Extreme Transients in Time-Delay Autonomous Boolean Networks

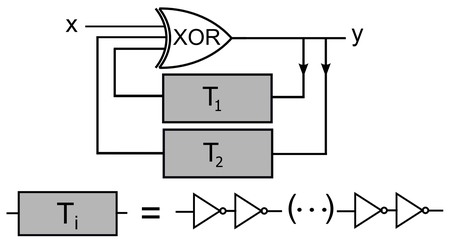
Nicholas D. Haynes, Otti D'Huys and Daniel J. Gauthier
Department of Physics, Duke University, Durham, NC, USA

Duke UNIVERSITY

Introduction

- Time-delay autonomous Boolean networks (TDABNs) are common models for gene regulatory networks and neural networks
- Simple TDABNs were shown theoretically [1, 2, 3] to display complex switching behavior between "on" and "off" states
- We demonstrate an experimental TDABN that displays extremely long chaotic transients

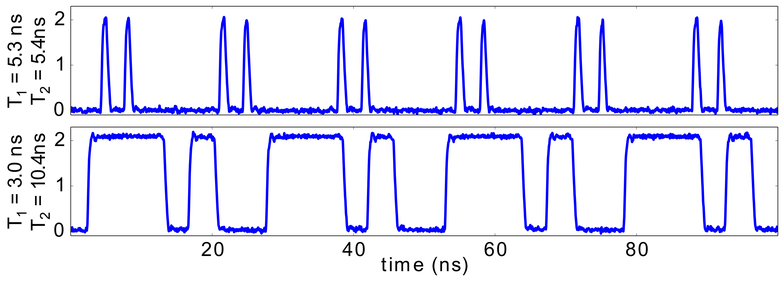
Experimental system



x(t)	x(t-T_1)	x(t-T_2)	y(t)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

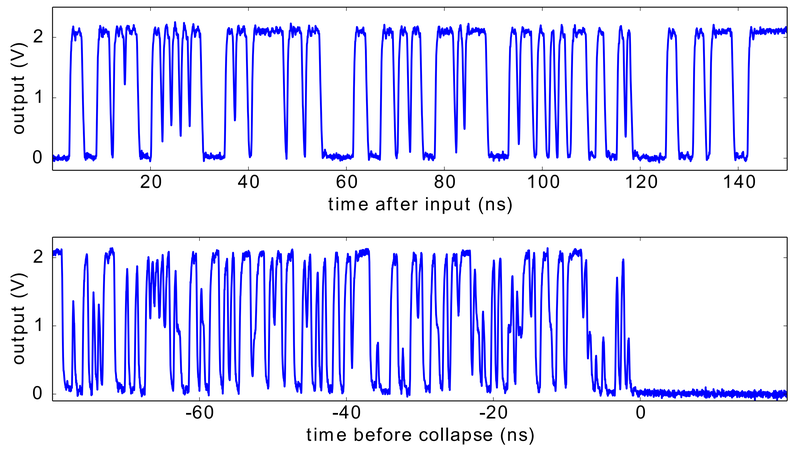
- Delay dynamical system is a **single unclocked Boolean XOR gate** with 2 time-delay feedback lines
- Physical system built using Altera Cyclone IV FPGA with **characteristic time scale of** ~ 1 **ns**
- Time-delay feedback constructed using cascades of inverter gates

Periodic dynamics



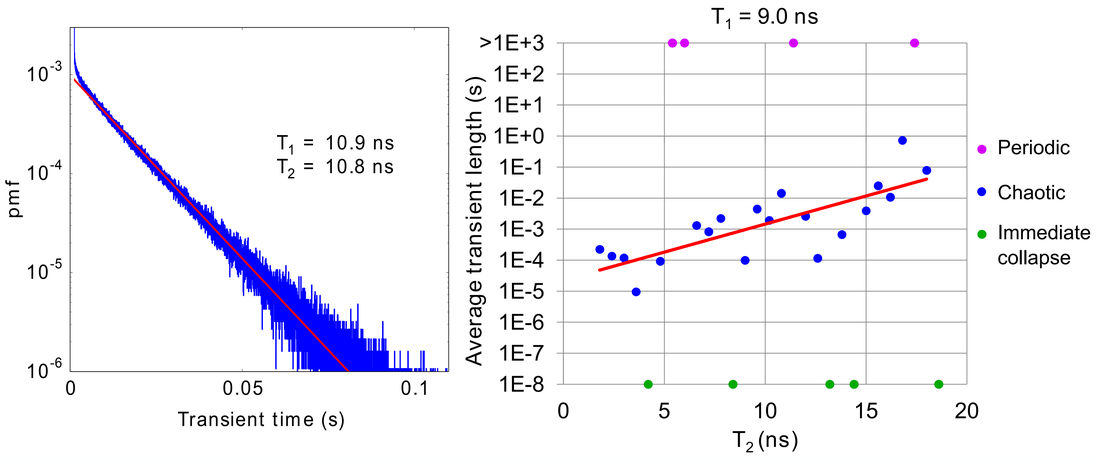
- For certain values of T_1, T_2, chaotic transient evolves into periodic orbit
- Periodic orbits are **extremely stable over time** – able to survive days without collapsing

Chaotic transient dynamics



- System is initialized in fixed point [XOR(0,0,0) = 0], then excited by an input voltage and allowed to evolve freely
- **Long, irregular transient outputs** are often observed
- After some time (typically 1 μs – 1 minute), dynamics suddenly collapse back to fixed point
- Outputs are **chaotic until collapse** – trajectories from similar outputs diverge exponentially [4]

Transient lengths



- Dynamics observed to **always eventually collapse** to fixed point or periodic orbit
- Transients can **survive an extremely long time** before collapse – between 10^3 and 10^9 characteristic time scales is common to observe
- For fixed values of T_1 and T_2, chaotic transient times are exponentially distributed
- Average transient time tends to increase with delay times

Conclusions and future work

- A simple, physical delay dynamical system can display a **wide variety of complex behaviors**
- **Fast characteristic time scale** allows for observation of chaotic transient collapse
- Model that predicts exponential distribution of transient times and dependence on T_1/T_2 is needed
- Possibility of predicting or preventing collapse may have implications for brain dynamics, stock market crashes, etc.

References

[1] Dee, D. and Ghil, M. *SIAM J. Appl. Math.* **44**(1), 111–126 (1984).

[2] Ghil, M. and Mullhaupt, A. *J. Stat. Phys.* **41**(1-2), 125–173 (1985).

[3] Cavalcante, H. L. D. d. S., Gauthier, D. J., Socolar, J. E. S., and Zhang, R. *Philos. Tr. R. Soc. A* **368**(1911), 495–513 (2010).

[4] Haynes, N. D. et al. *Phys. Rev. E* **91**, 020801 (2014).

The authors gratefully acknowledge the financial support of U.S. Army Research Office Grant #W911NF-12-1-0099 and National Science Foundation Grant #DGE-1068871.