Instructions: We'll use the same whiteboard process until, as a whole class, we decide to use some other process.

1&2

1

Investigations on Calculating and Using the GCD
Pre-class, Week 3

For the justification questions in this activity, you do not need to worry about official proofs. If a question asks for a justification, just think about how you would convince someone. If you can think of a general explanation (possible using specific numbers, but an explanation which can be generalized), that great. If you cannot think of a general explanation, then try to explain why it works for specific numbers. Maybe try specific numbers in two different ways. We will make things more general and official during class.

1. Justify why any factor of a and b should be a factor of a and $a-b$, (For example, any factor of 6799 and 6789 should be a factor of 6799 and 10.) We let d be factor of a and b. This means that $d \mid a$ and $d \mid b$ so by the definition of divides, $a = dq_1$ and $b = dq_2$ for some integers q_1 and q_2. By substitution we readily see

$$\begin{aligned} a - b &= dq_1 - dq_2 \\ &= d(q_1 - q_2). \end{aligned} \tag{1}$$

By closure of integers under subtraction and multiplication, we conclude that $(q_1 - q_2)$ from equation (1) is an integer. This means we have written $a-b$ in the form dk from some integer k.

2

2. Justify why any factor of a and $a-b$ should be a factor of a and b. (For example, any factor of 6799 and 10 should be a factor of 6799 and 6789.) For a factor $d \in \mathbb{Z}$, $d \mid a$ and $d \mid (a-b)$. This means that $a = dq_1$ for some integer q_1 and that $(a-b) = dq_2$ for some integer q_2. Using substitution and algebra we obtain

$$\begin{aligned} b &= a - dq_2 \\ &= dq_1 - dq_2 \\ &= d(q_1 - q_2). \end{aligned} \tag{2}$$

By the closure property of integers in equation (2) we have written b in the form dk for some integer k,showing $d \mid b$.

Cy

3&4

3:

gcd(a, b) is a common factor of a and a-b so gcd(a,b) <= gcd(a,a-b)

gcd(a,a-b) is a common factor of a and b so gcd(a,a-b) <= gcd(a,b)

Thus, gcd(a,b) = gcd(a-b)

I think you're using 1&2 but in a different way than I had thought.

I was thinking of using 1&2 to say "Any factor of a and b is also a factor of a and a-b, and vice versa, so the set of factors is same; since the set is same, the greatest factor is same as well."

4:

gcd(123456,123476) = gcd(123456,20)

The factors of positive 20 are 10, 5, 4, 2, 1. The largest of these that divides 123456 is 4.

So gcd(123456, 123476) = 4

-Nick

5

The gcd(b,r) tells us we have a factor, d, that divides both b and r. So we know that b=dm and r=dn, for integers m and n.

We assume that a=bq+r, for some integer q. So with some substitution we get, a=dmq+dn, which can be simplified to, a=d(mq+n).

By definition we know that d|a, showing that the gcd(a,b)=gcd(b,r)

I feel so far we only have "gcd(b,r) | gcd(a,b)" (because if gcd(b,r) divides both a, b and Theorem 4 of the Week 2 Class activity says anything that divides both a,b divides their gcd).

But I think we still need to show gcd(a,b) divides gcd(b,r).

Michael

6

23024709= 122(188727) + 15

188727= 12581(15)+12

15=1(12)+3

12=4(3)

Could this be shortened a bit? (For everyone)

For the problem at hand: Using substitution we obtain

$$\begin{aligned} 15 &= 23024709 - 188727 \cdot 122 \\ &= 3(7674903) - 3(7674898) \\ &= 3(7674903 - 7674898). \end{aligned}$$

So $m = n = \gcd(a, b) = \gcd(b, r) = 3$.

GCD(23024709, 188727)=3

Cian

7

Find gcd(527176, 35039)

First we find the remainder of a mod b, which we call r. In this case r = 1591, so now our problem becomes finding gcd(35039, 1591). This is still a very difficult calculation.

method, we can further simplify our problem. Another iteration, with a = 35039, b = 1591, gives us r = 37. Now our problem is to find gcd(1591, 37). As 37 is prime, we know it must either be 1 or 37, but we can check for this using one more

Using a = 1591, b = 37, we find that r = 0. This means that b|a, so gcd(1591, 37) = 37. This means that gcd(527176, 35039) = 37.

-Miah

8
a,b,c

A) $2(-2)+3(4)=8$

B) $2(8)+3(-1)=13$

C) $2(2c)+3(-c)=c$

Joe

8
d,e,f

d. $2x + 4y = 105$ No integer solution because an the product of two even integers is even, and the sum or difference of two even integers is always an even integer.

$$2x + 4y = 2(x + 2y).$$

e. $6x + 9y = 24$ Let n be an integer.

n	x	y	$6x$	$9y$	$6x + 9y$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
-4	-11	10	-66	90	24
-3	-8	8	-48	72	24
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$3n + 1$	$-2n + 2$	$6(3n + 1)$	$9(-2n + 2)$	24

For $6x + 9y = 24$ there exist integer solutions in the form

$$6(3n + 1) + 9(-2n + 2) = 24.$$

Neat! A general solution!

f. $6x + 9y = 11$

For similar reasons for 8d, there are no integer solutions.

$$6x + 9y = 3(2x + 3y).$$

So the solution would have to be divisible by 3 and $3 \nmid 11$.

Cy

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