Midterm 2 Study Guide

Note: Before I begin there is material/examples that I do not cover in here much because I have previously uploaded PDF's onto Icon that focus on those things. So before going over this study guide I would also suggest printing out those PDF's to supplement this study guide

Convert Between Degrees and Radians:

$$\pi \text{ radians} = 180^\circ$$

Examples:

1. **Degrees to Radians:**

$$135^\circ \cdot \frac{\pi}{180} = \frac{3\pi}{4} \text{ radians}$$

2. **Radians to Degrees:**

$$\frac{5\pi}{6} \text{ radians} \cdot \frac{180}{\pi} = 150^\circ$$

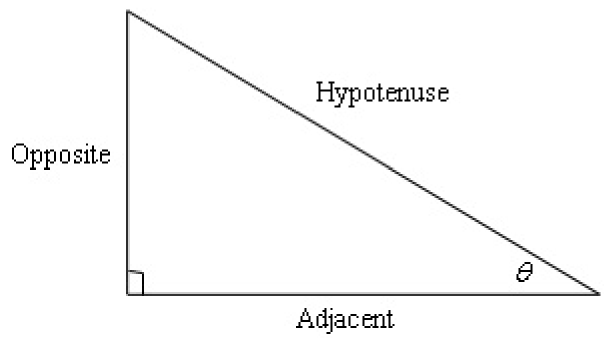
Length of a Circular Arc: (θ in radians)

$$L = r \cdot \theta$$

Area of a Circular Sector: (θ in radians)

$$A = \frac{1}{2} \cdot r^2 \cdot \theta$$

Trig Functions Defined via Right Triangle:



$$\sin(\theta)=\frac{\text{Opp}}{\text{Hyp}} \qquad \cos(\theta)=\frac{\text{Adj}}{\text{Hyp}} \qquad \tan(\theta)=\frac{\text{Opp}}{\text{Adj}}$$

$$\csc(\theta)=\frac{\text{Hyp}}{\text{Opp}} \qquad \sec(\theta)=\frac{\text{Hyp}}{\text{Adj}} \qquad \cot(\theta)=\frac{\text{Adj}}{\text{Opp}}$$

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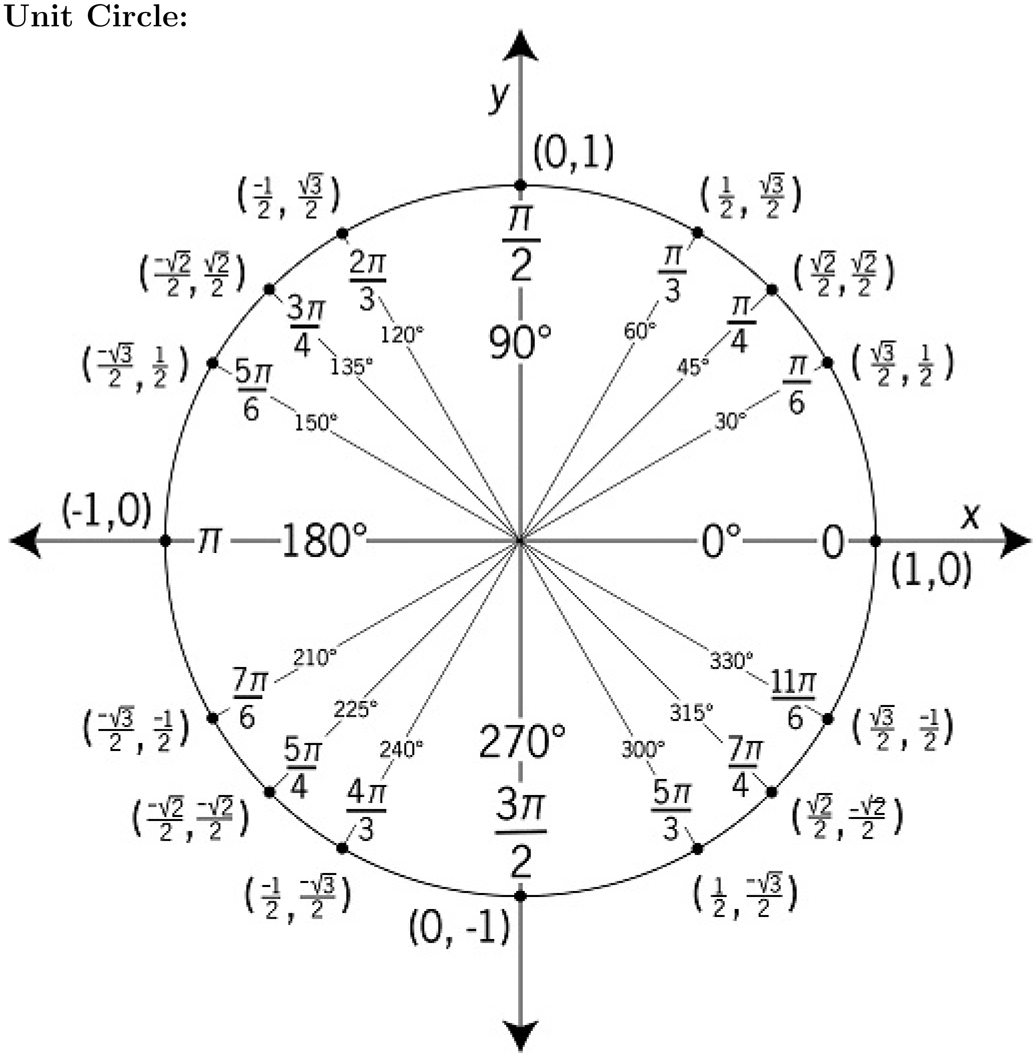
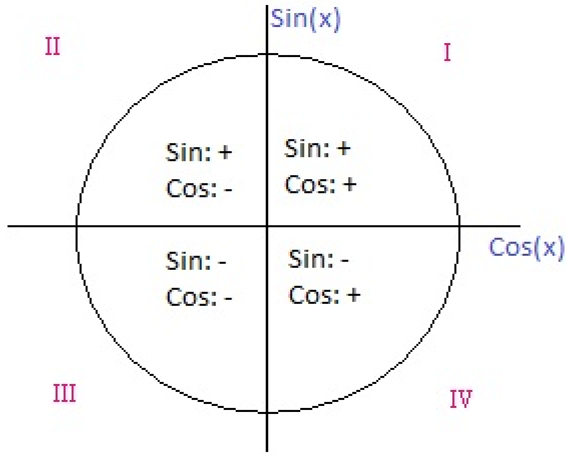
Table of Common Values - Trig Functions:

Degrees	0	30	45	60	90
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Unit Circle:


y
x
(0,1)
(1,0)
(-1,0)
(0, -1)
0
0°
π
180°
π/2
90°
3π/2
270°
π/6
30°
π/4
45°
π/3
60°
2π/3
120°
3π/4
135°
5π/6
150°
7π/6
210°
5π/4
225°
4π/3
240°
5π/3
300°
7π/4
315°
11π/6
330°
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(-1/2, √3/2)
(-√2/2, √2/2)
(-√3/2, 1/2)
(-√3/2, -1/2)
(-√2/2, -√2/2)
(-1/2, -√3/2)
(1/2, -√3/2)
(√2/2, -√2/2)
(√3/2, -1/2)

Signs of Trig Functions in Quadrants:



Even and Odd:

1. **Cos(x)** is an **EVEN** function: $\cos(-x) = \cos(x)$

   Graphically, $\cos(x)$ is symmetric about the y-axis

2. **Sin(x)** is an **ODD** function: $\sin(-x) = -\sin(x)$

   Graphically, $\sin(x)$ is symmetric about the origin

Reference Number - for an real number t, the **reference number** r associated with t is the shortest distance along the unit circle from t to the x-axis. For any t, the reference number r is in $\left[0, \frac{\pi}{2}\right]$

Example: Determine the value of $\sin(t)$ and $\cos(t)$ for a) $t = \frac{29\pi}{6}$ and b) $t = \frac{41\pi}{4}$

a) First we find the reference number by finding the closest number to 29 that evenly divides by 6. We can use 30. Then

$$\frac{29\pi}{6} = \frac{30\pi}{6} - \frac{\pi}{6}$$

So the reference number $r = \frac{\pi}{6}$. This helps us determine the value of sin(t) and cos(t). We know $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$. We now just need to find what quadrant we are in to determine the correct sign.

So what quadrant are we in? $\frac{30\pi}{6} = 5\pi$ which places us at $(-1, 0)$ on the unit circle. We then must move BACK (clockwise) $\frac{\pi}{6}$ radians which places us in the second quadrant where sin(t) is positive and cos(t) is negative.

So $\sin\left(\frac{29\pi}{6}\right) = \frac{1}{2}$ and $\cos\left(\frac{29\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

b) First we find the reference number by finding the closest number to 41 that evenly divides by 4. We can use 40. Then

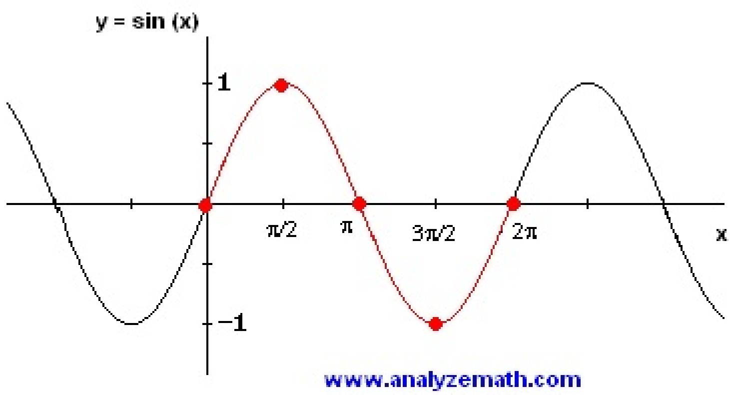
$$\frac{41\pi}{4} = \frac{40\pi}{4} + \frac{\pi}{4}$$

So the reference number $r = \frac{\pi}{4}$. We know $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ and $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

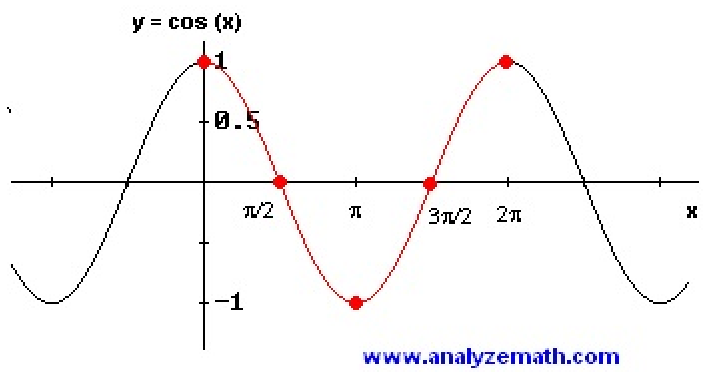
Now what quadrant are we in? $\dfrac{40\pi}{4} = 10\pi$ which places us at $(1, 0)$ on the unit circle. We then must move FORWARD (counterclockwise) $\dfrac{\pi}{4}$ radians which places us in the first quadrant where sin(t) is positive and cos(t) is positive.

So $\sin\left(\dfrac{41\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$ and $\cos\left(\dfrac{41\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

Graph of Sine:



Graph of Cosine:



Graphs of $\mathbf{y = A\sin(Bx + C) + D}$ and $\mathbf{y = A\cos(Bx + C) + D}$

- Amplitude: $|A|$
- Period: $\dfrac{2\pi}{B}$
- Phase Shift: $\dfrac{|C|}{B}$

$$\frac{C}{B} < 0 \implies \text{shift right}$$
$$\frac{C}{B} > 0 \implies \text{shift left}$$

- Interval: $\dfrac{2\pi}{4B}$
- Vertical Shift: $|D|$

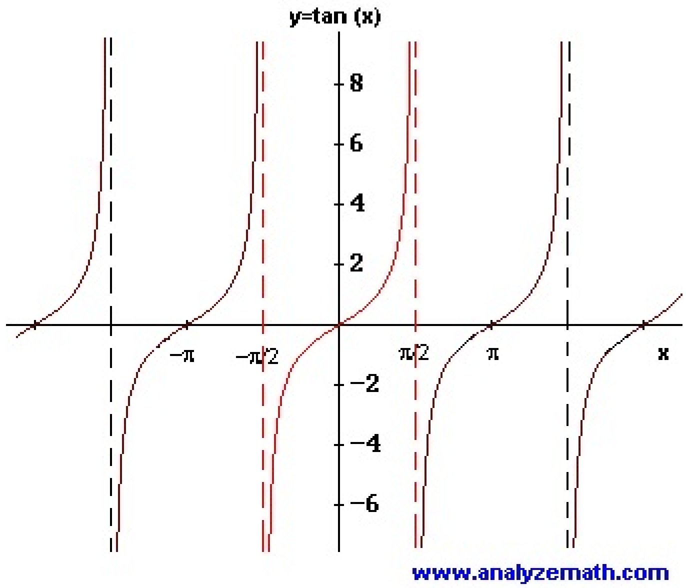
$$D < 0 \implies \text{shift down}$$
$$D > 0 \implies \text{shift up}$$

Note: When graphing sine/cosine **phase shift** tells you how far left/right to move the first critical point. Then the **interval** tells you how far along the x-axis you go until the next critical point occurs. The **amplitude** tells you how far up/down you go for the maximum/minimum of the graphs. See notes on graphing sine and cosine uploaded onto Icon.

The Other Trig Functions:

1. $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$
2. $\csc(x) = \dfrac{1}{\sin(x)}$
3. $\sec(x) = \dfrac{1}{\cos(x)}$
4. $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$

Graph of Tangent:



Inverse Trig Functions:

1. $\arcsin(x)$

   (a) $\sin\Big(\arcsin(x)\Big)$

   i. x in $[-1, 1] \implies \sin\Big(\arcsin(x)\Big) = x$

   ii. x not in $[-1, 1] \implies$ **no solution**

   (b) $\arcsin\Big(\sin(x)\Big)$

   i. x in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \implies \arcsin\Big(\sin(x)\Big) = x$

   ii. x not in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \implies$ find the corresponding radian measure that is inside $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

2. $\arccos(x)$

   (a) $\cos\Big(\arccos(x)\Big)$

i. x in $[-1,1] \implies \cos\Big(\arccos(x)\Big) = x$

ii. x not in $[-1,1] \implies$ **no solution**

(b) $\arccos\Big(\cos(x)\Big)$

i. x in $[0,\pi] \implies \arccos\Big(\cos(x)\Big) = x$

ii. x not in $[0,\pi] \implies$ find the corresponding radian measure that is inside $[0,\pi]$

3. $\arctan(x)$

(a) $\tan\Big(\arctan(x)\Big)$

i. $\tan\Big(\arctan(x)\Big) = x$ ALWAYS

(b) $\arctan\Big(\tan(x)\Big)$

i. x in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \implies \arctan\Big(\tan(x)\Big) = x$

ii. x not in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \implies$ find the corresponding radian measure that is inside $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$$

Law of Sines:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Note: Remember when working with Law of Sines/Law of Cosines, the side opposite α is a, the side opposite β is b and the side opposite γ is c.

a $\to \alpha$

b $\to \beta$

c $\to \gamma$

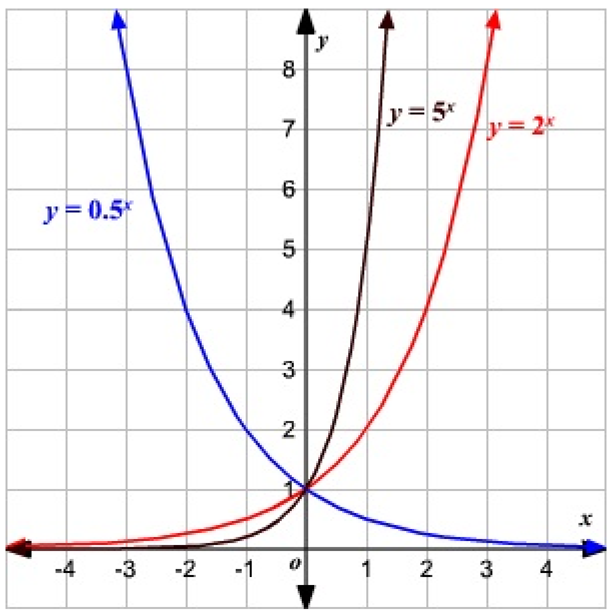
Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{Perimeter}{2}$$

Exponential Function

$$f(x) = a^x \qquad\qquad a \neq 1$$



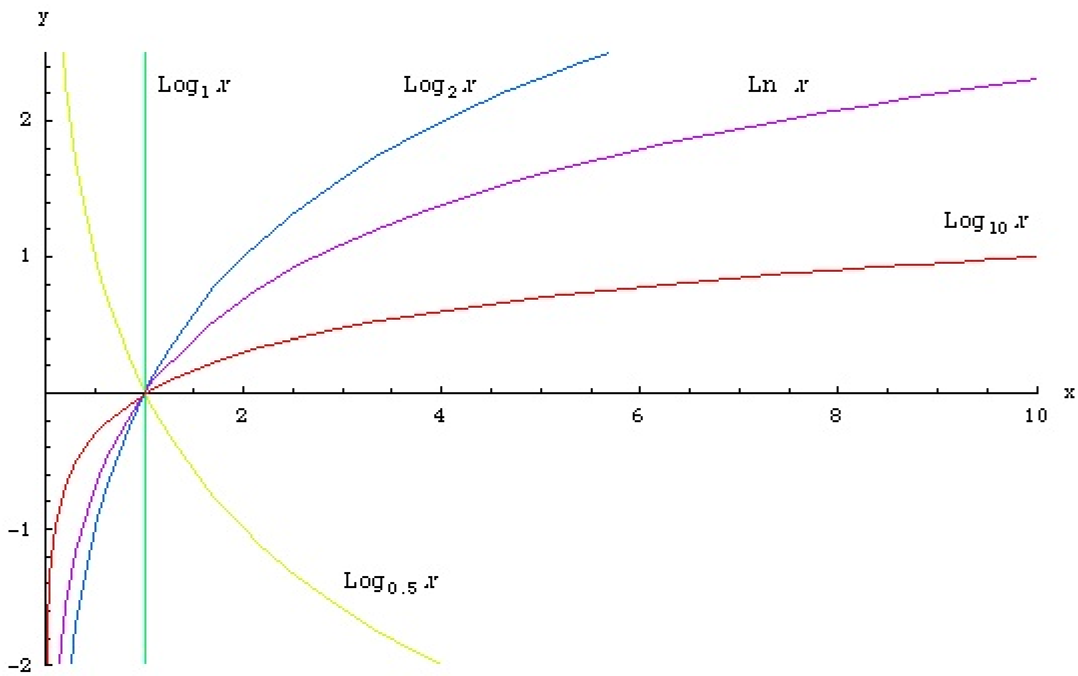
We say a is the **base** of the exponential function.

There is a special kind of exponential function that we single out because of its significance and we call it the **Natural Exponential Function**. It is the function

$$f(x) = e^x$$

Logarithm Function

$$f(x) = \log_a(x) \qquad\qquad a \neq 1$$



Again, we call a the **base** of the logarithm function.

There is a special kind of logarithm function that we single out becaue of its significance. It is the **Natural Log Function**. It is the function

$$f(x) = \ln(x)$$

The Natural Log Function is the logarithm function with **base e** ($f(x) = \log_e(x)$) and it is the **inverse** of the natural exponential function $f(x) = e^x$

Note: If you see something like $f(x) = \log(x)$ with no base, then we assume it is **log base 10**, or written out, we assume $f(x) = \log_{10}(x)$

Evaluating Logarithms:

Evaluate the following expressions:

1. $\log_2(8)$

2. $\log_{32}(2)$
3. $\log_7(-3)$
4. $6^{\log_6(8)}$

(1) We ask ourselves the following question: When does $2^y = 8$? Well we know $2^3 = 8$ so our answer is $\log_2(8) = 3$

(2) We ask ourselves the following question: When does $32^y = 2$? Well we know $32^{\frac{1}{5}} = 2$ so our answer is $\log_{32}(2) = \frac{1}{5}$

(3) We ask ourselves the following question: When does $7^y = -3$? If we think about it we should realize this can never happen so there is no solution

(4) There is a property in the book that says that $a^{\log_a(x)} = x$ so our answer is $6^{\log_6(8)} = 8$

Important Properties of Logs:

1. $\log(x \cdot y) = \log(x) + \log(y)$
2. $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$
3. $\log(x^p) = p \cdot \log(x)$

Graph Translations:

Vertical Shifts: Let $c > 0$

1. $y = f(x) + c$ shifts $f(x)$ c units up
2. $y = f(x) - c$ shifts $f(x)$ c units down

Horizontal Shifts: Let $c > 0$

1. $y = f(x + c)$ shifts $f(x)$ c units left
2. $y = f(x - c)$ shifts $f(x)$ c units right

Reflections:

1. $y = -f(x)$ reflects $f(x)$ about the x-axis
2. $y = f(-x)$ reflects $f(x)$ about the y-axis

Vertical Stretch & Shrink:

Let $c > 0$. We consider $y = cf(x)$

1. $c > 1 \quad \rightarrow$ stretch $f(x)$ vertically by a factor of c
2. $0 < c < 1 \rightarrow$ shrink $f(x)$ vertically by a factor of c

Horizontal Stretch & Shrink:

Let $c > 0$. We consider $y = f(cx)$

1. $c > 1 \quad \rightarrow$ shrink $f(x)$ horizontally by a factor of c
2. $0 < c < 1 \rightarrow$ stretch $f(x)$ horizontally by a factor of c

Compount Interest Formulas:

- **Non-Continuous Compounding**

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

- **Continuous Compounding**

$$A = A_0 e^{rt}$$

$A =$ Amount

$A_0 =$ Principal

$r =$ Interest rate

$n =$ Number of times compounded per year

$t =$ Time