Heriot-Watt University

Masters Thesis

Thesis Title

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Declaration of Authorship

I, John SMITH, declare that this thesis titled, 'Thesis Title' and the work presented in it is my own. I confirm that this work submitted for assessment is my own and is expressed in my own words. Any uses made within it of the works of other authors in any form (e.g., ideas, equations, figures, text, tables, programs) are properly acknowledged at any point of their use. A list of the references employed is included.

Signed: ______________________________

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"Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism."

Dave Barry

Abstract

The Thesis Abstract is written here (and usually kept to just this page).

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor :)

Contents

List of Figures

List of Tables

Abbreviations

LAH	**L**ist **A**bbreviations **H**ere

Symbols

Unless otherwise specified, the following naming conventions apply.

Scalar constants

N	The number of nodes in a graph
T	The number of time points considered
M	The number of explanatory variables
Q	The number of queries

Scalar variables

α	An autocorrelation regularisation parameter
β	A hyperparameter characterising a graph filter
γ	A precision parameter
λ	An eigenvalue *or* ridge regression penalty parameter
μ	The mean of a random variable
θ	AR(1) autocorrelation parameter
σ^2	The variance of a random variable

Matrices

$\mathbf{A}$	The graph adjacency matrix
$\mathbf{D}$	A diagonal matrix
$\mathbf{E}$	The prediction residuals
$\mathbf{F}$	A predicted graph signal
$\mathbf{G}$	A graph filter
$\mathbf{H}$	A Hessian matrix
$\mathbf{I}$	The identity matrix

$\mathbf{K}$	A kernel (Gram) matrix
$\mathbf{L}$	The graph Laplacian
$\mathbf{S}$	A binary selection matrix
$\mathbf{U}$	Laplacian eigenvector matrix
$\mathbf{V}$	Kernel eigenvector matrix
$\mathbf{X}$	Data matrix of explanatory variables
$\mathbf{Y}$	(Partially) observed graph signal
$\mathbf{\Lambda}$	A diagonal eigenvalue matrix
$\mathbf{\Sigma}$	A covariance matrix
$\mathbf{\Phi}$	Auxiliary eigenvector matrix
$\mathbf{\Psi}$	Auxiliary eigenvector matrix
$\mathbf{\Omega}$	Log marginal variance matrix

Vectors/tensors

$\mathbf{e}$	The prediction residuals
$\mathbf{f}$	The predicted graph signal
$\mathbf{s}$	A binary selection vector/tensor
$\mathbf{x}$	A vector of explanatory variables
$\mathbf{y}$	The observed graph signal
$\boldsymbol{\alpha}$	A flexible intercept vector/tensor
$\boldsymbol{\beta}$	A graph filter parameter vector *or* vector of regression coefficients
$\boldsymbol{\theta}$	A aggregated coefficient vector $[\boldsymbol{\alpha}^\top, \boldsymbol{\beta}^\top]^\top$

Functions

$g(\cdot)$	A graph filter function
$p(\text{statement})$	The probability that a statement is true
$\pi(\cdot)$	A probability density function
$\xi(\cdot)$	Optimisation target function
$\kappa(\cdot, \cdot)$	A kernel function

Operations

$(\cdot)^\top$	Transpose of a matrix/vector
$\|\| \cdot \|\|_\mathrm{F}$	The Frobenius norm

$\mathrm{vec}(\cdot)$	Convert a matrix to a vector in column-major order
$\mathrm{vec}_{\mathrm{RM}}(\cdot)$	Convert a matrix to a vector in row-major order
$\mathrm{mat}(\cdot)$	Convert a vector to a matrix in column-major order
$\mathrm{mat}_{\mathrm{RM}}(\cdot)$	Convert a vector to a matrix in row-major order
$\mathrm{diag}(\cdot)$	Convert a vector to a diagonal matrix
$\mathrm{diag}^{-1}(\cdot)$	Convert the diagonal of a matrix into a vector
$\otimes$	The Kronecker product
$\oplus$	The Kronecker sum
$\circ$	The Hadamard product

Miscellaneous

$\hat{(\cdot)}$	The estimator of a matrix/vector/tensor
$O(\cdot)$	The runtime complexity
x_i	A vector element
$\mathbf{X}_i$	A matrix column
$\mathbf{X}_{ij}$	A vector element

For/Dedicated to/To my…

Chapter 1

Introduction

1.1 Welcome and Thank You

[Arnold et al., 1998]

Chapter 2

Outline and Fundamentals

2.1 Graph Signal Processing

2.1.1 A broad overview of the field

2.1.2 The graph Laplacian

2.1.3 Graph filters

2.2 Regression and Reconstruction

2.2.1 Graph Signal Reconstruction

2.2.2 Kernel Graph Regression

2.2.3 Regression with Network Cohesion

2.3 Thesis overview

Chapter 3

Kernel Generalized Least Squares Regression for Network Data

3.1 Kernel Graph Regression with Missing Values

3.2 GLS Kernel Graph Regression

3.2.1 A Gauss-Markov estimator

3.2.2 AR(1) processes

3.2.3 Experiments

Chapter 4

Regression and Reconstruction on Cartesian Product Graphs

4.1 Introduction

Hello

4.2 Graph Signal Reconstruction on Cartesian Product Graphs

Hello

4.2.1 A stationary iterative method

Hello

4.2.2 A conjugate gradient method

Hello

4.2.3 Convergence properties

Hello

4.2.4 Image processing experiments

Hello

4.3 Kernel Graph Regression with Unrestricted Missing Data Patterns

Hello

4.3.1 Cartesian product graphs and KGR

Hello

4.3.2 Convergence properties

Hello

4.4 Regression with Network Cohesion

Hello

4.4.1 Regression with node-level covariates

Hello

4.4.2 Convergence properties

Hello

4.5 Multi-Dimensional Cartesian Product Graphs

Hello

4.5.1 Fast computation with d-dimensional Kronecker products

Hello

4.5.2 Signal reconstruction

Hello

4.5.3 Kernel Graph Regression

Hello

4.5.4 Regression with Network Cohesion

Chapter 5

Signal Uncertainty: Estimation and Sampling

5.1 Introduction

5.2 Posterior Estimation

5.2.1 Log-variance prediction

5.2.2 Estimation models

5.2.3 Query strategies

5.2.4 Comparison and analysis

5.3 Posterior Sampling

5.3.1 Perturbation optimization

5.4 Estimation vs Sampling

5.4.1 Experiments

Chapter 6

Working with Binary-Valued Graph Signals

6.1 Logistic Graph Signal Reconstruction

6.2 Logistic Kernel Graph Regression

6.3 Logistic Regression with Network Cohesion

6.4 Approximate Sampling via the Laplace Approximation

Chapter 7

Conclusions

7.1 Main Section 1

Appendix A

Appendix Title Here

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Bibliography

Arnold, A. S., Wilson, J. S., and Boshier, M. G. (1998). A simple extended-cavity diode laser. *Review of Scientific Instruments*, 69(3):1236–1239.