Cryptanalysis of Round Reduced KECCAK

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Hash Function

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- It takes as input an arbitrary string (of 0's and 1's) and outputs a fixed size (say n) string.

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- It is used in many cryptographic applications e.g., Authentication, Digital Signatures and Integrity etc..

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  - **Efficiency**: Given a message m, it is easy to compute its hash i.e. $H(m)$.
  - **Preimage Resistance**: Given $H(m)$, it is computationally hard to find the message m.
  - **Second-preimage Resistance**: Given a message m, it is computationally hard to find another message m' such that $H(m) = H(m')$.
  - **Collision Resistance**: It is computationally hard to find two messages m and m' such that $H(m) = H(m')$.

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- Hash functions having the above properties are referred to as cryptographic/secure hash functions.

Need For SHA-3

- MD5, SHA-1, SHA-2 are very popular hash functions and are widely used.
- In year 2005, first practical collision attacks were found on:
  - MD5, SHA-0 and SHA-1 by Xiaoyun Wang *et al.* .
- National Institute of Standards and Technology (NIST) was worried about the security of hash functions.
- Though by that time SHA-2 family of hash functions was standardized.
- SHA-2 was also based on Merkle-Damgard construction like SHA-0, SHA-1.
- There was a possibility that it could also be attacked in a similar fashion.

SHA-3 Competition

- In the year 2006, NIST decided to hold a competition for the next secure hash function.
- In 2008, NIST announced a competition for the Secure Hash Algorithm-3 (SHA-3).
- In the year 2012, NIST announced KECCAK as the winner of the competition among the five finalists viz. BLAKE, Grøstl, JH, KECCAK and Skein.
- Since 2015, KECCAK has been standardized as SHA-3 by NIST.

Keccak

- Keccak hash function is based on sponge construction.
- Sha-3 family of hash functions is based on Keccak.
- The Sha-3 family provides four hash functions:
  - Sha3-224, Sha3-256, Sha3-384 and Sha3-512.

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- The Sha-3 family provides four hash functions:
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- Keccak's excellent resistance towards crypt-analytic attacks is one of the main reasons for its selection by NIST.
- The algorithm is a good mixture of linear as well as non-linear operations.

Sponge Construction

- A sponge construction consists of:
  - Permutation function f,
  - Parameter "rate" r, and
  - Padding rule pad.
  - This construction produces a sponge function that takes as input a bit string M and generates a string of length l.

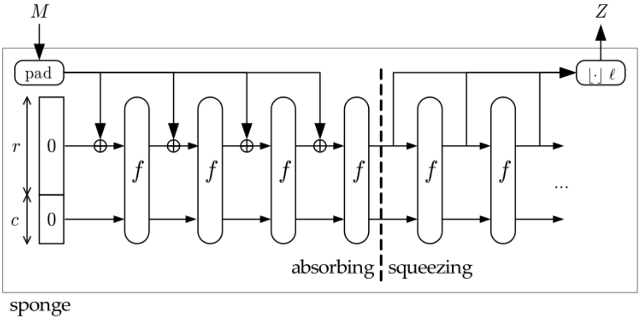

Figure: The sponge construction

KECCAK-p Permutation

- The function f in the sponge construction is denoted by KECCAK-$f[b]$.
- b is the length of input string.
- Internally, KECCAK-$f[b]$ consists of a round function p which is applied n_r number of times.
- KECCAK-$f[b]$ function is specialization of KECCAK-$p[b, n_r]$.

Keccak State

- The state input to Keccak-$f[b]$ consists of b bits.
- The state is divided into slices.
- Each slice is of fixed size i.e., 25 bits.
- A state S, which is a b-bit string, in Keccak is usually denoted by a 3-dimensional grid of size $(5 \times 5 \times w)$.

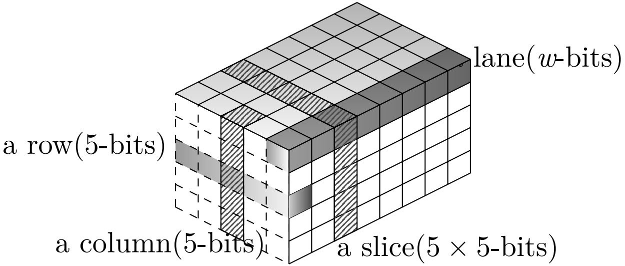

Figure: The Keccak State

Round Function of KECCAK-p

- The round function p in KECCAK comprises of 5 step mappings.
- These step mappings are called $\theta,\ \rho,\ \pi,\ \chi\ and\ \iota$.
- These transformations are applied in sequence.

θ step mapping

- XOR each bit in the state with the parities of two neighboring columns.

θ step mapping

- XOR each bit in the state with the parities of two neighboring columns.
- If we have A as the input state to θ, then the output state B is:

$$B[x,\ y,\ z] = A[x,\ y,\ z] \bigoplus P[(x-1) \bmod 5,\ z] \bigoplus P[(x+1) \bmod 5,\ (z-1) \bmod w] \tag{1}$$

where $P[x,\ z]$ represents the parity of the column $(x,\ z)$ i.e.,

$$P[x,\ z] = \bigoplus_{y=0}^{4} A[x,\ y,\ z]$$

ρ step mapping

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ρ step mapping

- ρ (**rho**): This step rotates each lane by a constant value towards the MSB.
- If we have A as the input state to ρ, then the output state B is given by:

$$B[x,\ y,\ z] = A\left[x,\ y,\ z + \rho\,(x, y) \bmod w\right],$$

  where $\rho\,(x,\ y)$ is the constant for a given lane $(x,\ y)$.

- ρ is a linear step mapping.

π step mapping

- π (**pi**): It permutes the position of lanes.
- The new position of a lane is determined by a matrix,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}, \qquad (2)$$

  where $(x',\ y')$ is the position of lane $(x,\ y)$ after π step.

- π is also a linear step mapping.

χ step mapping

- χ (**chi**): Each bit in the original state is XOR-ed with a non-linear function of next two bits in the same row.

$$B[x,\ y,\ z] = A[x,\ y,\ z] \oplus \\ ((A[(x+1) \bmod 5,\ y,\ z] \oplus 1) \cdot A[(x+2) \bmod 5,\ y,\ z])\,. \quad (3)$$

- χ is the only non-linear operation among the 5 step mappings in Keccak.

ι step mapping

- ι (**iota**): This step mapping only modifies the (0, 0) lane depending on the round number.
- If we have A as the input state to ι, then the output state B is:

$$B[0,\ 0] = A[0,\ 0] \oplus RC_i, \tag{4}$$

  where RC_i is round constant that depends on the round number.
- The remaining 24 lanes remain unaffected.
- All the rounds are identical but the symmetry is destroyed by this step due to the addition of a round constant to a particular lane.

Specification of KECCAK-$p[b, n_r]$

- Round in KECCAK is given by:
  - $\texttt{Round}(A,\ i_r) = \iota(\chi(\pi(\rho(\theta(A)))),\ i_r)$
- It consists of n_r number of iterations of $\texttt{Round}(A,\ i_r)$.
- KECCAK-$p[b,\ n_r](S)$
  - Convert S into a state array A
  - For i_r from 0 to $n_r - 1$, let $A = \texttt{Round}(A,\ i_r)$
  - Convert A into string S' of length b
  - Return S'

SHA-3 Hash Function

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- Where $r = 1600$-c and the capacity c is chosen to be twice the size of hash output d.

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- When the value of $b = 1600$, we have $l = 6$.
- Thus, the f function in SHA-3 is KECCAK-$p\,[1600,\ 24]$.
- Instances of KECCAK are denoted by KECCAK $[r,\ c]$.
- Where $r = 1600$-c and the capacity c is chosen to be twice the size of hash output d.
- We set $c = 2 \cdot d$, to avoid generic attacks with expected cost below 2^d.
- The hash function with output length d is denoted by:

$$\text{KECCAK-}d \quad = \quad \text{KECCAK}\,[r := 1600 - 2 \cdot d,\ c := 2 \cdot d] \qquad (5)$$

pad10*1

- The padding rule followed by KECCAK is **pad10*1**.
- According to the rule, the input string is appended with a 1 bit followed by some number of 0 bits and followed by 1 bit.
- The asterisk in the padding rule indicates that 0 bit is either not present or is repeated as required so that the length of output string after padding is a multiple of the block length (i.e. r).

KECCAK[r:=800-384, c:=384]

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- KECCAK $[r := 800 - 384, c := 384] =$ KECCAK-$p[800, 24]\,[r := 800 - 384, c := 384]$.
- 2-round KECCAK $[r := 800 - 384, c := 384] =$ KECCAK-$p[800, 2]\,[r := 800 - 384, c := 384]$.

Observations

- **Observation 1:** If we know all the bits of a row, then we can invert χ for that row. It is depicted below.

$$\begin{array}{|c|c|c|c|c|} \hline a_0 & a_1 & a_2 & a_3 & a_4 \\ \hline \end{array} \xrightarrow{\chi^{-1}} \begin{array}{|c|c|c|c|c|} \hline a_0' & a_1' & a_2' & a_3' & a_4' \\ \hline \end{array}$$

Figure: Computation of χ^{-1} for full row

$$a_i' = a_i \oplus (a_{i+1} \oplus 1) \cdot (a_{i+2} \oplus (a_{i+3} \oplus 1) \cdot a_{i+4}) \tag{6}$$

- **Observation 2:** When only one output bit is known after χ step, then we can fix the first output bit to be the same as the input bit and the second bit as 1.

$$\begin{array}{|c|c|c|c|c|} \hline a_0 & * & * & * & * \\ \hline \end{array} \xrightarrow{\chi^{-1}} \begin{array}{|c|c|c|c|c|} \hline a_0 & 1 & * & * & * \\ \hline \end{array}$$

Figure: Computation of χ^{-1} when only 1-bit is known in row

Observations

- **Observation 3:**
- $a'_i,\ a_i$ are the input and output bits of χ respectively.
- Guo *et al.* observed that when 4 out of 5 output bits of χ are known, then we can obtain 4 linear relations in terms of a'_i.

$$a'_i = a_i \oplus (a_{i+1} \oplus 1) \cdot (a_{i+2} \oplus (a_{i+3} \oplus 1) \cdot a_{i+4}) \tag{7}$$

- If the values of $a_0,\ a_1,\ a_2,\ a_3$ are known using the Equation 7, we can eliminate the expression a_4 from the rest of the equations.
- Hence, we obtain 4 linear equations on the input bits.

Notations

- The KECCAK state is represented by 25 lanes.
- Each lane is represented by a variable which is a 32-bit array.
- A variable with a number in round bracket "$(\cdot)$" represents the shift of the bits in array towards MSB.
- A variable with a number in square bracket "$[\cdot]$" represents the bit value of the variable at that index.
- If there are multiple numbers in the square bracket, then it represents the corresponding bit values.

2 rounds of KECCAK[r:=800-384, c:=384]

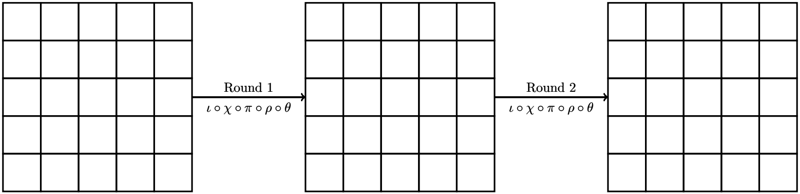

Figure: Two rounds of KECCAK[$r := 800 - 384$, $c := 384$]

- We will discuss a preimage attack on above structure.

Final State of 2-round KECCAK[r:=800-384, c:=384]

- $c = 384 \rightarrow d = 192 \rightarrow$ hash of 6 lanes

y					
4	$\star$	$\star$	$\star$	$\star$	$\star$
3	$\star$	$\star$	$\star$	$\star$	$\star$
2	$\star$	$\star$	$\star$	$\star$	$\star$
1	h_5	$\star$	$\star$	$\star$	$\star$
0	h_0	h_1	h_2	h_3	h_4
	0	1	2	3	4
	x				

Figure: The Final Hash State for KECCAK[$r := 800 - 384$, $c := 384$]

Initial State of 2-round KECCAK[r:=800-384, c:=384]

- $r = 800 - 384 \rightarrow r = 416 \rightarrow$ Message block of 13 lanes

0	0	0	0	0
0	0	0	0	0
a_1	b_1	c_2	0	0
a_2	b_2	c_1	d_1	e_1
a_0	b_0	c_0	d_0	e_0

Figure: Setting of Initial State in the Attack

Attack

- Our aim is to find the values of $a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,\ c_0,\ c_1,\ c_2,$ $d_0,\ d_1$ and $e_0,\ e_1$ variables in the initial state.

Attack

- Our aim is to find the values of a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2, d_0, d_1 and e_0, e_1 variables in the initial state.
- Such that, they lead to a final state having first six lanes as h_0, h_1, h_2, h_3, h_4 and h_5.

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- Such that, they lead to a final state having first six lanes as h_0, h_1, h_2, h_3, h_4 and h_5.
- We follow the basic idea of the attack given by Naya *et al.* in 2011.
- 2 rounds of KECCAK$[r := 800 - 384,\ c := 384]$
  - Best-known attack, has a time complexity of $O(2^{64})$.
  - It is based on the idea of linear structures given by Jian Guo *et al.* in 2016.

First round θ step mapping

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- We add the following conditions to make column parity zero:

$$\begin{aligned} &a_2 = a_0 \oplus a_1, \quad b_2 = b_0 \oplus b_1, \quad c_2 = c_0 \oplus c_1 \\ &d_1 = 0, \quad d_0 = 0 \quad \text{and} \quad e_1 = e_0. \end{aligned} \tag{8}$$

Effect of θ

0	0	0	0	0
0	0	0	0	0
a_1	b_1	c_2	0	0
a_2	b_2	c_1	d_1	e_1
a_0	b_0	c_0	d_0	e_0

State 1

$\xrightarrow{\theta}$

0	0	0	0	0
0	0	0	0	0
a_1	b_1	c_2	0	0
a_2	b_2	c_1	d_1	e_1
a_0	b_0	c_0	d_0	e_0

Figure: Effect of θ on initial state

State 1 to State 2

0	0	0	0	0
0	0	0	0	0
a_1	b_1	c_2	0	0
a_2	b_2	c_1	d_1	e_1
a_0	b_0	c_0	d_0	e_0

State 1

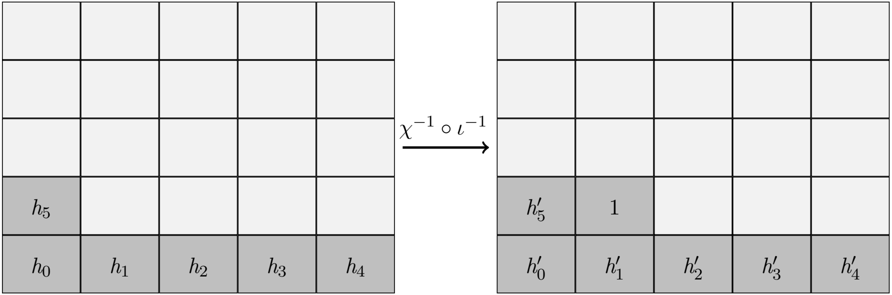
$\xrightarrow{\theta, \pi, \rho}$

$c_0(30)$	$d_1(23)$	0	0	0
$e_0(27)$	$a_2(4)$	$b_1(10)$	0	0
$b_0(1)$	$c_1(6)$	0	0	0
$d_0(28)$	$e_1(20)$	$a_1(3)$	0	0
$a_0(0)$	$b_2(12)$	$c_2(11)$	0	0

State 2

Figure: Preimage attack on 2-round KECCAK$[r := 800 - 384,\ c := 384]$

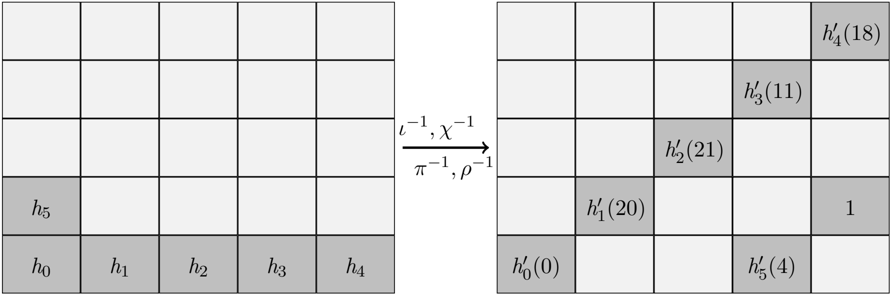
χ and ι inverse



State 4

Figure: Inversion of hash through $\chi^{-1} \circ \iota^{-1}$

State 4 to State 3



State 4 **State 3**

State 1 to 4

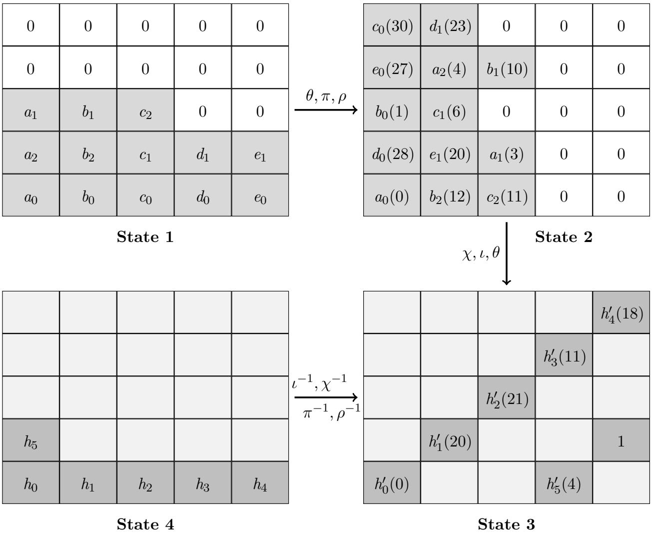


Figure: Preimage attack on 2-round KECCAK$[r := 800 - 384,\ c := 384]$

State 2 to State 3

$c_0(30)$	$d_1(23)$	0	0	0
$e_0(27)$	$a_2(4)$	$b_1(10)$	0	0
$b_0(1)$	$c_1(6)$	0	0	0
$d_0(28)$	$e_1(20)$	$a_1(3)$	0	0
$a_0(0)$	$b_2(12)$	$c_2(11)$	0	0

State 2

$$\xrightarrow{\chi, \iota, \theta}$$

				$h'_4(18)$
			$h'_3(11)$	
		$h'_2(21)$		
	$h'_1(20)$			1
$h'_0(0)$			$h'_5(4)$	

State 3

Figure: Intermediate States in 2-round preimage attack on KECCAK$[r := 800 - 384,\ c := 384]$

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- The number of variables and the number of conditions are equal.
- So, we expect a solution.

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- First, we find the set of input message bits which satisfy the small collection of consecutive slices of state 3.
- Then, we merge the solutions to find message bits which satisfy large collection of consecutive slices.

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  - Merge consecutive 3 slices to get solutions for 6 slices i.e. 4 groups of 6 slices.
  - Merge consecutive 6 slices to get solutions for 12 slices i.e. 2 groups of 12 slices.

Attack Layout

- We need to find values of variables in initial state for all the 32 slices.
- We solve for first 24 slices and then solve for the remaining 8 slices.
- Solving for first 24 slices:
  - Find solutions for 8 groups of 3 slices.
  - Merge consecutive 3 slices to get solutions for 6 slices i.e. 4 groups of 6 slices.
  - Merge consecutive 6 slices to get solutions for 12 slices i.e. 2 groups of 12 slices.
  - Merge the two groups of 12 slices to get solutions for 24 slices.

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- Solving for remaining 8 slices:
  - Find solutions for the first 6 slices.
  - Find solutions for the remaining 2 slices.
  - Merge the above two groups to obtain solutions for the 8 slices.
- The last step of the attack is to merge solutions of the 24 slices and 8 slices.
- Finally, we obtain a solution for all the 32 slices.

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  - $a_0[0, 1, 2]$, $a_1[3, 4, 5]$, $a_2[4, 5, 6]$
  - $b_0[1, 2, 3]$, $b_1[10, 11, 12]$, $b_2[12, 13, 14]$
  - $c_0[30, 31, 0]$, $c_1[6, 7, 8]$, $c_2[11, 12, 13]$
  - $e_0[27, 28, 29]$, $e_1[20, 21, 22]$

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  - $c_0[30, 31, 0]$, $c_1[6, 7, 8]$, $c_2[11, 12, 13]$
  - $e_0[27, 28, 29]$, $e_1[20, 21, 22]$
- Once we fix these message bits in the state 2, the slice 1 and slice 2 of state 3 get fixed.

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- It contains the following message bits:
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  - $b_0[1,2,3]$, $b_1[10,11,12]$, $b_2[12,13,14]$
  - $c_0[30,31,0]$, $c_1[6,7,8]$, $c_2[11,12,13]$
  - $e_0[27,28,29]$, $e_1[20,21,22]$
- Once we fix these message bits in the state 2, the slice 1 and slice 2 of state 3 get fixed.
- Furthermore, there is no dependency between these message bits.

Possible solutions for groups of 3 slices

- Consider a group of 3 slices (for example take the first 3 slices).
- It contains the following message bits:
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- Once we fix these message bits in the state 2, the slice 1 and slice 2 of state 3 get fixed.
- Furthermore, there is no dependency between these message bits.
- Thus, the total number of possible solutions for this 3-slice are $2^{33-2\cdot 7} = 2^{19}$.

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- Consider, for example, the first two 3-slices (first 6 slices).
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  - $a_0[0-5]$, $a_1[3-8]$, $a_2[4-9]$
  - $b_0[1-6]$, $b_1[10-15]$, $b_2[12-17]$
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- During merging, we get to compute the bit values of slice 3 of the state 3 as well.
- We already have the correct bit values of slice 3 of the state 3, and there is dependency between the above message bit variables.
- The total number of possible solutions are $2^{2\cdot 19-2-7} = 2^{29}$.
- There is dependency between bits $a_0[4,5]$, $a_1[4,5]$ and $a_2[4,5]$.

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- Similar to 6-slice, the bit variables are dependent.
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- Similar to 6-slice, the bit variables are dependent.
- The bit variables $a_0, a_1, a_2[6-9]$, $b_0, b_1, b_2[12]$, and $e_0, e_1[27-31]$ are dependent.
- Hence, the total number of possible solutions are $2^{2\cdot 29-10-7} = 2^{41}$.

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- For example, consider the first 24 slices i.e.,
  - $a_0[0-23], \quad a_1[3-26], \quad a_2[4-27]$
  - $b_0[1-24], \quad b_1[10-1], \quad b_2[12-3]$
  - $c_0[30-21], \quad c_1[6-29], \quad c_2[11-2]$
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  - $c_0[30-21]$, $c_1[6-29]$, $c_2[11-2]$
  - $e_0[27-18]$, $e_1[20-11]$
- This is very much similar to the 12 slice solution.
- In this case, we get 34 dependencies.
- The total number of possible solutions are $2^{2\cdot 41-34-7} = 2^{41}$.

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- Final solution space is obtained by merging the solution space of first 24 slices and the last 8 slices.
- In merging, we can compute the θ mapping of the remaining two slices, in turn, we get the additional restriction of $2 \cdot 7$ bits.
- In this case, we get 61 dependencies.
- Total number of solutions are $2^{41+34-61-2\cdot 7} = 2^0 = 1$.

Attack Complexity

- Space complexity of the attack $= 2^{42}$
- Time complexity of the attack $= 2^{44}$
- Also, we can find second preimages by setting d_0, d_1 to a constant such that it satisfies $d_0[i] = d_1[i]$ and then repeating the attack for this setting.

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- This is a practical attack with attack complexity of 2^{44}.
- Future work: Variant(s) of this attack for more rounds of KECCAK.

Questions?

Thank You