Announcements: 1) If you'd like to register for this course for the fall semester, send the forms I emailed you (after filling out the Word Doc form yourself) to the department head: Dr. Esther Billings at billinge@gvsu.edu

If you have any questions about the course (what work you need to complete, what your expected grade is based on current work, how many credits can you get, etc.), email me!

2) If you'd like to listen to some serious math talks, check out this page: https://researchseminars.org/

Word of caution: If you're not studying in that area, you will likely get lost after a few minutes of the talk. The number goes up to five minutes if the speaker is good and you're a Ph.D. mathematician in another field.

3) Midway evaluation (link posted in the deadlines document) has only one response so far. Any other feedback?

4) A handout putting together the pieces of "there are infinite primes of the form 4k+3" proof is posted under Week 4 folder. It also mentions Dirichlet's theorem. The handout is from last year's class.

5) I was hoping to fit the Chinese Remainder Theorem into this week's materials, but the activity was getting to be too long, so that went into next week.

6) Homework 4 solution is posted under Week 4 materials.

7) Projects topics and instructions will be listed on the deadlines document (at the end of the document). Project is due by anytime by the end of the Fall semester.

Theorem 1 (fill in the blanks)

Theorem 1: Let n be a positive integer. Then the following properties hold:

i. (Reflexivity) $a \equiv a \pmod{n}$ for any a.
ii. (Symmetry) If $a \equiv b \pmod{n}$, then $b \equiv a \pmod{n}$.
iii. (Transitivity) If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.

proof: You proved (i) in problem pre-class, where we had $n = 7$.

To prove (ii), which is an 'if' statement, we assume the hypothesis and prove the conclusion. Assume $a \equiv b \pmod{n}$. By definition this means $n|(a-b)$, i.e. $a - b = kn$ for some k. In that case, $b - a = -kn = (-k)n$ where $-k$ is an integer, therefore $n|b-a$ and $b \equiv a \pmod{n}$.

To prove (iii), assume $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$. We want to show $a \equiv c \pmod{n}$. From the assumption, $n|a-b$ and $n|b-c$. In other words, $a-b = k_1 n, b-c = k_2 n$ for some k_1, k_2. Adding these two expressions, we find that $a - c = (k_1 + k_2)n$, therefore $a \equiv c \pmod{n}$. □

1

Find a solution for the congruence equation $4x+5 \equiv 6 \pmod{11}$ using the simplification process.

First we subtract 5 from both sides so that we're looking for x such that $4x \equiv 1 \pmod{11}$, then we're like "oh, 1 mod 11?well 12 is 1 mod 11, and its divisible by 4" so we end up with $x=3$.

- Miah :)

2

4x+9==3(mod11)
4x==-6(mod11)
4x==5(mod11)
4x==16(mod11)   x=4.
25==3(mod11)
11|(25-3)

Michael

3

8x+9 ==
3mod18
8x == -6mod18
8x ==12mod18

8x ==30mod18
8x ==48mod18
x ==6mod18
so x=6 works

and
8(6) + 9 =
18 (3) + 3

Cian

4
a,b,c

Some integers in [3] mod 7 are: -11, -4, 3, 10, 17

These are numbers of the form 3+7k

Some integers in [10] mod 7 are: -11, -4, 3, 10, 17

These are numbers of the form 10+7k which is the same as 3+7(k+1)

Mod 7, [1]=[36], [2]=[16], [6]=[13], and [25] is not equal to anything we were given.

-Nick

5 a. Show that $\{0, 1, \ldots, 6\}$ is a complete residue system modulo 7. This system is called the **least residue system**.

To show that $\{0, 1, \ldots, 6\}$ is a complete residue system, we need to show two things:
i) For every integer n, $n \equiv r \pmod 7$ where $0 \leq r \leq 6$, ii) If $n \equiv i \pmod 7$ and $n \equiv j \pmod 7$ with $0 \leq i, j \leq 6$, then $i = j$.

The first property follows from the division algorithm as any integer n is equivalent to the remainder r after dividing by 7, and r satisfies $0 \leq r \leq 6$.

We will prove ii) using contradiction. Suppose $i \neq j$ satisfy the conditions. Assume without loss of generality $i > j$. Then consider $i - j$. Because both i, j are equivalent to n mod 7, we have

$$i - j \equiv n - n \pmod 7 \quad \longrightarrow i - j \equiv 0 \pmod 7$$

This means $i - j \geq 7$. But $i \leq 6$ and $j \geq 0$, so $i - j \leq 6$, which leads to a contradiction. Therefore, $i = j$.

b. Find another complete residue system modulo 7.

$\{1, 2, \ldots, 7\}$, $\{0, 8, 16, 24, 32, 40, 48\}$, $\{0, -1, -2, \ldots, -6\}$