Math for Bio Formula and Definitions Sheet

Spring 2014

Distance Between Two Numbers:

$$d(x_1, x_2) = |x_2 - x_1| = |x_1 - x_2|$$

Midpoint Between Two Numbers:

$$m(x_1, x_2) = \frac{1}{2} \cdot |x_2 - x_1| = \frac{1}{2} \cdot |x_1 - x_2|$$

Distance Between Two Points:

$$d\Big((x_1, y_1), (x_2, y_2)\Big) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Between Two Points:

$$d\Big((x_1, y_1), (x_2, y_2)\Big) = \left(\frac{1}{2} \cdot (x_1 + x_2), \frac{1}{2} \cdot (y_1 + y_2)\right)$$

Standard Equation for a Circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Center: (h, k) **Radius:** $\sqrt{r}$

Symmetries

Y-Axis Symmetry - symmetrical about the y-axis. Graphically, you could reflect the function over the y-axis and it would remain the same. Some examples of functions with y-axis symmetry are $y = x^2$ and $x^2 + y^2 = r^2$ (a circle centered at the origin)

X-Axis Symmetry - symmetrical about the x-axis. Graphically, you could reflect the function over the x-axis and it would remain the same. Some examples of functions with x-axis symmetry include $x = y^2$ and $x^2 + y^2 = r^2$

Origin Symmetry - symmetrical about the origin. Graphically, you could rotate the function 180 degrees about the origin and it would remain the same. Some examples include $y = x^3$ and $x^2 + y^2 = r^2$

Even & Odd Functions

Even Function - a function is *even* if $f(-x) = f(x)$ Even functions always have y-axis symmetry

Odd Function - a function is *odd* if $f(-x) = -f(x)$ Odd functions always have origin symmetry

Intercepts

Y-intercept - the point where a graph passes through the y-axis. To find the y-intercept we set $x = 0$ and solve for y

X-intercept - the point where a graph passes through the x-axis. To find the x-intercept we set $y = 0$ and solve for x

Domain - the set of numbers that you can input into your function

Note: Usually you need to be careful of two types of functions:

1. **square root function**

   The function $f(x) = \sqrt{x}$ has a domain $x \geq 0$.

   You may also see something like $f(x) = \sqrt{(x-1)(5-x)(x+7)}$ and be asked to find the domain.

   You then need to solve $(x-1)(5-x)(x+7) \geq 0$ to find the domain

2. **rational function**

   A rational function is of the form $f(x) = \dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial. The domain of a rational function is all reals except for $Q(x) = 0$.

   For example the domain of $f(x) = \dfrac{x^2+3x+1}{x-1}$ is $(-\infty, 1) \cup (1, \infty)$

Sometimes you might see a combination of both.

For example, find the domain of $f(x) = \dfrac{1}{\sqrt{(3-x)(2+x)}}$ We do this by solving $(3-x)(2+x) > 0$

Graphically, you can determine the domain by scanning along the x-axis and seeing what x-values the function is not defined for.

Range - the set of numbers that are output from a function. Often harder to determine than the domain, we can determine the range graphically by scanning along the y-axis and seeing what y-values the function is not defined for.

Slope of a Line: Given two different points (x_1, y_1) and (x_2, y_2) the *slope* of the line connecting these two points is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-Slope Equation of a Line: The line with slope m passing through (x_1, y_1) is given by

$$y - y_1 = m(x - x_1)$$

Slope Intercept Equation of a Line: The line with slope m has a slope-intercept form of

$$y = mx + b \qquad \text{where b is the y-intercept}$$

Parallel Lines: two lines are parallel if they have the same slope

Perpendicular Lines: two lines are perpendicular if the slope of one line is the **negative reciprocal** of the slope of the other line

Quadratic Function:

$$y = ax^2 + bx + c \qquad a \neq 0$$

Vertex-Form of a Quadratic Function:

$$y = a(x - v_1)^2 + v_2$$

where (v_1, v_2) is the x, y coordinate of the vertex. We've also called this the standard form of a quadratic function

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vertical Shifts: Let $c > 0$

1. $y = f(x) + c$ shifts $f(x)$ c units up
2. $y = f(x) - c$ shifts $f(x)$ c units down

Horizontal Shifts: Let $c > 0$

1. $y = f(x + c)$ shifts $f(x)$ c units left
2. $y = f(x - c)$ shifts $f(x)$ c units right

Reflections:

1. $y = -f(x)$ reflects $f(x)$ about the x-axis
2. $y = f(-x)$ reflects $f(x)$ about the y-axis

Vertical Stretch & Shrink:

Let $c > 0$. We consider $y = cf(x)$

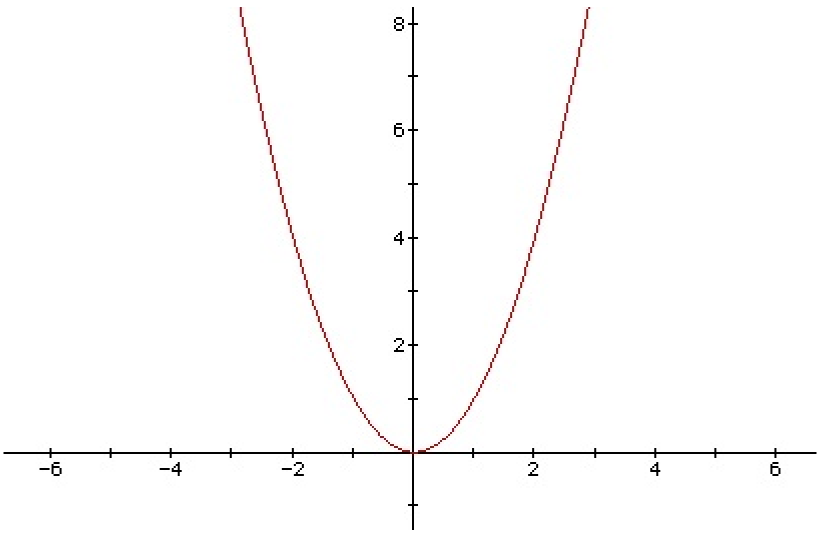
1. $c > 1 \rightarrow$ stretch $f(x)$ vertically by a factor of c
2. $0 < c < 1 \rightarrow$ shrink $f(x)$ vertically by a factor of c

Horizontal Stretch & Shrink:

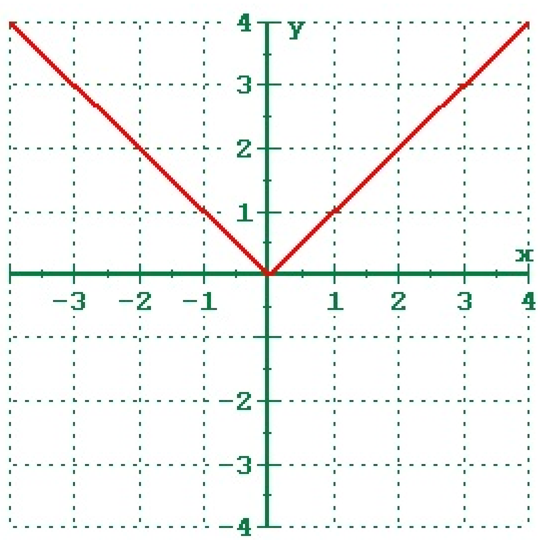
Let $c > 0$. We consider $y = f(cx)$

1. $c > 1 \rightarrow$ shrink $f(x)$ horizontally by a factor of c
2. $0 < c < 1 \rightarrow$ stretch $f(x)$ horizontally by a factor of c

Graph of $\mathbf{y = x^2}$



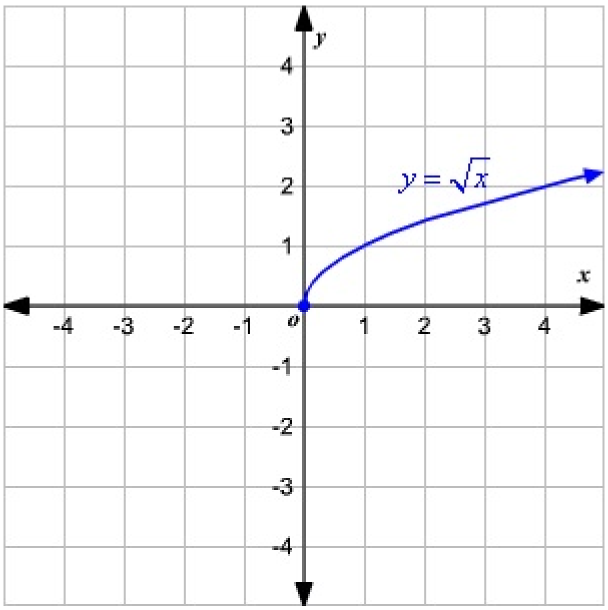
Graph of $\mathbf{y = |x|}$



Note: Remember the following two properties when solving absolute value inequalities:

1. $|x| < a \implies -a < x < a$
2. $|x| > a \implies x < -a$ and $x > a$

Graph of $\mathbf{y = \sqrt{x}}$



Arithmetic Combinations of Functions:

1. $(f+g)(x) = f(x) + g(x)$

   Example: Let $f(x) = x^2 + 3x + 7$ and $g(x) = -x + 2$

   Then $(f+g)(x) = (x^2 + 3x + 7) + (-x + 2) = x^2 + 2x + 9$

2. $(f-g)(x) = f(x) - g(x)$

   Example: Let $f(x) = x^2 + 3x + 7$ and $g(x) = -x + 2$

   Then $(f+g)(x) = (x^2 + 3x + 7) - (-x + 2) = x^2 + 4x + 5$

3. $(f \cdot g)(x) = f(x) \cdot g(x)$

   Example: Let $f(x) = x^2$ and $g(x) = \dfrac{1}{x}$

   Then $(f \cdot g)(x) = (x^2) \cdot \left(\dfrac{1}{x}\right) = \dfrac{x^2}{x} = x$

4. $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$

Example: Let $f(x) = x^2 + 3x$ and $g(x) = \frac{1}{x^2}$

Then $(\frac{f}{g})(x) = \frac{x^2+3x}{\frac{1}{x^2}} = x^2(x^2+3x) = x^4 + 3x^3$

Function Composition:

$(f \circ g)(x) = f\Big(g(x)\Big)$

Example: Let $f(x) = 3x^2 - 2$ and $g(x) = \frac{x+2}{x-1}$

Then $f \circ g = 3\Big(\frac{x+2}{x-1}\Big)^2 - 2$

Note: Be careful when finding the domain of a composition of functions.

Example: Let $f(x) = \sqrt{x-12}$ and $g(x) = x^2 - 4$

Find

a) $f \circ g$

b) $g \circ f$

and state the domain of each

Answer: a) $f \circ g = \sqrt{(x^2-4)-12}$

$= \sqrt{x^2-16}$

Domain: $x^2 - 16 \geq 0 \implies x \leq -4$ and $x \geq 4$

So the domain in interval notation is $(-\infty, -4] \cup [4, \infty)$

Answer: b) $g \circ f = \Big(\sqrt{x-12}\Big)^2 - 4$

$= x - 12 - 4$

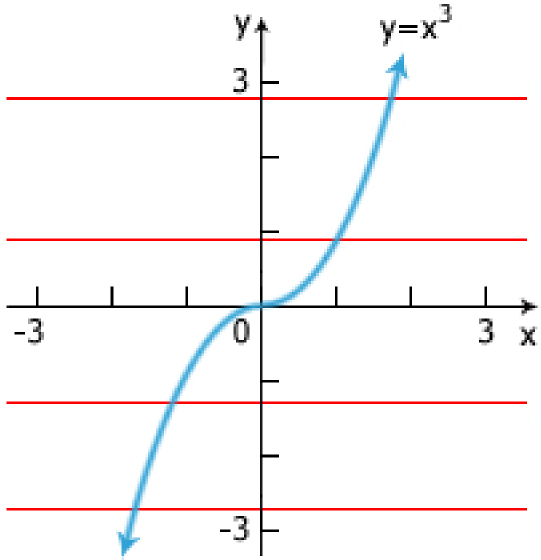
$= x - 16$

Domain: $x - 12 \geq 0 \implies x \geq 12 \implies [12, \infty)$

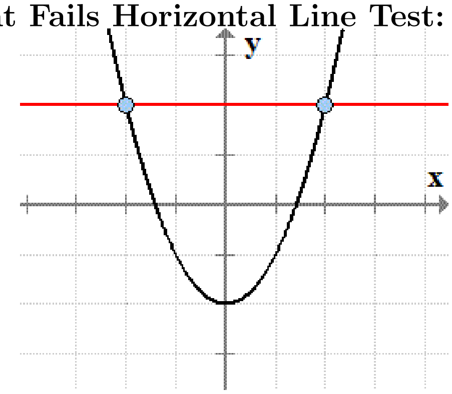
One-to-One: - a function is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$

Horizontal Line Test: - a nice, easy, graphical test to determine if a function is one-to-one. It says that a function is **one-to-one** if every horizontal line intersects a graph **at most once**

Function That Passes the Horizontal Line Test:

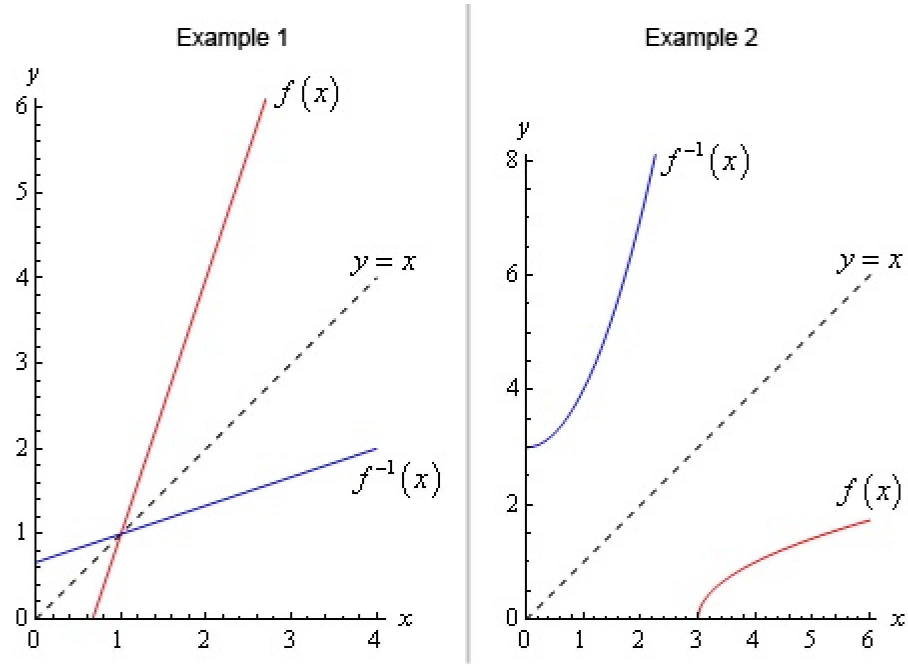


Function That Fails Horizontal Line Test:



Inverse Function: - if a function f is one-to-one then the **inverse function** denoted by $f^{-1}(x)$ is the function that "undoes" the original function f so that $f^{-1}\Big(f(x)\Big) = x$

- The domain of f is the range of f^{-1}
- The range of f is the domain of f^{-1}
- Graphically, the inverse function, $f^{-1}(x)$, is the graph of the original function, $f(x)$, reflected about the line $y = x$



Note: Notice how in Example 2 that the range of $f^{-1}(x)$ is the domain of $f(x)$

Finding the Inverse Function:

1. Set $y = f(x)$
2. Solve for x in terms of y
3. Swap x and y
4. Done!