Implementation of a One-Dimensional Electromagnetic Wave Solver in Initial Plasma Formation Modeling

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Abstract

The modeling of initial plasma formation (IPF) provides a pathway for understanding the ultrafast dynamics of laser–material interactions. A one-dimensional electromagnetic wave solver has been developed using the finite-difference time-domain (FDTD) method to accurately model laser propagation in metals and dielectrics. Laser absorption is modeled by the collisional damping of the light wave, which is determined by the dielectric function of the materials. A two-temperature model is employed to describe the heat transfer between electrons and lattices/ ions. Differences in the heat transfer dynamics to electrons and ions modeled by the wave solver and a ray trace approach are compared. The objective of this approach is to enhance the modeling accuracy of laser-matter interactions in inertial confinement fusion (ICF) codes.

Background

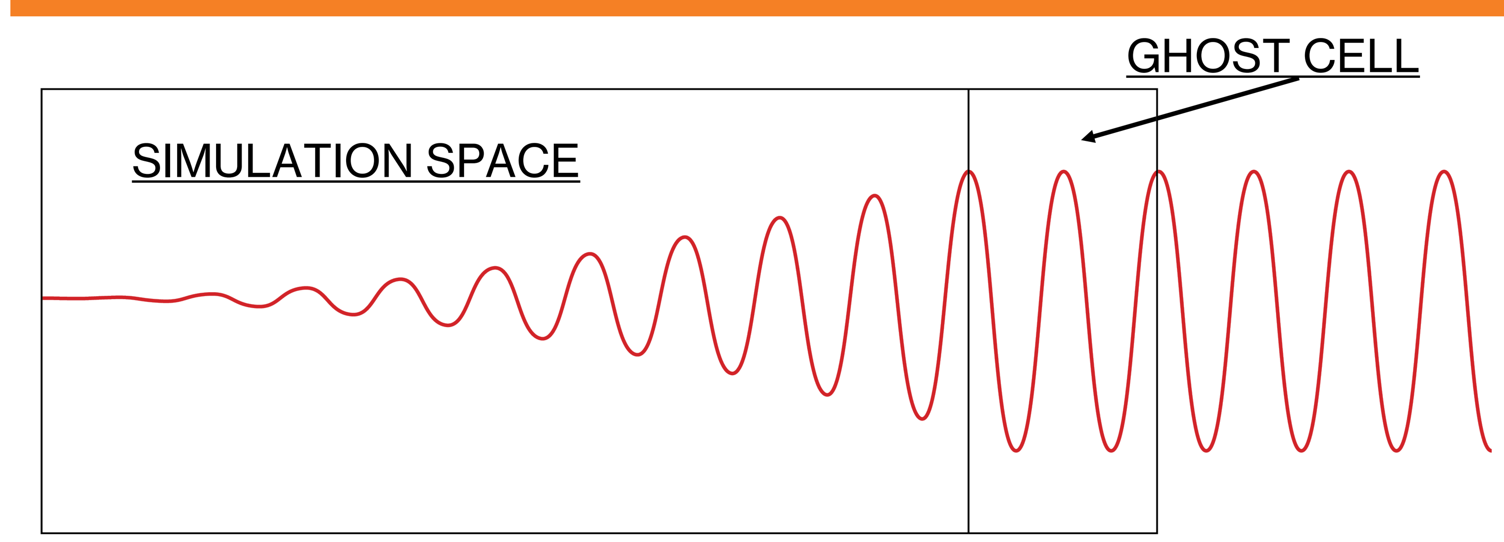

Figure 1: Diagram of laser propagation and its interaction with material. This was the outcome we were looking for. The modeling equations are given below.

$$\partial_{tt}E = (c/n)^2 \partial_{xx}E \quad \textit{... Eq. (1)}$$

$$c_{ve}\frac{dT_e}{dt} = -G(T_e - T_i) + S \quad \textit{... Eq. (2)}$$

$$c_{vi}\frac{dT_i}{dt} = G(T_e - T_i) \quad \textit{... Eq. (3)}$$

Equations (1) – (3) contribute to our 1-D initial plasma formation (IPF) modeling. In the first equation, E is the electric field, c is the speed of light, n is the complex refractive index where its real part accounts for reflections and transmissions while the imaginary part accounts for absorptions. In Eqs. (2) - (3) c_{ve} and c_{vi} are electron heat capacities and ion heat capacities at the constant volume, respectively. G is the energy exchange rate between the electron temperature T_e and the ion temperature T_i. The Drude model is used to model the rate of laser energy absorption caused by the collisional damping of the light wave, which is determined by the source term S. Modeling of multi-photon ionization can be found in Ref [1].

Methods and Figures

The explicit Finite-Difference Time-Domain (FDTD) scheme was implemented to model the electromagnetic wave propagation in medium. The explicit scheme imposes a constraint on the time-step size "dt" so that the wave propagation distance within an allowed time-step size cannot exceed the grid size "dx" according to the Courant–Friedrichs–Lewy (CFL). The allowed wave propagation step size within a grid size "dx" is controlled by the CFL number.

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P_{NL}}{\partial t^2} + \mu\sigma\frac{\partial E}{\partial t} \quad \textit{... Eq. (4)}$$

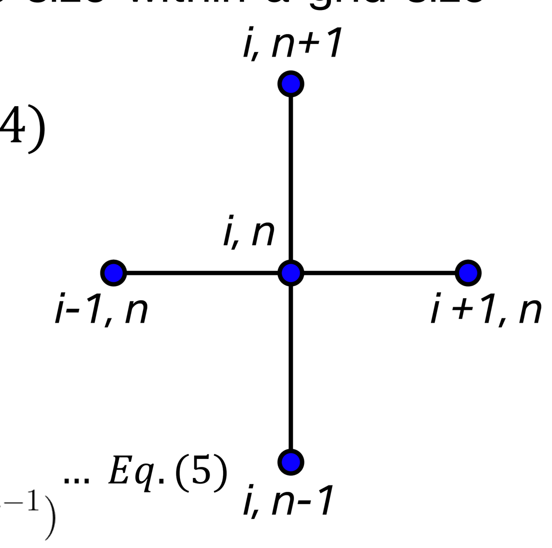

Figure 2: Stencil diagram of the explicit finite difference scheme

$$E_i^{n+1} = \left(\frac{\Delta t\, c}{\Delta x\, n}\right)^2 (E_{i+1}^n - 2E_i^n + E_{i-1}^n) - (-2E_i^n + E_i^{n-1}) \quad \textit{... Eq. (5)}$$

Here P_{NL} is the nonlinear polarization in media, ϵ_0 is the permittivity of free space, μ is the permeability of free space and σ is the electric conductivity. Eq (4) describes the propagation of light waves in material. The light wave interacts with electrons and sets them into oscillations along the electric field vector direction. The oscillating electrons then collide with neighboring atoms. Consequently, the light wave's energy is then transferred from photons to electrons and then ions. This process is known as the collisional damping of light waves and is uniquely described by the dielectric function $\varepsilon = n^2$. In our 1-D IPF modeling, the non-linear polarization term P_{NL} and the transient current term driven by the conductivity σ are neglected in Eq (5). The incident electric field is described by:

$$E_L = E_0 e^{-i(\omega t - kx)} = E_0\cos(kx - \omega t) + iE_0 \sin(kx - \omega t) \textit{... Eq. (6)}$$

The complex-valued incident electric field provides the initial boundary condition in the ghost cells, whereas the electric field inside the computational domain is initialized as zeroes. The Fourier analysis is applied to model the amplitude of the electric field vector. In 1-D, the reduced wave equation becomes:

$$\partial_{xx}E = -\frac{\omega^2 n^2}{c^2}E \quad \textit{... Eq. (7)}$$

Equation (7) accepts an exponentially decaying or growing solution. A positive sign in the refractive index corresponds to a growing wave amplitude whereas the negative sign corresponds to a decaying wave amplitude. In the absence of wave amplification processes caused by the nonlinear polarization effect, the negative sign is the correct solution to describe an attenuating wave for laser propagations in material. During the numerical update, the selection of the sign depends on the boundary condition.

$$E(x) = E_0 e^{-(\omega n_I/c)x} = E_0 e^{-k_I x} \quad \textit{... Eq. (8)}$$

Conclusion and Results

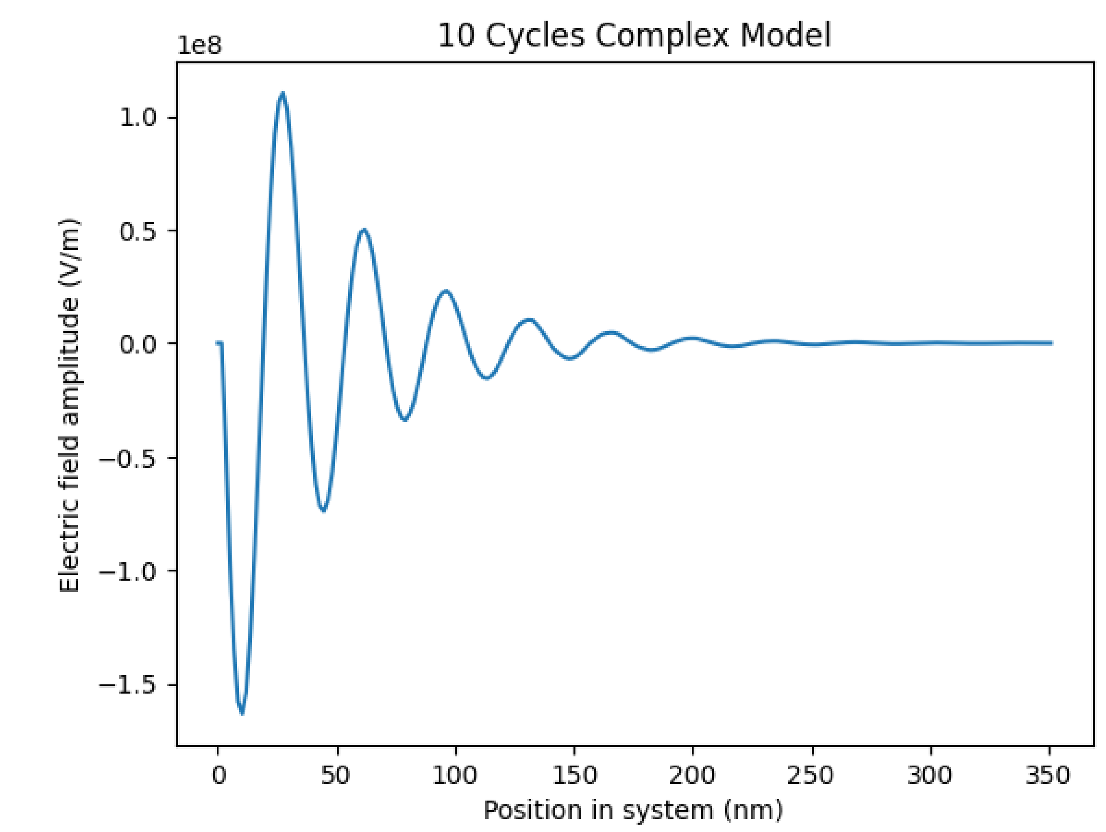

Figure 3: Simulation of light wave propagating into material undergoing collisional damping.

The 1-D electromagnetic wave solver was implemented over 10 cycles for a complex refractive index n = n_R + n_I = 1 + 0.125 and wavelength lambda = 351 nm. As shown in Fig. (3) we were able to accurately model the attenuation of the 1-D light wave propagating into a material. We are currently implementing a time-skipping strategy to speed up the code. Eventually will be implemented into ICF code and run by supercomputers.

Future Work

For future work, we can extend our solver into 2-D and 3-D to model the multi-dimensional wave propagation. We also plan to employ a two-temperature model to study and compare the heat transfer dynamics in material modeled by the wave solver and the ray tracing approach.

References

[1] L. V. Keldysh "Ionization in the Field of a Strong Electromagnetic Wave and modern physics of atomic interaction with a strong laser field" Journal of Experimental and Theoretical Physics, 122, 449–455 (2016).

Acknowledgments

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