Cryptanalysis of Round Reduced KECCAK

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- Hash functions having the above properties are referred to as cryptographic hash functions.

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- Though by that time SHA-2 family of hash functions was standardized.
- SHA-2 was also based on Merkle-Damgard construction like SHA-0, SHA-1.
- There was a possibility that it could also be attacked in a similar fashion.

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- In the year 2012, NIST announced KECCAK as the winner of the competition among the five finalists viz. BLAKE, Grøstl, JH, KECCAK and Skein.
- Since 2015, KECCAK has been standardized as SHA-3 by NIST.

Keccak

- Keccak hash function is based on sponge construction.
- Sha-3 family of hash functions is based on Keccak.
- The Sha-3 family provides four hash functions:
  - Sha3-224, Sha3-256, Sha3-384 and Sha3-512.

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- The Sha-3 family provides four hash functions:
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- Keccak's excellent resistance towards crypt-analytic attacks is one of the main reasons for its selection by NIST.
- The algorithm is a good mixture of linear as well as non-linear operations.

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  - Padding rule pad.
  - This construction produces a sponge function that takes as input a bit string M and generates a string of length l.

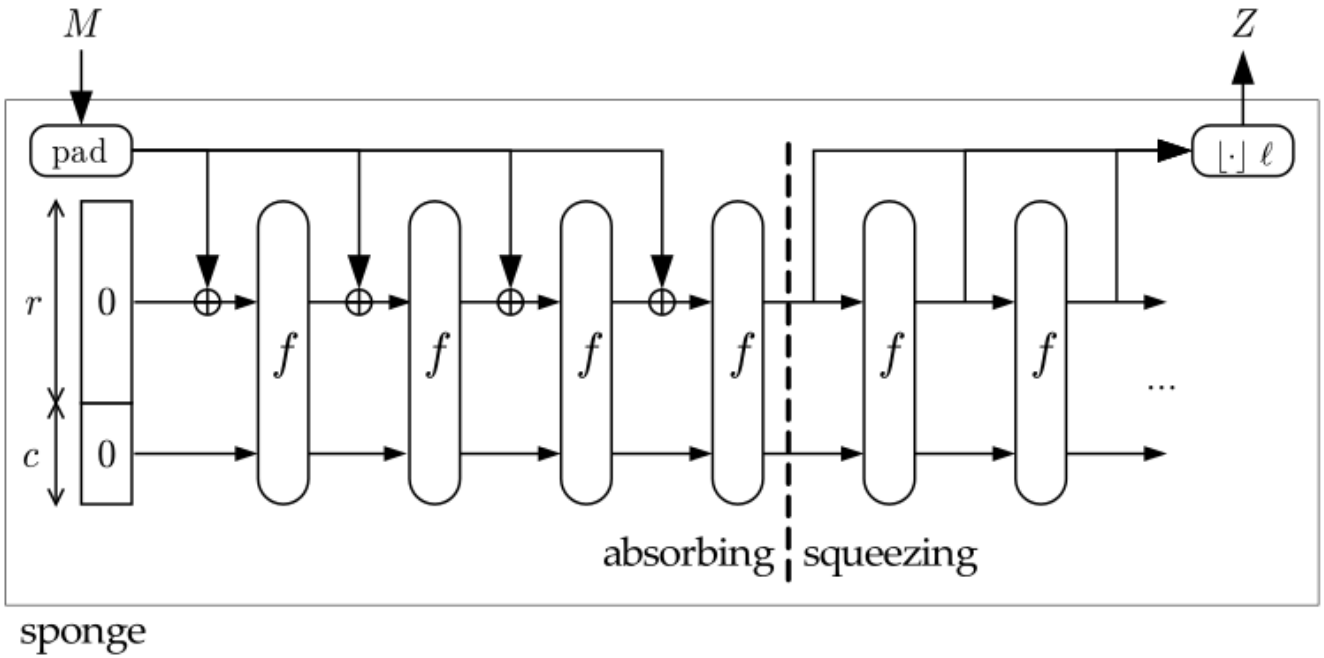

Figure: The sponge construction

KECCAK-p Permutation

- The function f in the sponge construction is denoted by KECCAK-$f[b]$.
- b is the length of input string.
- Internally, KECCAK-$f[b]$ consists of a round function p which is applied n_r number of times.
- KECCAK-$f[b]$ function is specialization of KECCAK-$p\,[b, n_r]$.

Keccak State

- The state input to Keccak-$f[b]$ consists of b bits.
- The state is divided into slices.
- Each slice is of fixed size i.e., 25 bits.
- A state S, which is a b-bit string, in Keccak is usually denoted by a 3-dimensional grid of size $(5 \times 5 \times w)$.

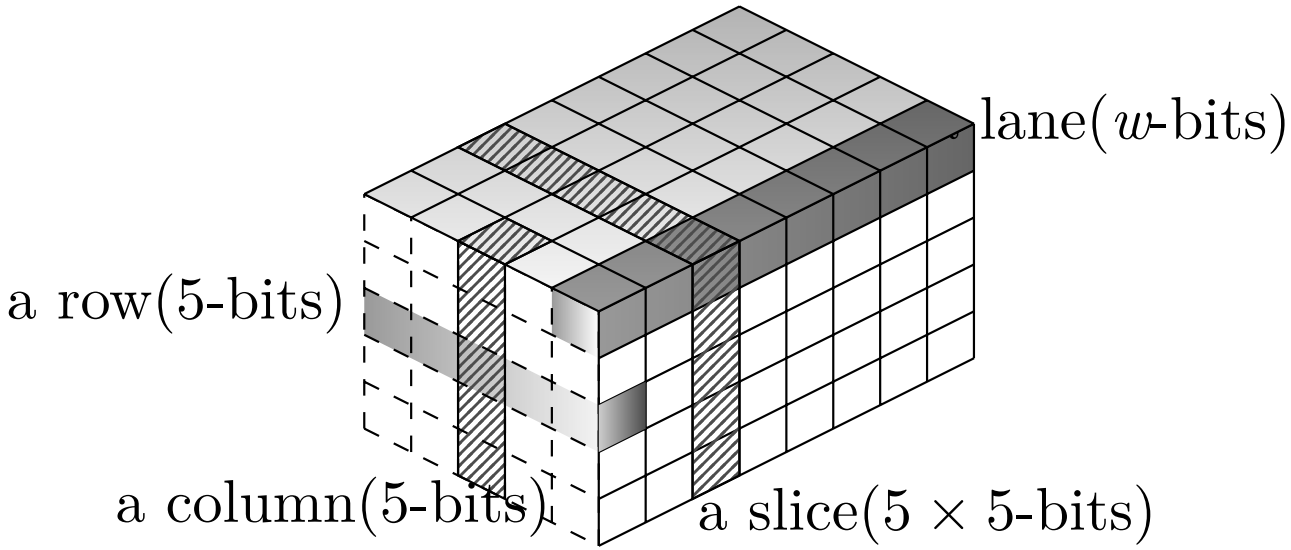

Figure: The Keccak State

Round Function of Keccak-p

- The round function p in Keccak comprises of 5 step mappings.
- The Keccak state undergoes some transformations specified by the step mapping.
- These step mappings are called $\theta,\ \rho,\ \pi,\ \chi\ and\ \iota$.
- These transformations are applied in sequence.
- Now, we will describe these 5 step mappings in detail.

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- If we have A as the input state to θ, then the output state B is:

$$B[x,\ y,\ z] = A[x,\ y,\ z] \bigoplus P[(x-1) \bmod 5,\ z] \bigoplus P[(x+1) \bmod 5,\ (z-1) \bmod w] \quad (1)$$

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- $P[x,\ z]$ represents the parity of the column $(x,\ z)$.

$$P[x,\ z] = \bigoplus_{y=0}^{4} A[x,\ y,\ z]$$

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- The constant value $\rho(x,\ y)$ is specified for each lane in the construction of KECCAK.

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- ρ is a linear step mapping.

π step mapping

- π (**pi**): It permutes the position of lanes.
- The new position of a lane is determined by a matrix,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}, \tag{2}$$

  where $(x',\ y')$ is the position of lane $(x,\ y)$ after π step.
- π is a linear step mapping.

χ step mapping

- χ (**chi**): Each bit in the original state is XOR-ed with a non-linear function of next two bits in the same row.

$$B[x,\ y,\ z] = A[x,\ y,\ z] \oplus \\ ((A[(x+1) \bmod 5,\ y,\ z] \oplus 1) \cdot A[(x+2) \bmod 5,\ y,\ z]) \,. \tag{3}$$

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- χ is the only non-linear operation among the 5 step mappings in KECCAK.

ι step mapping

- ι (**iota**): This step mapping only modifies the $(0,\ 0)$ lane depending on the round number.
- If we have A as the input state to ι, then the output state B is:

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- The remaining 24 lanes remain unaffected.
- All the rounds are identical but the symmetry is destroyed by this step due to the addition of a round constant to a particular lane.

Specification of KECCAK-$p\,[b, n_r]$

- Round in KECCAK is given by:
  - $\texttt{Round}\,(A,\ i_r) = \iota\,(\chi\,(\pi\,(\rho\,(\theta\,(A)))),\ i_r)$
- It consists of n_r number of iterations of $\texttt{Round}\,(A,\ i_r)$.
- KECCAK-$p\,[b,\ n_r]\,(S)$
  - Convert S into a state array A
  - For i_r from 0 to $n_r - 1$, let $A = \texttt{Round}\,(A,\ i_r)$
  - Convert A into string S' of length b
  - Return S'

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- Instances of KECCAK are denoted by KECCAK $[r,\ c]$.
- Where $r = 1600$-c and the capacity c is chosen to be twice the size of hash output d.
- We set $c = 2 \cdot d$, to avoid generic attacks with expected cost below 2^d.
- The hash function with output length d is denoted by:

$$\text{KECCAK-}d \quad = \quad \text{KECCAK}\,[r := 1600 - 2 \cdot d,\ c := 2 \cdot d] \qquad (5)$$

pad10*1

- The padding rule followed by KECCAK is **pad10*1**.
- According to the rule, the input string is appended with a 1 bit followed by some number of 0 bits and followed by 1 bit.
- The asterisk in the padding rule indicates that 0 bit is either not present or is repeated as required so that the length of output string after padding is a multiple of the block length (i.e. r).

Keccak[r:=800-384, c:=384]

- $\textsc{Keccak}\,[r := 800 - 384, c := 384] =$
  $\textsc{Keccak-}p\,[800, 24]\,[r := 800 - 384, c := 384]$.

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- 2-round Keccak $[r := 800 - 384, c := 384] =$
  Keccak-$p\,[800, 2]\,[r := 800 - 384, c := 384]$.

Observations

- **Observation 1:** If we know all the bits of a row, then we can invert χ for that row. It is depicted below.

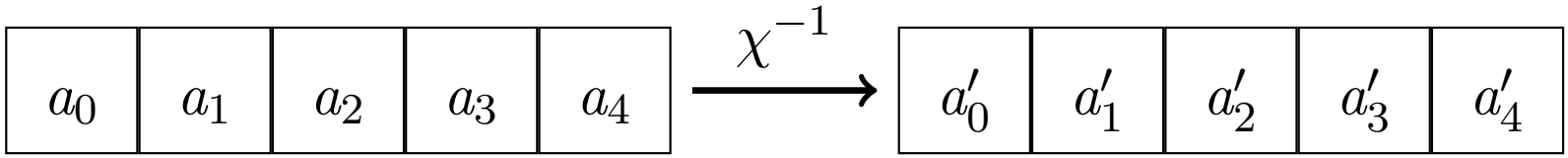

Figure: Computation of χ^{-1} for full row

$$a'_i = a_i \oplus (a_{i+1} \oplus 1) \cdot (a_{i+2} \oplus (a_{i+3} \oplus 1) \cdot a_{i+4}) \tag{6}$$

- **Observation 2:** When only one output bit is known after χ step, then we can fix the first output bit to be the same as the input bit and the second bit as 1.

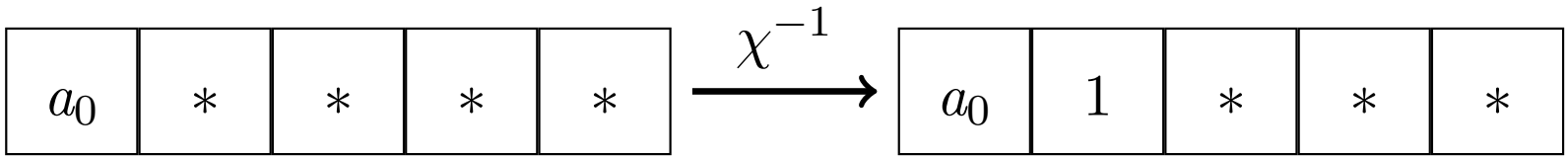

Figure: Computation of χ^{-1} when only 1-bit is known in row

Observations

- **Observation 3:**
- a'_i, a_i are the input and output bits of χ respectively.
- Guo *et al.* observed that when 4 out of 5 output bits of χ are known, then we can obtain 4 linear relations in terms of a'_i.

$$a'_i = a_i \oplus (a_{i+1} \oplus 1) \cdot (a_{i+2} \oplus (a_{i+3} \oplus 1) \cdot a_{i+4}) \tag{7}$$

- If the values of a_0, a_1, a_2, a_3 are known using the Equation 7, we can eliminate the expression a_4 from the rest of the equations.
- Hence, we obtain 4 linear equations on the input bits.

Notations

- The KECCAK state is represented by 25 lanes.
- Each lane is represented by a variable which is a 32-bit array.
- A variable with a number in round bracket "$(\cdot)$" represents the shift of the bits in array towards MSB.
- A variable with a number in square bracket "$[\cdot]$" represents the bit value of the variable at that index.
- If there are multiple numbers in the square bracket, then it represents the corresponding bit values.

2 rounds of KECCAK[r:=800-384, c:=384]

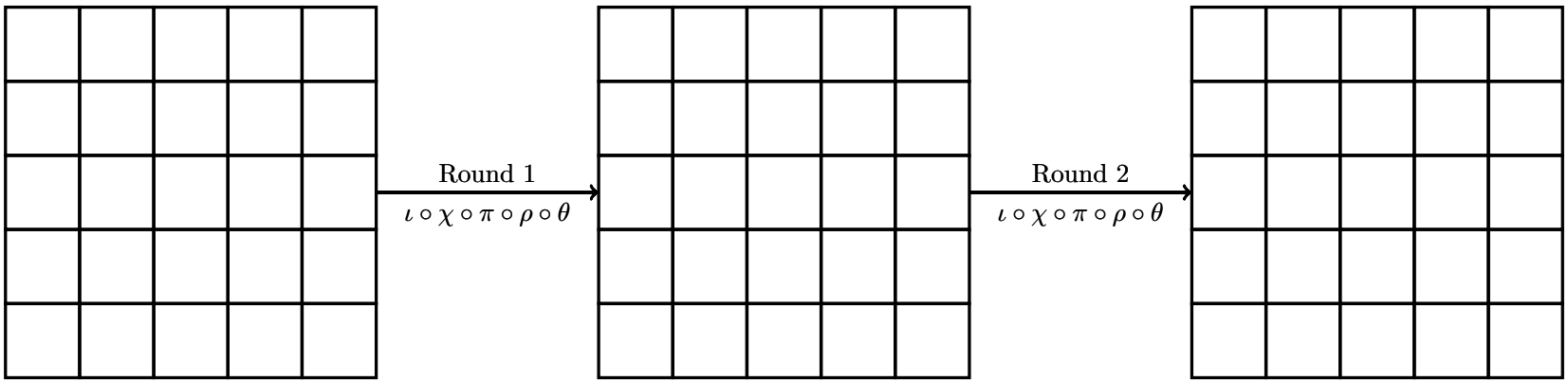

Figure: Two rounds of KECCAK$[r := 800 - 384, c := 384]$

- We will discuss a preimage attack on above structure.

Final State of 2-round KECCAK[r:=800-384, c:=384]

- $c = 384 \rightarrow d = 192 \rightarrow$ hash of 6 lanes

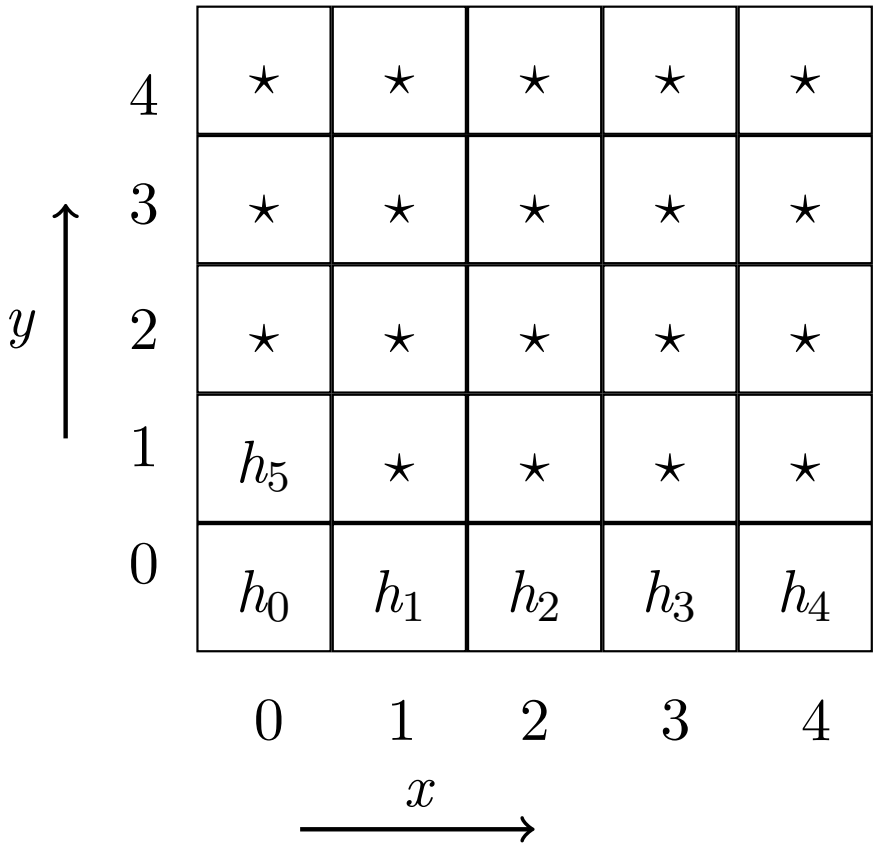

Figure: The Final Hash State for KECCAK[$r := 800 - 384, c := 384$]

Initial State of 2-round KECCAK[r:=800-384, c:=384]

- $r = 800 - 384 \rightarrow r = 416 \rightarrow$ Message block of 13 lanes

0	0	0	0	0
0	0	0	0	0
a_1	b_1	c_2	0	0
a_2	b_2	c_1	d_1	e_1
a_0	b_0	c_0	d_0	e_0

Figure: Setting of Initial State in the Attack

Attack

- Our aim is to find the values of $a_0,\ a_1,\ a_2,\ b_0,\ b_1,\ b_2,\ c_0,\ c_1,\ c_2,\ d_0,\ d_1$ and $e_0,\ e_1$ variables in the initial state.

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- Such that, they lead to a final state having first six lanes as $h_0,\ h_1,\ h_2,\ h_3,\ h_4$ and h_5.
- We follow the basic idea of the attack given by Naya *et al.* in 2011.
- 2 rounds of KECCAK$[r := 800 - 384,\ c := 384]$
  - Best-known attack, has a time complexity of $O(2^{64})$.
  - It is based on the idea of linear structures given by Jian Guo *et al.* in 2016.

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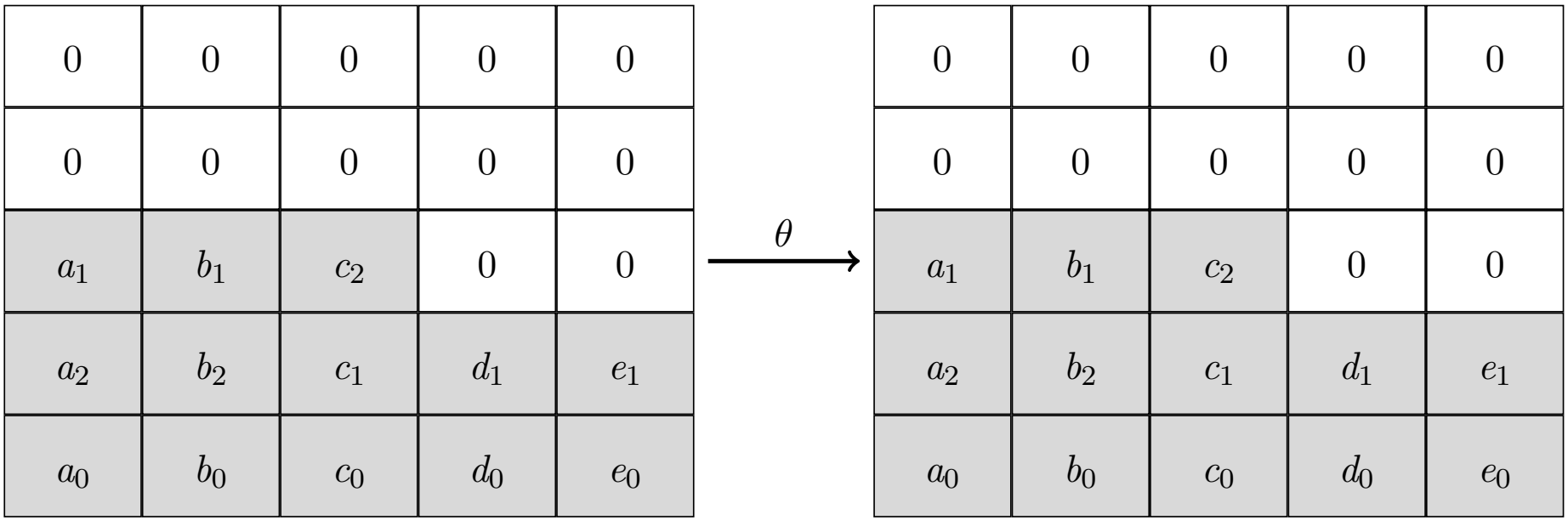
- In this step, our aim is to control the diffusion by θ.
- This can be done by adding constraints on message bits.
- We add the following conditions to make column parity zero:

$$\begin{aligned} &a_2 = a_0 \oplus a_1, \quad b_2 = b_0 \oplus b_1, \quad c_2 = c_0 \oplus c_1 \\ &d_1 = 0, \quad d_0 = 0 \quad \text{and} \quad e_1 = e_0. \end{aligned} \tag{8}$$

Effect of θ



State 1

Figure: Effect of θ on initial state

State 1 to State 2

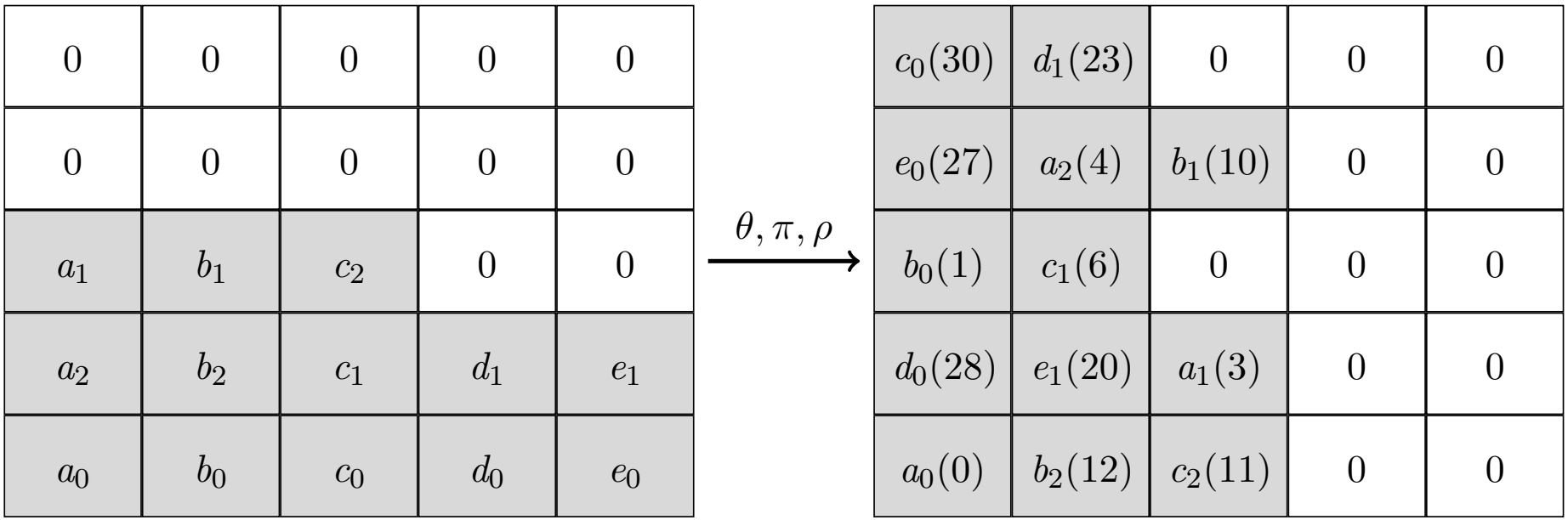
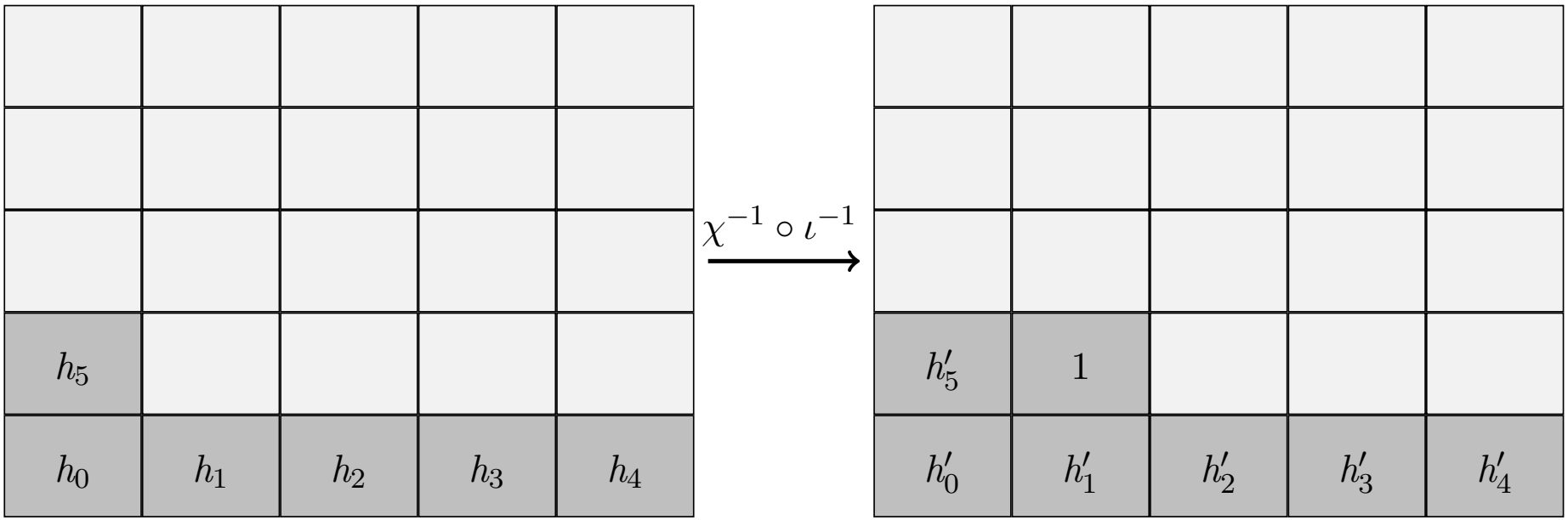


Figure: Preimage attack on 2-round KECCAK$[r := 800 - 384,\ c := 384]$

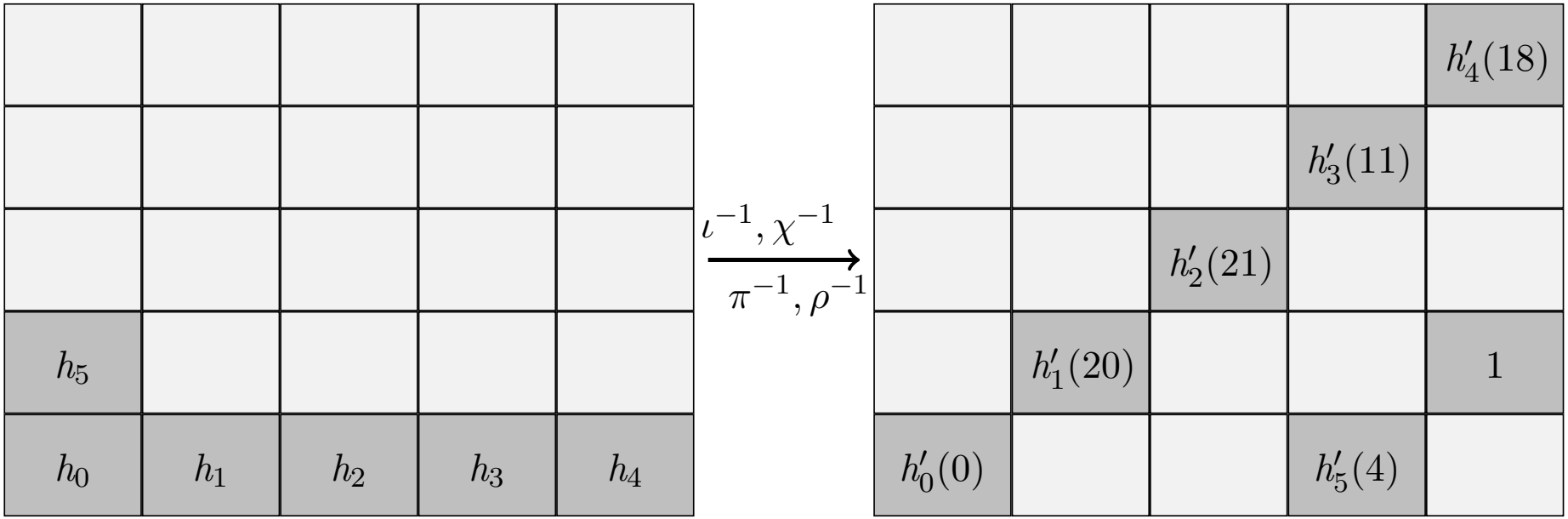
χ and ι inverse



State 4

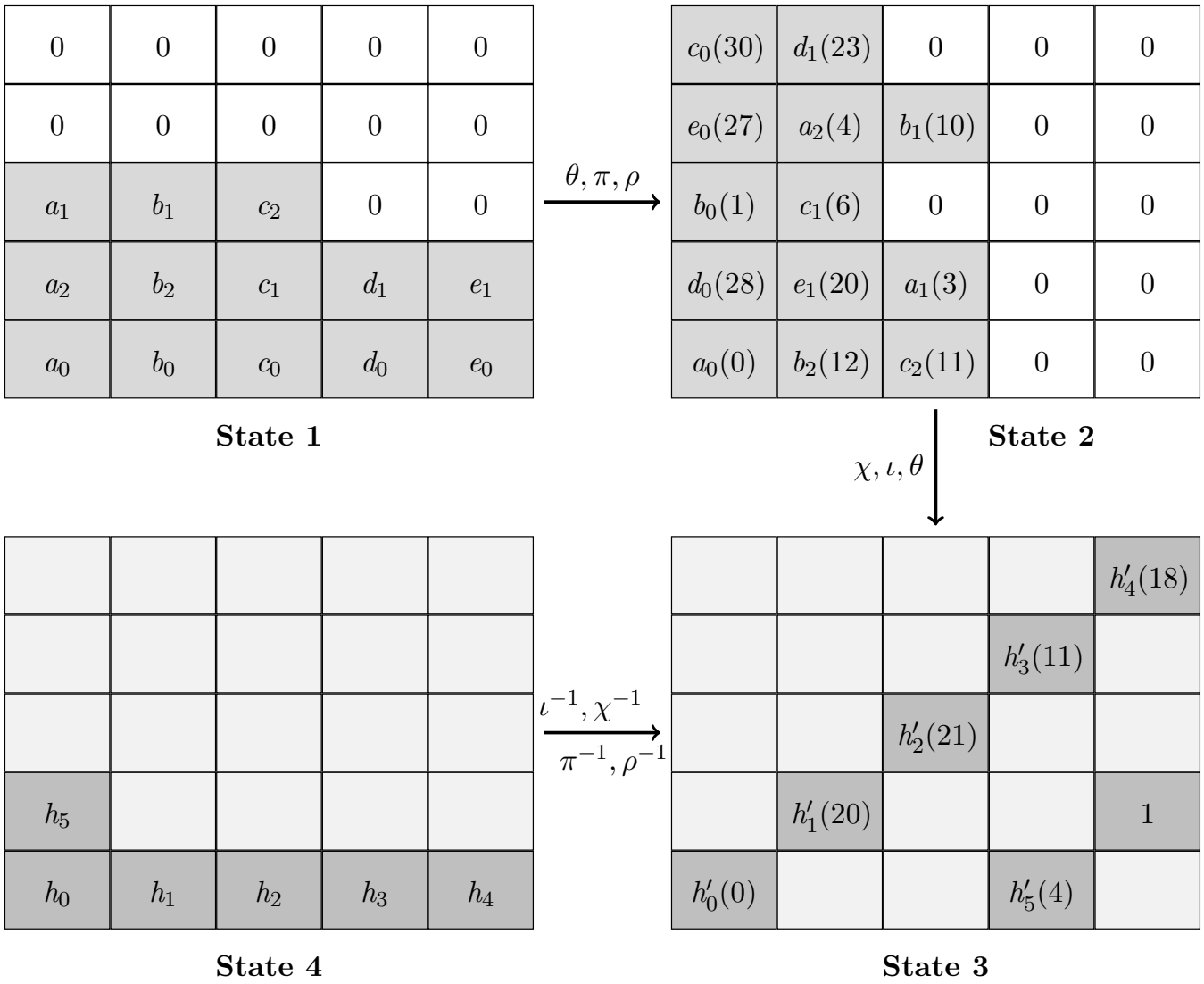
Figure: Inversion of hash through $\chi^{-1} \circ \iota^{-1}$

State 4 to State 3



State 4 **State 3**

State 1 to 4



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State 2 to State 3

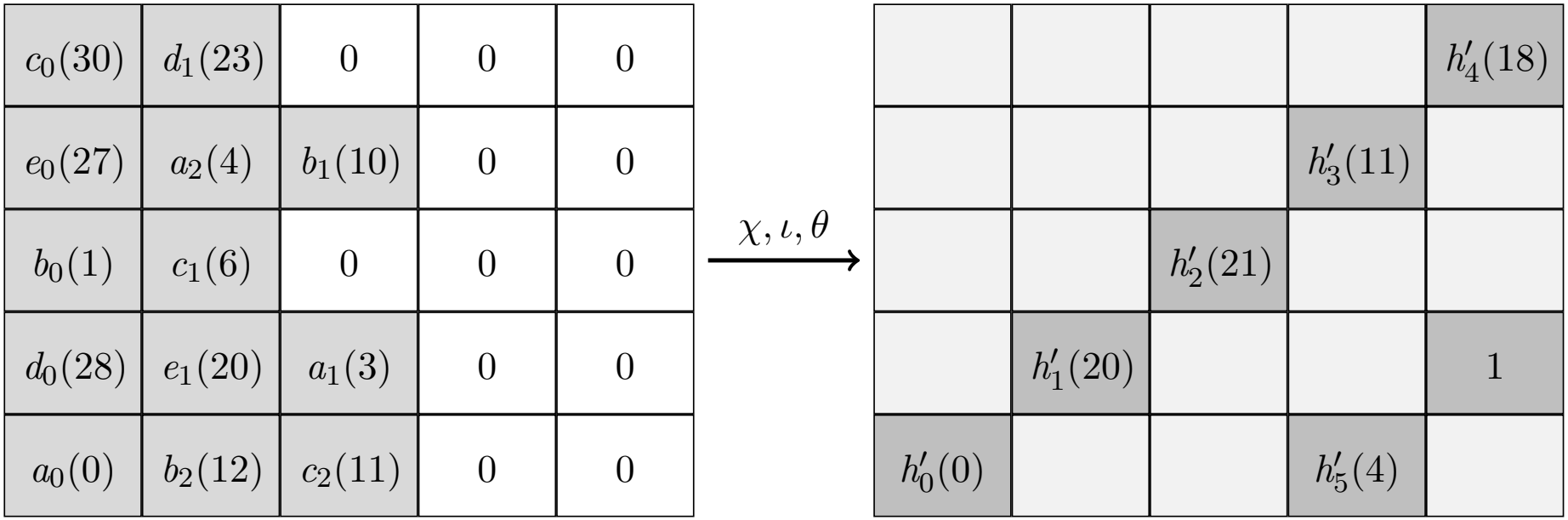

Figure: Intermediate States in 2-round preimage attack on KECCAK$[r := 800 - 384,\ c := 384]$

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- In state 2, we initially have $13 \cdot 32$ variables.
- In the state 3, there are 7 lanes whose values are fixed, this will impose a total of $7 \cdot 32$ conditions.
- We also set 6 conditions on the initial state (Equation 8). This will further add $6 \cdot 32$ conditions.

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- In state 2, we initially have $13 \cdot 32$ variables.
- In the state 3, there are 7 lanes whose values are fixed, this will impose a total of $7 \cdot 32$ conditions.
- We also set 6 conditions on the initial state (Equation 8). This will further add $6 \cdot 32$ conditions.
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- So, we expect a solution.

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- Then, we merge the solutions to find message bits which satisfy large collection of consecutive slices.

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- Finally, we obtain a solution for all the 32 slices.

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- Furthermore, there is no dependency between these message bits.
- Thus, the total number of possible solutions for this 3-slice are $2^{33-2\cdot 7} = 2^{19}$.

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- The total number of possible solutions are $2^{2\cdot 19-2-7} = 2^{29}$.
- There is dependency between bits $a_0[4,5]$, $a_1[4,5]$ and $a_2[4,5]$.

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- Similar to 6-slice, the bit variables are dependent.
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- The bit variables $a_0, a_1, a_2[6-9], b_0, b_1, b_2[12]$, and $e_0, e_1[27-31]$ are dependent.
- Hence, the total number of possible solutions are $2^{2\cdot 29-10-7} = 2^{41}$.

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  - $b_0[1-24]$, $b_1[10-1]$, $b_2[12-3]$
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  - $e_0[27-18]$, $e_1[20-11]$
- This is very much similar to the 12 slice solution.
- In this case, we get 34 dependencies.
- The total number of possible solutions are $2^{2\cdot 41-34-7} = 2^{41}$.

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- Final solution space is obtained by merging the solution space of first 24 slices and the last 8 slices.
- In merging, we can compute the θ mapping of the remaining two slices, in turn, we get the additional restriction of $2 \cdot 7$ bits.
- In this case, we get 61 dependencies.
- Total number of solutions are $2^{41+34-61-2\cdot 7} = 2^0 = 1$.

Attack Complexity

- Space complexity of the attack $= 2^{42}$
- Time complexity of the attack $= 2^{44}$
- Also, we can find second preimages by setting d_0, d_1 to a constant such that it satisfies $d_0[i] = d_1[i]$ and then repeating the attack for this setting.

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- This is a practical attack with attack complexity of 2^{44}.
- Future work: Variant(s) of this attack for more rounds of KECCAK.

Questions?

Thank You