

Judge's Comments

I enjoyed reading this paper, good work! I found your results generally believable. I would suggest some sensitivity analysis, though. How much do your results depend on possible noise in your data? Your support vector machine approach in part 1 is fairly novel. I haven't seen any other teams use this approach.

Good job summarizing your results. Say something about your approach to the problem.

Q1: Assumption 2 seems questionable. As battery price drops, sales would likely increase as prices get cheaper. Show data points on the curve to assess goodness of the fit visually.

Q2: Quantitatively assess factors contributing to growth. Probably environmental impact of clean energy? Perception of bikes as "cool" or "trendy"? etc.

Q3: Quantitatively plot how much carbon emission would be saved because of e-bikes.

That might make your paper more impactful.

Looking at your data, it appears cost per mile on electric bike seems to be going up? This makes one wonder if the ebikes will someday become more expensive than other modes of transport? Please clarify.

The Rise of Electric Bikes, *Why and How*

Executive Summary

Electric bicycles have begun as the next consumer epidemic following recent developments in electric vehicle technology, economic trends, and rising consumer interest in environmental protection. E-Bike sales have grown to encompass a market size of over 28 billion USD in the United States^[4], and as such, the cause and prospects of electric bicycles is growing ever-more intriguing to consumers and investors alike. As such, the purpose of this paper is to analyze the characteristics concerning the rise in popularity of electric bicycle sales throughout the United States and the United Kingdom, and to predict the economic growth of the electric bicycle trend.

In the first section of our paper, we evaluate the growth rate of the sale of e-bikes in the United Kingdom. We developed a mathematical model which predicts this growth rate in the near future, and then applied the model for the years 2025 and 2028. We found that in 2025, the number of e-bikes sold in the European Union will be approximately 76.922% greater than the sales in 2020. By 2028, the sales will be 133.537% greater than the sales in 2020. Based on our assumption that the United Kingdom approximately follows the trends of the European Union, we found that the number of e-bikes sold in 2025 will be approximately 300,767 in the United Kingdom. By 2028, this number will increase to approximately 397,013 according to our model. A key factor when considering the results, however, is the unpredictable nature of the Covid-19 Pandemic. When creating our model, we assumed that in the long run, the effects of the Pandemic will become negligible. Despite this, policy makers still need to consider how Covid-19 could affect the future of e-bikes.

In the second section of our paper, we analyze the various reasons as to why the trend of electric bicycle sales has begun in the first place through detailing the push factors away from traditional methods of transportation and the pull factors that being consumers to electric bicycles. We concluded that the three main facets that persuade a consumer to prefer an electric bicycle are the cost-efficiency of electric vehicles over engines due to lower energy prices and higher energy efficiency compared to the growing costs of gasoline and motor oil, the lower barrier of entry for low-and-middle-class people to purchase an electric bicycle over a car, and the far cheaper maintenance cost to operate an electric bicycle compared to a car.

In the third and final section of our paper, we analyze the impacts of growing electric bicycle usage on the everyday consumer and society at large through the facets of cost-effectiveness, distance-time efficiency, and environmental impact. We concluded that the switch to electric bicycles is resulting in an immediate improvement to global carbon footprint, and consumers that switch gain an increased advantage in lowered commute times and increased cost-effectiveness compared to common use traditional vehicles.

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Introduction

As environmental issues regarding the rapid progression of climate change, increasing carbon emissions, and the growing concerns over pollution as the result of fuel grow further in tandem with economic changes in consumer interests and changes in daily transportation due to COVID-19, consumers are quickly transitioning to alternative sources of transportation, be it public transport, carpooling and rideshare, or the growing use of electric vehicles. The foundational questions provided ask us to analyze the cause and predict the growth of the popularity of electric bicycles against previous forms of transport.

Part I: The Road Ahead

1.1 Restatement of the Problem

The first question asks us to create a mathematical model that predicts the growth of e-bike sales in the United Kingdom, and then apply the model to the years 2025 and 2028.

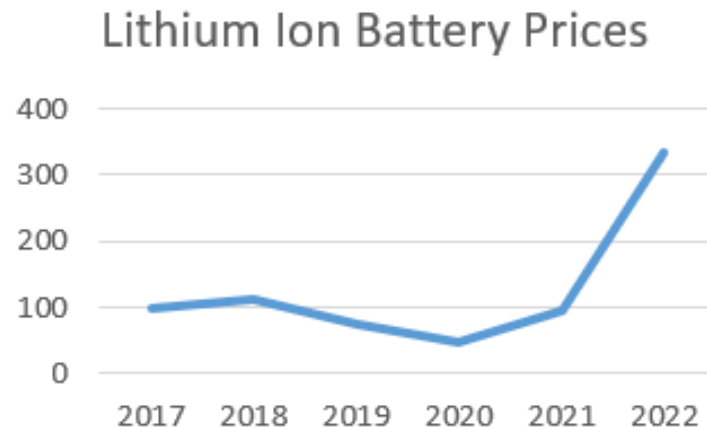
1.2 Assumptions

1. *There will be no immediate, drastic changes to the environment*

Justification: There is no way to accurately determine immediate changes to the environment several years into the future, and environmental change is far more likely to continue alongside the previous trend than to act sporadically.

2. *The price of lithium-ion batteries does not affect the rate at which e-bikes are sold*

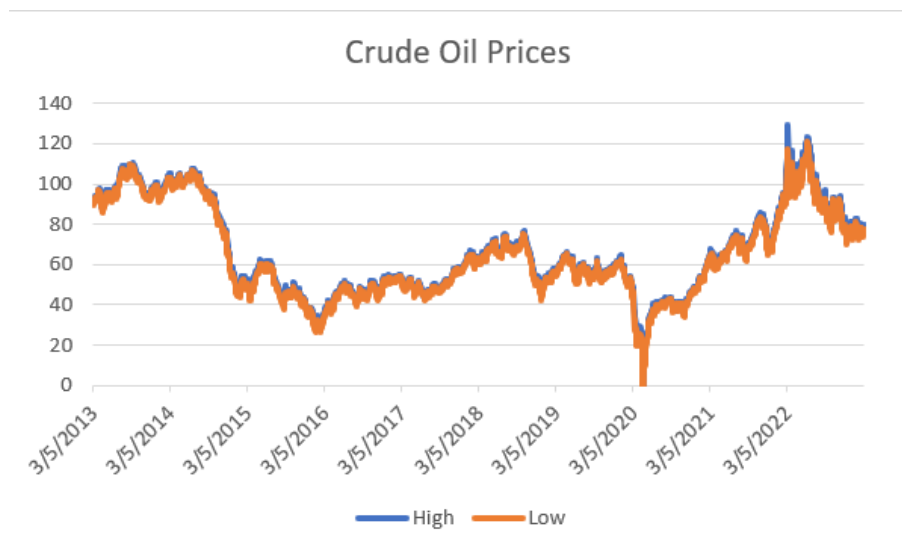
Justification: The price of lithium-ion batteries changes sporadically as can be seen in the graph below. Using a simple Least Squares Regression Line with the lithium-ion price as the independent variable, and the e-bike sales in Europe as the dependent variable, we found a coefficient of determination of 0.44. The same technique was used with the United States data, and we found the coefficient of determination to be 0.13. Both these coefficients of determination indicate that the lithium-ion price does not have a significant effect on the sale of e-bikes.



Lithium-Ion Battery Costs based on Battery Pack Prices ^[19]

3. *There will be no drastic changes to the price of oil*

Justification: The price of oil can be very difficult to predict as the result of multiple economic and political factors such as the stability of international governments, the continued efficiency of the supply chain, and more ^[16]. Such changes are unrealistic to predict without requiring an intense amount of effort.



Crude Oil Prices from 3/5/2013 until 3/3/2023 ^[17]

4. *There will be no new technologies which dramatically change the rate at which e-bikes are sold, and future technological improvements will match current trend*

Justification: There is no way to accurately predict the development of new technology. Technology is constantly being developed in both fields that would aid the production, marketing, and distribution of

electric bicycles and alternative competitors to electric bicycles. Such technology would include new developments in advertising, methods of producing cheaper fuels, vastly improved batteries, and more. As such, all conclusions in this research are based on all future technology meeting current technological trends.

5. *The United Kingdom follows approximately the same trend as the rest of Europe*

Justification: The United Kingdom is the second largest country in the European Union ^[29], which means that the United Kingdom will likely approximately follow the trends of the rest of the European Union. Additionally, countries in the European Union, including the United Kingdom, follow the same set of regulations and guidelines.

6. *The number of e-bikes in the United Kingdom will continue to make up only a small portion of the total bikes in the United Kingdom in the immediate future*

Justification: As e-bikes become a larger portion of the bikes sold, the rate at which e-bike sales are increasing, will become smaller. This would likely follow a logistic curve. However, because the question is only concerned with the next five years, and there are currently around 20 million bicycles sold in the European Union every year ^[15], we can assume that this effect will be negligible.

1.3 Model Development

To model the sales of E-bikes in Europe, we used a Linear Support Vector Machine regression algorithm from the Python Sci-kit Learn library. To train the model, we fitted it to the “E-bikes Sold in Europe” dataset ^[5] as shown below.

E-Bikes sold in Europe (1000s of units)	
Year	
2006	98
2007	173
2008	279
2009	422
2010	588
2011	716
2012	854
2013	907
2014	1139
2015	1364
2016	1637
2017	2074
2018	2767
2019	3397
2020	--
2021	--
2022	--

Training Data for SVM ^[6]

Because a linear support vector machine is a type of linear model, it makes predictions based on a linear function it creates from the given data. A linear Support Vector Machine creates the function so that the deviation for each point in the training data is no greater than the given parameter C , and so that the

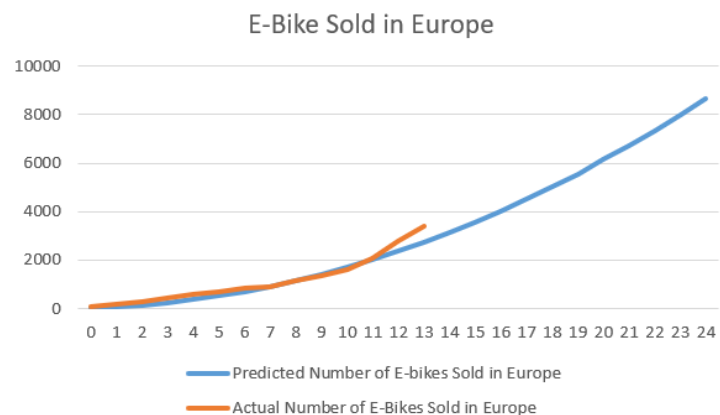
smallest possible coefficients for each feature are used^[13]. Because of the limited size of the dataset, we chose a C parameter of 1.0 to prevent overfitting or underfitting the data. This initial model, however, had a coefficient of determination (r^2) of 0.221, which is subpar. To correct this issue, we used the square roots of the sales amounts in our model. With this change, the coefficient of determination became 0.935 which indicates that the model is an accurate predictor of the sales of electric bikes based on the data.

1.4 Result

The model was applied to the years from 2006 until 2030. The results are shown in the table and graph below.

Years since 2006	Predicted Amount	Actual Amount
0	18.02768073	98
1	63.21553346	173
2	135.8562214	279
3	235.9497447	422
4	363.4961032	588
5	518.495297	716
6	700.947326	854
7	910.8521903	907
8	1148.20989	1139
9	1413.020425	1364
10	1705.283795	1637
11	2025	2074
12	2372.169041	2767
13	2746.790916	3397
14	3148.865628	
15	3578.393174	
16	4035.373555	
17	4519.806772	
18	5031.692825	
19	5571.031712	
20	6137.823435	
21	6732.067992	
22	7353.765386	
23	8002.915614	
24	8679.518678	

Output by SVM



Output of SVM graphed against actual sales

As the data indicates, in two years the number of e-bikes sold in Europe will increase to around 5,571,032. In five years, this number will have increased to 7,353,765 e-bikes sold in Europe.

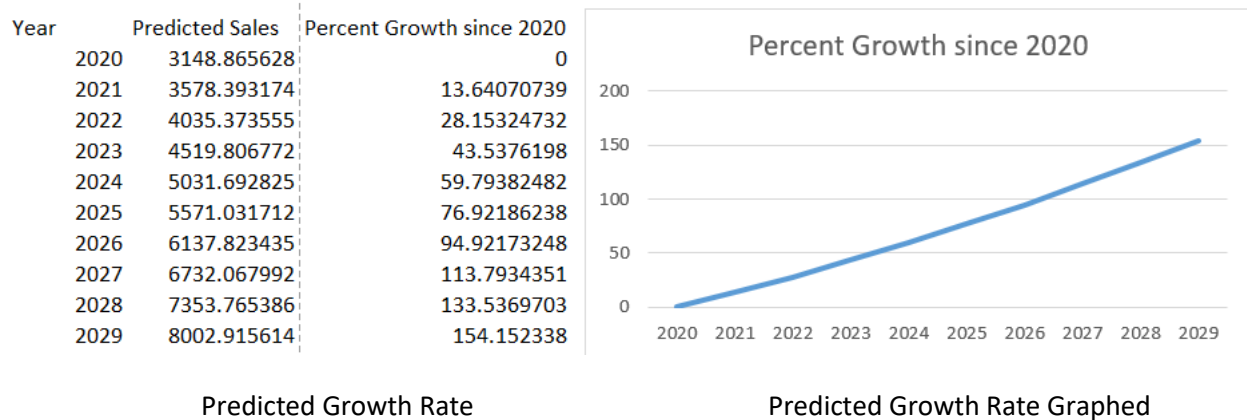
Next, the growth rate of the sale of e-bikes can be calculated using the following formula^[12]:

$$growth\ rate = \frac{current - base}{base} * 100$$

Which can be simplified to:

$$growth\ rate = \frac{current}{base} * 100 - 100$$

Using this formula, the growth rate for e-bike sales in Europe was predicted with 2023 as the base year. The result was then graphed, which is shown below.



Predicted Growth Rate

Predicted Growth Rate Graphed

The model indicates that by 2025 (two years from now), e-bike sales will have increased by 76.922% since 2020, and by 2028 (five years from now) e-bike sales will have increased by 133.537%.

Finally, to answer the initial question, we will use the percent growth in Europe to predict e-bike sales specifically for the United Kingdom. We will use the year 2020 as the base year, with e-bike sales estimated to be approximately 170,000 in the United Kingdom^[21]. Based on our assumption that United Kingdom e-bike sale trends approximately follows European trends, we can now simply multiply the growth rate from the European Data with the estimated e-bike sales data from the United Kingdom.

In 2025, the number of e-bikes sold in the United Kingdom is predicted to be:

$$\text{electric bikes sold in UK} = 170,000 * 1.76922 = 300,767$$

In 2028, the number of e-bikes sold in the United Kingdom is predicted to be:

$$\text{electric bikes sold in UK} = 170,000 * 2.33537 = 397,013$$

These numbers may seem surprising at first because sales are predicted to be smaller in 2025 (300,767) than they actually were in 2019 (339,700). However, this is likely because 2019 was an outlier. Because the model generalizes the data rather than matching the data exactly, the model is not significantly affected by this outlier.

We predict that two years from now, the number of e-bike sold in the United Kingdom will be approximately 300,767 e-bikes, and five years from now the number of e-bikes sold in the United Kingdom will be approximately 397,013 e-bikes.

1.5 Strengths and Weaknesses

One of the main benefits of using a linear regression algorithm is that it is easy to understand. The key components in linear regression are the weights and intercept, which can be easily graphed and visualized, unlike algorithms such as decision trees. Additionally, linear models are excellent at generalizing data so that temporary outliers do not affect the overall trendline.

The model, however, also had several weaknesses. One of the main weaknesses is that the model was trained with a limited dataset. The data only had thirteen 1-feature data points, while linear models work better for large datasets or datasets with many features. Even more limiting was the lack of data specifically for the United Kingdom. While in general, the United Kingdom approximately follows the same trends as the rest of Europe, the model would likely be more accurate if it were trained on data specifically for the United Kingdom, which may deviate slightly from the rest of Europe.

Our model assumes that over time, e-bike sale trends will eventually return to pre-pandemic rates, however it is impossible to accurately predict how the pandemic could affect e-bike sales. This is an important factor to consider while evaluating e-bike sales, however the effects cannot be quantified which is why our model does not take into consideration the effects of the Pandemic.

Part II: Shifting Gears

2.1 Restatement of the Problem

The second question asks us to analyze the cause of the rise of electric bicycle purchases.

2.2 Solution

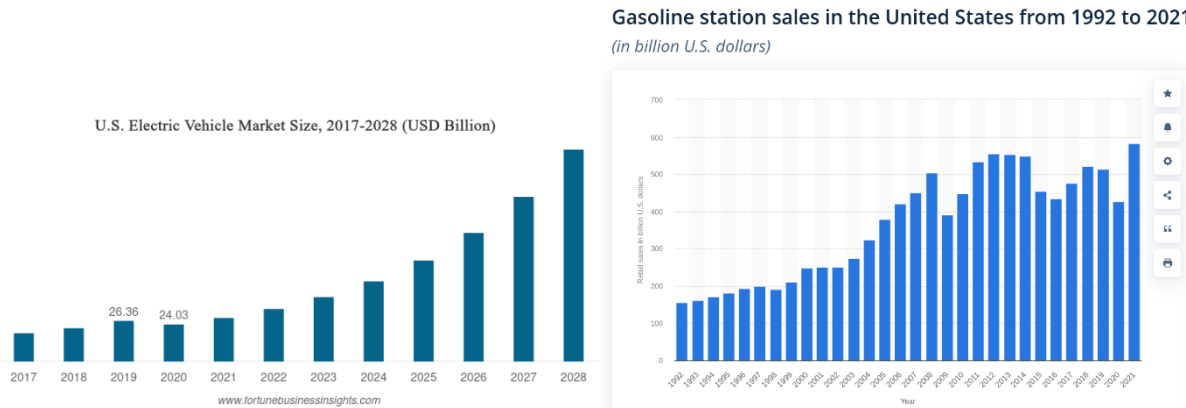
The growth of electric bicycles is most influenced by recent developments in the following factors: the increase of difficulty of consumers to access previous methods of transport due to fluctuations in gas prices as the result of changes in oil prices, the decrease in the cost of purchasing and upkeeping an electric bicycle due to the decrease of electricity, battery manufacturing, and battery maintenance costs, and the decrease of average commute distances as the result of COVID-19.

2.3 Assumptions and Definitions

1. *Traditional methods of transport include non-electric (gas or diesel) vehicles intended for short-range, daily travel which can be reasonably replaced by an electric bicycle, including cars, trucks, buses, and motorcycles.*

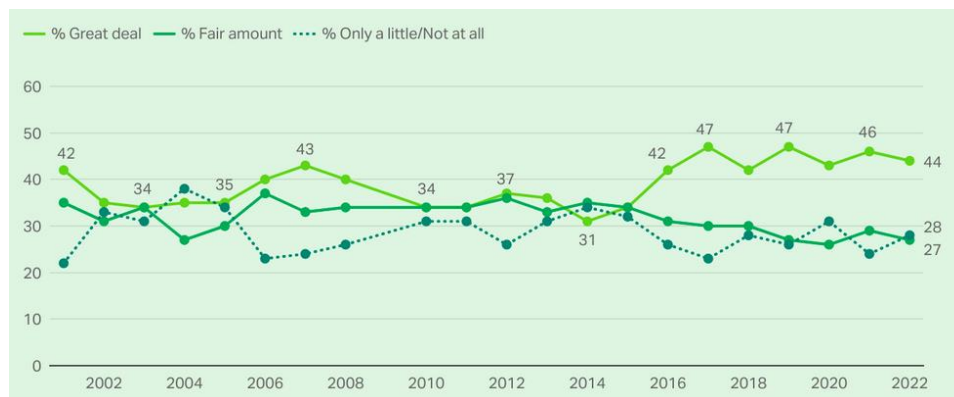
Justification: Electric bicycles are designed with the same purpose in mind as traditional, mechanical bicycles. Very long distances and specialization beyond the transportation of a singular person is out of the scope of what electric bicycles are created to serve.

Additionally, electric vehicles are excluded from traditional methods of transport as the rise of electric vehicles came alongside the rise of electric bicycles as they are both created to implement the same technology.

Electric Vehicle Market Size ^[4]Gas Station Sales ^[3]

Lastly, gasoline vehicles are to be considered the status quo due to the incredibly large market gap in market size between the gasoline industry and electric vehicles. The total gross revenue of gasoline stations alone in 2021 was about 428.08 billion USD ^[3] (about \$1,300 per person in the US) in the United States alone whereas the total market size of electric vehicles in the United States was about 28.24 billion USD ^[4] (about \$87 per person in the US).

2. Consumers are choosing to switch to electric bicycles strictly due to economic concerns and not environmental ones.

Polled Environmental Concern Levels by Year ^[6]

Justification: Interest in environmental concerns is largely stagnant in consumers. According to Gallup's yearly environmental poll as provided by M3, US residents have a largely stagnant environmental interest ^[5]. Although there is certainly a considerable level of concern towards environmental protection

among consumers, this interest is likely not the cause of the growth of electric bicycles as the interest in environmental concern did not rise to match the popularity of electric bicycles.

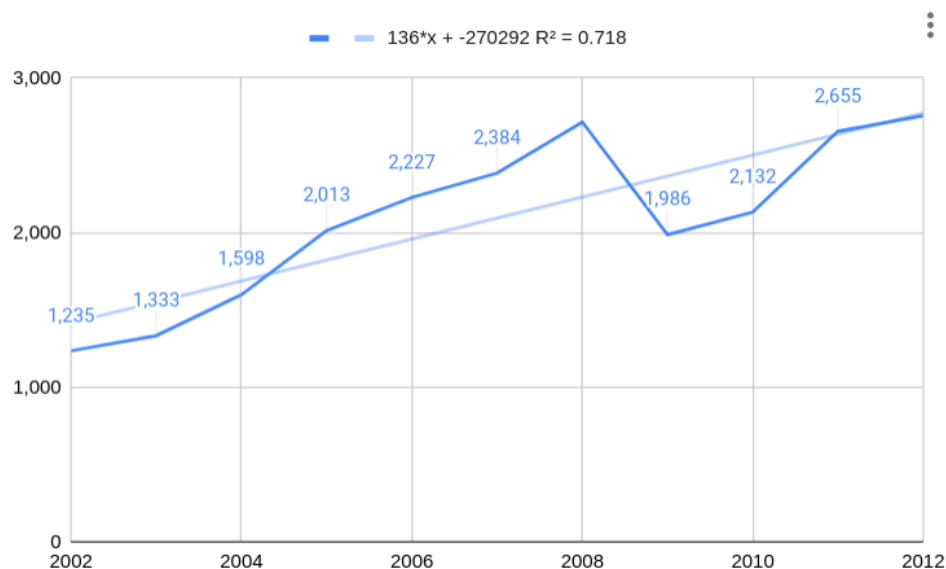
2.4 Discussion

To properly assess the rise of any commercial trend, the strengths and weaknesses of the product in question must first be analyzed. Consumers are more likely to switch between using one method of transportation to maximize the ratio between the comfortability, accessibility, and efficiency of the product compared to the initial and maintenance costs.

2.4.1 The Disadvantages of Traditional Transportation

1. Fuel Cost

The cost of fuel and motor oil, both vital to properly operating traditional vehicles, is consistent throughout using a vehicle and will add up quickly. From 2002 to 2012, the average yearly expense of gasoline and motor oil for drivers went from a low of 1,235 USD to a high of 2,756 USD in a rapidly increasing fashion.



The Average Yearly Cost of Gasoline and Motor Oil ^[7]

2. Maintenance Cost

The cost to maintain a traditional vehicle is one of the largest barriers of entry of car. Despite the already difficult initial cost to purchase a vehicle, the bulk of the money spent owning a car is repairs, insurance, and licensing. From 2002 to 2012, the average annual cost of maintenance (including insurance, repair, licensing, and other costs) for car owners went from a low of 2,335 USD to a high of 2,621 USD^[7].

3. Initial Cost

The initial cost of a traditional vehicle is incredibly expensive. From 2019 to 2021, the average price of a car rose by 11,320 USD, and the average price of a non-luxury vehicle hit 43,072 USD^[30] in 2022. In 2021, the average disposable income of US residents per capita was 48,219 USD^[5], thus meaning that the purchase of a singular vehicle would immediately use the vast majority of an entire family's savings for a year.

2.4.2 The Advantages of Electric Bicycles

1. Electricity (Fuel) Cost

The cost of electricity is steadily decreasing, and the efficiency of batteries is rapidly increasing. From 1995 to 2015, the average cost of lithium-ion batteries (in kW-hr) decreased from 3200 to 350, a reduction of nearly 85%^[5], and from 1991 to 2013, the storage capacity of a lithium-ion battery (in W-hr/kg) increased from 79 to 240, an increase of nearly 204%^[5]. In short, this means that the electricity demands of an electric bicycle are becoming easier to be met by consumers by the day as electrical technology rapidly progresses.

2. Maintenance Cost

The average yearly maintenance cost of an electric bicycle changes based on how old the bicycle is due to requiring new parts as certain parts (such as the battery) are worn out of operation. For the first three years of owning an electric bicycle, the average yearly maintenance cost is roughly 318 USD, and then for the next seven years, 518 USD^[31]. In comparison, this is still far less than the average cost of maintaining a traditional gasoline vehicle due to electric bicycles not requiring motor oil (as they do not have engines) or licensing (depending on the area in which it is driven).

3. Initial Cost

The barrier for entry to own an electric bicycle is very low in comparison to purchasing a personal vehicle. Of the most popular manufacturers in the US and UK, the minimum costs are only 899 USD in the US and 950 GBP in the UK, and the maximum costs are only 1,300 USD in the US and 8,200 GBP in the UK^[5].

Part III: Off the Chain

3.1 Restatement of the Problem

This question asks us to analyze the impacts of an increase in e-bike usage, including but not limited to carbon emissions, traffic, and health.

3.2 Assumptions

1. *Electric bicycles are charged at home.*

Justification: Since the average distance driven in the UK is 20 miles a day^[19] and the average range of an e-bike is at least 20 miles^[20], we assume that the battery in an e-bike is enough for it to be daily at home, and users typically do not travel long enough distances for the bikes to require a charge before the return to their homes.

2. *The lifespan of an electric bicycle is approximately three years.*

Justification: Without any replacement of parts, an electric bicycle lasts around three years before the lithium-ion battery begins to fail^[8]. As a result, to simplify calculations and avoid it can be estimated that the users will replace their e-bikes every three years to provide a worst-case for the costs and emissions of using e-bikes.

3. *An e-bike is only used to replace the function of a car, while not conflicting with the roles of HGVs, vans, buses, etc.*

Justification: As a result of the e-bike's relatively limited motor and carrying capacity, its use case is primarily to transport individuals to their nearby destinations. Thus, it is simply not practical for an e-bike to be able to replace the function of a bus or heavy load vehicle, and it can be assumed that despite the increasing adoption of e-bikes, the number of non-car or motorcycle-like vehicles will remain constant.

4. *If a user has an e-bike, they will use it for all their voyages.*

Justification: It is not needlessly complex to predict how many of a user's total driven miles account for their commute and how many are for road trips and other cases where an e-bike would be impractical to use.

5. *All cars driven in the UK are superminis.*

Justification: Of the 10 most popular car models in the UK, 9 are supermini or compact cars^[10], accounting for 8 million of the 32 million total cars registered^[11]. As a result, we assume most cars used for daily use or commute-like distances to be of this size (more specifically, we use the Ford Fiesta, the

most popular car in the UK, as a comparison). Moreover, their above-average fuel economy presents a best-case scenario to present against the worst-case scenario used for the e-bikes.

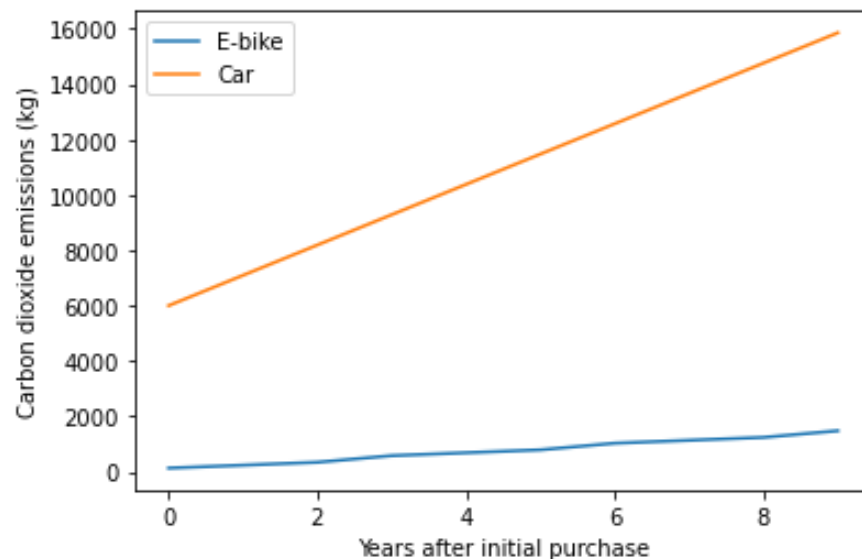
3.3 Model Development

To quantify the difference made by adopting e-bikes over standard cars, we chose to compare the case of a single user replacing their car with an e-bike for the purposes of commuting. This result can therefore be easily scaled to any number of e-bikes replacing cars over any designated period to effectively visualize the impact of their usage. This was done by finding comparable metrics as a factor of distance traveled or time, such as carbon footprint or charging costs.

3.4 Results

3.4.1 Carbon Footprint

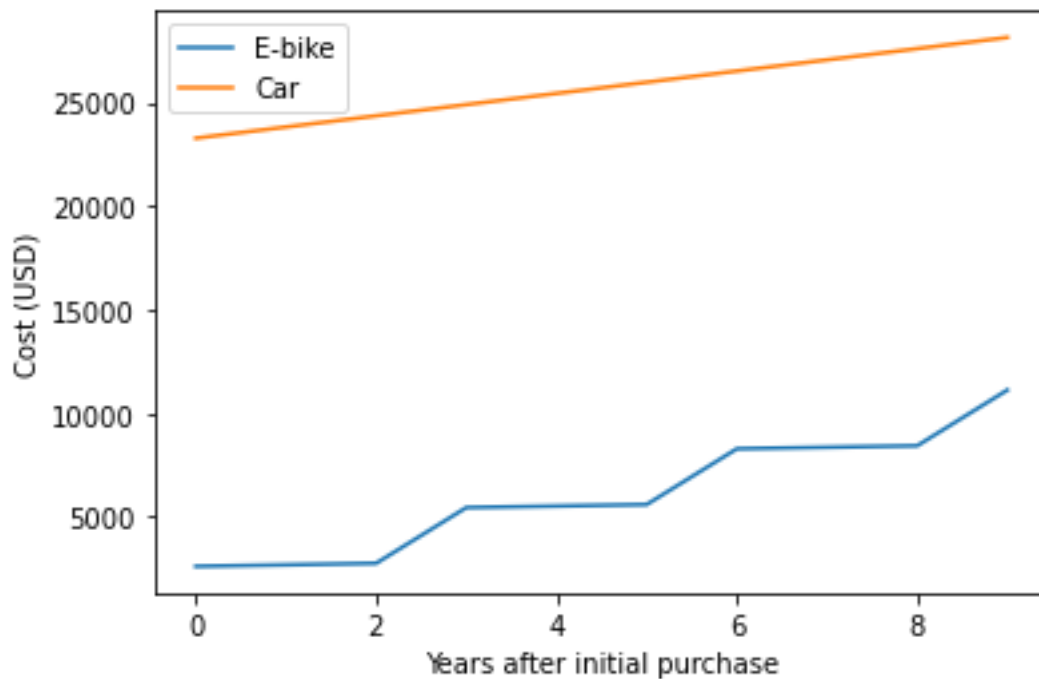
We take both the manufacturing and operating footprint into consideration here. A standard e-bike's manufacturing carbon cost is around 134 kilograms of CO₂ equivalent and emits approximately 14.4 g of CO₂ equivalent per mile (9 per kilometer) due to the electricity consumption needed for charging the battery^[14]. Comparatively, the Citroen C1, a supermini comparable to the Ford Fusion, produces around 6 tons of CO₂ equivalent due to manufacturing costs alone. The 2020 Ford Fusion also emits around 150 grams of CO₂ per mile (94 per kilometer) traveled. Using a yearly mileage of 7300 miles by assumption, we observe the following emissions for a supermini and an e-bike:



Total carbon footprint of an e-bike versus a Ford Fiesta, assuming the bike is replaced every three years

3.4.2 Cost Per Mile Driven

The average fuel economy of a Ford Fiesta is 30 mpg^[22]. Combined with current UK gas prices of 5.79 dollars per gallon, we reach an approximate driving cost of \$0.074 per mile. On an e-bike, the standard battery size is 500 watt-hours^[24], so using the current power cost of \$0.471 per kilowatt-hour, we reach a cost of \$0.21 per charge. Coupled with their 20-mile range, the ultimate cost of driving an e-bike is \$0.01 per mile. Compounded over 10 years, we see the following total price over ten years of driving, assuming an e-bike replacement of three years costing \$2,618^[25] and only an initial purchase of a Fiesta costing \$23,280.67^[23] (19330 GBP):



Cost of exclusively driving an e-bike or a car, measured in years

3.4.3 Commute Time

In 2019, the average distance travelled by person for commuting by car was approximately 782 miles, or 2.14 miles per day^[26]. In addition, the national average commute time in the UK is 27 minutes^[27]. Since e-bikes cannot exceed a velocity of 15.5 mph^[28], we assume an average velocity of a trip on an e-bike to be 10 mph to account for starts and stops, avoiding pedestrians, allowing cars to pass, and the like, resulting in a reduced commute time of 12.84 minutes.

3.5 Strengths and Weaknesses

These models and calculations are forced to make many generalizations because of the unpredictability of an average person's day. On weekends, for example, commute time may be greatly reduced, while total distance traveled may nevertheless increase as people take more leisure trips at larger distances

away, which would also require the use of a more powerful vehicle. However, by comparing the worst-case e-bikes and the best-case cars, we can find a lower bound for the advantages possessed by e-bikes. An e-bike with a more powerful motor, or a larger range and greater battery efficiency, or one that is cheap to maintain and has a long lifespan may change the figures to favor them even more.

Furthermore, the model is not able to account for technological innovation in the transportation sector, nor can it predict the fluctuations of gasoline prices. As a result, the discovery of a technology drastically improving fuel economy or reducing emissions could eventually make cars the favorable 15alternative, although it is an unlikely occurrence, as e-bike technology will also continue to grow.

Conclusion

The rise of electric bicycles is no doubt sudden, but it is not without due reason. The growth of electric bicycles as a means of daily transportation and a genuine, complete replacement from traditional methods of transportation comes as proof of an economic shift away from motor vehicles as the price to own and operate a car steadily increases, and technological advancements make electric bicycles increasingly more competitive to consumers. With this change, those that choose to utilize the electric bike over previous forms of transportation will notice improvements in their health, commute-time efficiency, carbon footprint, and most importantly, their funds.

References

- [1] - "Historical rate tables," <https://www.xe.com/currencytables>.
- [2] - Jeulin, Arnaud. "Pence Sterling (GBX) to US Dollar (USD) Converter," <https://www.mataf.net/en/currency/converter-GBX-USD?m1=1.00>.
- [3] - "Gasoline station sales in the United States from 1992 to 2021," <https://www.statista.com/statistics/197637/annual-gasoline-station-sales-in-the-us-since-1992/>
- [4] - "U.S. Electric Vehicle Market Size, Share & COVID-19 Impact Analysis, By Vehicle Type (Passenger Cars, Commercial Vehicles) and Regional Forecast, 2021," <https://www.fortunebusinessinsights.com/u-s-electric-vehicle-market-106396>.
- [5] - Ride Like the Wind Without Getting Winded: The Growth of E-bike Use, MathWorks Math Modeling Challenge 2023, <https://m3challenge.siam.org/node/596>.
- [6] - Saad, Lydia. "A Seven-Year Stretch of Elevated Environmental Concern," *Gallup News*, <https://news.gallup.com/poll/391547/seven-year-stretch-elevated-environmental-concern.aspx>.
- [7] - Pfirrmann-Powell, Ryan. "Americans' aging autos," *Beyond the Numbers: Prices & Spending*, vol. 3, no. 9 (U.S. Bureau of Labor Statistics, May 2014), <https://www.bls.gov/opub/btn/volume-3/americans-aging-autos.htm>
- [8] - "Used Car Price Trends", <https://www.cargurus.com/Cars/price-trends/>.
- [9] - Bonk, Lawrence, "How Long Do Electric Bikes Last?," <https://www.gadgetreview.com/how-long-do-electric-bikes-last>
- [10] - "How Common is Your Car | CompareTheMarket," <https://www.comparethemarket.com/car-insurance/content/how-common-is-your-car/>.
- [11] - Yurday, Erin. "Number of Cars in the UK 2023," *NimbleFins*, <https://www.nimblefins.co.uk/cheap-car-insurance/number-cars-great-britain>
- [12] - "How to Calculate Growth Rate (With formulas and examples)", Indeed.com. <https://www.indeed.com/career-advice/career-development/how-to-calculate-growth-rate>
- [13] - "Understanding Support Vector Machines Regression," MathWorks, Inc. <https://www.mathworks.com/help/stats/understanding-support-vector-machine-regression.html>.
- [14] - Stott, Seb. "How green is cycling? Riding, walking, ebikes and driving ranked," *BikeRadar*, <https://www.bikeradar.com/features/long-reads/cycling-environmental-impact/>.
- [15] - Minarik, Petr. "64 Incredible Bicycle Statistics and Facts (Updated in 2021)", [cyclistshub.com https://www.cyclistshub.com/bicycle-statistics-and-facts/](https://www.cyclistshub.com/bicycle-statistics-and-facts/)

- [16] - Perry, George. "Why is the Price of Oil so Hard to Predict", Brookings Institution. <https://www.brookings.edu/opinions/why-is-the-price-of-oil-so-hard-to-predict/>.
- [17] - "Crude Oil (CL:NMX)", NASDAQ, Inc. <https://www.nasdaq.com/market-activity/commodities/cl%3anmx/historical>
- [18] - "Technology Cost Trends and key material prices for lithium-ion batteries, 2017-2020", *International Energy Agency*, <https://www.iea.org/data-and-statistics/charts/technology-cost-trends-and-key-material-prices-for-lithium-ion-batteries-2017-2022>
- [19] - "What Average Mileage Is Good for a Used Car in the UK?," *CarAnalytics.co.uk*, <https://www.caranalytics.co.uk/guides/average-car-mileage/>
- [20] - "Electric Bike Range: How Far Can They Go?," *Pedal Electric*, <https://pedalelectric.com/blogs/journal/electric-bike-range-how-far-can-they-go>
- [21] - "UK electric bike sale value hits 23% of total, says Mintel", *Cycling Industry News*, <https://cyclingindustry.news/electric-bike-sales-value-bike-demand-rising/>
- [22] - "Ford Fiesta MPG – Actual MPG from 3,021 Ford Fiesta Owners," *Fuelly*, <https://www.fuelly.com/car/ford/fiesta>
- [23] - "Ford Fiesta Review & Prices 2023 | AutoTrader UK," *AutoTrader*, <https://www.autotrader.co.uk/cars/ford/fiesta?refresh=true>
- [24] - "Guide to e-cycle batteries | Cycling UK," *Cycling UK*, <https://www.cyclinguk.org/article/guide-e-cycle-batteries>
- [25] - "How Much is an Electric Bike? [E-BIKE COST ANALYSIS]," *Discerning Cyclist*, <https://discerningcyclist.com/electric-bike-cost/>
- [26] - "Average distance travelled per person for commuting purposes in England in 2019, by mode of transport," *Statista Research Department*, <https://www.statista.com/statistics/466097/average-distance-travelled-for-commuting-purposes-by-mode-england/>
- [27] - "Transport Statistics Great Britain: 2022 Domestic Travel," *United Kingdom Department for Transport*, <https://www.gov.uk/government/statistics/transport-statistics-great-britain-2022/transport-statistics-great-britain-2022-domestic-travel>
- [28] - "Electric bikes: licensing, tax and insurance," *United Kingdom Department for Transport*, <https://www.gov.uk/electric-bike-rules>
- [29] - "European Countries by Population," *Worldometer*, <https://www.worldometers.info/population/countries-in-europe-by-population/>

[30] - Tucker, Sean. "Average New Car Price Tops \$47,000," *Kelly Blue Book*, <https://www.kbb.com/car-news/average-new-car-price-tops-47000/>

[31] - "Electric Bike Maintenance Cost," *BicycleVOLT*, <https://bicyclevolt.com/electric-bike-maintenance-cost/>

Code Appendix

Part I

```
import numpy as np
from sklearn.svm import LinearSVR

price_dataset = np.array((
    (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13), # years since 2006
    (98, 173, 279, 422, 588, 716, 854, 907, 1139, 1364, 1637, 2074, 2767, 3397))) # Number of bikes sold in Europe

price_dataset[1] = np.sqrt(price_dataset[1]) # Use square root of the number of bikes sold to improve accuracy of model
model = LinearSVR(C=1.0, # use a C parameter of 1.0 so that the model does not over fit or under fit
    max_iter=10000) # use a max_iter parameter of 10000 so that the model does not "fail to converge"

model.fit(price_dataset[0].reshape(-1, 1), price_dataset[1]) # fit the linear SVM to the given data

print("R^2: {:.3f}\n".format(
    model.score(price_dataset[0].reshape(-1, 1), price_dataset[1]))) # print coefficient of determination

# use the model to predict for 2023.
# Make sure to square the result to undo the square root used to build the model
prediction_for_2023 = model.predict(np.array((2023 - 2006)).reshape(-1, 1))[0] ** 2
print("E-Bikes sold in Europe in 2023: {:.3f}".format(prediction_for_2023))

# use the model to predict for 2025.
# Again, make sure to square the result to undo the square root used to build the model
prediction_for_2025 = model.predict(np.array(2025 - 2006).reshape(-1, 1))[0] ** 2
print("E-Bikes sold in the Europe in 2025: {:.3f}".format(prediction_for_2025)) # Print predicted sales for 2025
print("Growth since 2023: {:.3f}%\n".format(
    prediction_for_2025 / prediction_for_2023 * 100 - 100)) # print the growth since 2023

# use the model to predict for 2028.
prediction_for_2028 = model.predict(np.array(2028 - 2006).reshape(-1, 1))[0] ** 2
print("E-Bikes sold in the Europe in 2028: {:.3f}".format(prediction_for_2028)) # Print predicted sales for 2028
print("Growth since 2023: {:.3f}%\n".format(
    prediction_for_2028 / prediction_for_2023 * 100 - 100)) # print the growth since 2023
```

Part III

```
import matplotlib.pyplot as plt

def bikeEmissions(year):
    if year == 0:
        return 134
    return ((year + 1) % 3 % 2) * 134 + 7300 * .0144 + bikeEmissions(year - 1)
#bikeEmissions(i) returns the total emissions from year 0 until i years after the initial purchase.

bike = [x for x in range(0, 10)]
for i in range(0, 10):
    bike[i] = round(bikeEmissions(i),2)

plt.plot([x for x in range(0, 10)], bike)
#calculate and plot e-bike emissions for ten years

def carEmissions(year):
    if year == 0:
        return 6000
    return 7300 * .15 + carEmissions(year - 1)
#carEmissions(i) uses a similar structure as bikeEmissions(), but assumes the car is never replaced

car = [x for x in range(0, 10)]
for i in range(0, 10):
    car[i] = round(carEmissions(i),2)
#calculate and plot car emissions for 10 years

plt.plot([x for x in range(0, 10)], car)

plt.xlabel("Years after initial purchase")
plt.ylabel("Carbon dioxide emissions (kg)")

plt.legend(['E-bike', 'Car'])

plt.show()
```

```
import matplotlib.pyplot as plt

def bikeCost(year):
    if year == 0:
        return 2618
    return ((year + 1) % 3 % 2) * 2618 + 7300 * .01 + bikeCost(year - 1)
#returns the total cost of purchasing and using an e-bike for a given number of years

bike = [x for x in range(0, 10)]
for i in range(0, 10):
    bike[i] = round(bikeCost(i),2)

plt.plot([x for x in range(0, 10)], bike)
#calculate and plot the values for an e-bike

def carCost(year):
    if year == 0:
        return 23280.67
    return 7300 * .074 + carCost(year - 1)
#finds the cost of purchasing and using a car for a given number of years, assuming no replacement

car = [x for x in range(0, 10)]
for i in range(0, 10):
    car[i] = round(carCost(i),2)
#calculate and plot the values for a car

plt.plot([x for x in range(0, 10)], car)

plt.xlabel("Years after initial purchase")
plt.ylabel("Cost (USD)")

plt.legend(['E-bike', 'Car'])

plt.show()
```