Investigations on Factors and Multiples of Numbers
Pre-class activity, Week 2

For the justification questions in this activity, you do not need to worry about official proofs. If a question asks for a justification, just think about how you would convince someone. If you can think of a general explanation (possibly using specific numbers, but an explanation which can be generalized), that's great. If you cannot think of a general explanation, then try to explain why it works for the given specific numbers. Maybe try specific numbers in two different ways. We will make things more general and official in the class activity.

The *Division algorithm* says that "For any integers a and b, with $b > 0$, there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < b$." The r is basically the same as the remainder in the long division, and q is the quotient.

1. A student, Leslie, did the long division of 123755417 by 9 by hand. She found the quotient to be 1375600 with the remainder being 17. How would you convince Leslie that the answer needs further modification? (Note: Let's avoid using a calculator. You can use smaller numbers in place of the given 123755417 in your explanation if that will help.)

The *greatest common divisor* (*gcd*) of two integers (not both 0) is the largest integer that divides them. For example, the gcd of 15 and 63 is 3, but gcd of 36 and 63 is 9.

2. Find the gcd of each of the following three pairs: $660, 270$; $1234 \cdot 3, 1234 \cdot 5$; $30, p$ where $p > 100$ is a prime.

3. Tim found the gcd of 40299 and 12255 to be 19. He also noticed that 3 divides both numbers. But since 19 is larger than 3, he thinks his answer is correct. Is that right? How can you convince Tim without calculating the gcd?

Over →

4. Can you find the gcd of 1234567890+2 and 1234567890?

5. If $\gcd(a, b) = 15$ for two unknown numbers, can you tell what $\gcd(a/15, b/15)$ is?

The *least common multiple* (*lcm*) of two integers (not both 0) is the smallest integer that is a multiple of both a and b. For example, the lcm of 12 and 14 is 84; the lcm of 6 and 18 is 18; and the lcm of 5 and 12 is 60.

6. Find the lcm of each of the following pairs: $660, 270$; $12345 \cdot 3, 12345 \cdot 5$; $30, p$ where $p > 100$ is a prime.