Rolle's Thm + Mean Value Thm

+1 Statement

Rolle's Thm Suppose f continuous on $[a,b]$ and differentiable on (a,b), $f(a)=f(b)=0$. Then $\exists$ at least one $c\in(a,b)$ such that $f'(c)=0$.

+1 setup

Pf. If $f\equiv 0$ then we are done. Thus suppose $f\not\equiv 0$. WLOG to assume $f>0$ by replacing f with $-f$ if necessary. +1 setup Thus by Maximum-Minimum thm (f continuous on $[a,b]$ $\Rightarrow$) f attains $\sup\{f(x) \mid x\in[a,b]\}>0$ for $c\in[a,b]$. As $f(a)=f(b)=0$, $c\in(a,b)$. So $f'(c)$ exists.

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As $f(c)=\sup f$, its a relative maximum. By interior extremum thm ($c\in \mathrm{Dom}(f)$ relative extremum $\wedge$ diff. $\Rightarrow$) $f'(c)=0$.

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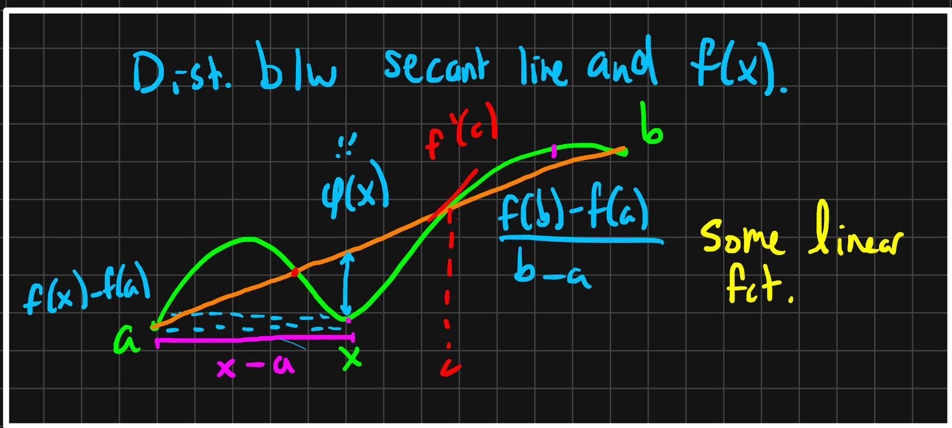
+1 statement

Mean Value Thm Suppose f continuous on $[a,b]$ and diff. on (a,b). Then $\exists$ at least one point $c\in(a,b)$ s.t. $f(b)-f(a)=f'(c)(b-a)$.

Pf. Consider $\varphi:[a,b]\to\mathbb{R}$ defined as

+2 Setup

$$\varphi(x)=\underbrace{\left(f(x)-f(a)\right)}_{\text{diff in } f \text{ val.}}-\left(\underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{secant line}}\underbrace{(x-a)}_{\text{dist. b/w } x \text{ and } a}\right)$$



As φ is a sum of a diff. fct and some scalar values, φ is cont. on $[a,b]$ and diff. on (a,b). By Rolle's, $\exists c\in(a,b)$: $\varphi'(c)=0$,

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i.e. $f'(c)-\frac{f(b)-f(a)}{b-a}=0 \Rightarrow f'(c)=\frac{f(b)-f(a)}{b-a} \Rightarrow f'(c)(b-a)=f(b)-f(a)$.

+0 for poor style
+1 for spotty style
+2 for good style