Instructions: Same as before. We don't have enough questions for everyone in this pre-class, so feel free to add a comment/thought on other question if you don't get your own question.

I filled in all the missing questions since some of these ideas will be used in the class activity. Just a reminder that the class activity work is due Monday 12 pm.

1

1. A student, Leslie, did the long division of 123755417 by 9 by hand. She found the quotient to be 1375600 with the remainder being 17. How would you convince Leslie that the answer needs further modification? (Note Let's avoid using a calculator. You can use smaller numbers in place of the given 123755417 in your explanation if that will help.)

Since there are a total of 9 digits in the number 123755417 we can rewrite the number using modular addition and multiplication as

$$\begin{aligned}[123755417] &= [(1\cdot 10^8)+(2\cdot 10^7)+(3\cdot 10^6)+(7\cdot 10^5)+(5\cdot 10^4)+(5\cdot 10^3)+(4\cdot 10^2)+(1\cdot 10^2)+(7\cdot 10^0)] \\ &= [([1]\odot[1])\oplus([2]\odot[1])\oplus([3]\odot[1])\oplus([7]\odot[1])\oplus([5]\odot[1])\oplus([5]\odot[1])\oplus([4]\odot[1])\oplus([1]\odot[1])\oplus([7]\odot[1])] \\ &= [1]\oplus[2]\oplus[3]\oplus[7]\oplus[5]\oplus[5]\oplus[4]\oplus[1]\oplus[7] \\ &= [1+2+3+7+5+5+4+1+7]. \qquad (1)\end{aligned}$$

This means that both the number 123755417 and the sum of its 9 digits, 35, in equation (1) $\equiv 8 \pmod 9$.

A simpler explanation which points out the problem without fixing it is that 17 is greater than 9 so 17 cannot be the remainder after division by 9. -Nick

Thanks, Nick. I was in such a hurry to employ a modular approach, I walked right by it! -Cy

But also, sometimes a longer approach could be something useful in another problem.

Cy

2

gcd(660,270)=30

660=2*2*3*5*11
270=2*3*3*3*5

The intersection of these lists is 2*3*5=30

Hehe... I have a meme for this (coming up in Week 3)! :) But could this process be shortened in some other way?

Euclids algorithm!

I was thinking more about: 10 is already a common factor clearly, so we can focus on finding the common factor of 66 and 27. Basically: Theorem 6 ii) in the class activity.

gcd(1234*3, 1234*5) = 1234

1234 is a factor of both while 3 and 5 share no factors so they contribute no common divisors.

gcd(30,p)=1 for prime p>100

Any prime p will have no positive divisors other than itself and 1. So no positive divisors other than 1 will be shared with 30.

-Nick

3

Since 3 and 19 both divide the number, and they are relatively prime, 3*19 should also divide the number and is a larger factor. So 19 cannot be the gcd.

The gcd is 2. Because both numbers differ by 2, anything that divides both should also divide their difference, 2. So gcd is 1 or 2. Since both numbers are even, 2 divides both, which makes it the gcd.

5

If gcd(a, b) = 15, then gcd(a/15, b/15) = 1. We know this because since 15 is the greatest common divisor of a and b, when a and b are divided by 15 they have no common divisors left except for one.

-Maggie

660, 270: They both have 10 as a common factor, so let's put that aside. 660=66*10, and 270=27*10. They also have 3 as common, 66=3*22, 27=3*9. So lcm = 3*10*9*22.

12345*3, 12345*5: Since both have 12345 as a factor, we need to have that in the lcm. But we also need 3 and 5. So lcm=12345*3*5.

30, p: There are no common factors in 30 and p, so lcm = 30*p.