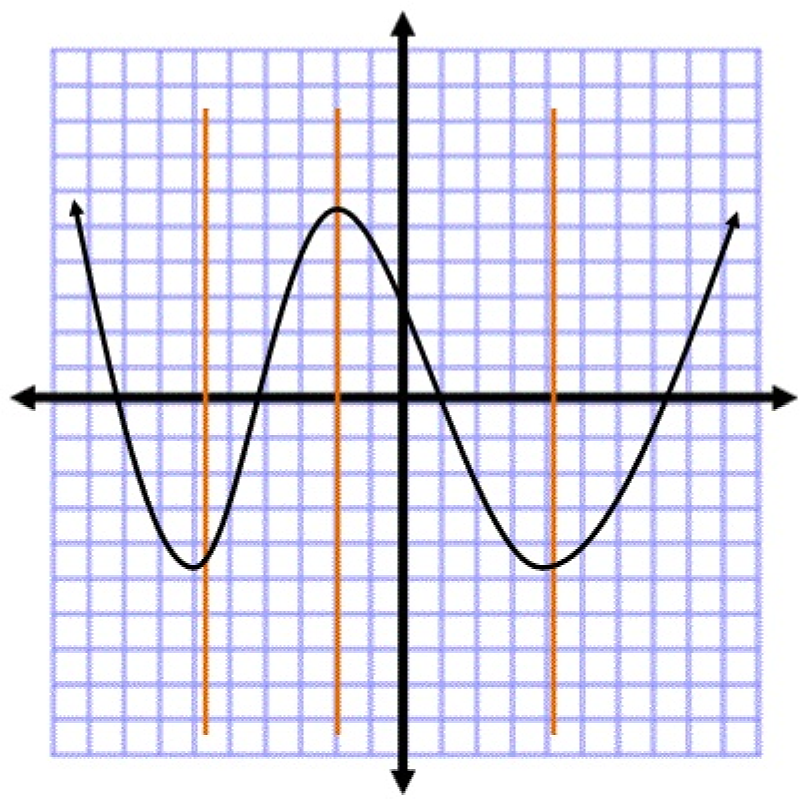
2-26-15

1 Functions, Function Notation, and the Graph of a Function (Section 2.4)

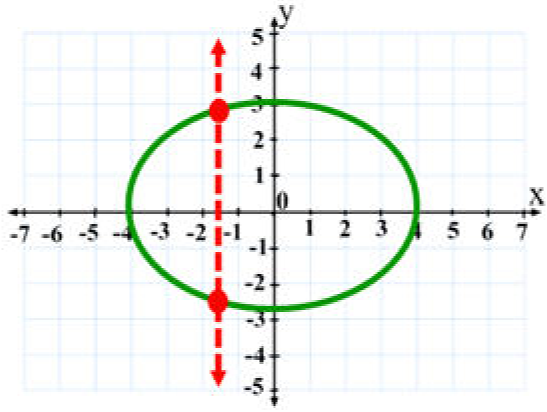
Function - a *function* is a set of ordered pairs (x, y) in which each first coordinate is paired with only one second coordinate

Vertical Line Test: - a nice, easy, graphical test to determine if a graph is a function. It says that a graph of an equation is a function if every vertical line intersects the graph **in at most one point.**

Graph That Passes the Vertical Line Test:



Graph That Fails the Vertical Line Test:



Domain - the set of numbers that you can input into your function

Note: Usually you need to be careful of two types of functions:

1. **Finding the Domain of the Square Root Function:** $f(x) = \sqrt{x}$

   If you have a function of the form $\sqrt{STUFF}$ you then solve the inequality

   $$STUFF \geq 0$$

   to find the domain of the function.

2. **Finding the Domain of a Rational Function:** $f(x) = \dfrac{P(x)}{R(x)}$

   A rational function is of the form $f(x) = \dfrac{P(x)}{R(x)}$ where $P(x)$ and $R(x)$ are polynomial. The domain of a rational function is all reals except for $R(x) = 0$.

   What this means is that to find the domain of a rational function you **set the denominator equal to 0 and solve that**. You then **exclude** those values from the domain of your function.

Note: Sometimes you might see a combination of both. For example, find the domain of $f(x) = \dfrac{-1}{\sqrt{x+8}}$. We do this by solving $x + 8 > 0$

Graphically, you can determine the domain by scanning along the x-axis and seeing what x-values the function is not defined for.

Range - the set of numbers that are output from a function. Often harder to determine than the domain, we can determine the range graphically by scanning along the y-axis and seeing what y-values the function is not defined for.

2 Analyzing the Graph of a Function (Section 2.5)

<u>**Even & Odd Functions**</u>

Even Function - a function is *even* if $f(-x) = f(x)$ Even functions are always symmetric about the y-axis.

Odd Function - a function is *odd* if $f(-x) = -f(x)$ Odd functions are always symmetric about the origin.

Increasing: A function f is *increasing* on an interval I if for any x_1, x_2 in I with $x_1 < x_2 \implies f(x_1) < f(x_2)$

Decreasing: A function f is *decreasing* on an interval I if for any x_1, x_2 in I with $x_1 < x_2 \implies f(x_1) > f(x_2)$

Constant: A function f is *constant* on an interval I if for all x_1, x_2 in $I \implies f(x_1) = f(x_2)$

3 The Toolbox Functions and Transformations (Section 3.1)

3.1 Transformations

Vertical Shifts: Let $c > 0$

1. $y = f(x) + c$ shifts $f(x)$ c units up
2. $y = f(x) - c$ shifts $f(x)$ c units down

Horizontal Shifts: Let $c > 0$

1. $y = f(x + c)$ shifts $f(x)$ c units left
2. $y = f(x - c)$ shifts $f(x)$ c units right

Reflections:

1. $y = -f(x)$ reflects $f(x)$ about the x-axis
2. $y = f(-x)$ reflects $f(x)$ about the y-axis

Vertical Stretch & Shrink:

Let $c > 0$. We consider $y = cf(x)$

1. $c > 1 \quad \rightarrow$ stretch $f(x)$ vertically by a factor of c
2. $0 < c < 1 \rightarrow$ shrink $f(x)$ vertically by a factor of c

Horizontal Stretch & Shrink:

Let $c > 0$. We consider $y = f(cx)$

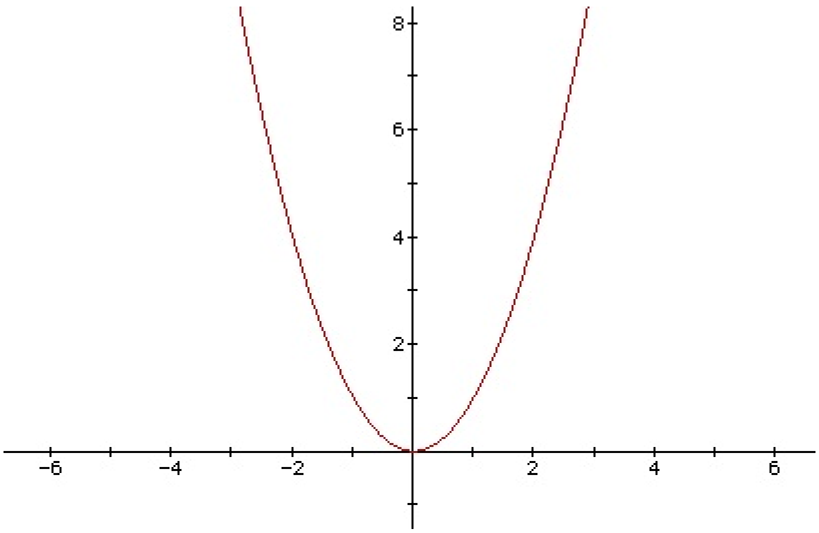
1. $c > 1 \quad \rightarrow$ shrink $f(x)$ horizontally by a factor of c
2. $0 < c < 1 \rightarrow$ stretch $f(x)$ horizontally by a factor of c

Order of Transformations:

1. Horizontal Shift
2. Stretch/Shrink
3. Reflections
4. Vertical Shift

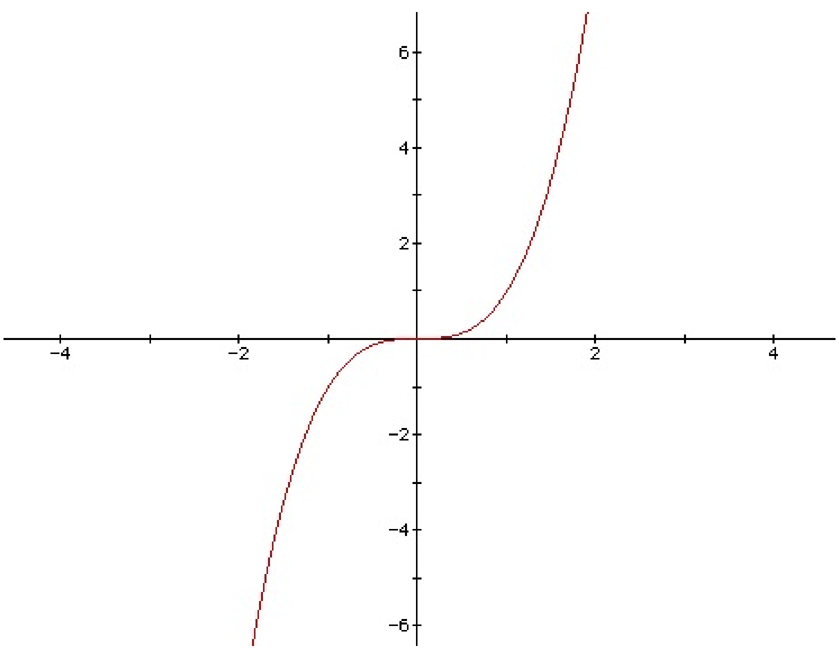
Graphs of Common Functions with Their Domain and Range

Graph of Quadratic Function $\mathbf{y = x^2}$



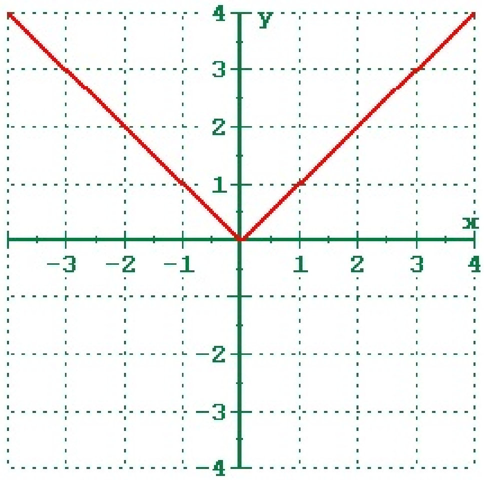
Domain: $(-\infty, \infty)$ **Range** $y \geq 0$

Graph of Cubic Function $\mathbf{y = x^3}$



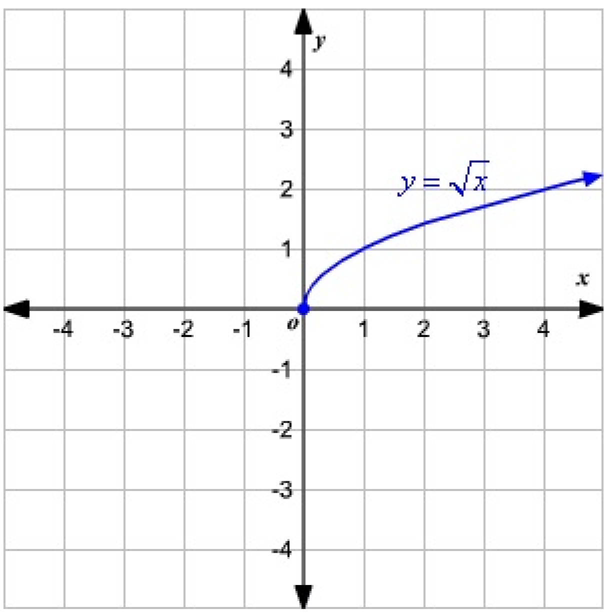
Domain: $(-\infty, \infty)$ **Range** $(-\infty, \infty)$

Graph of Absolute Value $\mathbf{y} = |\mathbf{x}|$



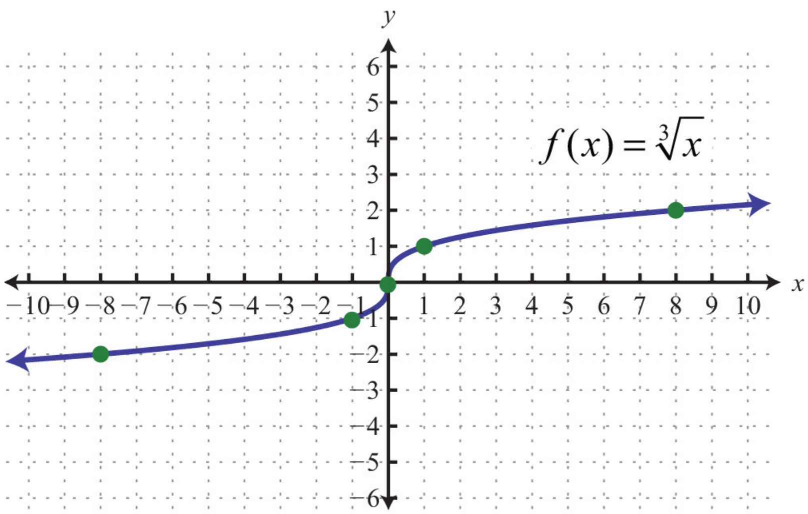
Domain: $(-\infty, \infty)$ **Range** $y \geq 0$

Graph of Square Root $\mathbf{y} = \sqrt{\mathbf{x}} = \mathbf{x}^{1/2}$



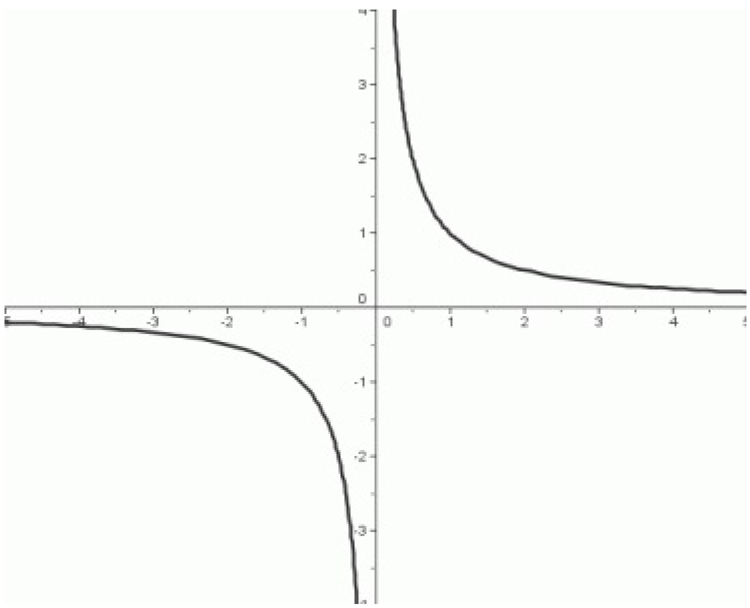
Domain: $x \geq 0$ **Range** $y \geq 0$

Graph of Cubic Root Function $\mathbf{y} = \sqrt[3]{x} = \mathbf{x}^{1/3}$



Domain: $(-\infty, \infty)$ **Range** $(-\infty, \infty)$

Graph of the Reciprocal Function $\mathbf{y} = \dfrac{\mathbf{1}}{\mathbf{x}}$



Domain: $(-\infty, 0) \cup (0, \infty)$ **Range** $(-\infty, 0) \cup (0, \infty)$