Instructions: Same process as in the pre-class. Each person takes a page and writes the answer. The others can comment/add another different solution/ask questions, etc.

I'll use yellow sticky notes. Each person who writes on a jamboard panel should use a different color so it's easier to follow thoughts.

Also, everyone but me write your name somewhere on that panel so that we can know what your color is and can follow who is asking what/answering what better.

Problem 1

If a|b then b = a * k for some integer k.
If b|c then c = b * n for some integer n.

Substituting our definition of b in to the second equation gives c = (a * k) * n.

Rearranging gives
c = a * (k * n). Since k and n are integers, k * n is an integer, call it q and so
c = a * q which is the definition of divides so a|c

I'd skip assigning a name to the k*n since we don't need to refer to q later. After saying "k*n is an integer", I'd simply say "so, by definition, a|c."

Ok!

Cian

2

If $a|b$ we will have $b = a*k$ for some integer k, if $a|c$ we will have $c = a*j$ for some integer j.

Using our new definitions for b and c we can see that $b+c = a*k + a*j$, which we can break up to $b+c = a(k+j)$. Since $k+j$ is an integer, by definition $a|(b+c)$

Michael

3

Can you determine whether 456123 is prime or not without using a calculator?

I approached this problem in two ways, one where I realized I used a calculator afterwards, and then another method which doesn't, but I'm not sure how practical the second method is.

I researched and utilized Fermat's Primality Test, which stated that if p is prime and a is not divisible by p, then the congruence [a^(p-1) ≡ 1 (mod p)] holds true.

a is chosen as any integer, and p= 456123. I chose a=2. Using WolframAlpha, I calculated the congruence (2^(456122) mod 456123), and obtained 4.

Since 4 does not equal 1, by the Primality test, I concluded that 456123 was composite. However, after I finished the test, I realized that I used a calculator.

So, I went with another method. I know that certain rules exist for determining if numbers are prime or not, and if they're divisible by a certain value.

Since 456123 isn't even, I know that the number isn't divisible by 2, 4, 6, or 8. Similarly, since 456123 doesn't end in 5 or 0, then the number isn't divisible by 5 or 10.

However, I know that if you add all up of the numbers and the resulting value is divisible by 3 or 9, then 456123 is divisible by 3 or 9.

4 + 5 + 6 + 1 + 2 + 3 = 21, which is divisible by 3, therefore 456123 is divisible by 3 (comes out to be 152,041). Therefore, 456123 is not prime.

Mirza :)

4

$f(n) = n^3 - n$.
$f(1)=0$, $f(2)=6$,
$f(3)=24$,
$f(4)=60$,
$f(5)=120$,
$f(6)=210$

Each of the values of $f(n)$ is divisible by 6. This is because $f(n)=n^3-n=(n-1)n(n+1)$. So $f(n)$ is just the product of 3 consecutive integers.

In any 3 consecutive integers there will be a multiple of 3 and at least one multiple of 2. So the product of the three numbers will be a multiple of 3*2=6.

-Nick

5

$$f_i \mid f_{in}$$

6

2 divides f_3n

Base case: f_3 = 2 and 2|2

assume 2|f_3n

then f_3(n+1) = f_3n+3

=f_3n+2 + f_3n+1 = f_3n + f_3n+1+ f+3n+1

=f_3n + 2(f_3n+1)

2 | f_3n and 2| 2(f_3n+1)

if a | b and a |c then a| (b+c)

Joe

7

Problem #7 from Week 1 Class Activity

7. Determine the ones' digit of $3, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, \ldots$ until you notice a pattern. Use this pattern and properties of modular arithmetic modulo 10 to determine the ones' digit of 3^{2019}.

The ones digits for $3, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, 3^8$ are $3, 9, 7, 1, 3, 9, 7, 1$ respectively. The ones' digits correspond to each of the values $3, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, 3^8 \pmod{10}$. Notice that the pattern repeats each of the 4 integers in the pattern $\{3, 9, 7, 1\}$ in order. Since the pattern of 4 repeated I decided to try looking at the exponent, we'll call $x \in \mathbb{N}$ as modulo 4. We notice that $x \equiv 1 \pmod 4$ always coincided with the 3 in the ones' place, $x = 2 \pmod 4$ always coincided with the 9 in the ones' place, $x = 3 \pmod 4$ always coincided with the 7 in the ones' place, and $x \equiv 0 \pmod 4$ always coincided with the 1 in the ones' place. This means that for 3^{2019}, we obtain

$$2019 \equiv 3 \pmod 4,$$

and this corresponds to a 7 in the ones place.

$$(3^4)^{504} \, 3^3 \equiv 1^{504} \, 3^3 \pmod{10}$$

Cy

8

0 is always a square

The remaining residue classes, about half of them will be square