

Comparing phonological representations

Nick Danis

nsdanis@wustl.edu

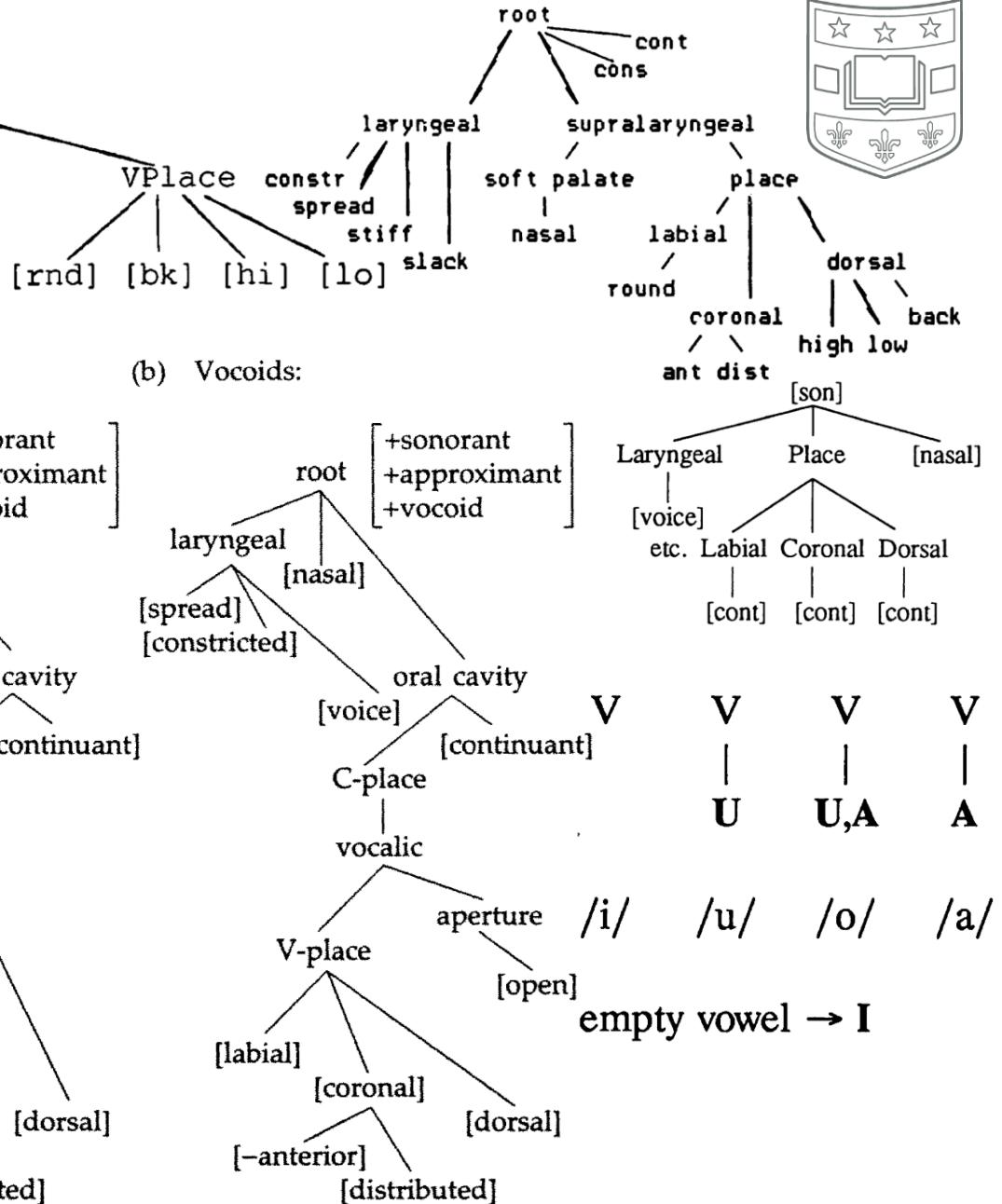
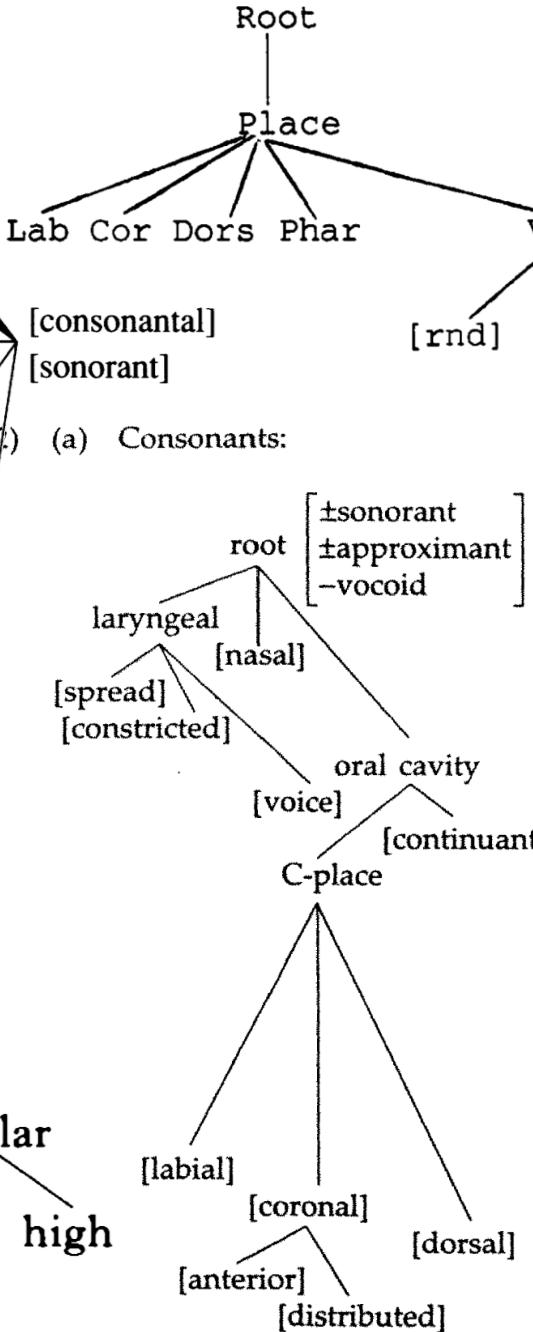
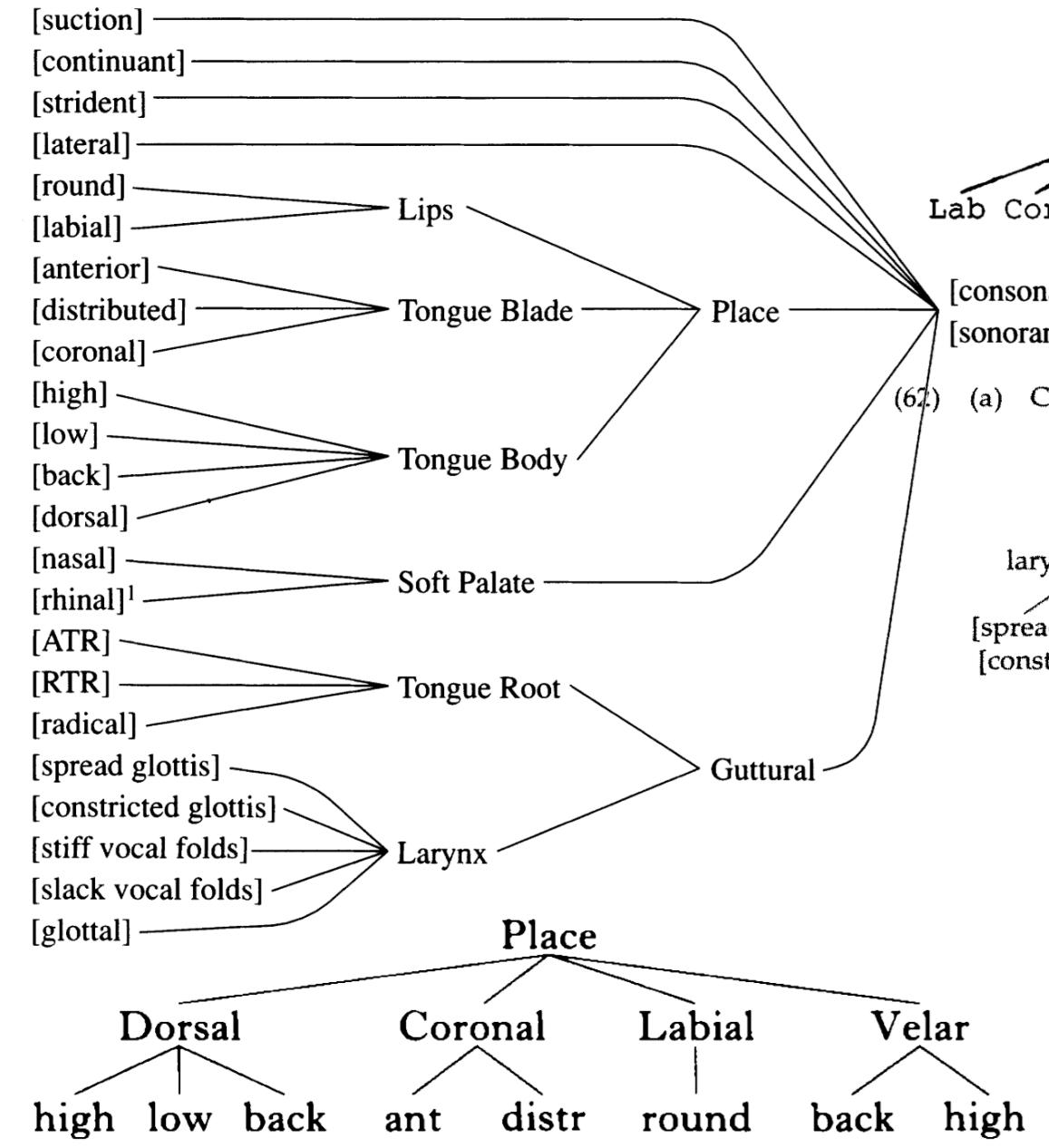
Graduate Linguistic Expo at Michigan State
(GLEAMS)

October 28-29, 2022



Washington University in St. Louis

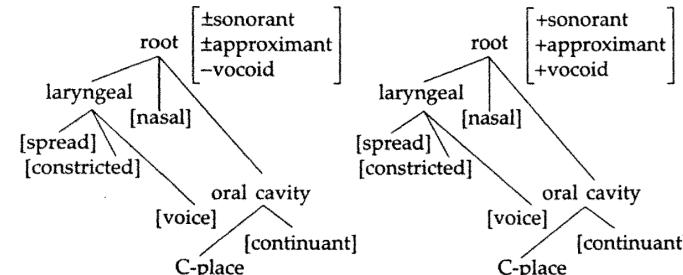




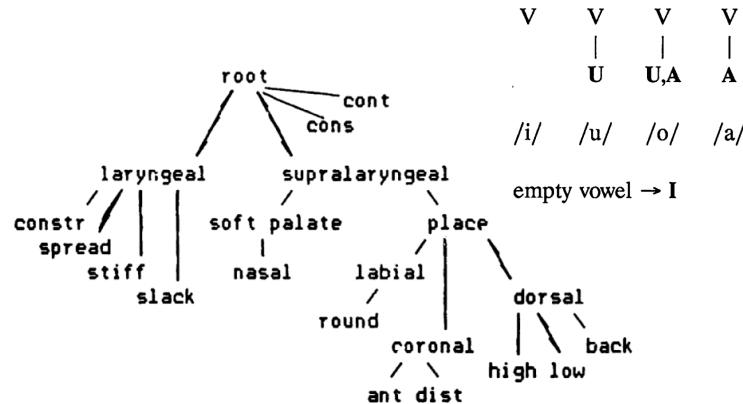
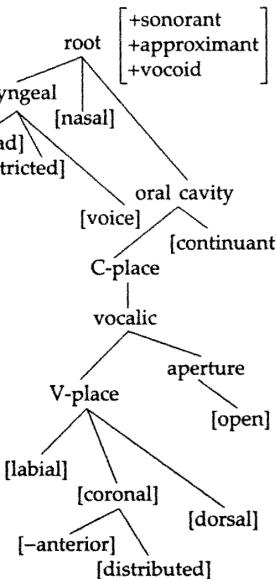


consonants and vowels probably should be natural classes

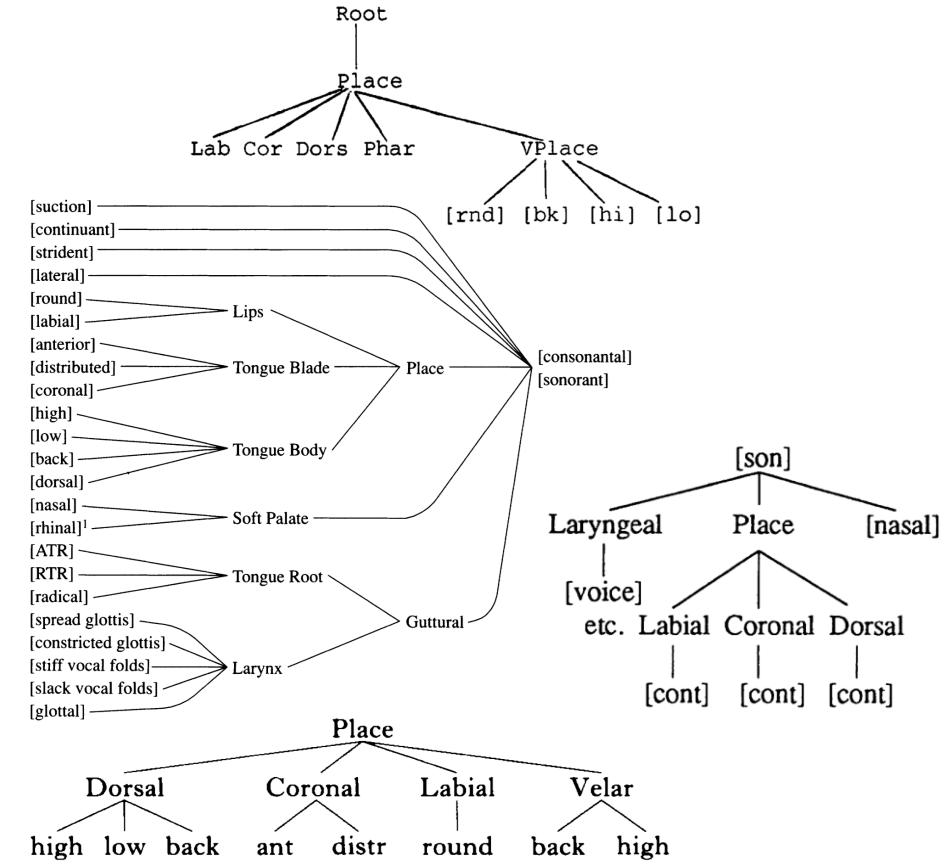
(62) (a) Consonants:



(b) Vocoids:



consonants and vowels probably *shouldn't* be natural classes





big questions

- how can we formally compare phonological representations?
- what can we learn from these comparisons?
- what do we care about as linguists?
- why care about anything?



medium answers

- two theories can be shown to be **formally equivalent** using logic and model theory
 - given two representations A and B, a **transduction** between A and B means that any linguistic rule given with structure A can be translated into structure B, and vice versa
 - Strother-Garcia (2019), Danis & Jardine (2019), Oakden (2020), a.o.

Figure 4.5: $\mathcal{M}_{\text{plenty}}^{\text{flat}}$

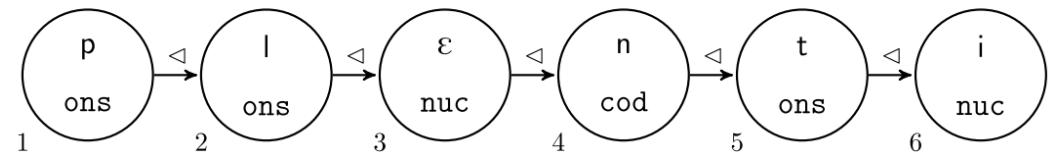
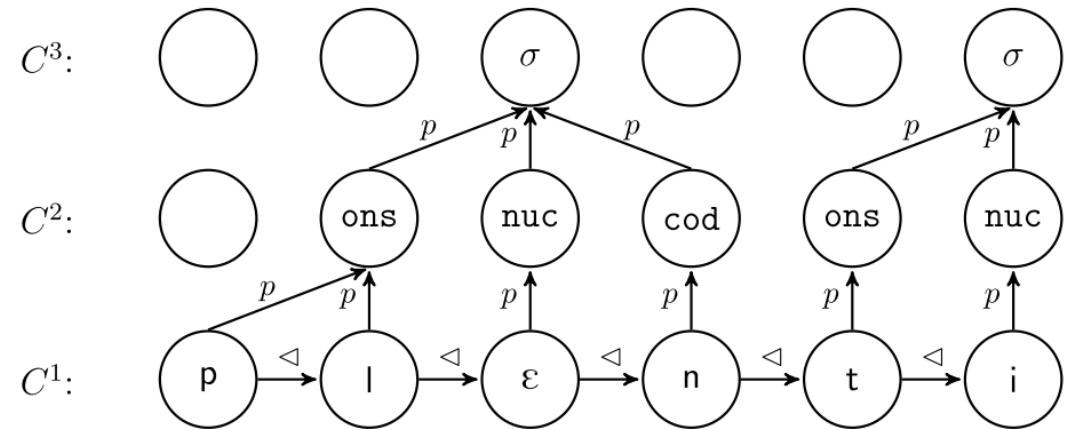


Figure 4.16: $\Gamma_{ft}(\mathcal{M}_{\text{plenty}}^{\text{flat}})$ fully specified

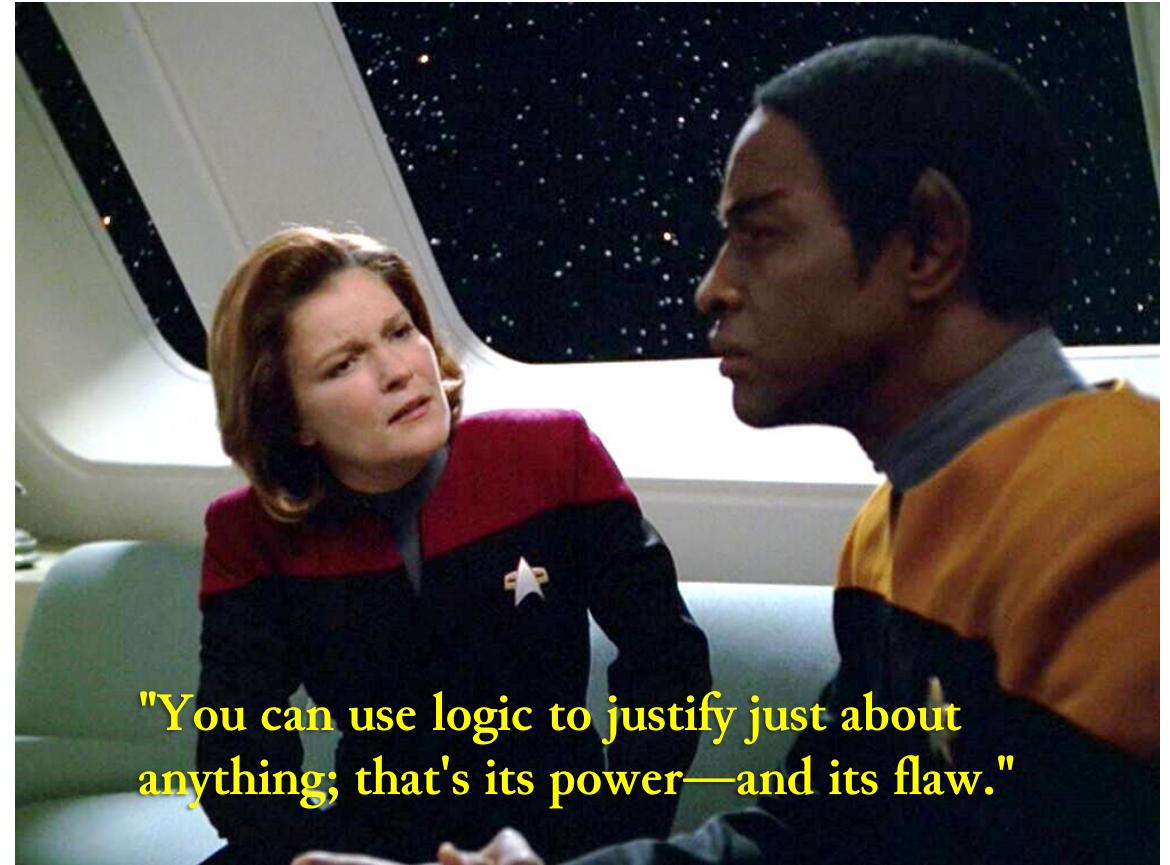


(Strother-Garcia 2019)



medium answers

- not every transduction preserves ideas of *linguistic* equivalence
 - process should respect natural classes, which may be lost in certain transductions
- the property of a **natural-class preserving transduction** is defined to find those logically equivalent representations that also share linguistic intuitions



introduction

foundations

example transduction

existing transductions

broader significance

conclusion



natural classes

- segments have structure
- some segments potentially share structure
- a **natural class** is a set of all segments that share some piece of structure





natural-class preserving transductions

- A transduction between two representational theories A and B is **natural-class preserving** ♣ iff the set of all natural class extensions of A exactly match those of B
 - A **natural class extension** is an exhaustive set of atomic segments that map to feature structures that share some common structural property

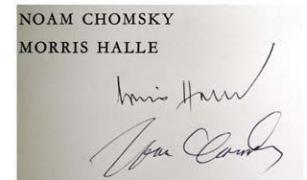
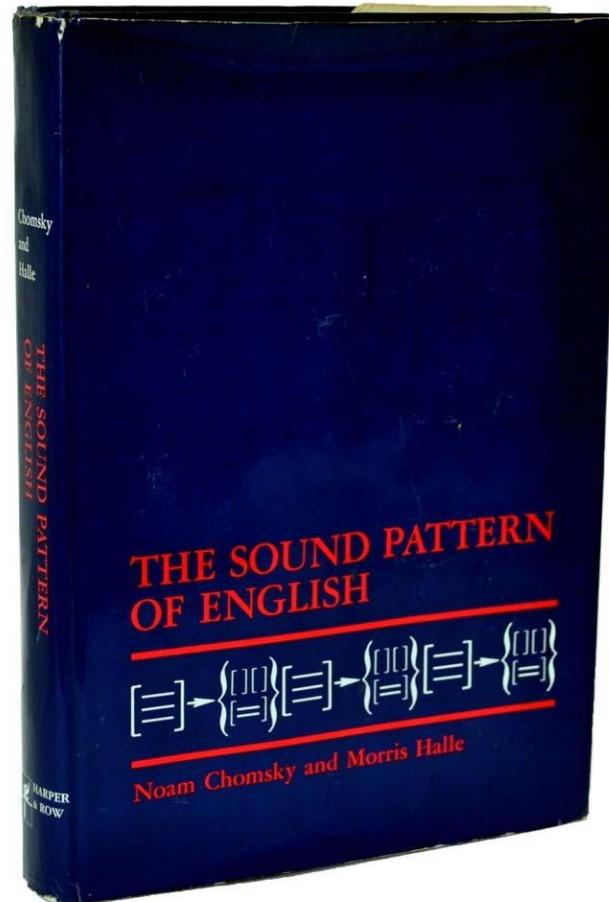
		[+front]	[-front]	[-front]
[-low]	[+high]	i		u
[-low]	[-high]	e		o
[+low]	[-high]		a	

shared structure: [+high]
natural class: $\left\{ \begin{bmatrix} -\text{low} \\ +\text{high} \end{bmatrix}, \begin{bmatrix} -\text{low} \\ +\text{high} \end{bmatrix} \right\}$
natural class extension: { i u }

natural classes

"In view of this, if a theory of language failed to provide a mechanism for making distinctions between more or less natural classes of segments, **this failure would be sufficient reason for rejecting the theory as being incapable of attaining the level of explanatory adequacy.**"

(Chomsky & Halle 1968: 355)





natural classes

"This combinability of features allows phonology to construct complex symbols from an inventory of simple parts, and provides an explanation for the so-called natural class behavior—**different structures can behave alike because they contain identical substructures.**"

" In Logical Phonology (see section 3), rules refer to natural classes by definition: a statement that cannot be formulated in terms of natural classes is not a rule."

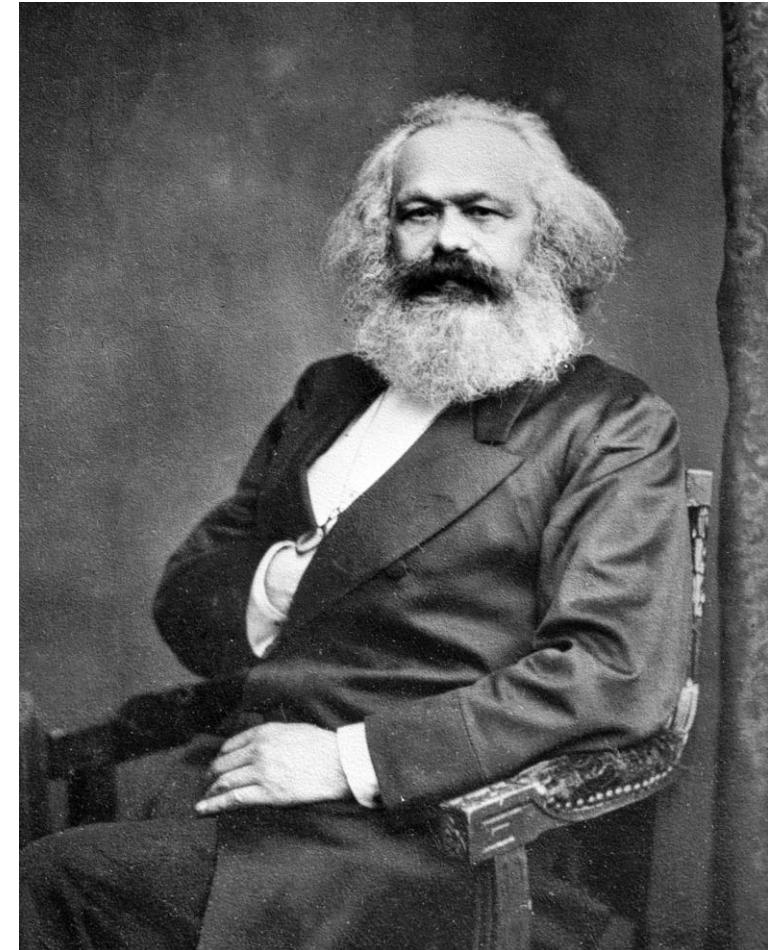
(Volenec & Reiss 2020: 22, 28)



natural classes

"...that consequently the whole history of mankind [...] has been a history of class struggles."

(Marx 1848:8)

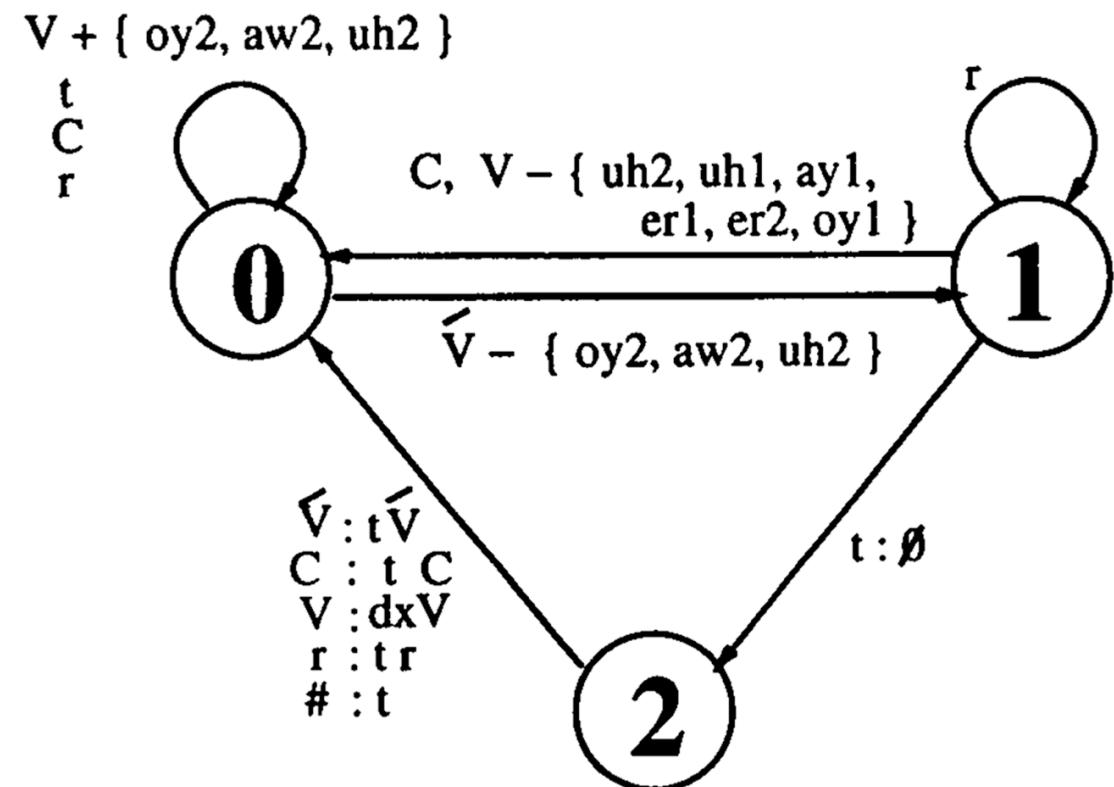




natural classes as a computational learning bias

"Without an ability to use knowledge about phonological features to generalize across phones, OSTIA's transducers have missing transitions for certain phones from certain states. This causes errors when transducing previously unseen words after training is complete."

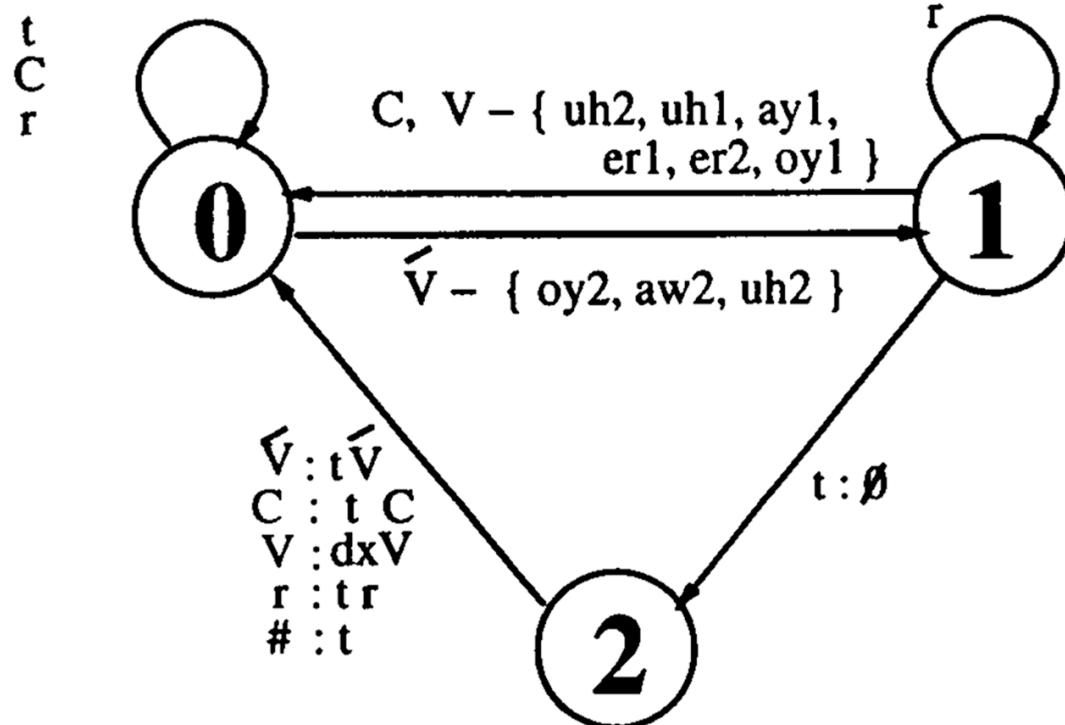
(Gildea & Jurafsky 1996)



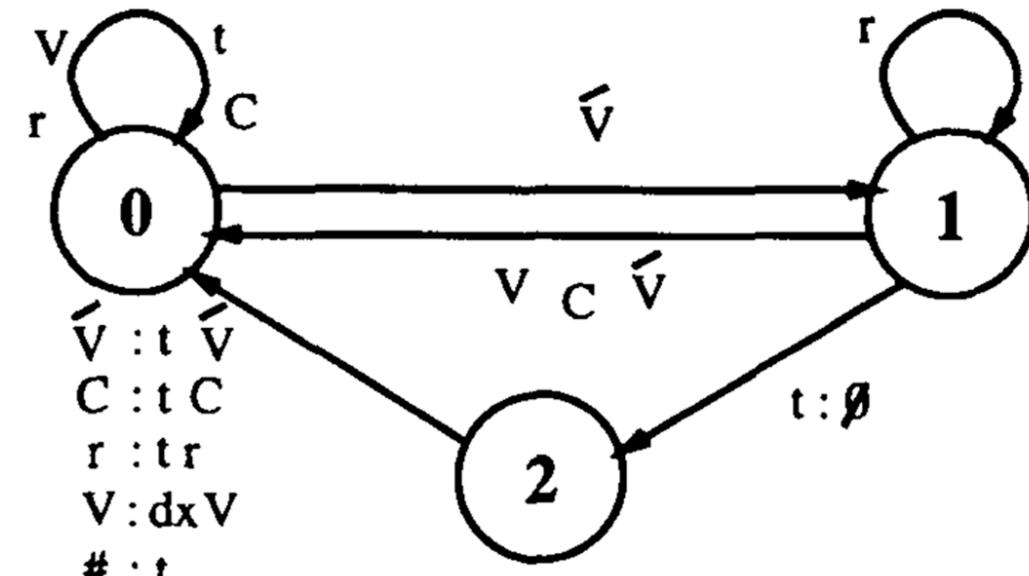


natural classes as a computational learning bias

$V + \{ oy2, aw2, uh2 \}$



transducer learned with no natural class knowledge



transducer learned with knowledge of natural classes

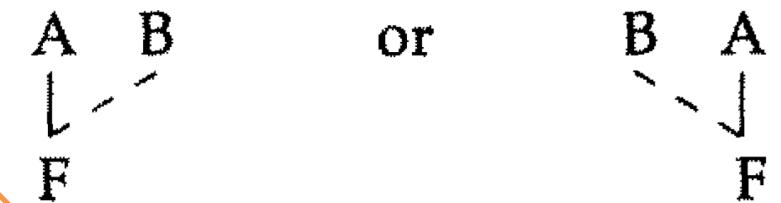


assimilation: act naturally

- assimilation operates over like things
 - Trubetzkoy (1969), Chomsky and Halle (1968), Hyman (1974), Hayes (1986), Clements & Hume (1995), me right now, a.o.

$$X \rightarrow [\alpha F] / [\alpha F]$$

Y



Sharing is Caring

the structural changes on the target of an assimilation process should be factors of the trigger



assimilation: sharing is caring

- Clements & Hume (1995):
 - "Phonological rules perform single operations only." (p. 250)
 - "In the present model, in contrast, assimilation rules are characterized as the association (or "**spreading**") of a feature or node F of segment A to a neighboring segment B..." (p. 258)
- If assimilation is the result of spreading (the addition of an association relation), then it directly follows from this that the resulting segments will have shared structure and therefore constitute a nontrivial natural class



the general argument

1. if we assume a nontrivial theory of segmental structure, and
2. if we assume for assimilation that **sharing is caring** 
3. then the range of possible assimilation processes is restricted

further:

4. if two theories are shown to be logically equivalent, and
5. if this transduction is not **natural-class preserving** 
6. then the two theories do not make the same empirical predictions (by 3)

introduction

foundations

example transduction

existing transductions

broader significance

conclusion



comparing theories

unified place theory

- consonants and vowels share representational primitives
 - e.g. LABIAL C-place, LABIAL V-place
- Sagey (1986), Clements & Hume (1995), a.o.

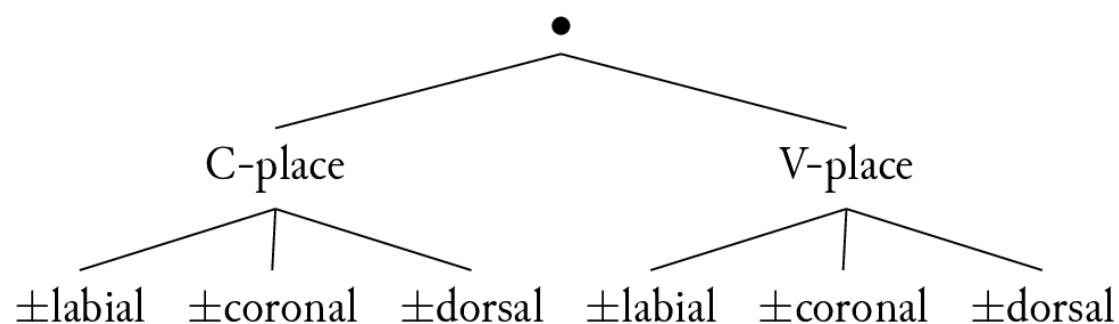
vowel features theory

- vowel place is largely defined by primitives not used to describe consonant place
 - e.g. [+back], [-round]
- Odden (1991), Ni Chiosain & Padgett (1993), Halle et al. (2000), a.o.

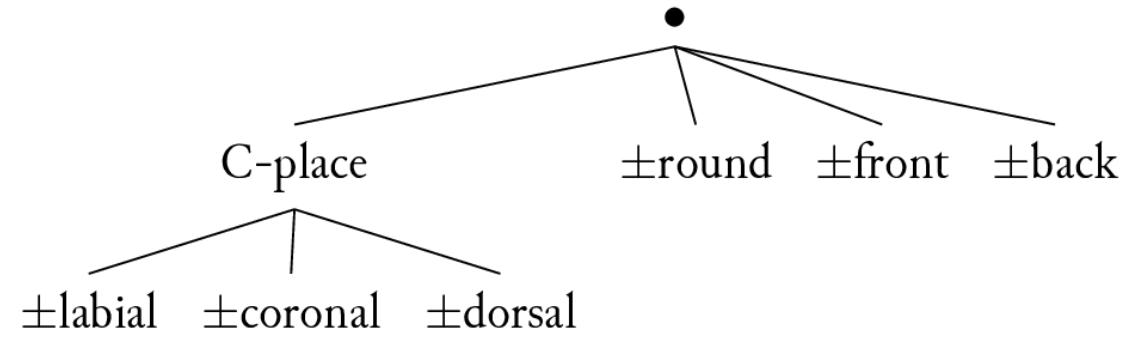


comparing theories

unified place theory



v-features theory



crucial difference: **unified** uses same features for
vocalic and consonantal contrasts

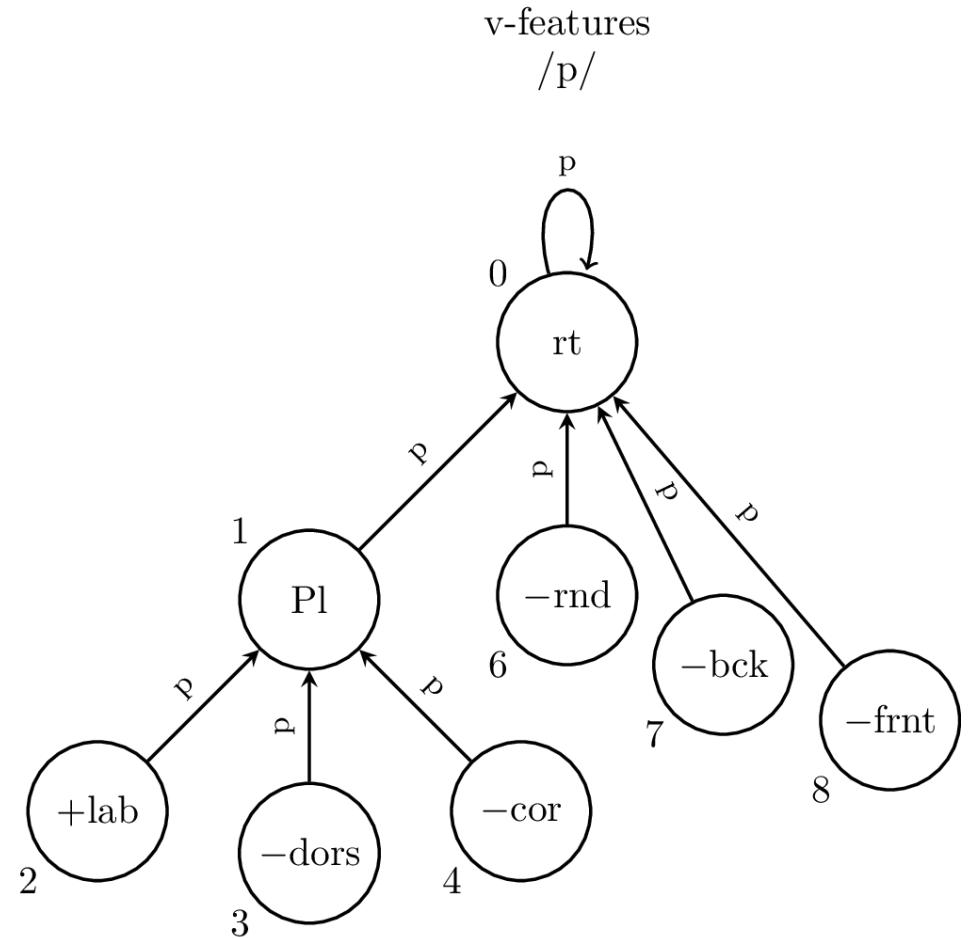
orthogonal issues:

- binary vs. privative features
- underspecification



comparing theories

- each theory is translated into a **finite model** defining the domain of nodes, relations, and functions in each
- each model defines a logical language for each theory of representation
- a **transduction** translates all relations & functions in one model to the other
- any sentence/rule/constraint expressible in one model is therefore expressible in the other





comparing theories

$$D = \{0, 1, 2, 3, 4, 6, 7\}$$

$$P_{rt} = \{0\}$$

$$P_{Pl} = \{1\}$$

$$P_{+lab} = \{2\}$$

$$P_{-dors} = \{3\}$$

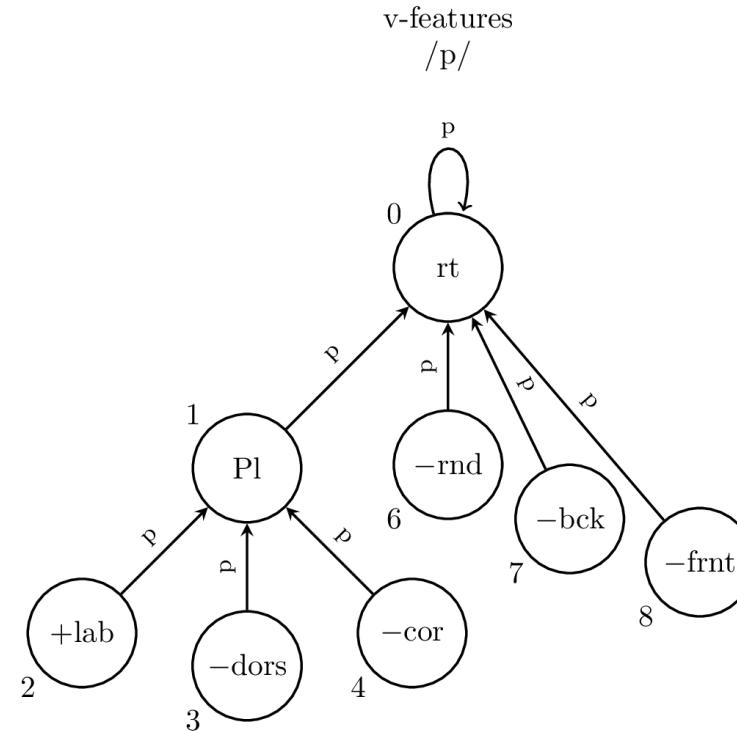
$$P_{-cor} = \{4\}$$

$$P_{-rnd} = \{6\}$$

$$P_{-bck} = \{7\}$$

$$P_{-frnt} = \{8\}$$

$$\text{parent}(x) = \begin{cases} 0 \Leftrightarrow x \in \{0, 1, 6, 7, 8\} \\ 1 \Leftrightarrow x = \{2, 3, 4\} \end{cases}$$

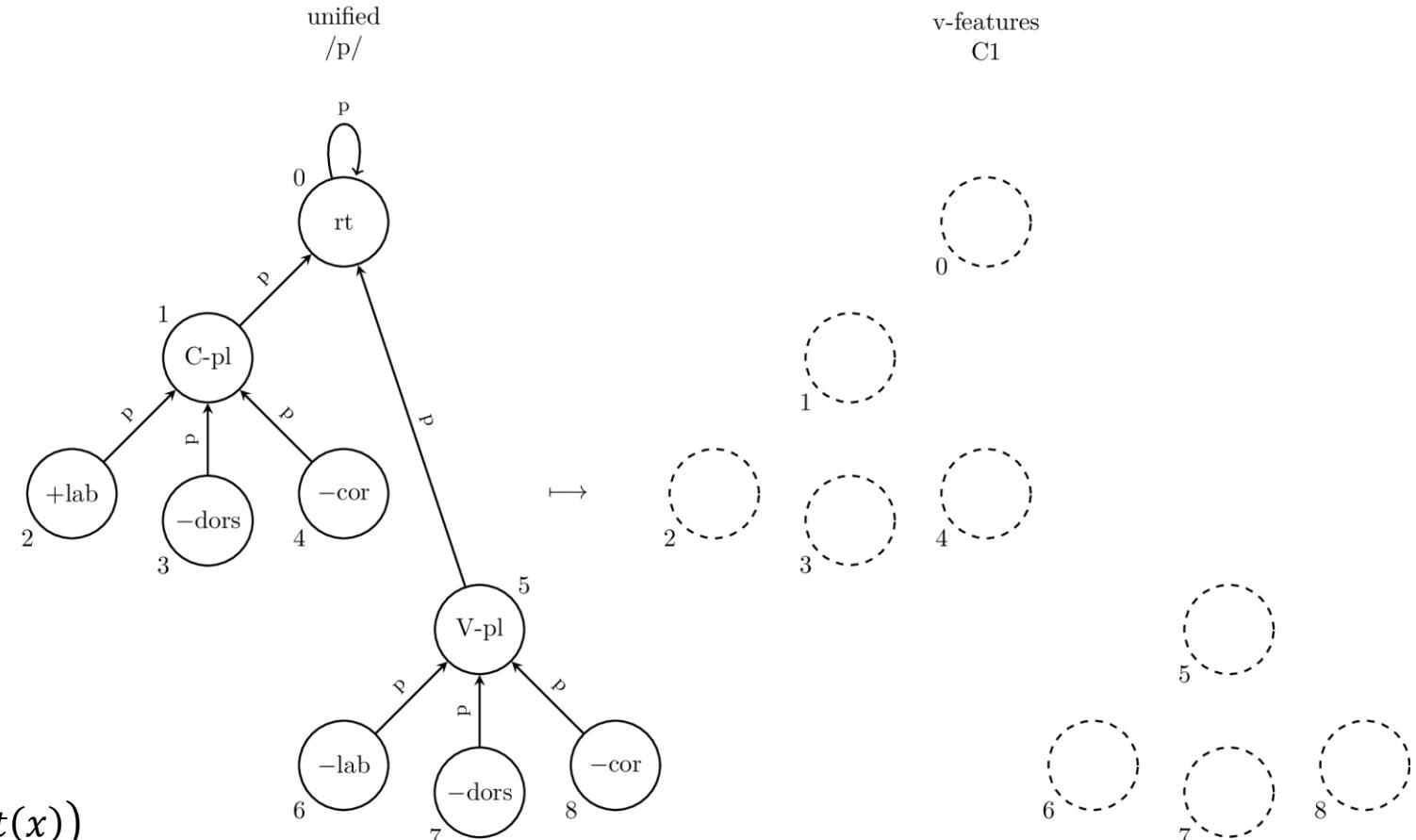


the following slides provide a transduction in quantifier-free first-order logic (QF) that translates between the unified model and the v-features model



the transduction: unified → v-features

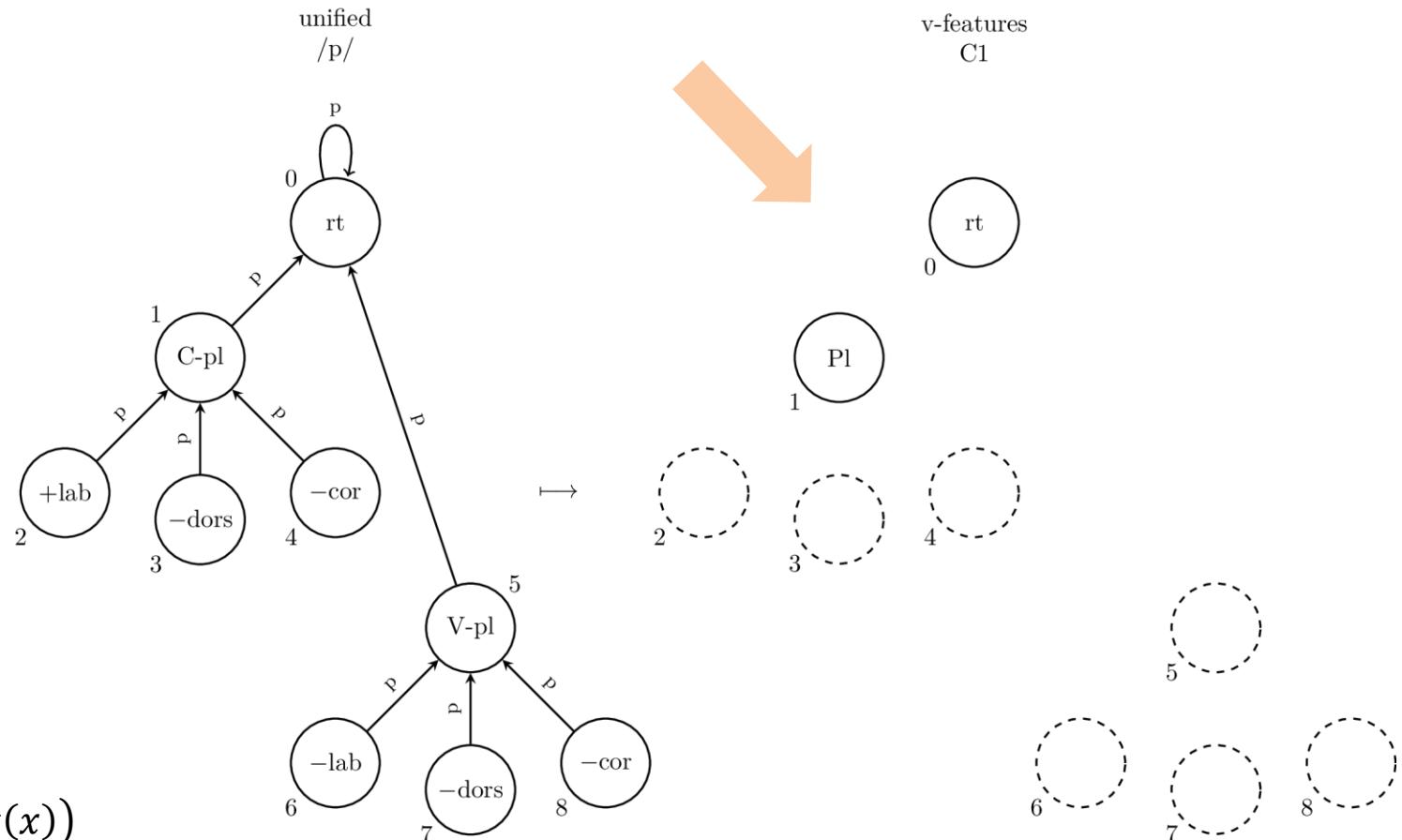
$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $\text{Place}(x^1) \stackrel{\text{def}}{=} C\text{-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $+coronal(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-labial(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $-coronal(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $+front(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $+back(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $-front(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-back(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $\text{parent}(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg V\text{-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow V\text{-place}(\text{parent}(x)) \end{cases}$





the transduction: unified → v-features

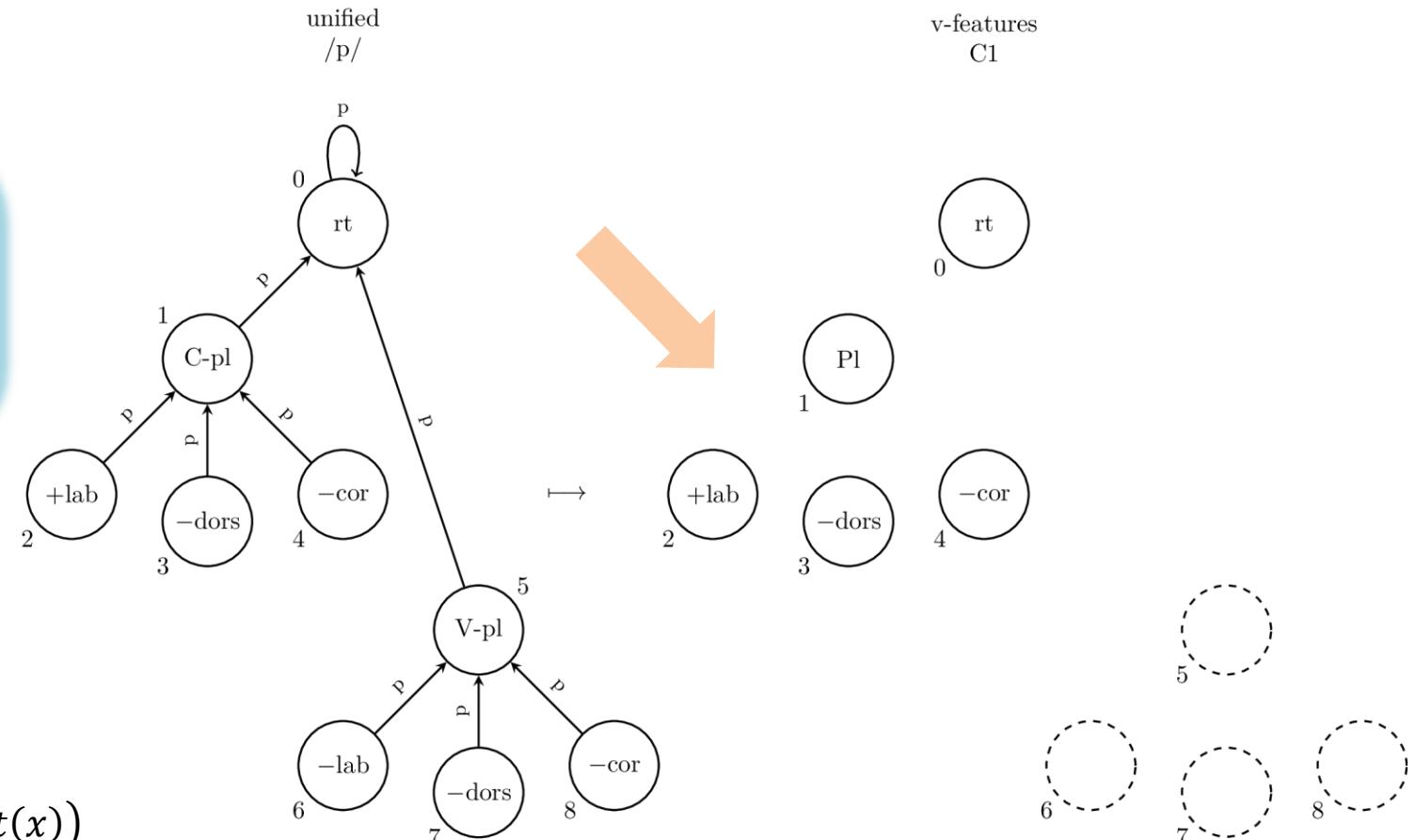
$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $\text{Place}(x^1) \stackrel{\text{def}}{=} \text{C-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +\text{labial}(x) \wedge \text{C-place}(\text{parent}(x))$
 $+\text{coronal}(x^1) \stackrel{\text{def}}{=} +\text{coronal}(x) \wedge \text{C-place}(\text{parent}(x))$
 $+\text{dorsal}(x^1) \stackrel{\text{def}}{=} +\text{cortal}(x) \wedge \text{C-place}(\text{parent}(x))$
 $-\text{labial}(x^1) \stackrel{\text{def}}{=} -\text{labial}(x) \wedge \text{C-place}(\text{parent}(x))$
 $-\text{coronal}(x^1) \stackrel{\text{def}}{=} -\text{coronal}(x) \wedge \text{C-place}(\text{parent}(x))$
 $-\text{dorsal}(x^1) \stackrel{\text{def}}{=} -\text{dorsal}(x) \wedge \text{C-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +\text{labial}(x) \wedge \text{V-place}(\text{parent}(x))$
 $+\text{front}(x^1) \stackrel{\text{def}}{=} +\text{coronal}(x) \wedge \text{V-place}(\text{parent}(x))$
 $+\text{back}(x^1) \stackrel{\text{def}}{=} +\text{cortal}(x) \wedge \text{V-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -\text{labial}(x) \wedge \text{V-place}(\text{parent}(x))$
 $-\text{front}(x^1) \stackrel{\text{def}}{=} -\text{coronal}(x) \wedge \text{V-place}(\text{parent}(x))$
 $-\text{back}(x^1) \stackrel{\text{def}}{=} -\text{dorsal}(x) \wedge \text{V-place}(\text{parent}(x))$
 $\text{parent}(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg \text{V-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow \text{V-place}(\text{parent}(x)) \end{cases}$





the transduction: unified \rightarrow v-features

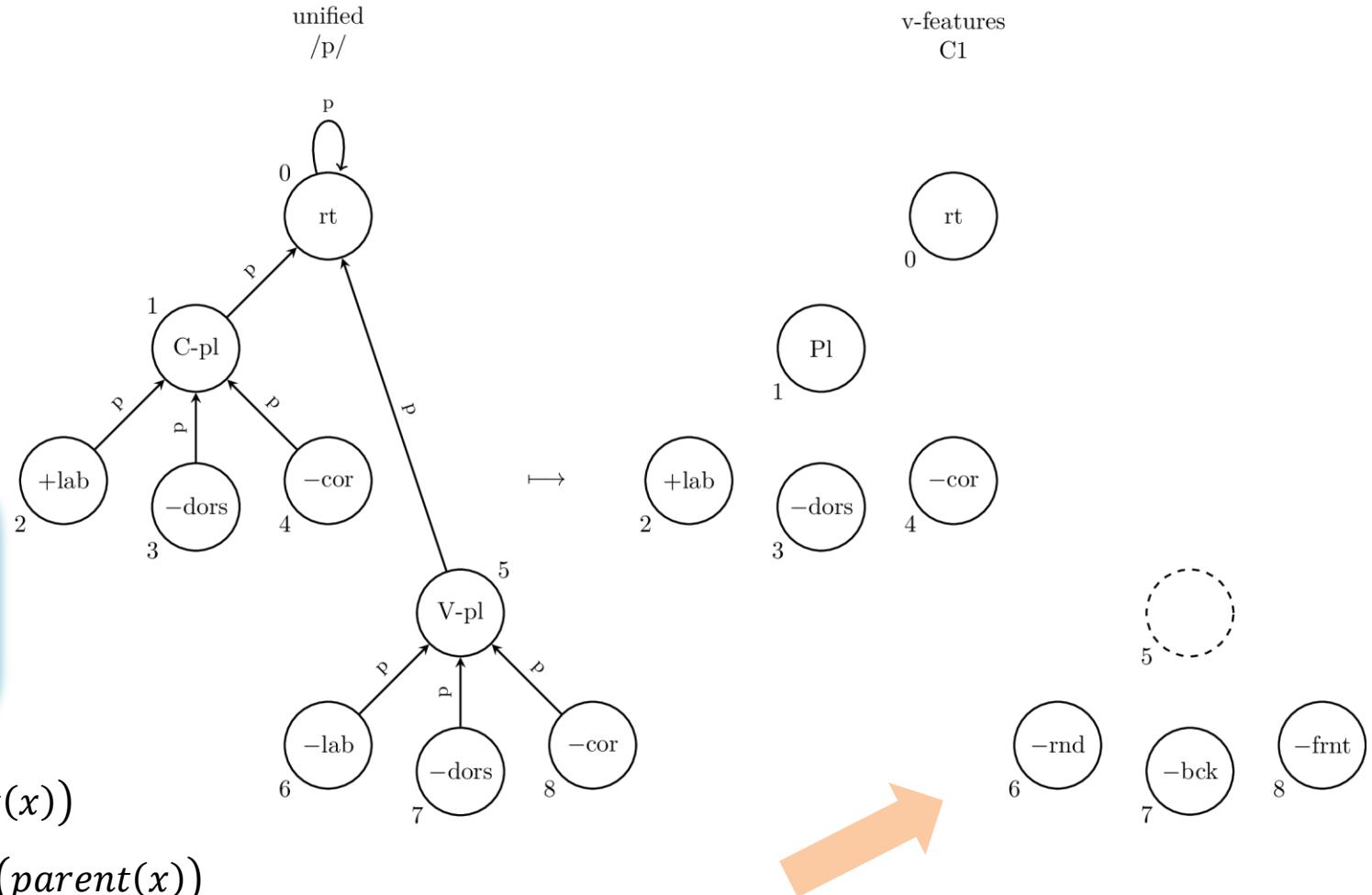
$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $Place(x^1) \stackrel{\text{def}}{=} C\text{-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $+coronal(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-labial(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $-coronal(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $+front(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $+back(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $-front(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-back(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg V\text{-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow V\text{-place}(\text{parent}(x)) \end{cases}$





the transduction: unified \rightarrow v-features

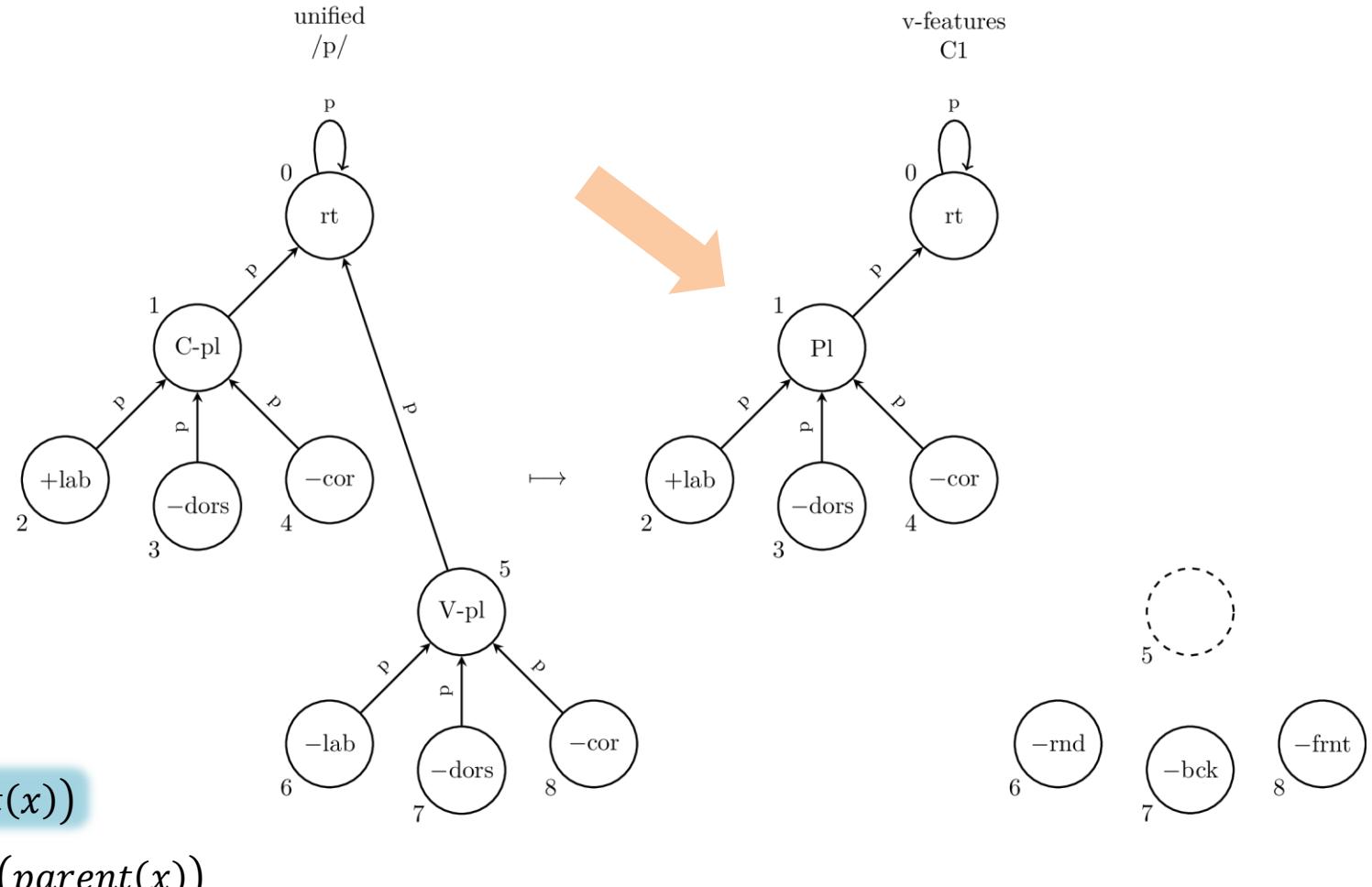
$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $Place(x^1) \stackrel{\text{def}}{=} C\text{-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $+coronal(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-labial(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $-coronal(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $+front(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $+back(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $-front(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-back(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg V\text{-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow V\text{-place}(\text{parent}(x)) \end{cases}$





the transduction: unified \rightarrow v-features

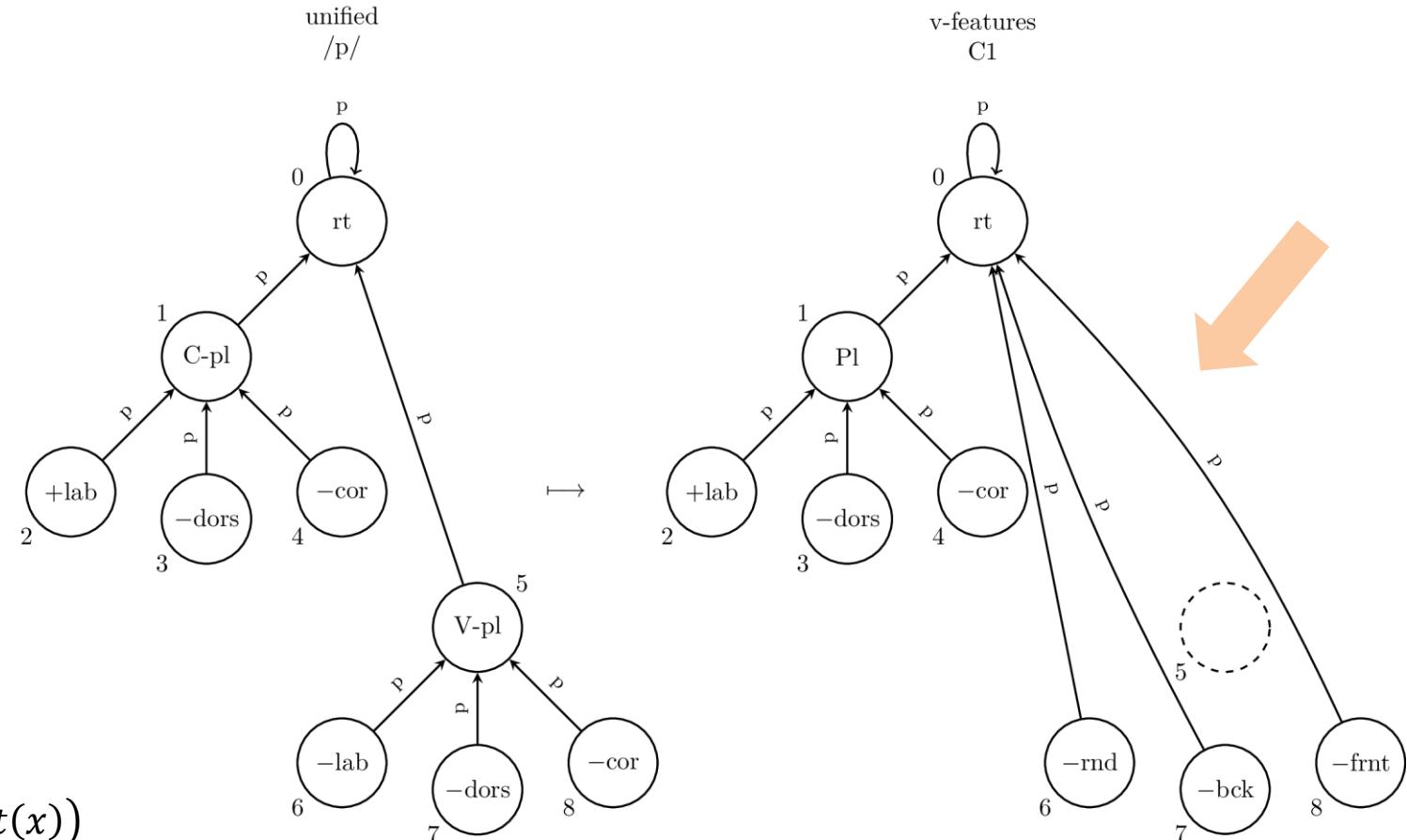
$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $Place(x^1) \stackrel{\text{def}}{=} C\text{-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $+coronal(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-labial(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $-coronal(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $+front(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $+back(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $-front(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-back(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg V\text{-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow V\text{-place}(\text{parent}(x)) \end{cases}$





the transduction: unified \rightarrow v-features

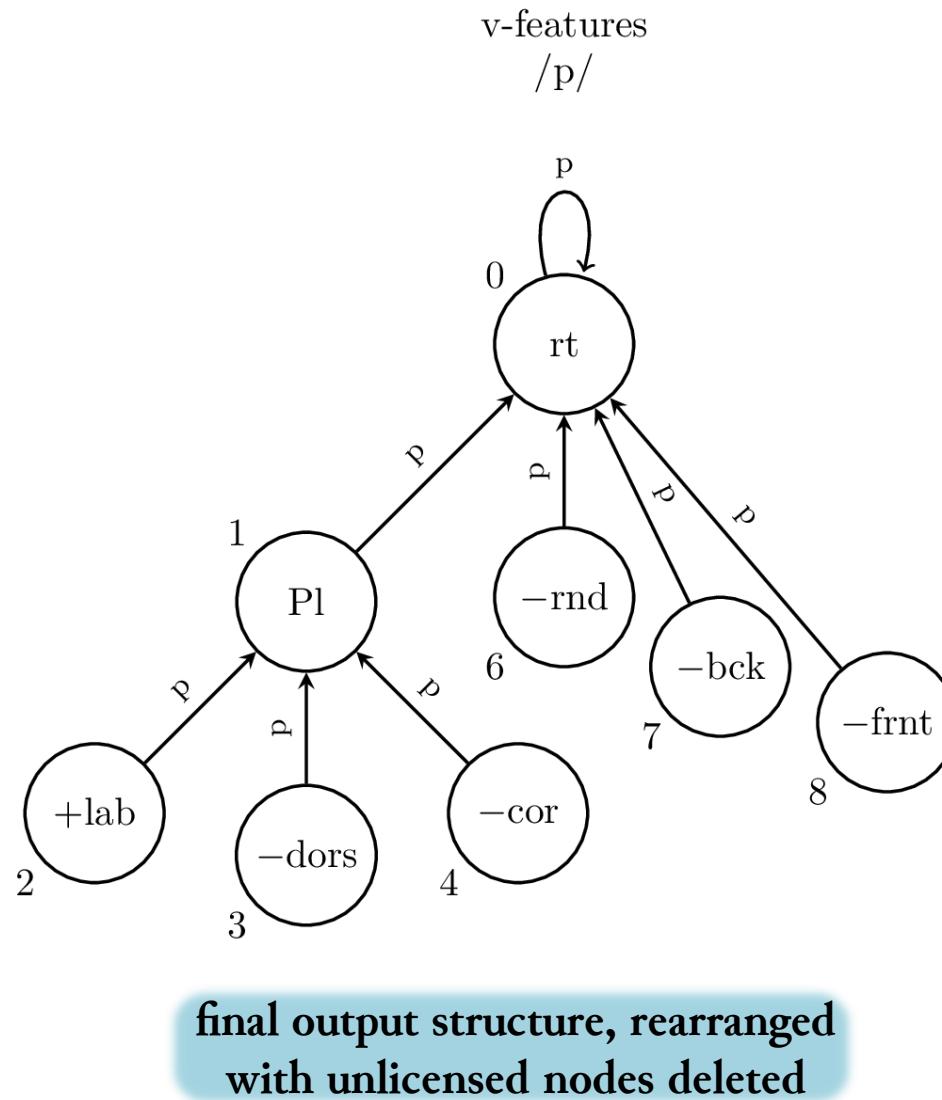
$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $Place(x^1) \stackrel{\text{def}}{=} C\text{-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $+coronal(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +corsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-labial(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $-coronal(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $+front(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $+back(x^1) \stackrel{\text{def}}{=} +corsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $-front(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-back(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg V\text{-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow V\text{-place}(\text{parent}(x)) \end{cases}$





the transduction: unified \rightarrow v-features

$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $Place(x^1) \stackrel{\text{def}}{=} C\text{-place}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $+coronal(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-labial(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge C\text{-place}(\text{parent}(x))$
 $-coronal(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge C\text{-place}(\text{parent}(x))$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge C\text{-place}(\text{parent}(x))$
 $+round(x^1) \stackrel{\text{def}}{=} +labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $+front(x^1) \stackrel{\text{def}}{=} +coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $+back(x^1) \stackrel{\text{def}}{=} +cortal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-round(x^1) \stackrel{\text{def}}{=} -labial(x) \wedge V\text{-place}(\text{parent}(x))$
 $-front(x^1) \stackrel{\text{def}}{=} -coronal(x) \wedge V\text{-place}(\text{parent}(x))$
 $-back(x^1) \stackrel{\text{def}}{=} -dorsal(x) \wedge V\text{-place}(\text{parent}(x))$
 $parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg V\text{-place}(\text{parent}(x)) \\ (\text{parent}(\text{parent}(x)))^1 \Leftrightarrow V\text{-place}(\text{parent}(x)) \end{cases}$

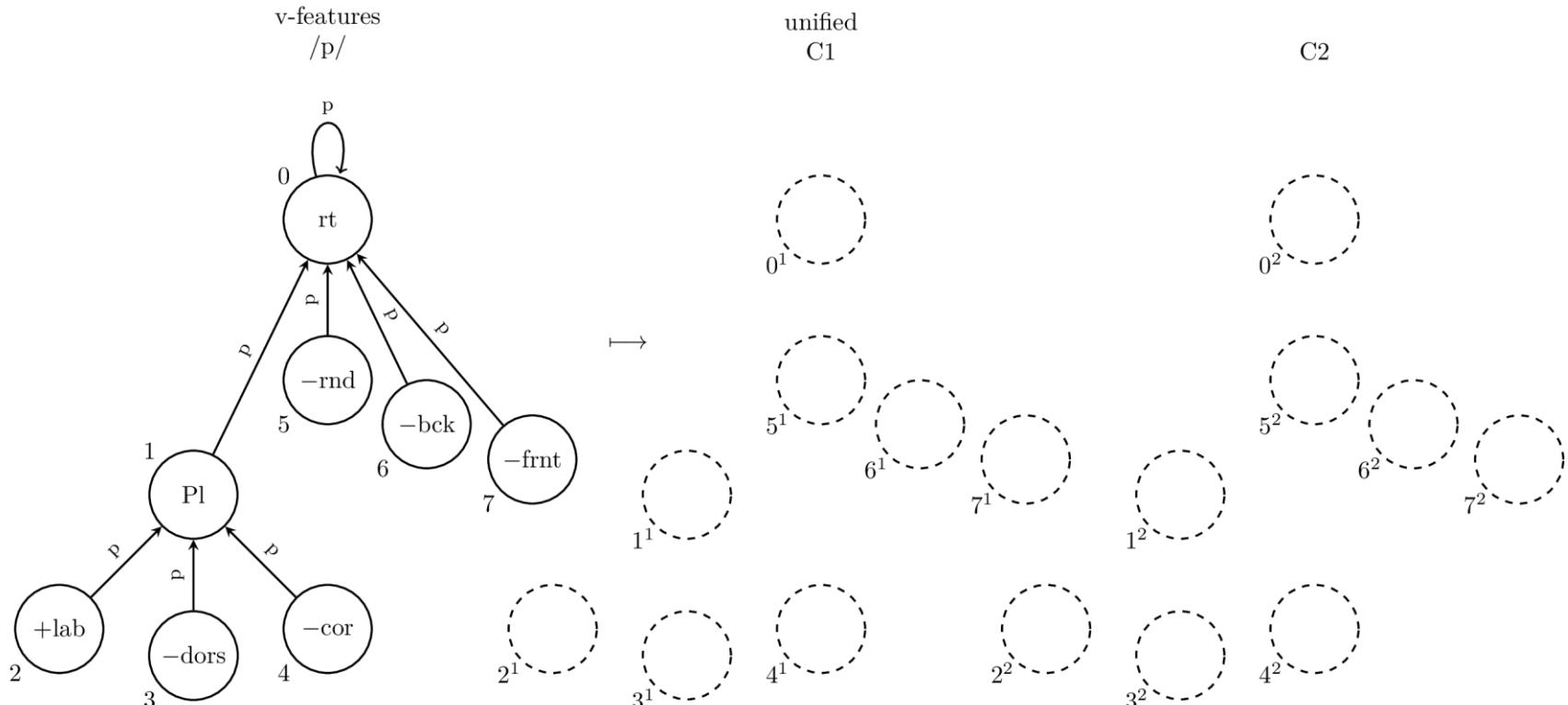




the transduction: v-features → unified

$$\begin{aligned}
 \text{rt}(x^1) &\stackrel{\text{def}}{=} \text{rt}(x) \\
 +\text{labial}(x^1) &\stackrel{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x) \\
 +\text{coronal}(x^1) &\stackrel{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x) \\
 +\text{dorsal}(x^1) &\stackrel{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x) \\
 -\text{labial}(x^1) &\stackrel{\text{def}}{=} -\text{round}(x) \vee \text{labial}(x) \\
 -\text{coronal}(x^1) &\stackrel{\text{def}}{=} -\text{front}(x) \vee \text{coronal}(x) \\
 -\text{dorsal}(x^1) &\stackrel{\text{def}}{=} -\text{back}(x) \vee \text{coronal}(x) \\
 \text{C-place}(x^1) &\stackrel{\text{def}}{=} \text{Place}(x) \\
 \text{V-place}(x^2) &\stackrel{\text{def}}{=} \text{rt}(x)
 \end{aligned}$$

$$\begin{aligned}
 \text{parent}(x^1) &\stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg \text{vowelFeature}(x) \\ (\text{parent}(x))^2 \Leftrightarrow \text{vowelFeature}(x) \end{cases} \\
 \text{parent}(x^2) &\stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \text{rt}(x)\}
 \end{aligned}$$

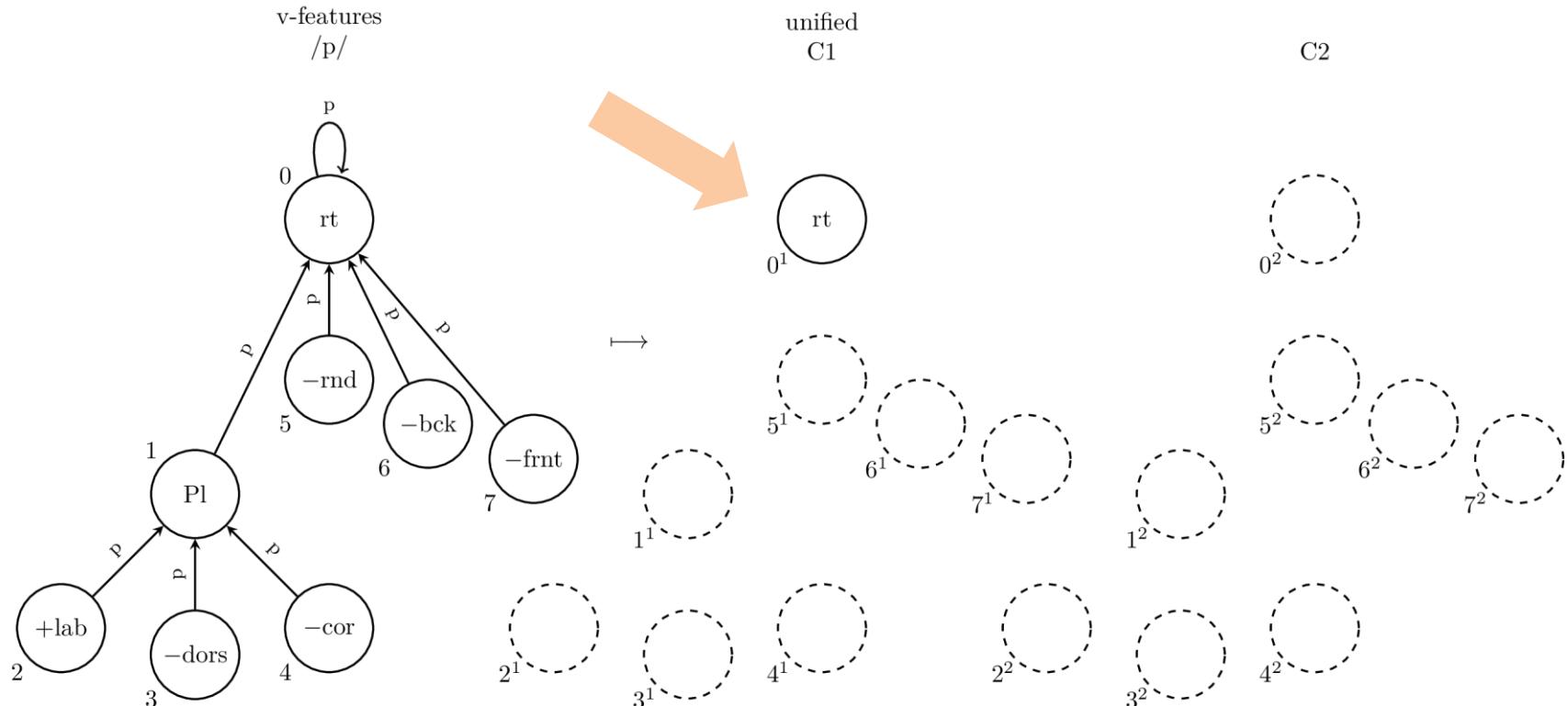




the transduction: v-features → unified

$\text{rt}(x^1) \stackrel{\text{def}}{=} \text{rt}(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x)$
 $+\text{coronal}(x^1) \stackrel{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x)$
 $+\text{dorsal}(x^1) \stackrel{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x)$
 $-\text{labial}(x^1) \stackrel{\text{def}}{=} -\text{round}(x) \vee \text{labial}(x)$
 $-\text{coronal}(x^1) \stackrel{\text{def}}{=} -\text{front}(x) \vee \text{coronal}(x)$
 $-\text{dorsal}(x^1) \stackrel{\text{def}}{=} -\text{back}(x) \vee \text{coronal}(x)$
 $\text{C-place}(x^1) \stackrel{\text{def}}{=} \text{Place}(x)$
 $\text{V-place}(x^2) \stackrel{\text{def}}{=} \text{rt}(x)$

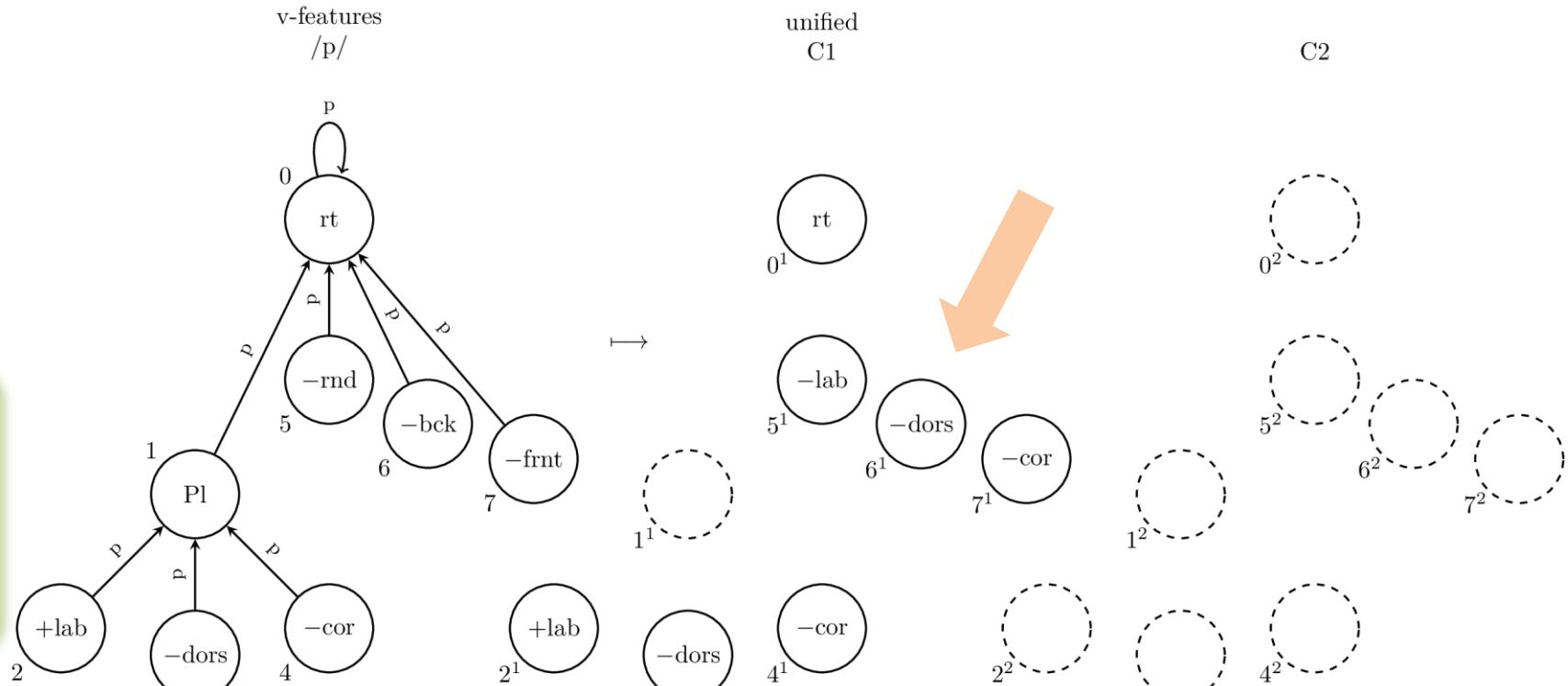
$\text{parent}(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 & \Leftrightarrow \neg \text{vowelFeature}(x) \\ (\text{parent}(x))^2 & \Leftrightarrow \text{vowelFeature}(x) \end{cases}$
 $\text{parent}(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \text{rt}(x)\}$





the transduction: v-features → unified

$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +round(x) \vee labial(x)$
 $+coronal(x^1) \stackrel{\text{def}}{=} +front(x) \vee coronal(x)$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +back(x) \vee coronal(x)$
 $-labial(x^1) \stackrel{\text{def}}{=} -round(x) \vee labial(x)$
 $-coronal(x^1) \stackrel{\text{def}}{=} -front(x) \vee coronal(x)$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -back(x) \vee coronal(x)$
 $C\text{-place}(x^1) \stackrel{\text{def}}{=} Place(x)$
 $V\text{-place}(x^2) \stackrel{\text{def}}{=} rt(x)$
 $parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 & \Leftrightarrow \neg vowelFeature(x) \\ (parent(x))^2 & \Leftrightarrow vowelFeature(x) \end{cases}$
 $parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow rt(x)\}$

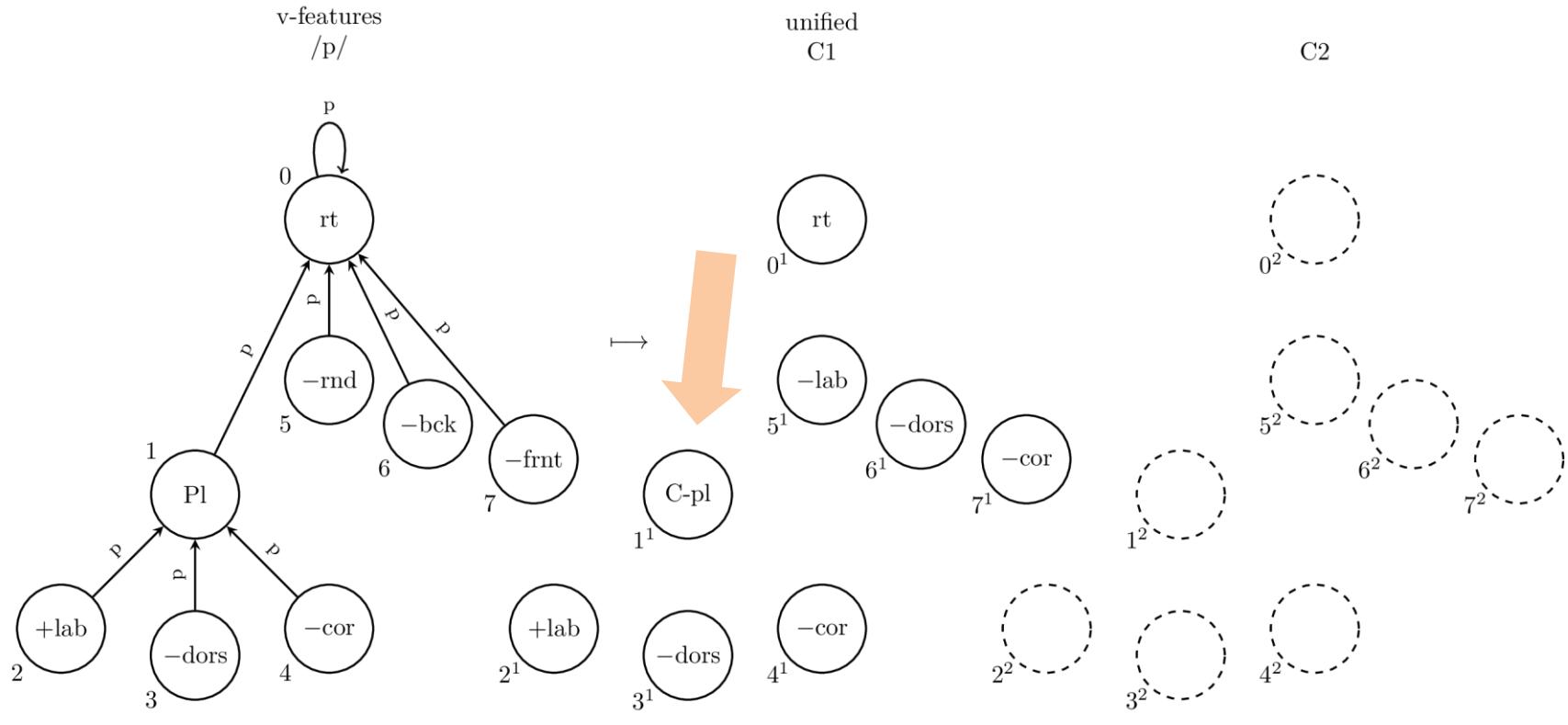




the transduction: v-features → unified

$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +round(x) \vee labial(x)$
 $+coronal(x^1) \stackrel{\text{def}}{=} +front(x) \vee coronal(x)$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +back(x) \vee coronal(x)$
 $-labial(x^1) \stackrel{\text{def}}{=} -round(x) \vee labial(x)$
 $-coronal(x^1) \stackrel{\text{def}}{=} -front(x) \vee coronal(x)$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -back(x) \vee coronal(x)$
 $C\text{-place}(x^1) \stackrel{\text{def}}{=} Place(x)$
 $V\text{-place}(x^2) \stackrel{\text{def}}{=} rt(x)$

$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 & \Leftrightarrow \neg vowelFeature(x) \\ (parent(x))^2 & \Leftrightarrow vowelFeature(x) \end{cases}$
 $parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow rt(x)$

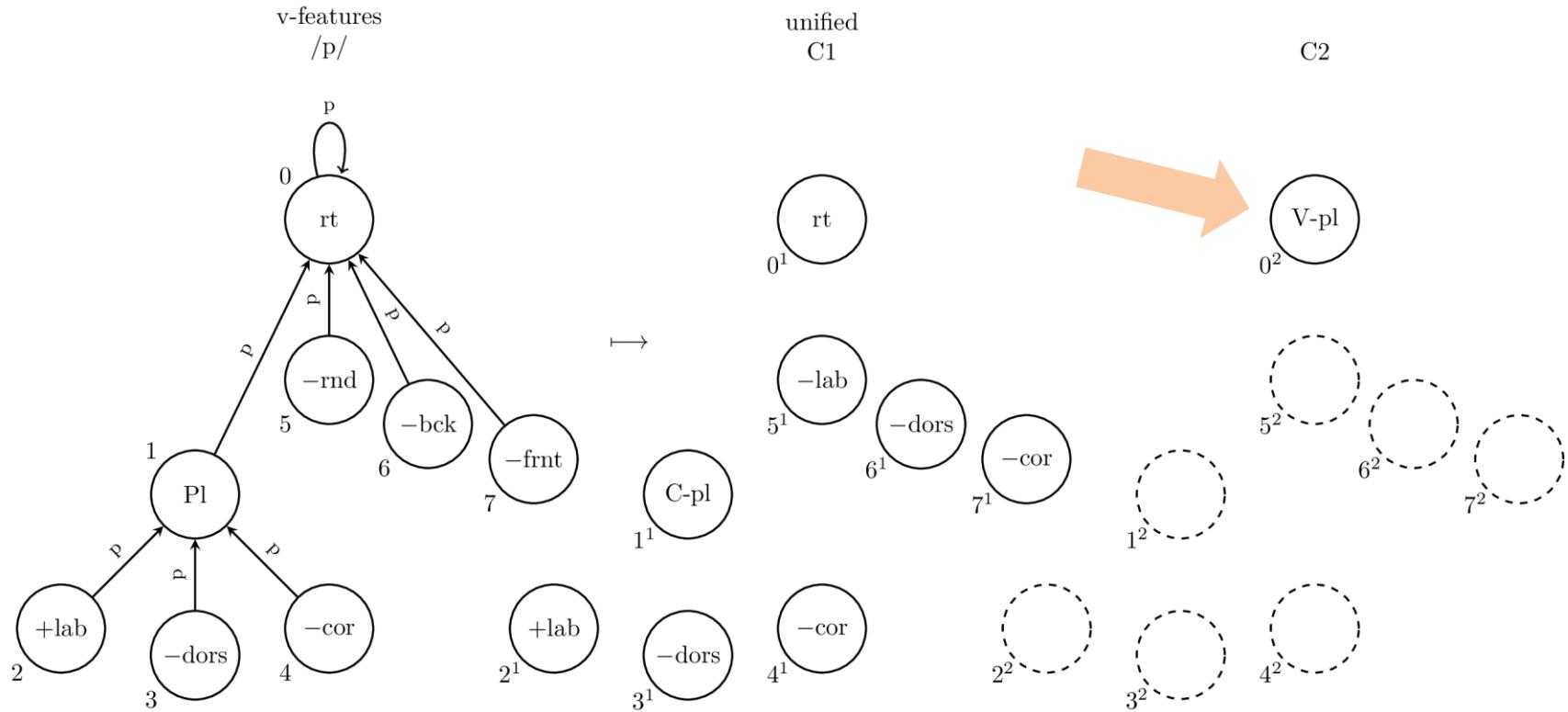




the transduction: v-features → unified

$rt(x^1) \stackrel{\text{def}}{=} rt(x)$
 $+labial(x^1) \stackrel{\text{def}}{=} +round(x) \vee labial(x)$
 $+coronal(x^1) \stackrel{\text{def}}{=} +front(x) \vee coronal(x)$
 $+dorsal(x^1) \stackrel{\text{def}}{=} +back(x) \vee coronal(x)$
 $-labial(x^1) \stackrel{\text{def}}{=} -round(x) \vee labial(x)$
 $-coronal(x^1) \stackrel{\text{def}}{=} -front(x) \vee coronal(x)$
 $-dorsal(x^1) \stackrel{\text{def}}{=} -back(x) \vee coronal(x)$
 $C\text{-place}(x^1) \stackrel{\text{def}}{=} Place(x)$
 $V\text{-place}(x^2) \stackrel{\text{def}}{=} rt(x)$

$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (parent(x))^1 & \Leftrightarrow \neg vowelFeature(x) \\ (parent(x))^2 & \Leftrightarrow vowelFeature(x) \end{cases}$
 $parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow rt(x)$





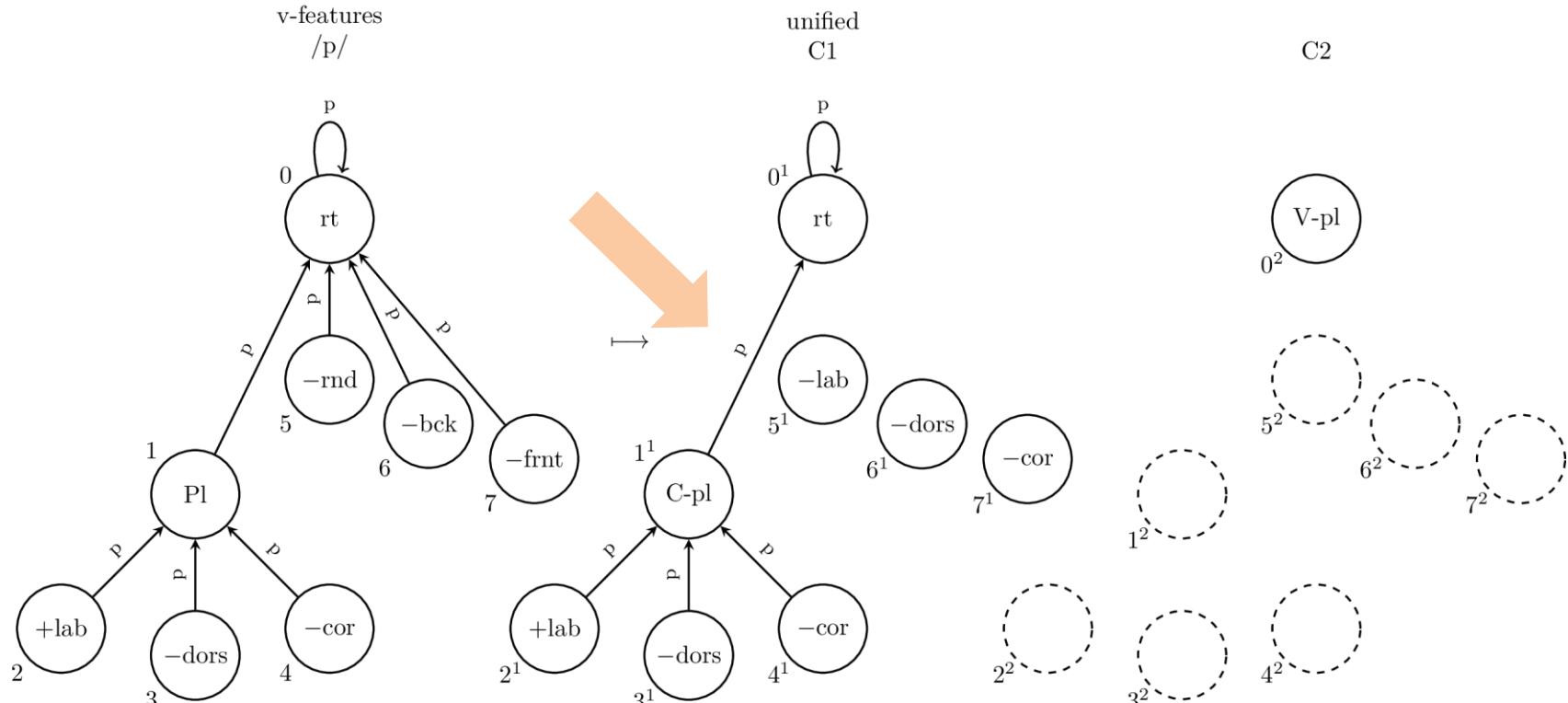
the transduction: v-features → unified

$$\begin{aligned}
 \text{rt}(x^1) &\stackrel{\text{def}}{=} \text{rt}(x) \\
 +\text{labial}(x^1) &\stackrel{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x) \\
 +\text{coronal}(x^1) &\stackrel{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x) \\
 +\text{dorsal}(x^1) &\stackrel{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x) \\
 -\text{labial}(x^1) &\stackrel{\text{def}}{=} -\text{round}(x) \vee \text{labial}(x) \\
 -\text{coronal}(x^1) &\stackrel{\text{def}}{=} -\text{front}(x) \vee \text{coronal}(x) \\
 -\text{dorsal}(x^1) &\stackrel{\text{def}}{=} -\text{back}(x) \vee \text{coronal}(x) \\
 \text{C-place}(x^1) &\stackrel{\text{def}}{=} \text{Place}(x) \\
 \text{V-place}(x^2) &\stackrel{\text{def}}{=} \text{rt}(x)
 \end{aligned}$$

$$\text{parent}(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg \text{vowelFeature}(x) \\ (\text{parent}(x))^2 \Leftrightarrow \text{vowelFeature}(x) \end{cases}$$

$$\text{parent}(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow \text{rt}(x)\}$$

$$\text{vowelFeature}(x) = +\text{round}(x) \vee -\text{round}(x) \vee +\text{front}(x) \vee -\text{front}(x) \vee +\text{back}(x) \vee -\text{back}(x)$$





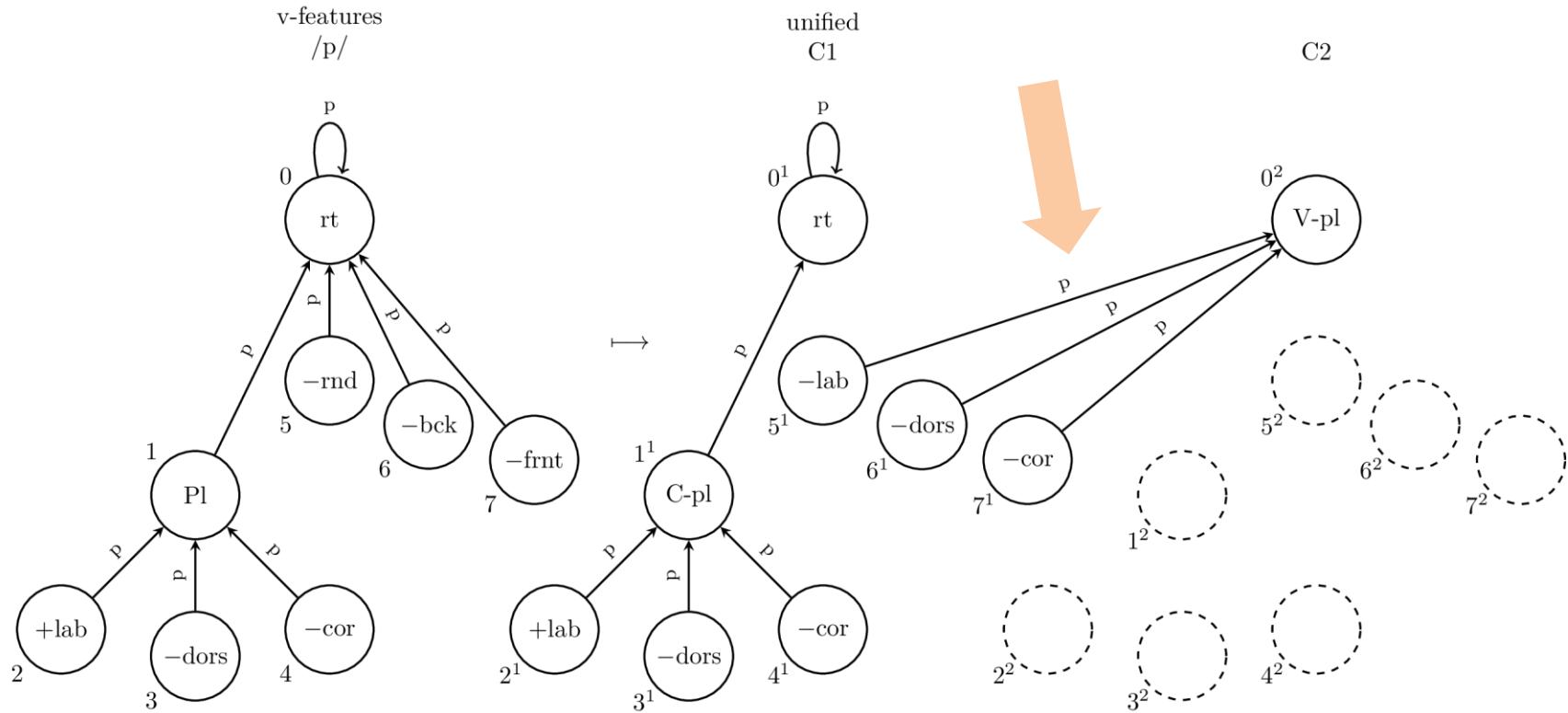
the transduction: v-features → unified

$$\begin{aligned}
 rt(x^1) &\stackrel{\text{def}}{=} rt(x) \\
 +\text{labial}(x^1) &\stackrel{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x) \\
 +\text{coronal}(x^1) &\stackrel{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x) \\
 +\text{dorsal}(x^1) &\stackrel{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x) \\
 -\text{labial}(x^1) &\stackrel{\text{def}}{=} -\text{round}(x) \vee \text{labial}(x) \\
 -\text{coronal}(x^1) &\stackrel{\text{def}}{=} -\text{front}(x) \vee \text{coronal}(x) \\
 -\text{dorsal}(x^1) &\stackrel{\text{def}}{=} -\text{back}(x) \vee \text{coronal}(x) \\
 C\text{-place}(x^1) &\stackrel{\text{def}}{=} \text{Place}(x) \\
 V\text{-place}(x^2) &\stackrel{\text{def}}{=} rt(x)
 \end{aligned}$$

$$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg \text{vowelFeature}(x) \\ (\text{parent}(x))^2 \Leftrightarrow \text{vowelFeature}(x) \end{cases}$$

$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow rt(x)\}$$

$$\text{vowelFeature}(x) = +\text{round}(x) \vee -\text{round}(x) \vee +\text{front}(x) \vee -\text{front}(x) \vee +\text{back}(x) \vee -\text{back}(x)$$





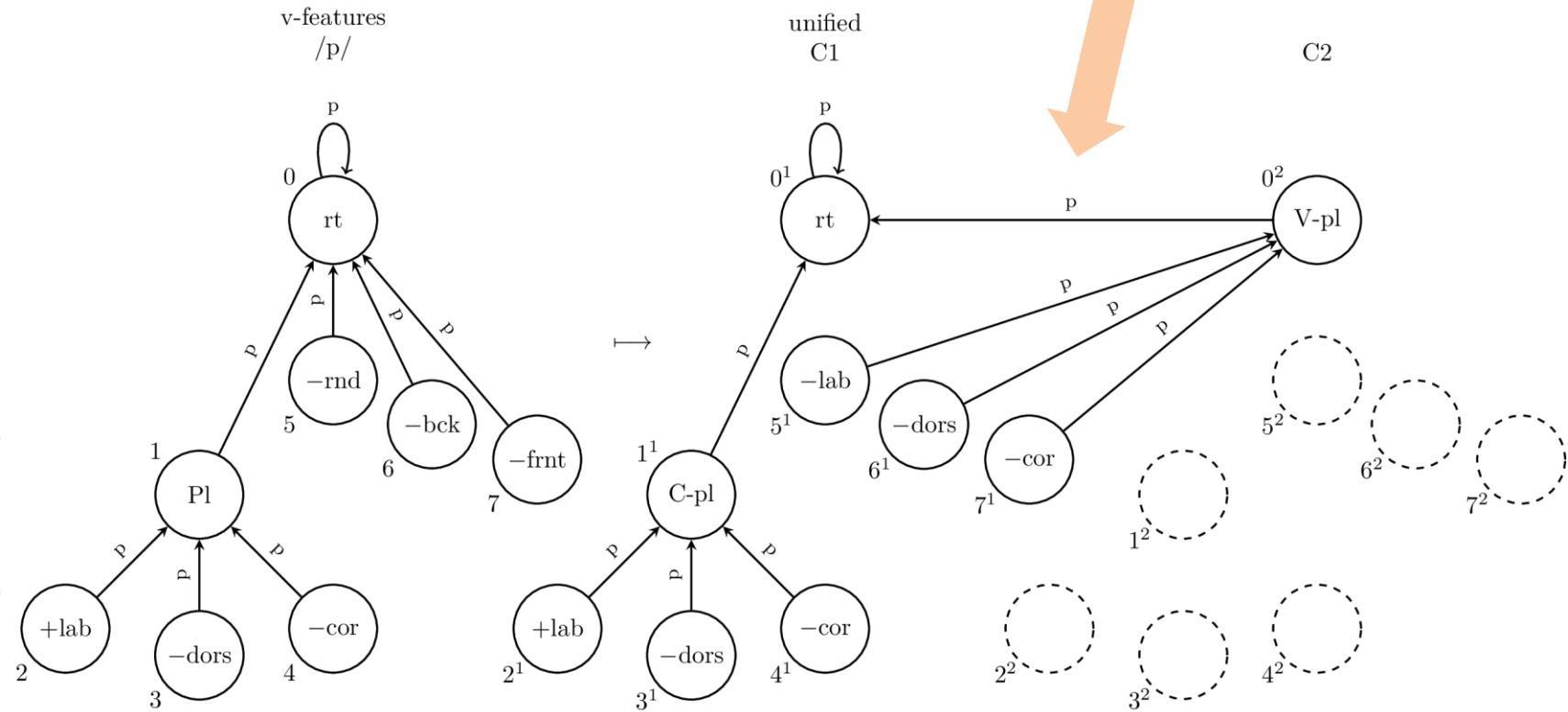
the transduction: v-features → unified

$$\begin{aligned}
 rt(x^1) &\stackrel{\text{def}}{=} rt(x) \\
 +\text{labial}(x^1) &\stackrel{\text{def}}{=} +\text{round}(x) \vee \text{labial}(x) \\
 +\text{coronal}(x^1) &\stackrel{\text{def}}{=} +\text{front}(x) \vee \text{coronal}(x) \\
 +\text{dorsal}(x^1) &\stackrel{\text{def}}{=} +\text{back}(x) \vee \text{coronal}(x) \\
 -\text{labial}(x^1) &\stackrel{\text{def}}{=} -\text{round}(x) \vee \text{labial}(x) \\
 -\text{coronal}(x^1) &\stackrel{\text{def}}{=} -\text{front}(x) \vee \text{coronal}(x) \\
 -\text{dorsal}(x^1) &\stackrel{\text{def}}{=} -\text{back}(x) \vee \text{coronal}(x) \\
 C\text{-place}(x^1) &\stackrel{\text{def}}{=} \text{Place}(x) \\
 V\text{-place}(x^2) &\stackrel{\text{def}}{=} rt(x)
 \end{aligned}$$

$$parent(x^1) \stackrel{\text{def}}{=} \begin{cases} (\text{parent}(x))^1 \Leftrightarrow \neg \text{vowelFeature}(x) \\ (\text{parent}(x))^2 \Leftrightarrow \text{vowelFeature}(x) \end{cases}$$

$$parent(x^2) \stackrel{\text{def}}{=} \{x^1 \Leftrightarrow rt(x)\}$$

$$\text{vowelFeature}(x) = +\text{round}(x) \vee -\text{round}(x) \vee +\text{front}(x) \vee -\text{front}(x) \vee +\text{back}(x) \vee -\text{back}(x)$$





unified and v-features are QF-bi-interpretable

and are therefore notational variants?

"The paper capitalises on structural similarities apparent in the Yip and Bao models to show that one can be freely translated into another, and *vice versa*. Such a translation does not result in any loss of the contrasts expressible by either theory. Given these two results, the main claim of the paper is that the two representational proposals do not constitute distinct theories, but are notationally equivalent."

(Oakden 2021: 258)

"A QF transduction is extremely restricted in the degree to which the output can differ from the input because QF is a weak logical language limited to local operations. QF-bi-interpretability can therefore be considered an indication of notational equivalence."

(Strother-Garcia 2019: 39)



enumerating natural class extensions

general procedure: (e.g. how I currently have it programmed in a python script)

initialize **natural_classes** as dict

- for each licit structure **s** in some theory **T**: *collect factors*
 - for each factor (connected substructure) **f** of **s**:
 - add **f** to **natural_classes** if **f** not in **natural_classes**
- for each factor **f** in **natural_classes**: *collect segments with each factor*
 - for each segment **s**
 - if **f** is a factor of **s**:
 - add **s** to **natural_classes[f]**



enumerating natural class extensions

- full range of contrasts considered:
 $\{p, t, k, pw, tw, kw, pj, tj, kj, py, ty, ky, kp, tp, kt, y, \emptyset, u, i, \dot{i}, w\}$

plain consonants

palatalized consonants

double articulations

unrounded vowels

labialized consonants

velarized consonants

rounded vowels

- **by design**, unified and v-features do not predict the same natural classes
- but we'll look anyway

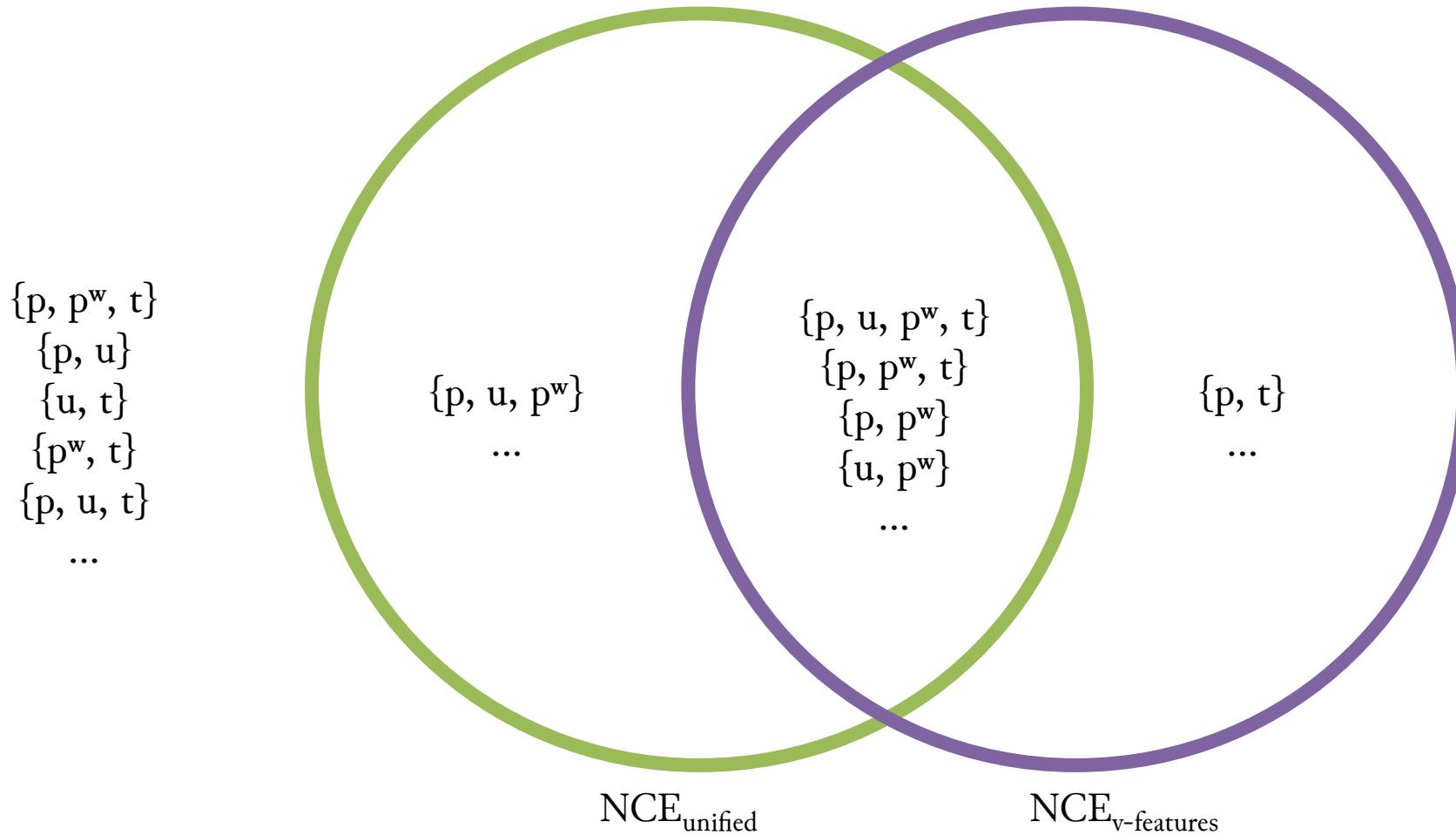


natural classes unique to unified

- [-dors]:
 - ('i', 'k', 'kj', 'kp', 'kt', 'kw', 'p', 'pj', 'pw', 'py', 't', 'tj', 'tp', 'tw', 'ty', 'u', 'y', 'i', 'w', 'u')
- [+dors]:
 - ('k', 'kj', 'kp', 'kt', 'kw', 'ky', 'py', 'ty', 'u', 'w')
- [-cor]:
 - ('i', 'k', 'kj', 'kp', 'kt', 'kw', 'ky', 'p', 'pj', 'pw', 'py', 't', 'tp', 'tw', 'ty', 'u', 'y', 'i', 'w', 'u')
- [+cor]:
 - ('i', 'kj', 'kt', 'pj', 't', 'tj', 'tp', 'tw', 'ty', 'y')
- [+lab]:
 - ('kp', 'kw', 'p', 'pj', 'pw', 'py', 'tp', 'tw', 'u', 'y', 'u')
- [-lab]:
 - ('i', 'k', 'kj', 'kp', 'kt', 'kw', 'ky', 'p', 'pj', 'py', 't', 'tj', 'tp', 'tw', 'ty', 'u', 'y', 'i', 'w', 'u')



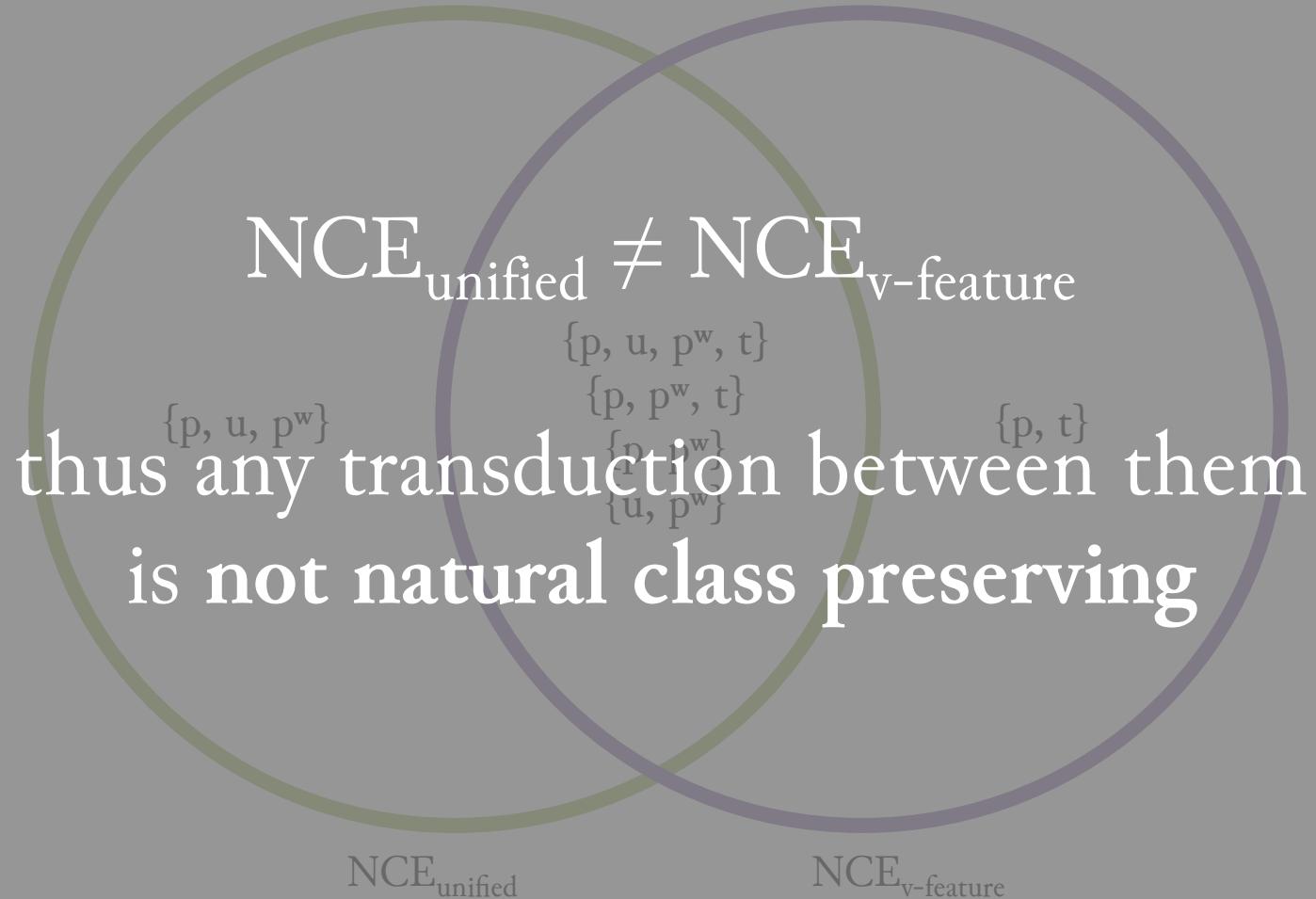
comparing natural class extensions





comparing natural class extensions

$\{p, p^w, t\}$
 $\{p, u\}$
 $\{u, t\}$
 $\{p^w, t\}$
 $\{p, u, t\}$

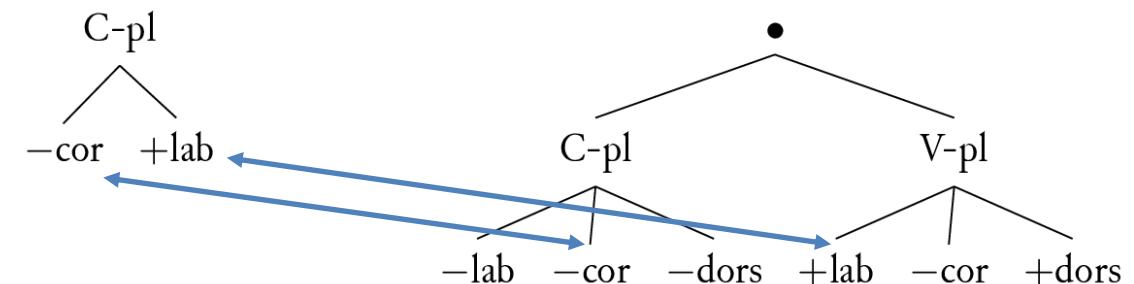
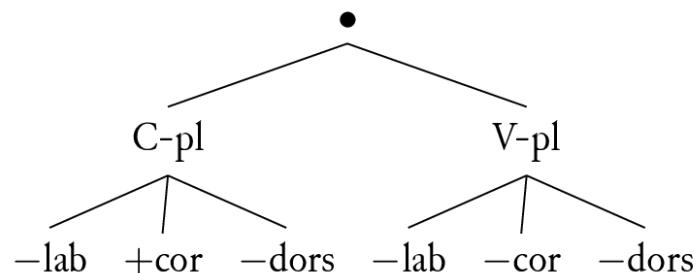


hypothetical assimilation

Sharing is Caring 

the structural changes on the target of an assimilation process should be factors of the trigger

unified



t

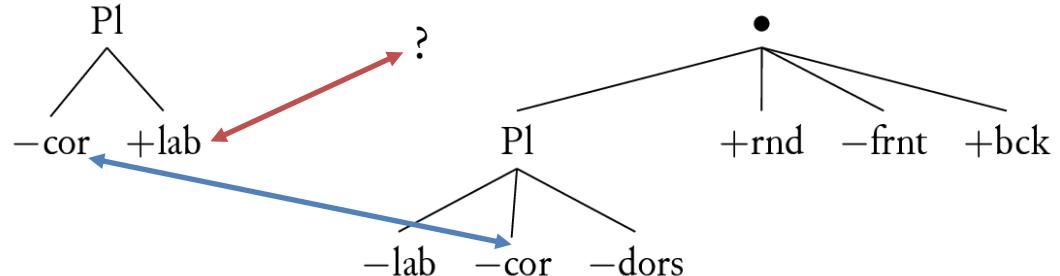
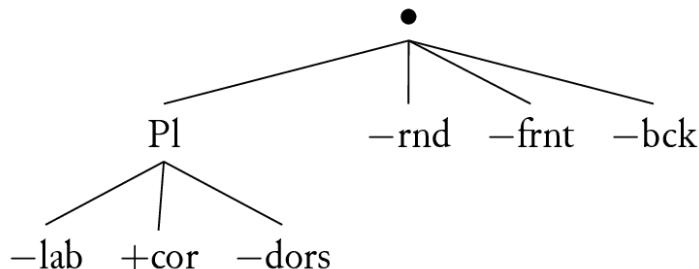
→

p

/

u

v-features





comparing theories

(1) *Conditions for notational equivalence*

- a. Two models do not differ in their empirical predictions.
- b. Two models represent the same set of abstract properties, differing only superficially.

(from Oakden 2021, summarizing Fromkin 2010)

- if we take seriously assumptions like sharing is caring  ,
then a QF-bi-interpretable contrast-preserving transduction is not enough to satisfy (1a) above

introduction

foundations

example transduction

empirical basis

existing transductions

conclusion



(some) existing transductions

transduction	logic	contrast preserving	natural class preserving
unified vs. v-features	QF	yes	no
Oakden (2021)	QF	yes	no
Danis & Jardine (2019)	FO	yes*	??
Cahill & Parkinson (1997)**	QF	yes	yes

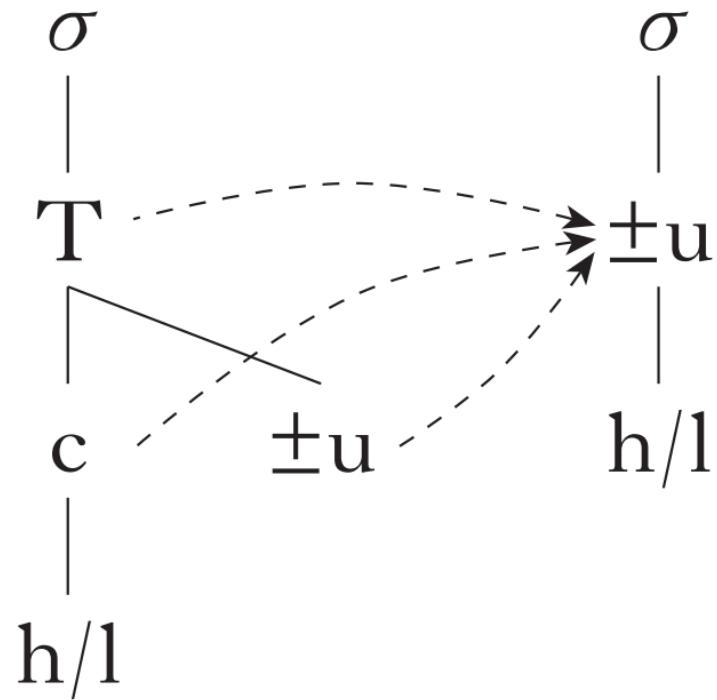
* the segments in the transduction are those that were optima in Shih & Inkela (2019), but the general set of contrasts are likely distinct

** this was not given as a transduction proper, but it is simple to construct one from their claim



Oakden (2021) & tonal geometry

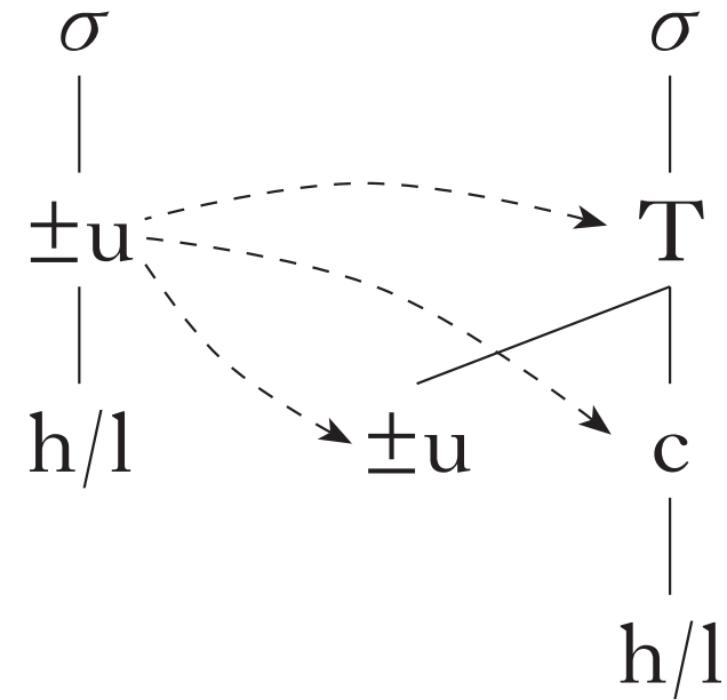
Bao



Yip

Yip

Bao

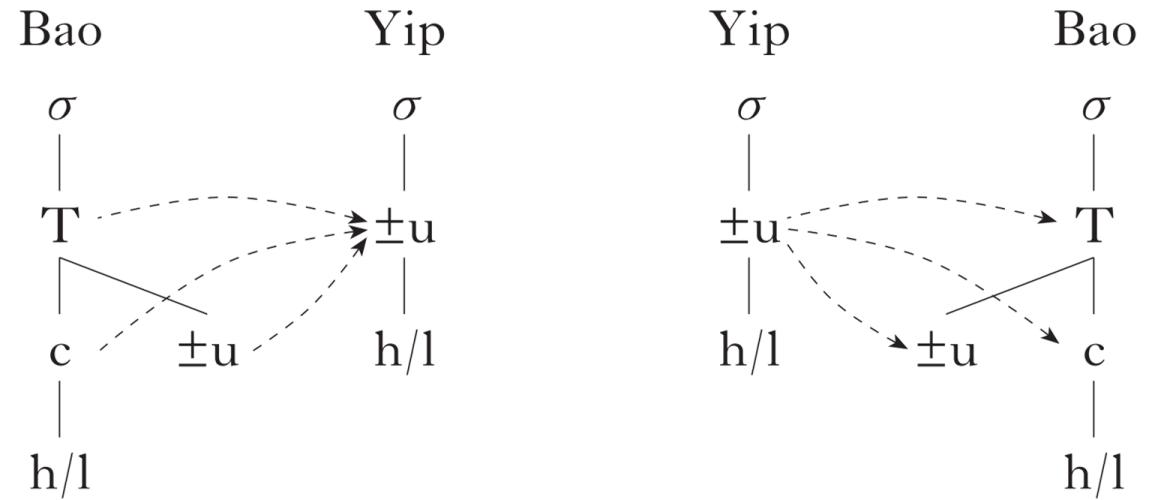




Oakden (2021) & tonal geometry

tone	Yip (1989)	Bao (1990)
low level L	σ -u l	σ T -u c l
high level H	σ +u h	σ T +u c h
mid level M	σ σ -u or +u h l	σ T -u c h or σ T +u c l
high falling HM	σ +u h l	σ T +u c h l

tone	Yip (1989)	Bao (1990)
high rising MH	σ +u l h	σ T +u c l h
low falling ML	σ -u h l	σ T -u c h l
low rising LM	σ -u l h	σ T -u c l h



- Oakden (2021) provides a non-size-preserving QF transduction (above) between two theories of tone sandhi (left), arguing for notational equivalence
- **is this transduction also natural class preserving?**

Table I

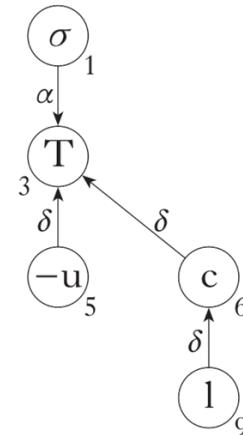
Level and contour tonal contrasts in Yip (1989) and Bao (1990).



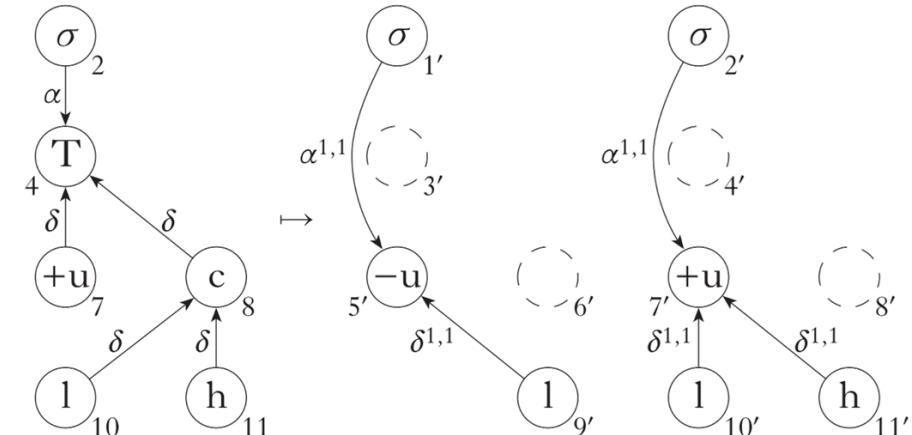
Oakden (2021) & tonal geometry

- Oakden (2021) provides a non-size-preserving QF transduction (above) between two theories of tone sandhi (left), arguing for notational equivalence
- **is this transduction also natural class preserving?**

(38) *separated*



bundled





Oakden (2021)

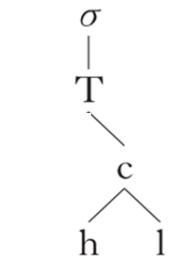
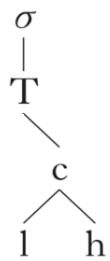
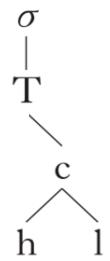
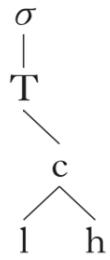
- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's

tone	Yip (1989)	Bao (1990)
high rising MH	$\begin{array}{c} \sigma \\ \\ +u \\ / \quad \backslash \\ l \quad h \end{array}$	$\begin{array}{c} \sigma \\ \\ T \\ / \quad \backslash \\ +u \quad c \\ \quad \\ l \quad h \end{array}$
low falling ML	$\begin{array}{c} \sigma \\ \\ -u \\ / \quad \backslash \\ h \quad l \end{array}$	$\begin{array}{c} \sigma \\ \\ T \\ / \quad \backslash \\ -u \quad c \\ \quad \\ h \quad l \end{array}$
low rising LM	$\begin{array}{c} \sigma \\ \\ -u \\ / \quad \backslash \\ l \quad h \end{array}$	$\begin{array}{c} \sigma \\ \\ T \\ / \quad \backslash \\ -u \quad c \\ \quad \\ l \quad h \end{array}$
high falling HM	$\begin{array}{c} \sigma \\ \\ +u \\ / \quad \backslash \\ h \quad l \end{array}$	$\begin{array}{c} \sigma \\ \\ T \\ / \quad \backslash \\ +u \quad c \\ \quad \\ h \quad l \end{array}$



Oakden (2021)

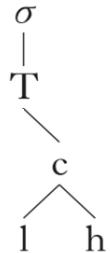
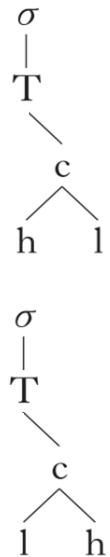
- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's





Oakden (2021)

- Oakden's transduction is **not** natural class preserving
- the tone contours HM, LM, MH, ML form a natural class in Bao's model, but not in Yip's





Cahill & Parkinson (1997) & geometric relations

- autosegmental phonology/feature geometry:
 - segments are trees which organize features into constituents
 - constituents predict spreading behavior
- Feature Class Theory (Padgett 1995a; Padgett 2002; Padgett 1995b):
 - segments have trivial structure
 - features are contained in nested sets
 - violable constraints predict class behavior



Cahill & Parkinson (1997) & geometric relations

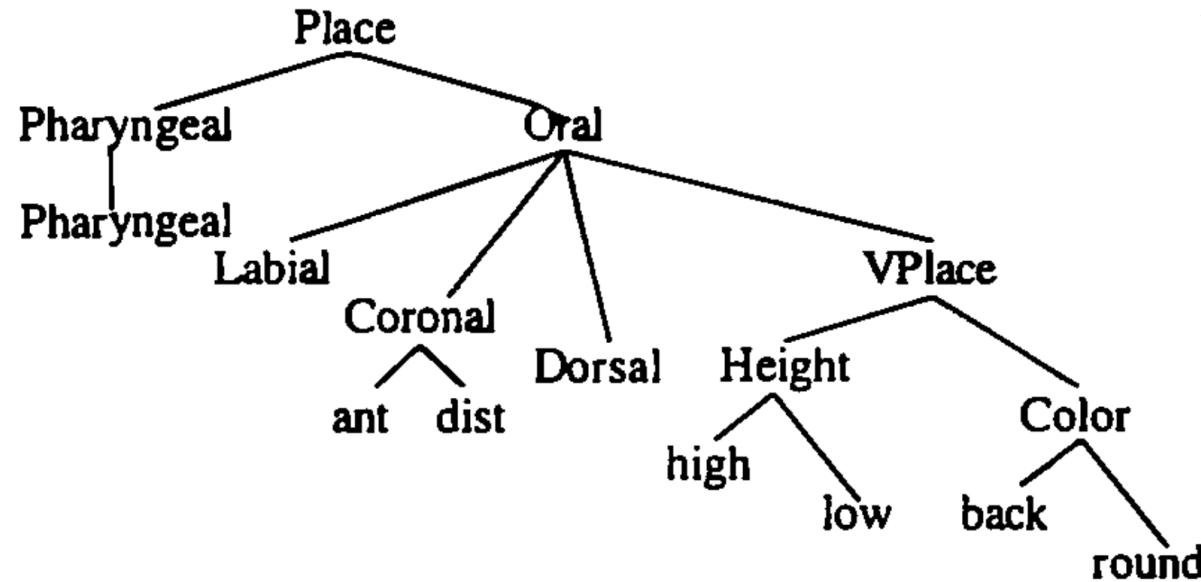
(3) The *Place* class as a set of sets.

$$\left\{ \begin{smallmatrix} _{PL} \{ _{Ph} \text{Phary} \} \{ _o \text{Lab, Cor, Dor, ant, dist} \{ _{VP} \{ _H \text{high, low} \} \{ _C \text{back, round} \} \} \} \end{smallmatrix} \right\}$$

The transition from (3) to (4) is one of notation only.

(4) Feature Geometry (Padgett 1995:398).

$$\begin{aligned} \text{LABEL}(x) &\stackrel{\text{def}}{=} \text{LABEL}(x) \\ \text{parent}(x) &\stackrel{\text{def}}{=} \text{included-in}(x) \end{aligned}$$





Cahill & Parkinson (1997) & geometric relations

(3) The *Place* class as a set of sets.

$\{_{PL}\{_{Ph} Phary\}\{_o La$

does not generalize:

The transition fro

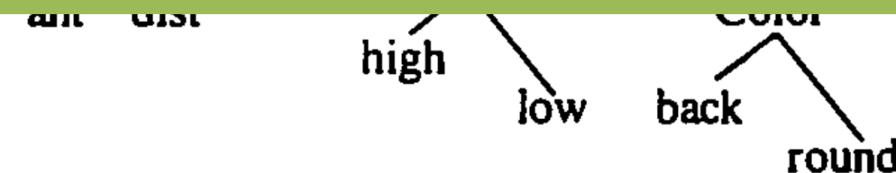
FCT requires constant definition for *included-in* function
across all models w.r.t. node labels

(4) Feature Geometry (P

Pharyngeal
Pharyngeal

ABEL(x)
included-in(x)

unified disobeys this axiom
[cor] in [C-pl] or [V-pl], depending on segment



introduction

definitions

example transduction

existing transductions

broader significance

conclusion



strong generative capacity

" The study of strong generative capacity is related to the study of descriptive adequacy, in the sense defined. A grammar is descriptively adequate if it strongly generates the correct set of structural descriptions. A theory is descriptively adequate if its strong generative capacity includes the system of structural descriptions for each natural language; otherwise, it is descriptively inadequate."

(Chomsky 1969: 60)



strong generative capacity

- ...in syntax:
 - Chomsky's definition often criticized (see Miller 1999 and references therein)
 - Miller (1999) reworks definition of SGC for syntax in robust model theory
- ...in morphology:
 - Dolatian et al. (2021) define and show divergence of WGC and SGC for various morphological processes and their transductions
- ...in phonology:
 - "In morphology and phonology, there are fewer debates on generative capacity. We speculate that this is due to two issues. First, morphology and phonology have comparatively restrictive WGC. Second, it is unclear what external basis (grounding) should be used for SGC, and thus what diagnostics or metrics to use."
(Dolatian et al. 2021: 229)



strong generative capacity

- **natural class preservation** should be in the set of diagnostics for evaluating the SGC of phonological theories
- **contrast preservation** is a weaker notion, entailed by natural class preservation
 - **Proof:** assume two theories are natural class preserving. if they are natural class preserving, they have the same extensions of atomic segments (by def.). if these elements are flattened to a single set for both theories, then $S_1 = S_2$. this is the definition of contrast preserving. therefore the two theories are contrast preserving.
- contrast preservation might be an indicator of the WGC

introduction

definitions

example transduction

existing transductions

broader significance

conclusion

summary



- assumptions about subsegmental structure predicts sets of segments that share structure (**natural classes**)
- assumptions about computation include the desire for processes to **target natural classes**, and for certain processes like assimilation to have further restrictions on natural classes (**sharing is caring** 🤝)
- logical equivalence between **representations** might ignore these assumptions about **computation**
- **natural class preservation** serves as a proxy for how computation behaves with respect to representation, and is a criterion for a stronger notion of logical & *linguistic* equivalence

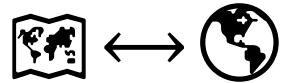


going forward

- the definition of *natural class preserving* is based on the representation themselves—can this property be identified by investigating the transduction rules alone?
 - disjunctive labeling
 - loss of labels
- how *else* can transductions themselves be compared and evaluated from a linguistic standpoint?
- **how strongly should our metatheoretical assumptions and expectations about linguistic processes be formalized? (or even said aloud)**



thank you



and thank you Adam Jardine (for first helping with the transductions like 4 years ago)
& the audiences at the Stony Brook Workshop on Model Theory in Phonology on Sept. 25
& the Wash U Linguistics Brown Bag on Sept. 16