

What are all
the primitive
roots modulo
13?

a	1	2	3	4	5	6	7	8	9	10	11	12
order of a mod 13	1	12	3	6	4	12	12	4	3	6	12	2
		★				★	★				★	

2, 6, 7, and 11
are all the
primitive roots
modulo 13.

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$$g=2$$

$$\begin{aligned}g^0 &= 1 \pmod{13} \\ g^1 &= 2 \pmod{13} \\ g^4 &= 3 \pmod{13}\end{aligned}$$

$$\begin{aligned}g^2 &= 4 \pmod{13} \\ g^9 &= 5 \pmod{13} \\ g^5 &= 6 \pmod{13}\end{aligned}$$

$$\begin{aligned}g^{11} &= 7 \pmod{13} \\ g^3 &= 8 \pmod{13} \\ g^8 &= 9 \pmod{13}\end{aligned}$$

$$\begin{aligned}g^{10} &= 10 \pmod{13}, \\ g^7 &= 11 \pmod{13}, \\ g^6 &= 12 \pmod{13}\end{aligned}$$

-Maggie

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$G=2$

the order of g^2 or 2^2 is 6,
Which is 2's
order $12 / 2$

the order of g^3 or 2^3 is 4,
Which is 2's
order $12 / 3$

In general, if g is a
primitive root mod
 p , the order of g^i is
the order of g
divided by the gcd
of the order of g and
 i

In other
words, $(\phi(n)) / \gcd\{\phi(n), i\}$

Cian

Combining Maggie and Nick's work we see that the primitive roots expressed as 2^i are:
 $2=2^1$, $6=2^5$,
 $7=2^{11}$, $11=2^7$.

Using Cian's work, we see that this makes sense because a primitive root will have order = $\phi(n)$ and hence $\gcd(\phi(n), i)$ has to equal 1 for the order of g^i not be smaller than $\phi(n)$.

The powers i for which i is relatively prime to $\phi(n)=12$ are 1, 5, 7, 11.

5

$a=1,3,4,5,9$

**Even powers
of the
primitive root
are the
squares.**

6

**a. 5 quadratic
residue, 5
non-residue (0
is not
counted).**

**b. 6 quadratic
residue, 6
non-residue.**

**c. $(p-1)/2$
quadratic
residue, $(p-1)/2$
nonresidue.**