Childcare Subsidies and the Child Penalty: A Life-Cycle Analysis

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Abstract

The “child penalty” of motherhood represents the reduction in women’s wages upon having children. This paper considers the question of how much of this reduction would be affected by a universal childcare subsidy. I do this by estimating a rich life-cycle model of women’s labour supply and savings choices, using data from Understanding Society. This considers the utility cost of work, the human capital accumulation process, and access to informal forms of childcare. Using this model, I estimate the effect of universal childcare on this child penalty, as well as on women’s wages over the long-run. I find that the cost to women’s earnings from having children is significantly reduced with a universal childcare policy, and that such a policy leads to a small increase in women’s wages later in life.

1 Introduction

The role of children in affecting female labour market outcomes is an important one. Studies have shown that the “child penalty” in earnings for women having children is large and takes the form of a reduction in wages and a reduction in labour market participation, at both the intensive and extensive margin over the long term (Kleven et al., 2019). One plausible driver of this child penalty are childcare costs, which add an additional cost to labour market participation for parents of young children. In this paper, I study how policies reducing childcare costs would affect this child penalty. I also examine how it would affect female labour market participation and long-run wages in the aggregate, across both mothers and non-mothers.

To do this, I build a life-cycle model of women's consumption and labour market decisions. This allows us to understand the determinants of women's labour supply decisions before, during and after childcare. The use of a dynamic model allows me to illuminate the role labour supply choice plays in human capital accumulation, the key mechanism behind the interaction between employment choices in the early years of motherhood and wages in the long run. The use of a structural model allows me to estimate three key sets of unobservable parameters using simulated method of moments: the non-pecuniary costs of work dependent on children and marital status, the parameters governing human capital accumulation, and the proportion of women who need to use formal channels to acquire childcare.

Estimating these parameters allows me to examine how counterfactual childcare subsidies would impact female labour market participation. I simulate these counterfactuals and estimate how they would affect employment during the early years of motherhood. By estimating the human capital accumulation process, and by studying labour market outcomes across women's working life, I then consider how long-run wages are affected by the subsidy. I measure this both in terms of the effect on the gap between mothers and non-mothers, and in terms of the aggregate impact on wages across all women.

I use survey data from Understanding Society to estimate the labour market and saving choices of non-graduate women in the United Kingdom, as well as how those choices interact with future labour market outcomes. I do this by estimating a model of choices over the course of women's working life, using simulated method of moments to match key facts in the data to equivalent elements of the simulated model. This allows me to estimate important unobservable parameters. Using these unobservable parameters, I then run the same simulated model while including various childcare subsidy scenarios, and observe the change in labour market outcomes, both in terms of aggregate outcomes across all women over the life cycle and in terms of the child penalty.

The results show that the government completely subsidising the cheapest types of market childcare would have large effects on life-cycle labour market outcomes. I find that a full childcare subsidy funded by income tax can increase mean employment for non-graduate women by as much as 7 percentage points at certain points of the life cycle. I also find that it can have a positive impact on long-run mean wages. The results are driven by the subset of the population that does not have access to any other type of childcare, such as from grandparents. This is because only their labour market participation would be affected by the cost of market childcare.

The results also show significant impact on the child penalty, which I evaluate by comparing the labour market outcomes in a comparison between mothers and non-mothers of the same age. I find that the penalty in total labour earnings from having children, which is greater than 30% for much of motherhood and remains above 20% 20 years after becoming a mother, is reduced by between a quarter and a half in every year from becoming a mother to the first child turning 20. This is driven by increases on all three margins of total earnings: employment, hours worked and wages.

Various strands of the life-cycle literature have modelled women's labour market outcomes over the life cycle. Adda et al. (2017) look at how having children affects human capital accumulation through the lens of occupation choice. Eckstein et al. (2019) use a life cycle model to examine how childbirth decisions have affected changing wages in the post-war period. Similarly, Attanasio et al. (2008) look at how childcare cost changes in the second half of the 20th century changed the gender pay gap. Meanwhile, Blundell et al. (2016) use labour market participation choices to look at the role of Working Tax Credit reforms in the United Kingdom.

The life-cycle model literature has not covered the impact of how childcare policy changes could impact labour market outcomes, but there have been examinations of natural experiments arising from policy changes. Significant reduced-form work has shown that policy changes reducing childcare costs and increasing availability have had significant positive impact on mothers' labour market participation in Spain (Nollenberger and Rodríguez-Planas, 2015) and Italy (Carta and Rizzica, 2018). These results approach mine, but I go on to show how this effect would stretch out by pushing up mothers' wages in the long run.

The rest of the paper is as follows. Section 2 outlines the life-cycle model, showing the structure of utility, wages and human capital accumulation that drives decision making in the model. Section 3 outlines the data used to estimate the model. Section 4 outlines the estimation process. This includes details of the estimation of exogenous parameters, and an outline of the method and moments used to match the unobservable parameters in the model. Section 4 also includes the results of this estimation, outlaying the estimates of these unobservable parameters. Section 5 demonstrates the results from the counterfactual analysis, through measurements of employment, wages, the child penalty, and a welfare analysis. Section 6 concludes.

2 Life-Cycle Model

The model in this paper follows the saving and labour supply decisions of adult women over the course of their working life. At the beginning of each period, women observe a series of shocks, including productivity, fertility, and marital shocks. Upon observing these shocks, they make decisions how much to work, and how much money to save and consume. If they have children, their labour supply decision has consequences for their use of childcare, and for how much they pay for childcare. The model also accounts for human capital accumulation, meaning that labour supply decisions affect future productivity. The aim of the model is to examine how women adjust their labour supply decisions when the cost of childcare changes, and to think about the implications of this for women's long-term earnings.

The period modelled is from age 20 to 65. Each period t in the model represents their consumption and saving decisions at age t. At age 65, women retire and, after that point, can no longer make any decision to work, only a consumption decision. [1]. After retirement, women live for ten more years from their accumulated savings.

[1]There is no possibility of change of marital status after 65, so women either stay married or stay single until death. There are also no bequests in the model, so women spend all their savings before they die.

2.1 Problem of the individual

At age t, women make decisions on how much to work ($P_{i,t} \in \{0, 0.5, 1\}$), and how much to consume ($c_{i,t}$). The different values of P represent proportions of a 40-hour week spent working. Marriage and childbirth are determined by exogenous shocks. The decisions made by the individual take place after observing marital status, childbirth, and productivity shocks. These decisions are taken based on the individual's preferences, which are represented by $u(P_{i,t}, c_{i,t})$. Additional state variables savings ($A_{i,t,}$) and human capital ($h_{i,t}$) are dependent on the consumption and labour supply decisions made. If the woman i is single, then her budget constraint is as follows:

$$\frac{A_{i,t+1}}{1+r} = A_{i,t} - c_{i,t} + (w_{i,t} - Paid_i * CC_{i,t})P_{i,t} + B_{i,t} \tag{1}$$

$$A_{i,t+1} \geq 0 \tag{2}$$

where t is her age. The wage rate is determined by a persistent stochastic process, and by levels of human capital, as outlined below. $B_{i,t}$ is the level of benefits received, which is given by a formula that follows the UK benefits system in 2011. It is a function of number of children, age of children, employment status, income, and assets. $CC_{i,t}$ is also dependent on the number and age of children.

Meanwhile, if the woman is married, then the decisions to be made are how much the woman works, and how much the household consumes. The budget constraint for the married household is therefore as follows:

$$\frac{A_{i,t+1}}{1+r} = A_{i,t} - c_{i,t} + (w_{i,t} - Paid_i * CC_{i,t})P + w^M_{i,t} + B_{i,t} \tag{3}$$

$$A_{i,t+1} \geq 0 \tag{4}$$

where $w^M_{i,t}$ is the income of her spouse. I follow much of the female labour supply life-cycle literature (Blundell et al., 2016; Low et al., 2018; Adda et al., 2017) and set male labour supply to be exogenous, as well as assuming that men always have some form of income, whether through labour or benefits. In doing so, I effectively take the gender dynamics that may drive the gender wage gap and child penalty as a given, and consider policies that might improve these outcomes under these norms.

2.2 Childcare

Childcare is represented by $CC_{i,t}$ in the model. This is paid for if the woman works, and if they have no alternate option for childcare. The binary variable $Paid_i$ represents whether an individual has access to an alternative option for childcare, such as a relative or a neighbour. If this is 0, they can work without paying for childcare at all. If it is 1, then they pay for childcare according to the age and number of children, and depending on whether the work is full-time or part-time.

Childcare costs depend on whether children are at the age of pre-school, nursery, or school age. If they are of pre-school or nursery age, then part-time work halves the amount of childcare needed, halving the cost. Meanwhile, in the model those who work part-time do not have to pay for childcare for children of school age.[2]

It is assumed that everyone in the model pays for the cheapest type of childcare, so there is no variation in childcare quality. This is done so as to capture fully the question of how the additional need to pay for childcare affects the labour supply decision of mothers. The choice to pay for higher quality childcare, rather than pay for the cheapest form of childcare, can therefore be treated as consumption in this model. [3]. As a result, only childcare that is necessary for the a mother to work the number of hours she does is considered in the model.

2.3 Maximisation Problem

The state space for women is $\Omega_{i,t} = \{t, A_{i,t}, \mu^F_{i,t}, h_{i,t}, \mu^M_{i,t}, m_{i,t}, age^K_{i,t}, n_{i,t}, Paid_i, \mathcal{O}_{i,t}\}$. Here, $m_{i,t}$ is a binary variable indicating whether a woman is married, $n_{i,t}$ is the number of children they have, and $age^K_{i,t}$ is the age of their youngest child. $\mu^F_{i,t}$ and $\mu^M_{i,t}$ represent the persistent productivity shocks of the woman (F) and, if applica-

[2]In the case of school age children, childcare costs refer to the cost of before-school and after-school care that would be needed by mothers working full-time.

[3]The choice to use childcare when you have access to non-market childcare, or when the mother does not work, can also be treated as consumption in the model

ble, their partner (M). $\mathcal{O}_{i,t}$ represents a binary variable, which is 1 in period t if the individual has faced an unemployment shock in that period, and cannot find work, and 0 otherwise.

The value function of the woman at age t is therefore given by:

$$V_t(\Omega_{i,t}) = max_{q_{i,t}}\{u(P,C) + \beta E_t(V_{t+1}(\Omega_{i,t+1}))\} \tag{5}$$

where $q_{i,t} = \{P_{i,t}, c_{i,t}\}$ represents the choices an individual makes each period. The expectation encompasses the probability that a woman will be married next period, the probability that they will have a child, and the possible distribution of their and their partners future wages.

Within-period utility is given by the following function, for both single and married women:

$$u(P,c) = \frac{(\frac{c}{k}e^{\eta P})^{1-\gamma}}{1-\gamma} \tag{6}$$

where k represents an equivalence scale dependent on marriage and the number of children, so to represent the portion of household consumption that benefits the woman.[4] γ can be interpreted as the level of risk aversion, and η can be interpreted as the utility cost of work. η is determined by a non-parametric function:

$$\eta_{i,t} = \eta(m_{i,t}, n_{i,t}, age^K_{i,t}) \times g(P_{i,t}, m_{i,t}) \tag{7}$$

where

$$g(P_{i,t}) = \begin{cases} \psi_m & \text{if } P_{i,t} = 0.5 \text{ and } m_{i,t} = 1 \\ \psi_s & \text{if } P_{i,t} = 0.5 \text{ and } m_{i,t} = 0 \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

that is, the utility cost of work is dependent on marital status, number of children, age of youngest child, and whether work is part-time or full-time.[5]

[4] k is set equal to 1 for single women. 0.4 is added for each child, and 0.6 is added if the woman is married. This follows the results of Adda et al. (2017)

[5] It is worth emphasising that the utility function includes the multiplying of utility cost of

2.4 Wage Processes

The female wage process is determined by the following equation:

$$\log(w_{i,t}^F) = w_{initial}^F + f_i^F + \log(h_{i,t}) + \mu_{i,t}^F \tag{9}$$

so wages are driven by a combination of human capital and productivity shocks, as well as initial heterogeneity f_i^F, which is normally distributed with mean 0. There exists a minimum wage in the model. Thus, if $w_{i,t}$ is determined to be lower than minimum wage, the individual nevertheless earns the minimum wage if they work. The human capital accumulation process is given by:

$$\log(e_{i,t}) = \log(e_{i,t-1}) + \alpha_0 I(P_{i,t-1} = 1) + \alpha_1 I(P_{i,t-1} = 0.5) + \alpha_2 I(P_{i,t-1} = 0) - \delta t \tag{10}$$

Through this mechanism, the labour participation decision affects the level of human capital, and so impacts future wages. A lower bound is placed on human capital, so that $e_{i,t}$ cannot decay beyond its initial value, which is the value of every individual's human capital when they first enter the workforce. Meanwhile, the persistent shock $\mu_{i,t}^F$ is given by:

$$\mu_{i,t}^F = \nu^F \mu_{i,t-1}^F + \zeta_{i,t}^F \tag{11}$$

where $\zeta_{i,t}^F$ is normally distributed with mean zero.

The male income process of the spouse of woman i of age t is given by:

$$\log(w_{i,t}^M) = w_{initial}^M + f_i^M + \gamma_1 t - \gamma_2 t^2 + \mu_{i,t}^M \tag{12}$$

where $\mu_{i,t}^M$ is determined analogously to $\mu_{i,t}^F$.

work by labour supply, so a value of $\psi_m = 1$ would mean that the exponent in the utility function was half as big in the part-time case as in the full-time case.

2.5 Exogenous Variables

Women face three other types of shocks, in addition to the persistent productivity shock. The first of these is an unemployment shock. In each period, there is a probability of being unemployed which is correlated only with age, and is not autocorrelated. If women face an unemployment shock, they receive unemployment benefits, and do not face a labour supply decision in that period, as they cannot work.

Women also face shocks to their marital status and fertility, which are both determined exogenously.[6] The probability of being married in at age $t+1$ is given by:

$$Pr(m_{i,t+1}|m_{i,t}, t) \tag{13}$$

while the probability of having a child at age $t+1$ is given by:

$$Pr(k_{i,t+1}|m_{i,t}, n_{i,t}, t) \tag{14}$$

so the likelihood of getting married or getting divorced is conditional on age, while the probability of having a child is conditional on marital status, number of children and age.

Finally, the value of $Paid_i$ is exogenously determined for each woman. This is revealed when women have their first child, and is constant from then on.

2.6 Solution

The solution of the model consists of policy functions for consumption and labour supply conditional on age, earnings, family demographics and each other element of the state space. There are no analytical expressions for these policy functions. Therefore, I solve the model numerically, beginning with the consumption decisions from age 65 onwards, and then iterating backwards, solving for the consumption and labour supply decision in each period. A detailed description of the solution

[6] I treat fertility exogenously because reduced form research into the effects of childcare subsidies and other child benefit reforms on fertility have found no significant effects (Nollenberger and Rodríguez-Planas, 2015; Low et al., 2018).

method can be found in Section B of the Appendix.

3 Data

I use data from ten panels of the Understanding Society panel survey, taken from 2009 to 2018. Understanding Society is a representative sample of UK adults with data on a wide range of demographic and labour-market information, which interviews respondents annually. I restrict my sample to women aged between 20 and 65, who did not attend university. The survey contains data on their job status, employment history, income, income of their (if applicable), and children and their ages, as well as on their use of childcare. Women who are self-employed, and those who only appear in one panel, are dropped from the survey. Due to their not being accounted for in the model, those with more than three children and those with severe disabilities are also removed from the data. Because they are also not accounted for in the model, and to reduce measurement error, I remove those whose imputed hourly wage is less than the minimum wage. This leaves an unbalanced panel of 17,522 women, with 88,982 yearly observations in total. Statistics on family makeup of those in the data are presented in Table 1, and statistics of employment rate of women in each group can be found in Table 2.

Table 1: Distribution of Family Makeups, Age 20-40

Number of Children	Married	Single	Total
No Children	0.253	0.105	0.358
One Child	0.118	0.142	0.261
Two Children	0.080	0.199	0.279
Three Children	0.025	0.078	0.103

Source: Understanding Society data from 2009 to 2018.

Table 2: Women's Employment by Family Makeup, Age 20-65

Number of Children	Married	Single	Total
No Children	0.756	0.810	0.792
One Child	0.798	0.666	0.747
Two Children	0.719	0.476	0.655
Three Children	0.471	0.349	0.444
Adult Children	0.703	0.757	0.720

Source: Understanding Society data from 2009 to 2018. "Adult Children" applies if a family's youngest child is over 15 years old.

Data on employment experience is taken from Understanding Society's employment history module, from which data on total years spent working can be constructed. Data on women's income is calculated in the form of an hourly wage, using data on annual labour income and on hours worked per week. Because there is no labour market participation choice for men in the model, male income is measured on an annual basis, and combines all sources of income.[7] While this paper refers to marriage, the term is also used to apply to those who are cohabiting in the data. Data on childcare usage in Understanding Society consists of a question to those who work, what type of childcare they use, and how many hours of that childcare they use. [8].

Understanding Society does not have data on the cost of childcare or expenditure on childcare. Therefore, I obtain data on the cost of childcare directly. For this, I use the Childcare Cost Survey from the Daycare Trust (2011), which was a survey of daycare and out-of-school club costs in local authorities across Britain.

4 Estimation

The parameters of this model are estimated in three different ways. Some are set using standard values from the literature and other sources, some are estimated

[7] I adjust all income for inflation, adjusting to 2011 price levels.

[8] For the purposes of the measurement of moments, I treat someone working full-time with a child under 5 using more than 20 hours of childcare a week as using full-time childcare

directly from the Understanding Society data, and the rest are estimated using Simulated Method of Moments (SMM).

4.1 Externally Set Parameters

Some parameters are not taken from the data or the model, but from external sources. The coefficient of relative risk aversion γ is set to 1.5, following much of the literature, including Blundell et al. (1994) and Attanasio and Weber (1995). I also follow Attanasio et al. (2008) in setting the discount rate β to 0.98 and the interest rate r to 1.5%.

The benefits received are taken from UK legislation in 2011. The benefits accounted for are Income Support, Child Benefit, Working Tax Credit, Child Tax Credit and Jobseekers Allowance, an unemployment benefit. The working tax credit includes a childcare subsidy for children who are 3 or 4, which is also included. [9] Each of these can be calculated with information on the variables included in the state space, including those that constitute income, facts about children, assets and employment status.

4.2 Directly Estimated Parameters

4.2.1 Childcare Costs

Childcare costs are estimated from the Childcare Survey 2011. This gives the average weekly cost of childcare for nursery for children under 2, children over 2, and out-of-school clubs. From this, annual costs of childcare are imputed. The cost of childcare is scaled up according to the number of children, with a cost of childcare 1.5 times higher for women with two children relative to having one child, and 2 times higher for women with three children.[10]

[9] This is the main existing childcare subsidy in the UK. In it, 70% of childcare costs within this age are included in the base elements in the Working Tax Credit, which are tapered away as income increases.

[10] Childcare for an additional childcare costs less than for the first child for two reasons: the first is that it can be cheaper to enter multiple children into the same childcare system. The second is that the state space does not include the ages for older children, and so an additional child may be older by enough that they require cheaper childcare.

4.2.2 Marriage Probabilities

Marriage probabilities are estimated from the Understanding Society data. In order to generate a sample representative of the 10-year snapshot of the population in the data, the marriage probabilities and divorce probabilities are generated to match the frequency of divorce transitions in the data, and the proportion of married people in 5-year age groups in the data. From this, implied marriage probabilities are derived.

4.2.3 Fertility Probabilities

Each household can have up to three children, and the probability of having a child is taken from frequency of childbirth given certain characteristics in the same data. Factors taken into account are 5-year age group, marital status, and number of children. The resulting simulated dataset closely matches the proportion of single and married women in each age group with each number of children.

4.2.4 Unemployment Probabilities

The probability of becoming unemployed is also taken from the data. Unemployment probability is allowed to vary with age, and is taken from the proportion of those in the labour force who are unemployed in each age group.

4.2.5 Male Wage Process

In the model, it is assumed that men always receive some income, so I do not correct for selection. This income may be from labour income, benefits, or some other source. It is estimated as annual earnings.

The values of γ_1 and γ_2 are estimate by a linear regression of age and age squared on log annual earnings. The residuals are used to estimate the other parameters, which are $\sigma^2_{f^M}$, ν^M and $\sigma^2_{\zeta^M}$, through GMM, taking into account measurement error.

The results indicate a high variance of permanent shocks, at 0.039, with fairly low levels of persistence, with ν^M at 0.6. Initial heterogeneity is also high; the initial variance of log annual income is 0.159. [11]

4.2.6 Initial Conditions

Some women have children, have a partner, and have worked before the age of 20. To account for this, I estimate initial conditions based on women aged 20 in the data. The probability of being married is estimated in combination with the probability of having one or two children, as well as the distribution of youngest children. The distribution of number of years worked before age 20 is also taken from the data, and corresponding levels of human capital are estimated for each point in this distribution.

4.3 Estimation by Simulated Method of Moments

The remaining parameters are estimated using SMM. I minimise:

$$\min_{\Pi}(\hat{\phi}_{data} - \phi_{sim}(\Pi))'\mathcal{F}(\hat{\phi}_{data} - \phi_{sim}(\Pi)) \tag{15}$$

The vector Π contains the rest of the unknown parameters. This includes the disutility of work for different marital status, number of children, and age of youngest children, which make up the possible outputs of the function $\eta(m, n, age^K)$. The adjustment factors made for the utility cost of part-time work, ψ_m and ψ_s, are also included. Π also includes the parameters determining the female wage process. This is both those determining the productivity shock, ν^F and $\sigma^2_{\zeta^M}$, those that determine experience accumulation, α_0, α_1, α_2 and δ, and the others that determine wages, $w^F_{initial}$ and $\sigma^2_{f^F}$. Finally, the proportion of women who do not have access to non-market childcare, λ, is also estimated by SMM.

The empirical moments $\hat{\phi}_{data}$ are calculated using the Understanding Society data. I use moments that relate to wages, extensive and intensive margins of employment, marital status, number and age of children, and childcare use. Simulated moments ϕ_{sim} are calculated by running the full model, and obtaining equivalent statistics from the simulated data. The weighting matrix $\mathcal{F}$ is a diagonal matrix,

[11] Further details of this estimation process, and the estimation of marriage and fertility probabilities, can be found in Section B of the Appendix.

containing the inverse of the standard deviation of each moment. These standard deviations are calculated from the data, using the bootstrap method.

The moments used can be divided into four categories. There are those that are used to extract the utility cost parameters, those that inform the human capital accumulation process, income moments that define other income parameters, and a single moment to draw out the proportion of women who have access to non-market childcare.

4.3.1 Utility Cost Moments

The utility cost moments derive from data on employment rates. Most of the moments derive from a probit regression of a dummy variable representing different values of marital status, number of children, and age of youngest child on employment, controlling for age. The coefficients for each value of the dummy variable are used as moments. Since single childless women are used as the baseline in the probit regression, I also include a moment representing the mean employment rate of single childless women in their late 20s, as a baseline to help determine the levels of utility cost of work across the whole population. Finally, to differentiate the utility cost of part-time and full-time work, I use four moments that represent the ratio of full-time workers to part-time workers, divided by marital status and age group.

Table 3: Utility Cost of Work Parameters by Number of Children, Age of Children and Marital Status

	Married			Single		
	Age of Youngest Child			Age of Youngest Child		
	0-5	5-10	10-15	0-5	5-10	10-15
1 Child	-0.76	-0.57	-0.68	-1.56	-1.95	-1.85
	(0.04)	(0.02)	(0.05)	(0.03)	(0.02)	(0.02)
2 Children	-0.91	-0.52	-0.47	-1.62	-1.44	-1.41
	(0.02)	(0.02)	(0.01)	(0.04)	(0.03)	(0.02)
3 Children	-0.98	-0.52	-0.71	-1.41	-1.17	-1.06
	(0.02)	(0.01)	(0.03)	(0.05)	(0.03)	(0.02)
No Children	-0.84			-2.17		
	(0.03)			(0.05)		
Adult Children	-0.70			-1.72		
	(0.01)			(0.01)		
Multiplier for Part-Time Work	0.949			1.369		
	(0.016)			(0.013)		

Standard errors listed under parameter estimates in brackets.

Table 3 gives the estimates of parameters relating to the utility cost of work. Note that more negative values mean that work is less attractive. For the part-time multipliers, higher values mean that part-time work is less attractive relative to full-time work, after taking into account the reduction in hours worked.[12].

The key takeaways from these estimates are that the utility cost of work is higher when single than when married, and that the utility cost of work without children is higher than with children. The intuition for these findings is as follows. Married women have access to some household income, regardless of whether or not they work, whereas single women do not. However, their employment rates are fairly similar, implying a greater non-pecuniary cost to work for single women than

[12]It should be remembered that the utility cost of work is not the only thing to affect the mental cost of work: because consumption and labour are non-separable, the equivalence scale of consumption will also affect the marginal cost of work. However, this does not change the comparison between types significantly

for married women. [13] Similarly, the rate of employment among mothers, given the increased pecuniary costs of working with children, is higher than one would expect it to be if their utility cost of work was equivalent to that of non-mothers, implying a higher utility cost of work. Furthermore, in general, the utility cost of work is decreasing in number of children. This implies that childcare costs and government benefits such as the child tax credit and income support can be taken as the drivers of reduced labour market participation from mothers, rather than decreased preferences for work.

4.3.2 Income Moments

To estimate the wage process, I re-specify the wage process outlined in Equation (9) to include mean-zero measurement error. This means the wage process equation is now as follows:

$$\log(w_{i,t}^F) = w_{initial}^F + f_i^F + \log(h_{i,t}) + \mu_{i,t}^F + \epsilon_{i,t} \tag{16}$$

This does not change the decisions made by agents in the model, as the measurement error does not affect them, but instead changes only the observations of income in the simulated data. Its variance is then estimated from the real data. This allows for the capturing of other relevant parameters using moments which will be affected by measurement error in the data. Since women can work part-time or full-time in each year in the model, wages are measured in the data at an hourly rate, net of tax, and then annualised.

The income moments are a combination of means, variances and covariances. They include the mean log annual income of 20-year-old women, in order to ascertain the initial wage level. They also include the standard deviation of log annual income at five-year intervals from age 20 to age 60. Finally, they include the autocovariance of log annual income, the standard deviation of log income growth rates, and the autocovariance of log income growth rates. In combination, these identify the parameters determining the wage process, which are the variance and persistence of shocks, the size of measurement error, and the variance of initial

[13] The explanation for what this smaller utility cost parameter for married might be capturing are explored in Jones et al. (2015).

heterogeneity in wages. [14]

Table 4: Income Process Parameters

Parameter		Estimate	Standard Error
Initial log wage	$\log(w^F_{Initial})$	9.425	0.006
s.d. of wage shocks	σ^F_ζ	0.121	0.005
s.d. of initial heterogeneity	σ^F_F	0.164	0.005
Persistence of wage process	ν^F	0.784	0.009

The estimates are shown in Table 4. They show reasonably large fluctuations in income, as well as large initial heterogeneity in income. However, the wage process is fairly persistent.

4.3.3 Experience Moments

The experience moments consist of the mean of log annual earnings at five year intervals of ages over 20. They also include parameters from two regressions. The first of these is of total years worked and total years worked squared on log annual earnings, controlling for fixed effects. The second is of a dummy variable denoting whether work work was full-time or part-time on wage growth into the next period, controlling for previous income level. Together, these help capture the relative impact of working on future human capital, and to differentiate the impact of part-time work from full-time work.

Table 5: Human Capital Parameters

Parameter		Estimate	Standard Error
Returns to full-time work	α_0	0.0311	0.0003
Returns to part-time work	α_1	0.0159	0.0004
Returns to not working	α_2	-0.0416	0.0038
Decline in experience from age	δ	0.00084	0.00001

[14] I estimate the standard deviation of the measurement error to be 0.114.

The estimates of the human capital accumulation parameters are presented in Table 5. They show a significant difference in the returns to full-time work and part-time work, and a much larger gap between the returns to part-time work and not working. This indicates that women have a strong incentive to participate in the labour force each year in some form, so as to avoid significant human capital decay. They also show a significant decline in wages with age.

4.3.4 Childcare Moment

The final moment relates to the proportion of women who do not have access to non-market childcare. Understanding Society only has data on non-market childcare access amongst working mothers, which can not be considered representative of all mothers, since some might only choose to work because they had access to non-market childcare. Therefore, I use the proportion of mothers in work with access to non-market childcare to identify the proportion of mothers in the population with access to non-market childcare. I do this by estimating the proportion of mothers working full-time with children under 5 who use more than 20 hours of childcare per week.

Table 6: Paid Childcare Probability

Parameter		Estimate	Standard Error
Probability of having to pay for childcare	λ	0.483	0.020

The proportion of women without access to non-market childcare is given in Table 6. The result implies that just over half of women have access to some other form of childcare. This is despite the fact that amongst mothers of young children working full-time, only 28% use childcare. This indicates that there are very large selection effects from having to pay for childcare, suggesting that those who do not have access to non-market childcare are heavily disinicentivised from working full-time.

4.4 Fit of the Model

The results of the model matches the moments used well, and aligns with other statistics not used for estimation that are relevant to the role of childcare costs in the female labour supply decision and the child penalty.

The moments that derive from a probit of categorical variables including marital status, number of children and age of your youngest child on employment match the data very closely, staying within two standard deviations of the data. The human capital regressions are matched similarly well by the model, staying close to the value in the data in each case. The variances and autocovariances of wages and wage growth also fit the data well. Due to the precise nature of the estimates of the first and second order income moments across the life cycle in the data, the confidence intervals are fairly small, so in some cases the data strays outside them, but stays fairly close. Finally, the proportion of working mothers using childcare in the model is matched almost perfectly to the data. A full outline of the moments used and their values in the data and the model can be found in Section C.1.

To further understand how well the model fits the data, I follow Honore et al. (2020) and evaluate the significance of each moment in estimating each parameter by evaluating how the standard error of the parameters change if a given moment is removed from the weight matrix. This allows me to find out which moments are driving the precision of the parameters. I find that the employment probit is very important for the utility cost parameters: in most cases, its removal would lead to the variance of the parameter estimate multiplying by a large magnitude. I also find that the ratios of full-time to part-time work are key to the estimation of the part-time work multipliers. I find that the precision of the wage process estimate is dependent on both the variance and autocovariance of wage growth, and on the progression of the variance of wages over the life cycle, while the persistence is mostly driven by the life cycle. Similarly, the returns to experience from work are driven by both the experience regressions and mean wages over the life cycle, while the decline in experience from age is mostly driven by the life cycle. Finally, the childcare use moment pins down the precision of the estimate of the proportion of women with access to childcare. Each of these runs in line with the intuition of the SMM estimation.[15].

The model also fits with key facts about the data that are not represented by the moments in the model. In order to compare the model to the data, I produce a measure of the "child penalty" in earnings that women suffer from not having children. It resembles that used by Kleven et al. (2019), measuring intensive and extensive margins of employment, wages, and total earnings, but I focus on the comparison between mothers and women without children, rather than the comparison between mothers and fathers that they use. The consequence of this is that the composition of the earnings drop is not split evenly between wages, the intensive margin of employment and the extensive margin. Instead, it is driven by a large drop in employment, both on the intensive and on the extensive margin. Meanwhile, due to selection bias, those who are in employment among mothers have a higher average wage than non-mothers in employment.

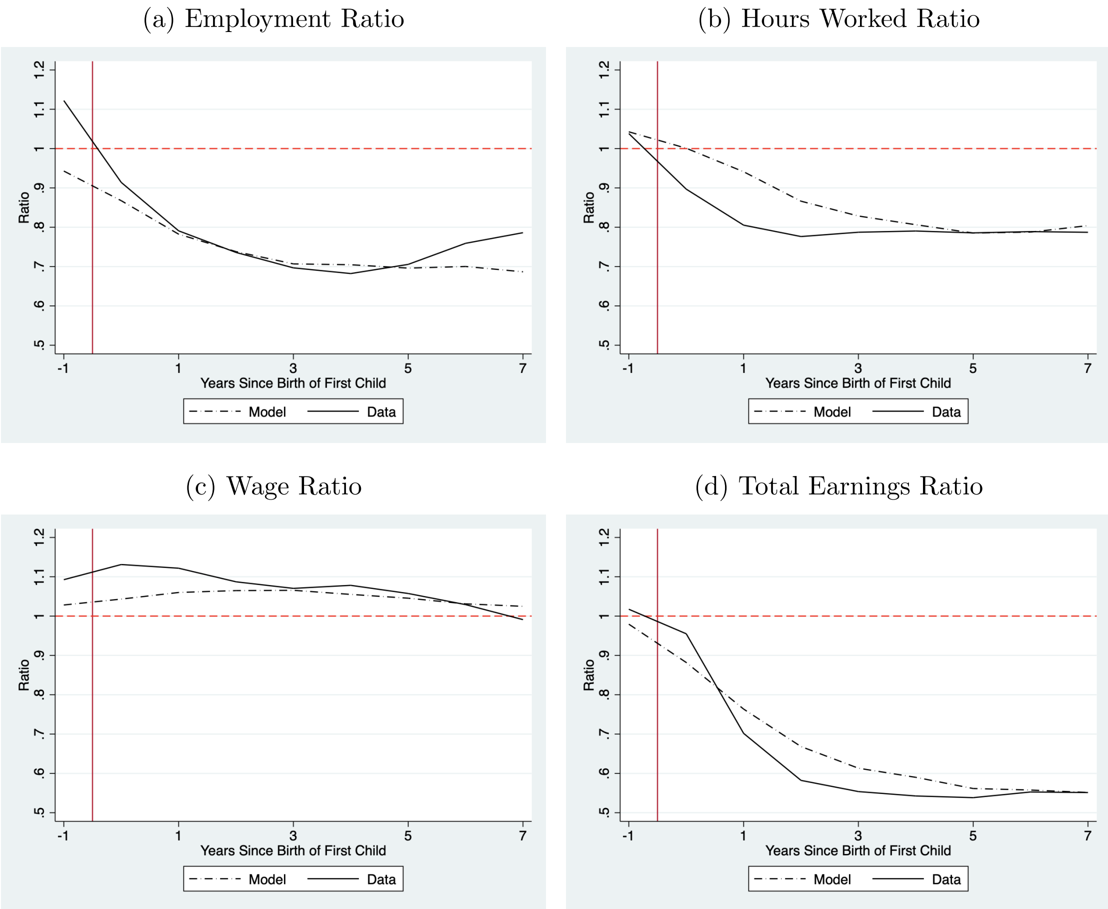
I demonstrate this using a subsample of the Understanding Society data, comprised of those who appear in every panel of the sample. I follow their labour market outcomes in the years just before and after having children, and compare them to the outcomes of women of the same age who have not had children. I use this data as a test of the fit of the model on a set of outcomes that are not used in the estimation of the model. [16]

Figure 1 shows the fit of the model to the data, and also demonstrates the short-run change in labour market outcomes that occurs upon becoming a mother. In each case, the model matches the data very closely. Both sets of lines demonstrate that, when measured relative to non-mothers, the child penalty in the short term is driven not by a drop in wages, but by a drop in labour supply. In fact, the model and the data both show an uptick in wages of working mothers relative to non-mothers after childbirth. This is likely driven by selection biases, as those who remain in the workforce upon having children are likely to have higher returns to working, due to having higher wages. Meanwhile, there is a significant drop in both the intensive and extensive margin of labour supply, with hours worked and

[15]A full table of the relevance of significant moments for certain parameters can be found in Section C.2 in the Appendix.

[16]The graph representing employment ratio bears some resemblance to some of the moments used, as they include the employment rates of mothers of young children and non-mothers, but there are no moments which represent the wages, hours worked or total earnings of mothers.

Figure 1: Ratio of Mothers' Labour Market Statistics relative to Non-Mothers of the same age in the model and the data



(a) Employment Ratio

(b) Hours Worked Ratio

(c) Wage Ratio

(d) Total Earnings Ratio

Source: Understanding Society for lines marked Data, simulations for estimated model for lines marked Model. Sample in Data is those who appear in every wave of the Understanding Society dataset and who have a child in the first three waves of the sample. Points are a moving average of three years. The red line indicates the point at which the first child is born. Hours Worked and Wage are conditional on Employment.

employment dropping by more than 20%, in both the data and the model. In combination, this adds up to a large drop in real earnings relative to non-mothers in the data, which is captured very well by the model.

The Understanding Society Panel only covers ten years, so the child penalty in the data is not shown any further in the above graph. In the Results section below, I use the measure above on the simulated data and extend it further into the future, allowing me to see how the child penalty develops over a longer period, as well as to examine how childcare cost subsidies can change the magnitude and shape of the child penalty.

5 Counterfactual Results

I use the estimated model to consider different policy scenarios, and their impact on employment, wages, and welfare. The main comparison I use is to a scenario where the government subsidises childcare costs completely, and funds this subsidy using a proportional tax on labour income. This allows me to see how the reduction in the cost of working for many young mothers affects their labour supply choices. The fact I model human capital accumulation allows the model to capture the long-run impact of childcare subsidies on wages, as well as on labour supply beyond the point of having young children.

5.1 Lifetime Labour Market Outcomes

First, I consider the impact of revenue-neutral childcare subsidies on labour market participation and log wages over the life cycle. The childcare subsidy I consider involves childcare costs being reduced to zero at all stages of early life, including daycare, nursery and after-school clubs. Therefore, those without access to other forms of childcare do not face any childcare costs. The cost to the government of this policy is dependent on take-up of the subsidy amongst those who *do* have access to another form of childcare, for which they do not pay. The model does not consider preferences for childcare, but it cannot be ruled out that those who previously used a grandparent or neighbour for childcare when nursery was expensive, might choose to use a nursery if it were free. This is because it would not be practically possible for childcare subsidies to be dependent on access to other forms of childcare.[17]

Therefore, I obtain the higher and lower bounds of this problem by considering

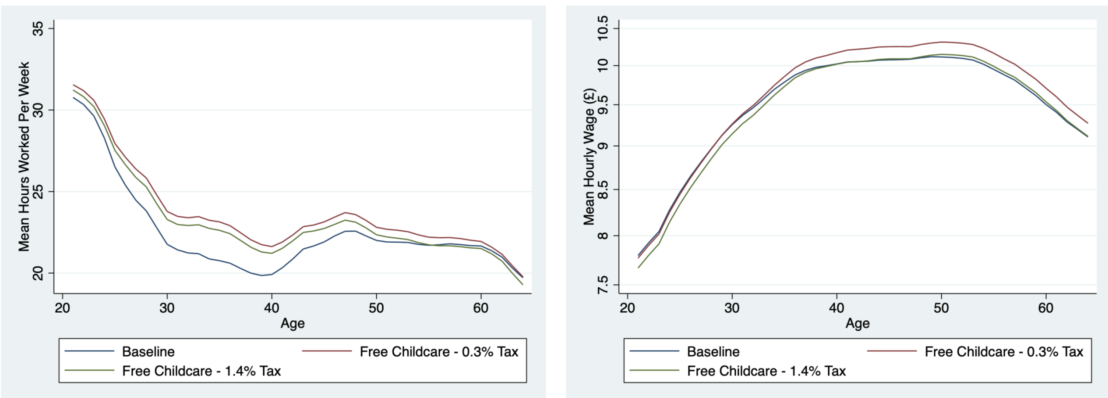
[17] I implement the condition that childcare subsidies are only available to those who work, either part-time or full-time. Therefore, I do not consider the possibility that women who do not work use subsidised childcare, which would change the cost to the government again.

the size of a proportional tax necessary to fund the childcare subsidy in two cases. The first of these is where no mothers with access to non-market childcare use the childcare subsidy, and the second is where all working mothers use the childcare subsidy, regardless of access to other forms of childcare. In the first case, the proportional tax required to make the policy revenue neutral is 0.3%, in the second case it is 1.4%. This tax is levied on all men and all women in the model, and the reductions in required benefit payments from the introduction of the childcare subsidies are factored into revenue-neutrality. To ensure that the subsidy does not involve any redistribution towards women in the model - so as not to impact the welfare analysis - I do not apply the tax to anyone not in the model, including single men. Therefore, the tax is only applied to non-graduate women and their spouses. In the figures below, I outline both the low-tax case and the high-tax case, to demonstrate the bounds of the policy, depending on the uptake of the subsidy amongst those not already paying for childcare.

Figure 2: Labour Supply and Income Over Time

(a) Mean Hours Worked Per Week

(b) Mean Post-Tax Log Hourly Salary



Simulations from the estimated model. Data is a moving average of three years.

Figure 2 demonstrates the impact. It shows that childcare subsidies would have a positive impact on both women's labour supply and, in the low-tax case, their wages. Labour supply is shown on the left, as mean hours worked per week, combining the intensive and extensive margins of labour supply. The impact on labour supply occurs mostly early in working life, coinciding with when women have children, by making it cheaper to have children at a young age. The increase in labour supply is over 10% for women in their thirties at either level of uptake. This increase continues beyond the point of having children, but fades out as women reach their fifties.

Wages are shown net of the proportional tax used to fund the childcare subsidy. In this case, the policy has a positive effect on wages only in the low-tax case, mostly occurring later in life. This is driven by the role of increased experience that comes from higher labour supply during the early years of motherhood, and its positive impact on wages over the long run. By the age of 45, this amounts to an average 1% increase in wages in the low-uptake case, which continues on for the rest of working life. In the high-uptake case, the policy leads to approximately a 1% decline in post-tax wages. Some of this difference in post-tax wages is mechanical, and the rest is driven by the reduced incentive effects earlier in life driving down long-run wages.

The change in labour supply and wages is a result of those who do not have anyone who can look after their children in an informal setting. The labour supply of those who do have someone is unaffected by a childcare subsidy, because although they may use the subsidy if they do work, it does not alleviate any pecuniary cost of working with young children, as they did not necessarily have such a cost anyway. Therefore, I next demonstrate the impact of the same counterfactual subsidy on only those women who do not have access to non-market childcare. The proportion of mothers is one of the parameters estimated by simulated method of moments, and amounts to about 48% of women who have children.

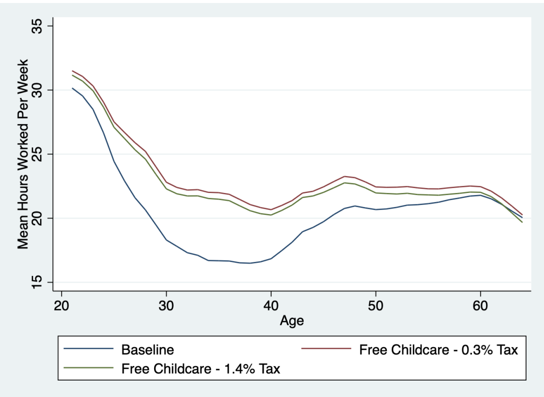
Figure 3 shows that the impact on these women is a lot higher than in the general population. The positive impact on their employment rate is over 25% in their late twenties, depending on the uptake of the subsidy and so on the tax rate. This drives a larger wage effect, reaching between a 4% and an 5% increase in salary from age 45 onwards. This, in turn, keeps a positive employment effect well beyond the point of having children needing childcare.

The results are therefore very different if you consider the proportion of women who need to use market childcare. In my sample, of non-university graduate women in the UK, I estimate this proportion to be fairly low, at just under half of mothers.

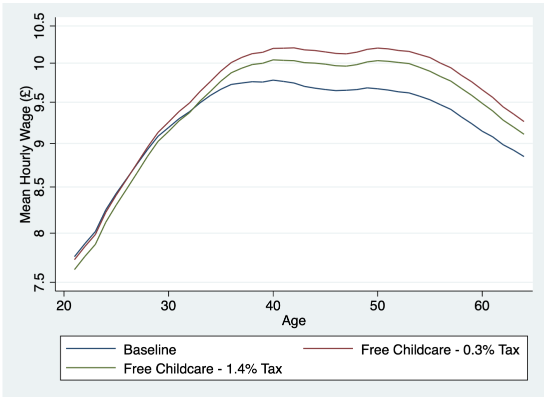
However, Figure 3 demonstrates how large an impact this has on these women, so much so that there is also a large positive impact on all working-age women, as shown by Figure 2. This indicates that were the proportion of women who did not have a grandparent or friend who could look after their child while they worked were to increase, the impact of childcare subsidies would increase drastically.

Figure 3: Employment and Income Over Time Amongst Those Who Have to Pay for Childcare

(a) Labour Market Participation



(b) Post-Tax Log Hourly Salary



Simulations from the estimated model. Data is a moving average of three years.

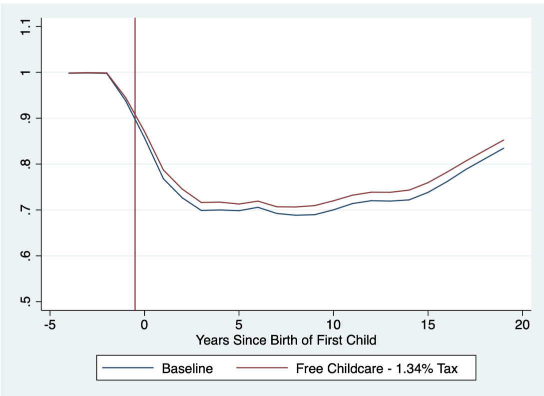
5.2 Child Penalty

I also consider the role of the counterfactual childcare subsidy on the child penalty faced by mothers. As in the model fit of Figure 1 I continue to measure this as differentiated from the labour market outcomes of women who do not have children. This is preferred to a comparison to the father of the children, as is sometimes used (Kleven et al., 2019), because the model does not account for male labour supply, so any counterfactual impact on men's labour market outcomes would not be captured by the model. Instead, I consider the former comparison, and extend it to longer-term outcomes than can be measured from the data. This allows me to examine the longer-run labour market penalties of having children, and consider, in the aggregate, how childcare costs impact them.

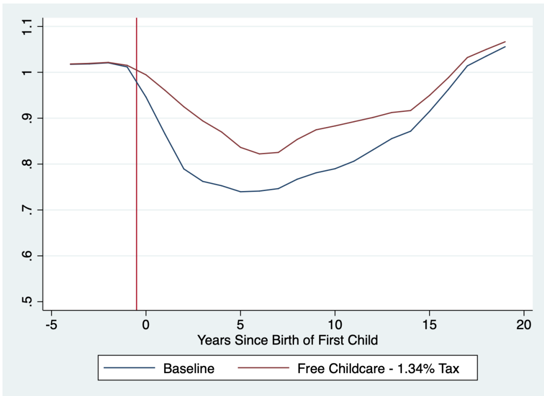
I consider four labour market outcomes: the extensive margin of employment, the intensive margin of employment, the wage of those working, and labour earnings, the product of the first three. In each case, I present the development of the child penalty over the course of the twenty years after the birth of a woman's first child, and include the counterfactual childcare subsidy outcomes. Because an increase in the tax rate affects both mothers and non-mothers, the impact of different tax rises on the outcome is small, and so I only include the low-tax case. The results are shown in Figure 4.

Figure 4: Ratio of Mothers' Labour Market Statistics relative to Non-Mothers of the same age

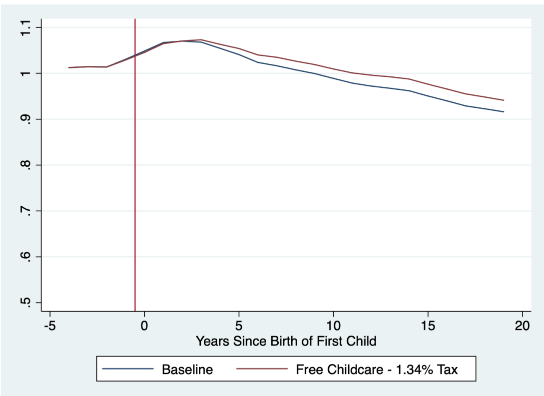
(a) Employment Ratio



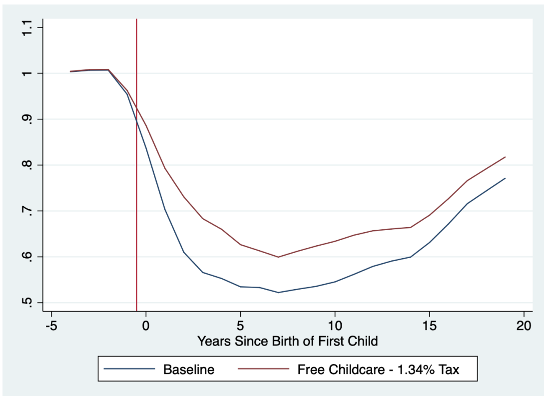
(b) Hours Worked Ratio



(c) Wage Ratio



(d) Total Earnings Ratio



Simulations from the estimated model. Points are a moving average of three years. The red line indicates the point at which the first child is born. Hours Worked and Wage are conditional on Employment.

These results give an indication of the size of the child penalty over the long run, and of how a childcare subsidy would change it. The first figure shows the gap in employment between mothers and non-mothers. It is clear that this is the main driver of the total earnings gap, particularly in the first years after childbirth. This gap reaches as high as a 30% drop in employment for much of the first decade after becoming a mother, before gradually shrinking. This graph shows that childcare subsidies would have only a very small impact on reducing this gap. It shrinks the gap by between 1 and 2 percentage points throughout motherhood, representing a reduction between 5% and 10% of the child penalty in the baseline case. The effect continues on over the long run, which can be explained by increased wages among mothers due to accumulated experience.

At the intensive margin of labour supply, the effect of the subsidy is much larger. The results indicate half of the child penalty in hours worked in the first few years of motherhood would be reduced with a childcare subsidy, with the effect deteriorating but not disappearing in the long run. This indicates that the reduction in the cost of working with young children is most strongly felt amongst those working part-time in the baseline case, who are enabled to work full-time by the reduced cost of childcare. This may be explained by the big difference in human capital outcomes between not working and working part-time, which means that those who do not work in the baseline case would need to be incentivised to work throughout motherhood in order for the effect on long-run wages to be worth it. However, the childcare cost subsidy enables those working part-time to shift to full-time work in some years but not others, since their human capital does not decay seriously in either case. Since full-time work has higher returns to experience than part-time work, this effect perpetuates itself, with higher wages driving more hours, driving up future higher wages.

Another explanation for the shift being concentrated in hours lies with the parents of children in primary school. At that age, with childcare costs, there is no cost of childcare for those working part-time, but there is a cost of childcare for those working full-time. So amongst that group, those who were constrained by the cost of childcare would already be working part-time. Since they are the ones constrained, the benefit of a childcare subsidy amongst mothers of older children would display itself in an increase in hours worked, not in entrances to the labour market. [18]

The wage ratio itself follows a very different trajectory. As demonstrated in the data, wages of mothers are higher than wages of non-mothers in the immediate aftermath of childbirth. In the model, this can be explained by selection effects, as the drop in employment means that mostly those with higher wages stay in the labour force, and thus earn higher wages on average than non-mothers. However, this effect peters out within a decade, and the effect of motherhood on wages becomes negative, as more mothers return to the workforce, reducing the selection effect, and the effect of lower human capital amongst mothers kicks in. By twenty years after the birth of their first child, the wages of mothers are on average almost 10% less than those of non-mothers. The childcare subsidy decreases that gap by a third, indicating a large effect of the childcare subsidy on the long run wage penalty of having children.

These three labour market outcomes combine to create the child penalty in terms of total earnings. In the baseline model, this is driven in the short and medium-run by the drop in employment and hours worked. However, in the long run, it can also be attributed to the wage penalty of having children. The cumulative gap between total earnings of mothers and non-mothers is almost as high as 50%. The childcare subsidy has a significant effect in reducing it: in each of the first 15 years after becoming a mother, the childcare subsidy reduces the total earnings gap by between 20% and 30%, which is a large impact on the child penalty. Beyond this point, when the main effect of the childcare subsidy is the small long-run impact on wages, the impact of the subsidy becomes smaller, but remains a significant reduction in the child penalty relative to the baseline case. This demonstrates that a childcare subsidy would go a long way to reducing the disparity in labour market outcomes between those women who have children and those who do not.

There is some question of what else constitutes this child penalty besides childcare costs. This seems noteworthy given the results of the estimation in Table 3 indicate that the utility cost of work decreases upon having children, meaning that

[18] This latter explanation cannot fully explain the difference, however, as there is a large change in hours worked among mothers less than five years after the birth of their first child.

the decrease cannot be explained in an increased non-pecuniary cost of work upon becoming a mother. In the model, the explanation instead lies in the benefits system. There are three benefits that drive down employment upon childbirth. The child tax credit and child benefit provide a source of income besides labour upon having children, and are not dependent on employment. Furthermore, Income Support is given only to those who do not work, but is more limited in scope, applying only to single mothers of children under 5. In combination, these benefits provide an income stream that reduces the incentive to enter the labour force, particularly for those on lower incomes. Therefore, even without childcare costs, there is still some child penalty in income over both the short and long run, although significantly decreased relative to current childcare pricing.

5.3 Welfare Analysis

The final analysis of the policy counterfactual where childcare costs are eliminated is to consider the ex-ante welfare effects of the policy. I follow Low and Pistaferri (2015) and measure this by considering what percentage change in their annual consumption in the baseline case an individual would be willing to trade for the policy. Because the childcare subsidy is revenue neutral, and because the cost of making it revenue neutral is not exactly estimated in this paper, I provide the welfare analysis for both the low and high uptake bounds of the childcare subsidy. I measure welfare in the first period of the model, so before most people know whether or not what their family circumstances will be, or whether they will have access to non-market childcare. I focus on the median estimate of the welfare effects, but they are fairly uniform across wage heterogeneity and experience. [19]

In the low-uptake, low-tax scenario, where taxes are only raised to cover the subsidy only for those who cannot use any other type of childcare, the childcare subsidy is welfare-decreasing, and has the same ex-ante value as an annual 0.5% increase in consumption for the rest of their working-age life. In the case of higher uptake, requiring a higher tax rise to fund it, the impact of the policy becomes

[19] The only significant heterogeneous effects are for those who have already had children at age 20, for whom the benefits and costs of the policy are very dependent on if they currently need to use childcare.

equivalent to an annual 0.9% decrease in consumption for the rest of their working life.[20] This follows logically, as the difference in tax rises in the two scenarios is similar in magnitude to the difference in consumption value of the two scenarios.

This indicates that the direct welfare effects of this policy are to some degree dependent on its uptake, and the cost of funding it that would come with higher uptake. For the policy to be welfare-improving in all cases for non-graduate women, some redistribution towards them would have to occur in the funding for the policy. In all likelihood, any childcare subsidy would not be funded solely by taxes on women and married men, and thus it is possible that a real childcare subsidy might be welfare-improving for women, but not for men.

6 Conclusion

In this paper, I have considered the broad dynamic implications of a childcare cost subsidy in the UK, both in its effects on short-run labour supply decisions and on long-run labour market outcomes. The life-cycle model demonstrated in this paper allows for the estimation of unobservable variables regarding the utility cost of work, the accumulation of human capital, and access to non-market childcare. By obtaining an estimate of all three of these processes, each of which is a key determinant in the efficacy of a childcare subsidy, I can use the life cycle model to estimate the dynamic effects of a childcare subsidy.

I find that the subsidy does have a significant impact on labour market outcomes, through the mechanism of making it easier for mothers who do not have access to any form of other childcare besides market-rate childcare to work. This creates an 11% increase in female labour supply for women in their thirties, a large increase. This increased labour market participation has some pass through onto long-run wages, increasing post-tax wages by 1% in the long run. There is a stronger effect for the specific group of women whom it affects: a 31% increase in labour supply and a 5% increase in wages. These effects are hindered by the large increase in taxes on non-graduate women used in the model. In a scenario where

[20] It is worth noting that the welfare response in the high-uptake case does not take into account any welfare gains of families switching from non-market childcare to government-subsidised childcare.

a childcare subsidy was treated as a redistribution towards women, so that taxes were raised on everyone and not just non-graduate women, the increases in female employment and wages would likely be higher.

This paper also shows that a childcare subsidy would have an effect on reducing the child penalty, which I measure using a comparison between mothers and non-mothers of the same age. In particular, it would prompt a large reduction in the child penalty on the intensive margin of labour supply, by allowing a lot of mothers who previously worked part-time to work full-time. This, along with a small reduction of the penalty on the extensive margin of employment, has two consequences. The mechanical effect is to significantly reduce the child penalty in total earnings due to an increase in total hours worked. The second is to increase human capital accumulation, and so reduce the smaller child penalty in wages, thus extending the reduction in the child penalty into later life.

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A Solution of the Model

A.1 Use of the Endogenous Grid Method

The estimation and simulation of the model requires the solution of women's decisions across the model. Since there is no analytical solution to the problem, I solve the problem numerically using backwards recursion, starting from the end of life at age 75.

For efficiency, I use the endogenous grid method to estimate saving and consumption decisions, following Carroll (2006). However, the model involves simultaneously solving for consumption and labour supply choices, with discrete labour supply and continuous consumption choices. The use of endogenous grid method to solve models with both discrete and continuous choices is much harder, due to discrete choices creating kinks in the value function. First, it requires calculating the Euler equation for every possible discrete labour supply choice in the first instance. This removes the risk of kinks appearing in the same period as the simultaneous choice, as optimal saving decisions are made for each possible labour supply choice.

However, in this setup, you might also expect kinks in the marginal utility of consumption in future periods. The reason they might occur is that a change in present-day consumption might change future labour supply decision, which would in turn change the marginal utility of consumption in the opposite direction. This means the marginal utility of wealth is not smooth, which means that attempting to solve the model using Euler equations creates problems. To get around this, I follow the solution proposed by Fella (2014). This involves finding the bounds of the assets space in which the consumption choice involves kinks in the marginal utility of wealth, and removing the non-optimal solutions within those bounds. In this way, I obtain a solution to the model.

A.2 Details of Solution and Simulation

I obtain the solution by using a discrete grid to represent the state space, across which I then interpolate upon simulating the population. I use a grid of 13 points for assets, 9 points for female income shock and 9 points for male income shock.

I also use 4 points for human capital, with a grid that varies depending on the age of the individual, and the human capital parameters being estimated. I use 4 grid points for number of children, and 17 for the age of the youngest child, as well as 2 for the question of whether the individual has access to non-market childcare. Because they are mutually exclusive from other elements of the state space, I append one extra grid point on the female income shock and male income shock, which represent unemployment and being unmarried respectively.

In each period, for each plausible point in the grid, I calculate the optimal consumption choice for each possible labour supply choice.[21] This takes into account expectations for future periods, including over income, probability of marriage transition, probability of fertility, as well as knowledge of what their human capital next period will be given their labour supply decision in this period. [22] Having calculated optimal consumption decisions for each labour supply choice, I then choose the labour supply choice which maximises the value function for each given consumption choice. I then input this into the value function, and iterate backwards onto the period representing an individual being one year younger. I iterate this process starting from age 75 down to age 20 (though the labour supply decision and any shocks only begins at age 65, making the first ten-periods analytically solvable), until I have a solution grid of optimal consumption choices for each labour supply choice in the state space for each age, and the value of the value function for each of these labour supply choices at each point.

I use this set of optimal choices to simulate the model. First I generate a population of 12,000 individuals across 45 years, and apply shocks to marriage, fertility, income and unemployment to them. I then apply the solution grid to the simulations. I do this by interpolating across up to four dimensions (assets, experience, and productivity shocks for both wife and husband), imputing consumption and labour supply decisions from the shocks and state variables of the simulated individuals, by linear interpolation.

[21] Implausible grid points include those like having a 14-year-old youngest child at age 23, or having a 3-year-old child at age 60.

[22] In the case of income and human capital accumulation, this is done by interpolating across the grid-points in the next period. In the case of income, this also involves using Gauss-Hermite quadrature to estimate the distribution of expectations across normally distributed shocks.

B Estimation

B.1 Estimation of Male Income Process

I use GMM to estimate the male income process. I do this first by regressing annual earnings $w_{i,t}$ on age and age squared, as well as controls. To enable comparison with the model, I use the age of the man's wife, and not the man himself. The resulting regression is as follows:

$$\log(w_{i,t}^M) = log(w_{initial}^M) + \gamma_1 t - \gamma_2 t^2 + \xi Year_{i,t} + u_{i,t} \tag{B.1}$$

This gives an estimate of initial income, as well as the time-varying parameters. I then use the residuals u_t of this regression to estimate the other elements of the male income process, including the variance of initial heterogeneity σ_f^2, persistence ν, variance of shocks σ_ζ^2 and measurement error σ_m^2. I do this by GMM, using the following moments:

$$E[\Delta u_t^2] = 2\sigma_m^2 + \frac{1+\nu(\nu-1)^2(1-\nu^{2(t-2)})}{1-\nu^2}\sigma_\zeta^2(\nu-1) \tag{B.2}$$

$$E[\Delta u_t \Delta u_{t-1}] = \frac{1+(1-\nu)^2(1-\nu^{2(t-1)})}{1-\nu^2}\sigma_\zeta^2 - \sigma_m^2 \tag{B.3}$$

$$E[u_t^2] = \sigma_f^2 + \sigma_m^2 + \frac{1-\nu^{2t}}{1-\nu^2}\sigma_\zeta^2 \tag{B.4}$$

$$E[u_t u_{t-1}] = \sigma_f^2 + \frac{1-\nu^{2t}}{1-\nu}\nu\sigma_\zeta^2 \tag{B.5}$$

This gives the following results, shown in Table B.1:

Table B.1: Male Income Process Parameters

Parameter		Estimate	Standard Error
Initial log wage	$\log(w^M_{Initial})$	9.534	0.036
Variance of wage shocks	σ^{2M}_{ζ}	0.039	0.008
Variance of initial heterogeneity	σ^{2M}_{f}	0.159	0.009
Persistence of wage process	ν^M	0.603	0.009
Variance of initial heterogeneity	σ^{2M}_{m}	0.172	0.0013
Returns to age	γ_1	0.028	0.003
Returns to age squared	γ_2	0.00060	0.00006

B.2 Marriage Parameters

In the Understanding Society data, there is a mismatch between the proportion of women who are married in period t and not married in period $t-1$, and the number of people who are married in total in period t. This indicates that those who got married in the last year are less likely to appear in the Understanding Society data. To accurately match the proportion of people who are married at each age from the data, I therefore impute the proportion of people who transition into marriage from the total number of married people, and the number of people who transition from marriage to singlehood in the data. This gives the following transition matrix for marriage shown in Table B.3. It represents the probability for a woman in the given age group transitioning in any given year.

Table B.2: Marriage Probability Parameters

Age group	$P(Marriage)$	$P(Divorce)$
20-24	0.160	0.159
25-29	0.150	0.054
30-34	0.084	0.030
35-39	0.062	0.025
40-44	0.044	0.021
45-49	0.021	0.015
50-54	0.025	0.010
55-59	0.033	0.014
60-64	0.019	0.009

B.3 Fertility Probabilities

A similar issue to that of marriage transitions exists in the data on having children in the Understanding Society data. This is exemplified by the fact that there are a much larger number of women with 1-year-old children than women with 0-year-old children in the data. Therefore, I adjust the fertility probabilities so as to match the proportions of women at each age who have 1, 2, or 3 children, for each marital status. This gives the following probabilities, shown in **??**, of childbirth in any given age group for each year, for a given marital status and number of children pre-fertility.

Table B.3: Probability of Childbirth

Age Group	Married			Single		
	Number of Children					
	0	1	2	0	1	2
20-24	0.215	0.370	0.212	0.087	0.168	0.201
25-29	0.189	0.267	0.105	0.047	0.030	0.045
30-34	0.207	0.175	0.010	0.031	0.045	0.005
35-39	0.079	0.091	0.034	0.007	0.018	0.053

C Model Fit

C.1 List of Moments

In this section I detail the fit of the model, and the specification of the moments used. I outline the moments and their confidence intervals in the data, and the equivalent moment in the simulated data. In each case the confidence intervals come from bootstrapping the moments in the data.

The first moments are those based around the employment rate of women with different family backgrounds. This is dependent on marital status, number of children and age of children. So as to control for selection effects, including age, I use a probit model to estimate this. The specification of the probit is as follows:

$$Employment_{i,t} = \beta_1 Type_{i,t} + \beta_2 Age_{i,t} + \beta_3 Age_{i,t}^2 + \beta_4 X_{i,t} + \epsilon_{i,t} \tag{C.1}$$

where *Employment* is measured at the extensive margin, *Type* is a dummy variable representing category of family demographics, and X is a set of controls, including year and ethnicity. After running this probit, I convert it to marginal effects, so the results represent the change in probability from changing type. The results of the value of β_i in the marginal effects regression are reported in Table C.1 below. Note that the case of being single and having no children is used as a baseline, so no coefficient is reported.

I also use other moments to capture parameters involved in the estimation of the utility cost of work. The first of these concerns single childless women, who are used as a baseline above. Therefore, I include a moment that is the mean employment of single childless women in their late twenties. In order to estimate the utility cost of part-time work for married and single women, I also estimate the ratio of full-time to part-time workers by marital status, as well as grouping by whether women are above 40 or below 40. These are shown below in Table C.2:

In order to estimate the human capital accumulation moments, I use two panel

Table C.1: Employment Coefficient by Family Demographics

Type	Number	in Data	in Model	Number	in Data	in Model
		Married			Single	
No children	M_1	-0.005	-0.006		Baseline Case	
		(-0.021,0.011)				
1 child (Under 5)	M_2	-0.091	-0.084	M_{12}	-0.327	-0.329
		(-0.109,-0.073)			(-0.349,-0.305)	
1 child (5-10)	M_3	-0.010	-0.036	M_{13}	-0.195	-0.195
		(-0.034,0.014)			(-0.228,-0.162)	
1 child (10-15)	M_4	-0.054	-0.056	M_{14}	-0.088	-0.096
		(-0.072,-0.036)			(-0.108,-0.068)	
2 children (Under 5)	M_5	-0.241	-0.231	M_{15}	-0.543	-0.541
		(-0.257,-0.225)			(-0.574,-0.512)	
2 children (5-10)	M_6	-0.089	-0.098	M_{16}	-0.287	-0.301
		(-0.109,-0.069)			(-0.322,-0.252)	
2 children (10-15)	M_7	-0.033	-0.041	M_{17}	-0.088	-0.082
		(-0.055,-0.011)			(-0.123,-0.053)	
3 children (Under 5)	M_8	-0.437	-0.424	M_{18}	-0.590	-0.592
		(-0.464,-0.410)			(-0.627,-0.553)	
3 children (5-10)	M_9	-0.260	-0.294	M_{19}	-0.270	-0.279
		(-0.295,-0.225)			(-0.344,-0.196)	
3 children (10-15)	M_{10}	-0.343	-0.478	M_{20}	-0.136	-0.200
		(-0.419,-0.267)			(-0.283,0.011)	
Adult Children	M_{11}	-0.020	-0.014	M_{21}	0.000	0.010
		(-0.034,-0.006)			(-0.014,0.014)	

Confident intervals listed under data estimates in brackets.

data regressions, along with other moments. The two regressions are as follows:

$$log(w_{i,t}) = \tau_1 Experience_{i,t} + \tau_2 Experience^2_{i,t} + \tau_3 X_{i,t} + \gamma_i + \epsilon_{i,t} \quad \text{(C.2)}$$

$$log(w_{i,t} - w_{i,t-1}) = \chi_1 FullTime_{i,t-1} + \chi_2 log(w_{i,t-1}) + \chi_3 Age_{i,t} + \chi_4 Age^2_{i,t} + \chi_5 X_{i,t} + \epsilon_{i,t} \quad \text{(C.3)}$$

The first of these regresses experience on log wages. For experience, I use the total number of years spent working. The second regresses the type of work done last period, and whether it is part-time or full-time, on wage growth, controlling for previous wages and other observables. The baseline case is working part-time. The fit of these regressions is shown in Table C.3.

Table C.2: Remaining Employment Moments

Moment	Number	in Data	in Model
Mean Employment, Single Childless Women	M_{22}	0.842	0.871
		(0.817,0.877)	
Full-Part Time Ratio, Married Under 40	M_{23}	1.535	1.244
		(1.337,1.733)	
Full-Part Time Ratio, Married Over 40	M_{24}	1.911	1.844
		(1.711,2.111)	
Full-Part Time Ratio, Single Under 40	M_{25}	2.597	2.148
		(2.158,3.036)	
Full-Part Time Ratio,Single Over 40	M_{26}	3.351	3.843
		(2.818,3.884)	

Confident intervals listed under data estimates in brackets.

Table C.3: Human Capital Regressions

Moment	Number	in Data	in Model
τ_1	M_{27}	0.0096	0.0117
		(-0.0004,0.0196)	
τ_2	M_{28}	-0.00021	-0.00018
		(-0.00027,-0.00015)	
χ_1	M_{29}	0.0344	0.0409
		(0.0293,0.0395)	

Confident intervals listed under data estimates in brackets.

In order to estimate the accumulation of experience and the parameters of the income process, I also estimate the mean and standard deviation of log wages at five year intervals throughout the life-cycle. The fit of the model to these results is shown in Table C.4 below:

The final moments contribute to estimating the income process, and pin down the proportion of women with access to non-market childcare. In the case of non-market childcare, it should be noted that here, working means working full-time, and using childcare means using more than 20 hours of childcare with a child under the age of 4. The results are given in Table C.5 below:

Table C.4: Income Statistics

Age	Number	in Data	in Model	Number	in Data	in Model
		Mean			Standard Deviation	
20	M_{30}	9.517	9.489	M_{39}	0.212	0.214
		(9.501,9.533)			(0.192,0.232)	
25	M_{31}	9.557	9.602	M_{40}	0.212	0.259
		(9.545,9.569)			(0.200,0.224)	
30	M_{32}	9.661	9.665	M_{41}	0.275	0.273
		(9.647,9.675)			(0.265,0.285)	
35	M_{33}	9.710	9.709	M_{42}	0.306	0.289
		(9.696,9.724)			(0.296,0.316)	
40	M_{34}	9.730	9.712	M_{43}	0.330	0.302
		(9.718,9.742)			(0.320,0.340)	
45	M_{35}	9.722	9.706	M_{44}	0.320	0.308
		(9.708,9.736)			(0.314,0.326)	
50	M_{36}	9.709	9.703	M_{45}	0.313	0.314
		(9.699,9.719)			(0.303,0.323)	
55	M_{37}	9.671	9.678	M_{46}	0.299	0.318
		(9.661,9.681)			(0.287,0.311)	
60	M_{38}	9.652	9.662	M_{47}	0.298	0.311
		(9.634,9.670)			(0.284,0.312)	

Confident intervals listed under data estimates in brackets.

Table C.5: Other Moments

Moment	Number	in Data	in Model
$Cov(w_{i,t}, w_{i,t-1})$	M_{48}	0.0680	0.0701
		(0.0660,0.0700)	
$s.d(\Delta w_{i,t})$	M_{49}	0.1993	0.1995
		(0.1954,0.2032)	
$Cov(\Delta w_{i,t}, \Delta w_{i,t-1})$	M_{50}	-0.0143	-0.0146
		(-0.0155,-0.0131)	
Proportion of Working Mothers	M_{51}	0.277	0.278
using Childcare		(0.259,0.295)	

Confident intervals listed under data estimates in brackets.

C.2 Relevance of Moments

I now outline the importance of different moments for determining value of each parameter. I do this by using methods pioneered by Andrews et al. (2017) and Honore et al. (2020). In particular, I measure how sensitive the standard errors of the parameter values are to removing each moment from their estimation. By measuring the change in the standard errors when a moment is removed, I gain a sense of how significant a moment is to that parameter: if the standard error of the parameter would double were the moment not there, then that moment can be considered as very important for the estimation of that parameter. Due to space, I do not report every combination of 33 parameters and 51 moments, but only those that might be intuitively significant to each parameter.

Parameter	Variance	Moment Removed	Change in Variance
Utility cost of work when:			
Married, 1 child (Under 5)	0.0016	M_2	0.0029
Married, 1 child (5-10)	0.0004	M_3	0.0043
Married, 1 child (10-15)	0.0025	M_4	0.0714
Married, 2 children (Under 5)	0.0004	M_5	0.0030
Married, 2 children (5-10)	0.0004	M_6	0.0051
Married, 2 children (10-15)	0.0001	M_7	0.0.0027
Married, 3 children (Under 5)	0.0004	M_8	0.0228
Married, 3 children (5-10)	0.0001	M_9	0.0072
Married, 3 children (10-15)	0.0009	M_{10}	0.0006
Married, no children	0.0009	M_1	0.0008
Married, adult children	0.0001	M_{11}	0.00002
Single, 1 child (Under 5)	0.009	M_{12}	0.0153
Single, 1 child (5-10)	0.0004	M_{13}	0.0055
Single, 1 child (11-15)	0.0004	M_{14}	0.0021
Single, 2 children (Under 5)	0.0016	M_{15}	0.0027
Single, 2 children (5-10)	0.0009	M_{16}	0.0024
Single, 2 children (11-15)	0.0004	M_{17}	0.0087
Single, 3 children (Under 5)	0.0025	M_{18}	0.0004
Single, 3 children (5-10)	0.0009	M_{19}	0.00001

Single, 3 children (11-15)	0.0004	M_{20}	≈ 0
Single, adult children	0.0001	M_{21}	0.0002
Single, no children	0.0025	M_{22}	≈ 0
Part-time work multiplier, married	0.0003	M_{23}	0.00002
"	"	M_{24}	0.00004
Part-time work multiplier, single	0.0002	M_{25}	0.00003
"	"	M_{26}	0.00001
Initial log wage	0.00004	M_{30}	0.00003
s.d. of initial heterogeneity	0.00003	M_{39}	≈ 0
s.d. of wage shocks	3×10^{-5}	M_{49}	3×10^{-6}
		M_{50}	1×10^{-6}
		M_{40}	3×10^{-6}
		M_{41}	1×10^{-6}
		M_{42}	≈ 0
		M_{43}	1×10^{-6}
		M_{44}	≈ 0
		M_{45}	1×10^{-6}
		M_{46}	1×10^{-6}
		M_{47}	2×10^{-5}
Persistence of wage process	8×10^{-5}	M_{48}	≈ 0
		M_{40}	≈ 0
		M_{41}	6×10^{-6}
		M_{42}	2×10^{-5}
		M_{43}	2×10^{-5}
		M_{44}	8×10^{-6}
		M_{45}	≈ 0
		M_{46}	8×10^{-6}
		M_{47}	≈ 0
Returns to full-time work	9×10^{-8}	M_{27}	1×10^{-8}
		M_{31}	8×10^{-8}
		M_{32}	7×10^{-8}
		M_{33}	1×10^{-8}

		M_{34}	2×10^{-9}
		M_{35}	8×10^{-9}
		M_{36}	≈ 0
		M_{37}	≈ 0
		M_{38}	≈ 0
Returns to part-time work	2×10^{-7}	M_{29}	≈ 0
Decline in experience from age	1×10^{-10}	M_{28}	≈ 0
		M_{31}	7×10^{-11}
		M_{32}	≈ 0
		M_{33}	≈ 0
		M_{34}	≈ 0
		M_{35}	≈ 0
		M_{36}	7×10^{-11}
		M_{37}	1×10^{-11}
		M_{38}	2×10^{-11}
Returns to not working	1×10^{-5}	M_{27}	2×10^{-6}
Probability of non-market childcare	0.0004	M_{51}	0.0084