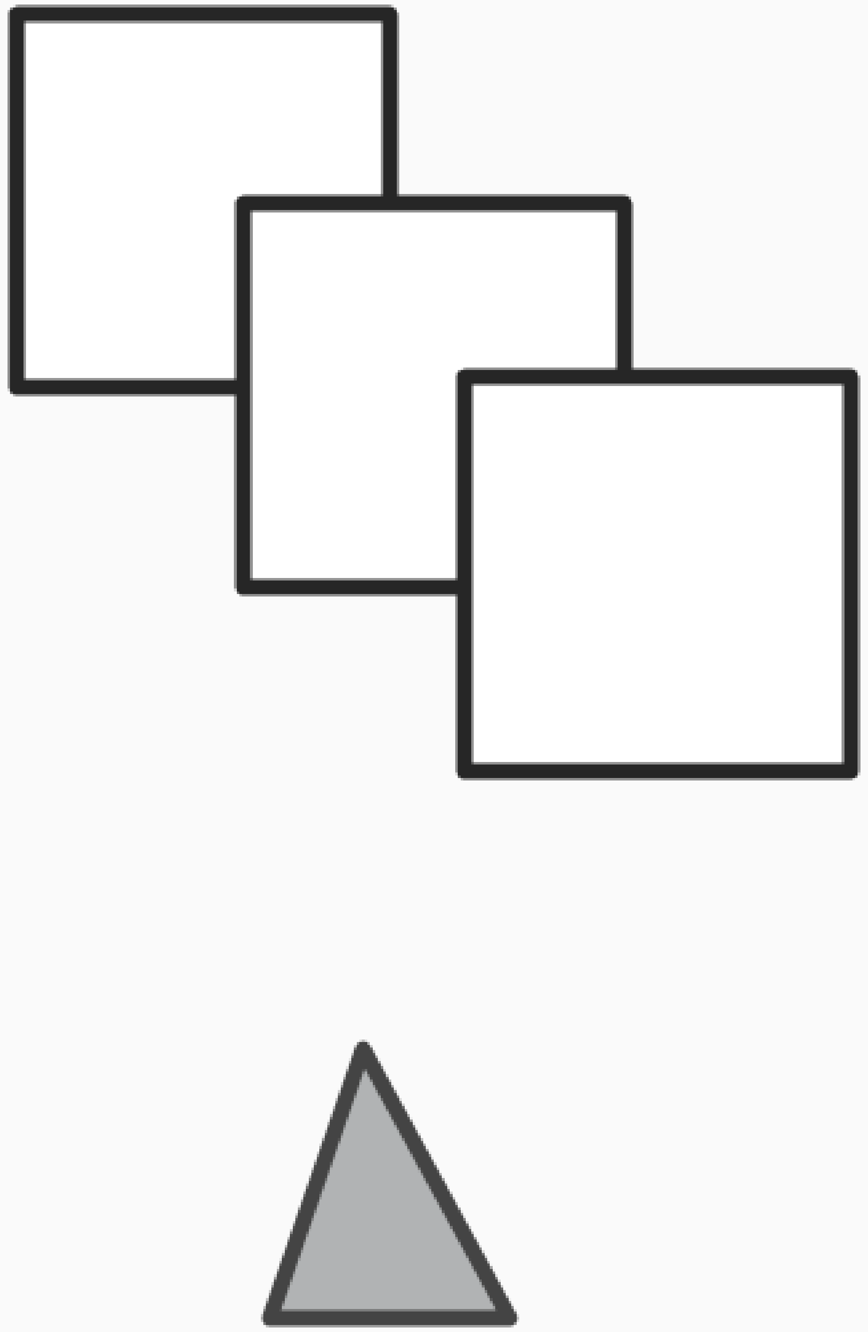
Just a reminder. This is the last pre-class and in-class. The homework due this Sunday, Homework 9 is the next-to-last homework. The last homework is Homework 10.

After we finish the activities of this week, I will add one more handout in this folder. That's the only activity we have not had a chance to cover, on the Jacobi symbol. I will post in case you'd like to look at it on your own later.



New gadgets in Jamboard! YAY!

No additional colors though. Sigh...

1 a

Solve the following equations, if possible, using the quadratic formula. 7x^2 - 4x + 2 = 0 (mod 11)

The quadratic formula for this equation is (14^-1)(4 +/- sqrt(16 - 56)) =

(3^-1)(4 +/- sqrt(-40)) =

4(4 +/- sqrt(4)) = 16 +/- 8 = 8 (mod 11), 24 = 2 (mod 11)

- Miah :) (hopefully I put this on the right thing this time)

1 b

Solve the following equations, if possible, using the quadratic formula.
7x^2 - 4x - 2 = 0 (mod 11)

The quadratic formula gives us x= (4+-sqrt(16+4*4*2))/14 = (4 +- sqrt(48))/14

Reducing mod 11 yields x = (4+-sqrt(6))/3

Unfortunately, x^2 = 6 has not solution mod 11 (checked with wolfram alpha) so this equation has no solution.

Hmm.. yeah, but if we use Wolfram Alpha to solve x^2=6, we can use it to solve 7x^2-4x-2=0 as well :) To make the whole method self-contained, let's use Euler's criterion for this part.

-Nick

1c

$3x^2 - 2x + 1 \equiv 0 \pmod{13}$

$$x = \frac{2 \mp \sqrt{4-12}}{6} = \frac{2 \mp \sqrt{5}}{6}$$

To find squares mod 13:

	1	2	3	4	5	6 (repeat after)
squares	1	4	9	3	12	10

So no square root of 5, which means no solution.

1d

$$2x^2 - 3x + 4 = 0$$

$$x = \frac{3 \mp \sqrt{9-32}}{4} = \frac{3 \mp \sqrt{3}}{4}$$

From previous part, we know $\sqrt{3} = 4$, so a solution exists.

$4 \times 3 = 12 \equiv -1 \pmod{13}$ so $4^{-1} \equiv -3 \pmod{13}$

$$x \equiv -3(3 \mp 4) \equiv 3, -21 \equiv 3, 5 \pmod{13}$$

2

$\left(\frac{0}{p}\right) = 0$ by definition

$\left(\frac{1}{p}\right) = 1$ because $1 \equiv 1^2 \pmod{p}$ for any p, so 1 is always a quadratic residue

$\left(\frac{4}{p}\right) = 1$ because $4 \equiv 2^2 \pmod{p}$

p	2	3	5	7	11	13	17	19
$\left(\frac{-1}{p}\right)$	1	-1	1	-1	-1	1	1	-1

It seems like 4k+1 primes are +1 and 4k-1 primes are -1

And well 2 is +1 in its special group as always

Cian

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