Decomposing the Government-Private School Performance Differential: Village Ethnic Politics and School Sorting

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Abstract

The emergence of rural, secular, affordable private schools across South Asia is one of the most promising developments in the education sector in decades. Yet the question of whether private schools are actually superior to government schools remains unsettled. Observational studies consistently show that private school students outperform government school students even when controlling for demographic characteristics and some unobservable heterogeneity. Nevertheless, it remains unclear whether this is because (a) private schools provide students with a better education, or (b) students attending private schools are more academically inclined in unobservable ways. Using data from the Learning and Educational Attainment in Pakistan Schools (LEAPS) survey, this paper sheds new light on this question by comparing private school performance in villages where students are known to sort on intelligence with villages where school choice is motivated by caste politics, not academic potential. It finds observational estimates of private school performance fall by half moving from villages with sorting on perceived intelligence to villages without sorting on perceived intelligence, suggesting at least 50% of the perceived difference between government and private school performance can be explained by differences in student composition, not teaching quality.

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1 Introduction

The rapid rise of affordable and purportedly high quality private schools in South Asian rural communities is one of the most exciting developments in the education sector in decades. Private schools account for an ever rising share of children attending school – in 2005, 33% of Pakistani primary school students and 20-24% of Indian rural primary school students attended a private school, and the students in these private schools consistently outperform their government school counterparts, even when controlling for observable student characteristics (Jimenez et al., 1991; Jimenez and Lockheed, 1995; Pratham, 2005; Andrabi et al., 2011; Desai et al., 2009; Tooley and Dixon, 2003; Alderman et al., 2003, 2001). This has given rise to the hope that private schools may someday circumvent reform-resistant government schools and finally deliver quality education to the hundreds of millions of children in the region.

Despite the promise of these developments, however, the true significance of the rapid emergence of private schools hinges critically on the question of whether these private schools are actually delivering superior educations, or whether they just attract students who are more academically inclined or come from families that prioritize educational attainment.

This paper leverages variation across villages in the factors that drive school choice to provide new insight into this question. In particular, it takes advantage of the fact that school choice in rural Pakistan is motivated by different factors in caste-homogeneous and caste-heterogeneous villages. This paper shows that in caste-diverse villages, school choice is driven by social considerations – high-status families send their children to private schools to keep them in homogeneous social settings. In caste-homogeneous villages where caste sorting is unnecessary, by contrast, school choice is driven by parental perceptions of academic potential – families send their more academically gifted children to private schools.

To take advantage of this variation, this analysis proceeds in three steps. First, this analysis estimates the performance differential between government and private schools using lagged-value-added models applied to a four-year panel of child test scores with demographic controls. This technique is currently considered to be the most rigorous method of studying observational education data (Gordon et al., 2006; McCaffrey et al., 2003; Hanushek, 2003), and is able to control not only for observable differences in child demographics, but also some non-observable differences.[1] This constitutes a baseline estimate of the government-private school performance differential using non-experimental data.

This analysis then compares estimates of the government-private performance differential in homogeneous villages with the performance differential in heterogeneous villages. Because school choice is primarily based on academic potential in homogeneous villages and not in heterogeneous villages, under a mild set of assumptions detailed below, differences in the performance differential between homogeneous and heterogeneous villages can be attributed to differences in academic sorting.

This analysis finds that while private schools outperform government schools in all villages, the amount they outperform government schools falls by half when moving from homogeneous villages to heterogeneous villages. This implies that at least half of the superior performance of private schools is due not to better teaching, but rather to unobservable differences in the quality of students in private schools that cannot be accounted for by lagged-value-added models.

This conclusion is supported by two other sets of results presented in this analysis. First, and

[1] Lagged-value-added models control for unobservable differences that affect test score *levels*, although they cannot control for unobserved heterogeneity in learning rates. These issues are discussed in more detail in Section 4.1.

most importantly, this analysis is unable to find any other differences between homogeneous and heterogeneous villages which might account for changes in performance differential. As shown in Section 2.2, for example, caste fragmentation does not appear to be well correlated with village median wealth, adult literacy, land inequality, the number of schools per household, or number of households. Moreover, other factors often cited as explanations for the government-private performance differential – like performance pay in private schools or differences in school resources – do not appear to vary systematically with village heterogeneity 5.

Second, at the level of villages there is no evidence that overall learning outcomes vary with village caste composition. As detailed in Section 4.3, the decrease in government-private school performance differential is the result of convergence between the two school types, consistent with a re-distribution of students rather than actual differences in teaching quality.

There are two nuances to the conclusions drawn here that are worth noting. First, the difference between homogeneous and heterogeneous villages is best interpreted as a *lower-bound* on the contribution of sorting to estimates of the government-private performance differential. There is likely still *some* sorting on potential in heterogeneous villages. Thus the comparison between heterogeneous and homogeneous villages is best understood as a comparison between villages with sorting and villages with *less* sorting, not a difference between villages with and without sorting.

Second, this analysis is motivated by the assumption that sending high-caste children to private schools and low-caste children to government schools does not constitute sorting on ability. For this to be true, it must be the case that residual academic potential – potential that cannot be accounted for in by things like parental education and wealth – must be equally distributed across different castes (or be distributed slightly in favor of lower status *biraderis*). As shown in Section 4.4, however, there is no evidence that those from higher social status *biraderis* have higher residual talent than those from low status *biraderis*; student caste does not appear to have any consistent effect on test scores.

It is difficult to overstate the potential importance of the answer to this question for education policy in the developing world. Not only do private schools constitute a substantial portion of current enrollments in South Asia, but enrollment is also growing explosively. From 2000 to 2005 in rural Pakistan, for example, the number of private schools in Pakistan rose from 32,000 to 47,000. (Andrabi et al., 2007, p. vi), and evidence suggests this growth continues today. Moreover, private schools deliver educations at a fraction of the cost of government schools by not requiring formal teacher training and by hiring local secondary-educated women as teachers rather than college-educated teachers who have to move to the villages where they teach (Andrabi et al., 2007). Thus the question of whether these lower-cost educations are of similar quality to government school educations also speaks to the importance of different teacher training and employment practices.

The findings of this paper are unlikely to put to rest the debate over whether private schools are superior to government schools. Indeed, private school students continue to outperform government school students even in the most heterogeneous villages, just by a dramatically smaller margin. But even after using some of the most sophisticated econometric methods available, this paper still establishes (as a "lower bound") that sorting explains a very large portion of the public-private test score gap. This should give analysts pause when examining other empirical results that claim to fully control for sorting.[2]

[2]It also bears noting that experimental studies of government-private school test differentials are, as currently implemented, not an empirical silver bullet. Randomizing school assignments is untenable, but in two major cases private school vouchers have been randomly assigned. Even these randomizations have proven problematic, however. Angrist

This paper is organized out as follows: Section 2 provides an overview of the rural Pakistan context from which data for this analysis is drawn. Section 3 details how the determinants of school choice vary with village caste heterogeneity. Section 4 then shows how the government-private test score gap varies with caste heterogeneity, and presents evidence this is due to differences in student sorting. Section 5 then examines and rules out a number of alternative possible explanations for this empirical regularity. And finally, Section 6 discusses the strengths and weaknesses of these findings and their interpretation.

2 Study Context

The focus of this paper is the rural, secular, primary school educational ecosystem of 112 randomly selected villages (*mauzas*) in the Punjab districts of Attock, Faisalabad, and Rahim Yar Khan in Pakistan. These 112 villages were the subject of the Learning and Educational Attainment in Punjab Schools (LEAPS) panel survey. LEAPS villages were selected through probability proportional to population sampling (stratified at the district level) from the universe of all villages in these districts with at least one private school. The survey ran from to 2003-2007, and includes detailed surveys of households, students, teachers, and both government and private schools in these villages. In addition, some data used in this analysis comes from a listing census conducted in sample villages prior to the start of the LEAPS survey that include basic demographic information on all households, allowing for the computation of accurate village level statistics.

The LEAPS survey is organized around two panels of students – one (initiated in 2003 with an initial population of 12,110 children) which followed students for four years, and one (initiated in 2005 with an initial population of 11,852 students) which followed students for two years. Each panel represents the universe of enrolled students in sample villages in Class 3 in both government and private schools. Both students and their teachers in both panels were administered annual exams in English, math, and Urdu by the LEAPS team, and these tests were subsequently standardized using Item Response Theory (IRT) methods.

2.1 Caste in Punjab

This paper focuses on how village-level caste politics influence the selection of students into private and government schools. Before directly examining this dynamic, however, some background on the nature of caste in Pakistan is warranted.

Caste – known variously as *biraderi* or *zaat* – is a central aspect of rural social identity in Pakistan, especially in Punjab. While *biraderi* is a somewhat distinct concept from the idea of "caste" in India, "it retains a very important feature of the [Indian subcaste] – that of an inherent, inbuilt hierarchy that governs social interactions. Society is hierarchically ordered with the Syeds at the top, followed by the landowning castes, then by the service castes or kammis, and finally by the Musallis, who occupy the lowest rung of the social ladder. This ordering dictates much of the social life in a Punjabi village and is most profound in the notions of community cooperation, where solidarity is strongest within a biraderi."(Gazdar and Mohmand, 2007, p. 29).

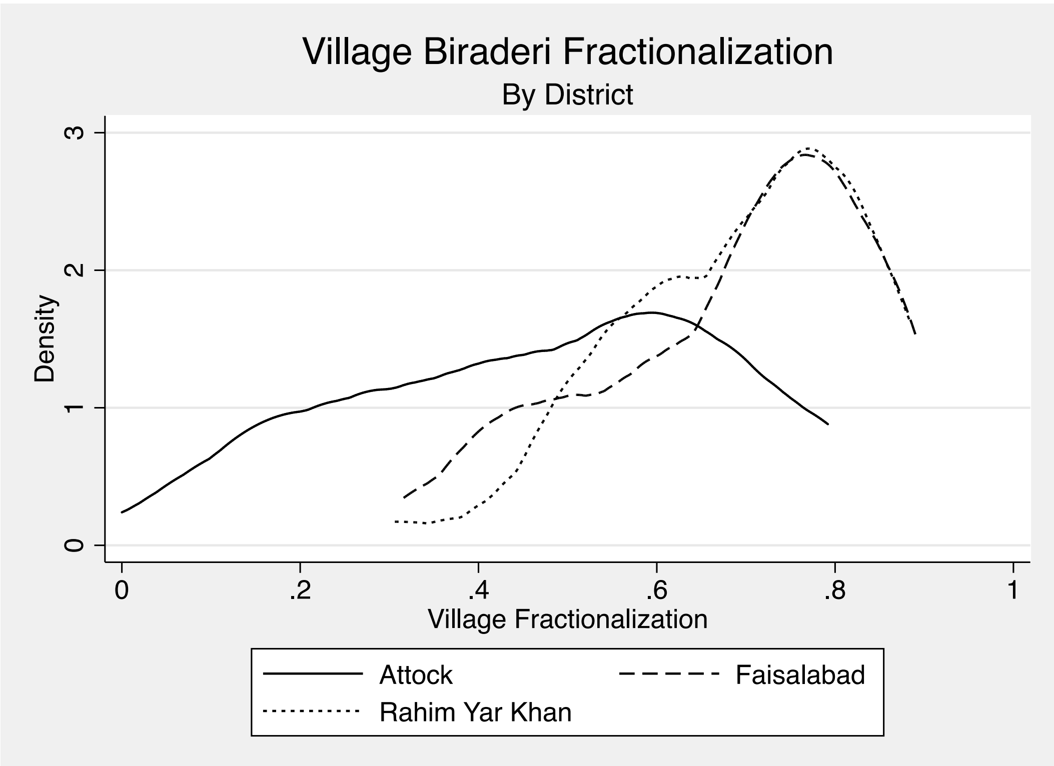
et al. (2002) examines a voucher lottery system in Colombia and finds a small positive effect of vouchers, but inference is clouded by the fact that voucher students who performed poorly were at risk of losing their vouchers, making it impossible to separate this incentive effect from the private school effect. And several studies have been conduced of a voucher system in Chile, but as Bellei (2008) notes, the slight private-school advantage these studies show may be down to the fact that private school admissions are selective and poorly performing students can be expelled from private schools, making it difficult to disentangle selectivity from school effects.

Biraderi is correlated with wealth, land holdings, and education, but it is not synonymous with economic class. As a related report observes, "while economic power is required to reinforce biraderi-based dominance, membership of a dominant biraderi can help mitigate some of the effects of being economically poor. As one respondent put it, 'the poorest Jatt is still better off than the richest kammi.'" (Gazdar and Mohmand, 2007, p. 13)

2.2 Village Caste Composition

While the centrality of caste politics is relatively universal across villages in Punjab, however, there is significant variation in village *biraderi* composition. Figure 1 below shows the density plot of villages of different levels of caste fractionalization, as measured using a simple herfindahl index.[3] As the figure shows, there is significant variation in the degree of fractionalization both within and across the three districts of the LEAPS survey.

Figure 1: Village Caste Fragmentation by District



As shown in Table 1, this variation in heterogeneity is not clearly related to village wealth, land inequality, or adult education. There is some relationship to primary school enrollment rates and village size, but on the whole fractionalization appears to be relatively independent of other compositional characteristics of villages.

[3]The herfindahl is a common measure of fractionalization equal to the probability that any two randomly selected individuals belong to the same group. Village fractionalization is computed using data from a village census conducted in 2002 to facilitate household sampling for the LEAPS survey which includes data on the *biraderi* of all households in LEAPS villages.

Table 1: Village Characteristics and Fractionalization

	(1) Median Wealth	(2) Adult Literacy	(3) Land Gini	(4) Enrollment Pct	(5) Schools per HH	(6) Log Num HH
Fractionalization	-314.1	-6.66	0.10	-14.1	0.0010	0.64*
	(146.8)	(15.0)	(0.074)	(13.3)	(0.0073)	(0.21)
District FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	112	112	112	112	112	112

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3 Caste Politics and School Sorting

The key to determining whether the government-private test score gap is caused by private schools delivering quality educations or by more academically-inclined students attending private schools is understanding what motivates parents to pick one type of school over the other. This Section provides an overview of how households make these choices, and how those choices vary by village caste composition.

3.1 Selection in Homogeneous Villages

As shown in Table 2 below, which regresses the choice to send a child to a private school on a number of characteristics, parental perceptions of child intelligence are an extremely strong predictor of whether a parent will send their child to a private school. Most notably in this table, this pattern holds even *within individual households*. As shown in Column 2, which includes household fixed effects, many parents send the child they perceive to be more intelligent to private school and the child they perceive to be less intelligent to government schools.

Indeed, this pattern also extends beyond school choice into other domains, such as household expenditure on educational materials and the amount of time parents spend helping their children with school work. In the words of the original LEAPS survey authors, "through their choices of whether to enroll a child, through the choice of school ([government] or private) and finally through the amount they chose to spend, households pick "winners" and try to carry them through." (Andrabi et al., 2007, p. 103)

The implications of this behavioral tendency for understanding the government-private school test gap is clear: if parents are choosing to send their more academically-inclined children to private schools, and if parents have more information about student quality than researchers are able to measure in surveys and control for statistically, then standard analyses are likely to systematically overstate the quality of private school educations.

3.2 Selection in Heterogeneous Villages

While the tendency for parents to invest in "winners" rather than distribute resources is a general tendency in the LEAPS data, it is not the only factor that shapes school choice. In more caste-heterogeneous villages, the *social* composition of schools becomes increasing salient and washes

Table 2: School Choice and Child Intelligence

	(1) Village FE	(2) HH FE
Mom Reports Child Above Average Intelligence	0.058***	0.043**
	(0.021)	(0.021)
Mom Has Some Schooling	0.077	-0.029
	(0.056)	(0.12)
Dad Has Some Schooling	0.080***	0.083
	(0.026)	(0.12)
PCA Wealth Index	-0.029	.
	(0.022)	.
Age	-0.0049	0.0066
	(0.023)	(0.029)
Age Squared	-0.00060	-0.0011
	(0.0012)	(0.0015)
Female	0.032	0.0016
	(0.022)	(0.030)
Observations	3346	3346

Standard errors in parentheses

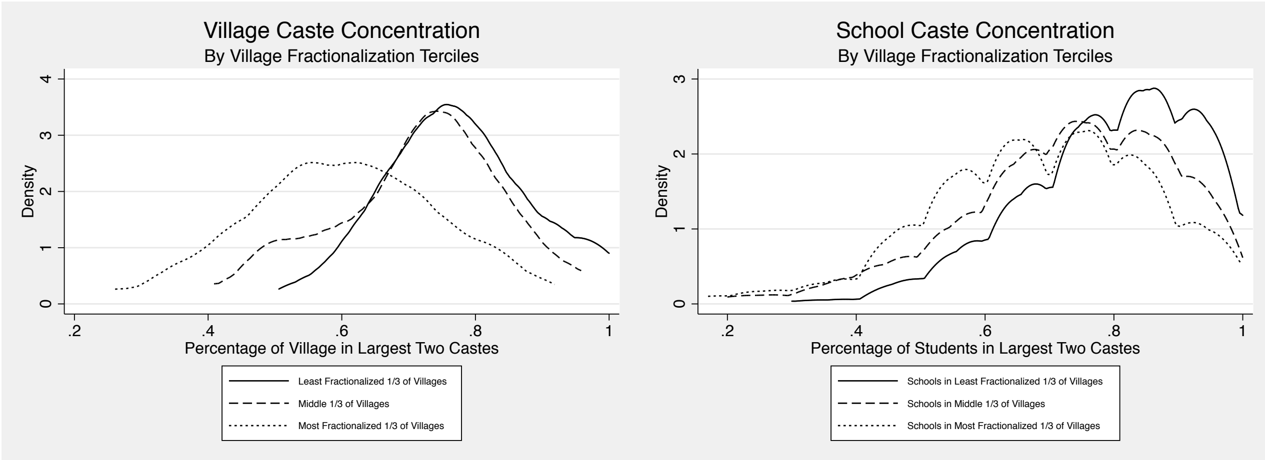
* p¡0.10, ** p¡0.05, *** p¡0.01

out much of this "sorting on intelligence." Instead, *all* children from "high status" *biraderis* – regardless of perceived academic potential – are sent to private schools to isolate them from lower-status families. And as a result, children from "low status" *biraderis* become concentrated in government schools. School choice, in other words, ends up being driven by social rather than academic considerations.

3.2.1 Evidence of Segregation

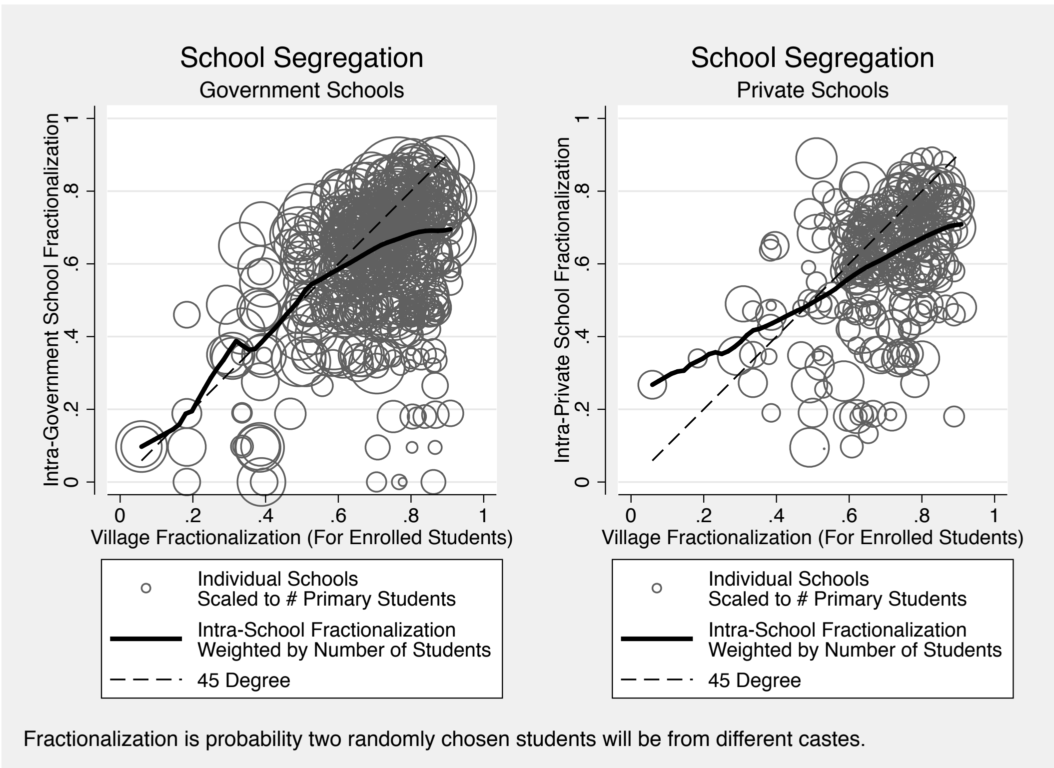
Figure 2 plots the population share of the two largest biraderis among students in the village at large and in each school level respectively. If students were uniformly distributed across schools, these figures would look nearly identical. In reality, however, in almost all schools the vast majority of students belong to only two (or fewer) castes, even in highly diverse villages.

Figure 2: Village and School Fractionalization



Segregation can also be seen in a comparison of village-level herfindahls and intra-school herfindahls. If schools were unsegregated, then we would expected the herfindahl indices computed *within* each school to track closely with herfindahl indices computed at the village level. Yet as shown in Figure 3, this is far from the case. Almost all schools are below the 45 degree line that would indicate school and village diversity moving one for one, and many are well below.

Figure 3: School Versus Village Fragmentation



3.2.2 Evidence of High Caste Private School Attendance

This data shows a clear pattern of segregation, but it does not provide an entirely clear picture of which groups are attending which schools. Grouping *biraderis* into "high" and "low" social status groupings allows for a better understanding of segregation patterns. The crudeness of these categorizations is unfortunate, but necessary – although *biraderis* are associated with strict hierarchies within villages, there does not exist an explicit global hierarchy of *biraderis* in Pakistan as in with the more familiar *varna* caste designations India. As a result, these hierarchies may vary somewhat from village to village, and as noted previously, this variation may not perfectly follow economic position.

To estimate the social status of different *biraderis*, Punjabi Pakistanis recruited on *oDesk.com* were asked to classify *biraderis* as having either "high" or "low" social status. Details of classifications can be found in Appendix B.[4]

As shown in Table 3, in villages with higher caste fractionalization, a larger share of private school students come from higher status *biraderis* and a larger share of government school students come from low *biraderis*. Private schools, in other words, become reservoirs of the social elite.

Table 3: Student Body Social Composition

	(1) Pct of Students High Status	(2) Pct of Students High Status
Private School	-0.097**	-0.11*
	(0.045)	(0.060)
Biraderi Fractionalization	-0.080**	-0.025*
	(0.037)	(0.014)
Fractionalization * Private	0.19**	0.22**
	(0.083)	(0.11)
Median Village Expenditure	-0.0000032	
	(0.0000027)	
Village: Pct Adults Literate	-0.00011	
	(0.00024)	
Log Village Size	-0.0058	
	(0.0050)	
Village: Pct High Status	1.03***	
	(0.024)	
District Fixed Effects	Yes	No
Village Fixed Effects	No	Yes
Observations	772	772

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

Demand for caste segregation is also manifest in the dramatically higher prices charged by segregated private schools. As shown in Table 4 below, moving from a perfectly non-fractionalized village to a perfectly fractionalized village is associated with a 600 Rupees increase in annual

[4]This work has avoided the Jacoby and Mansuri (2011) methodology – where castes are ranked on the basis of their land holding – due to input from numerous sources that social standing and land holding are not equivalent, and in this exercise *social*-status is of substantially more importance than *socio-economic* status.

school fees. Given that the average annual fee for all private schools in the LEAPS survey is 1191 Rupees, this is a very significant amount.[5]

Table 4: Annual Private School Fees

	(1) Weighted by School	(2) Weighted by School	(3) Weighted by Primary Students
Biraderi	375.7	390.2	409.2
Fractionalization	(251.5)	(257.7)	(307.5)
Village: Median		51.8	34.0
Expenditures		(65.5)	(58.9)
Expenditure Gini		87.3	95.2
		(224.2)	(244.1)
District Fixed Effects	Yes	Yes	Yes
Observations	296	296	295

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

Further, as shown in Table 5, none of these changes are driven by a change in the share of students in private schools. The percentage of students in private schools in almost perfectly stable, even when controlling for numerous village characteristics.

Table 5: Share of Enrolled Students in Private Schools

	(1) private_share	(2) private_share
Biraderi Fractionalization	0.18	0.14
	(0.14)	(0.14)
Median Village Expenditure		0.000070**
		(0.000026)
Village Land Gini		0.61**
		(0.27)
Village: Pct Adults Literate		0.00061
		(0.0025)
Log Num HHs		0.010
		(0.039)
District Fixed Effects	Yes	Yes
Observations	51	51

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

[5] Fees above the 95th percentile – 1900 Rupees – were adjusted down to 1900 Rupees. Without this adjustment, the coefficient on village fractionalization is approximately 950 Rupees with a t-stat of 2.09

4 School Sorting and Test Scores

Having established that the determinants of school choice vary dramatically between caste-homogeneous and caste-heterogeneous villages in Section 3, this section now turns to an analysis of how differences in sorting impact the performance differential between government and private schools. If private schools outperform government schools primarily due to differences in the quality of instruction, then the government-private performance differential should be relatively stable across villages with different caste compositions. If, however, private schools outperform government schools primarily due to differences in the composition of their students, then villages that are subject to different sorting processes should also see differences in the government-private differential. Specifically, the degree to which private schools out performance government schools should decline as one moves from caste-homogeneous villages (where students sort on academic potential) to caste-heterogeneous villages (where sorting is driven by considerations of caste politics).

4.1 Measuring Learning

To measure learning, this analysis employs a lagged-value-added model. Lagged-value-added models have increasingly become the norm in the education research (Gordon et al., 2006; McCaffrey et al., 2003; Hanushek, 2003) due to their potential to take into account not only observational differences between students, but also the potential to control for some unobserved differences, a subject discussed in more detail below.[6]

The lagged-value-added model incorporates the assumption that current knowledge is an additive function of all current and past inputs and an i.i.d. stochastic error term, and can be expressed as:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \epsilon_{i,t} \tag{1}$$

where $Y_{i,t}$ is child i's test scores at time t and $X_{t,i}$ is a vector of child, school, and village controls at time t (a full discussion of the lagged-value-added model and its assumptions can be found in Appendix A).

Note that while the inclusion of a lagged dependent variable effectively controls for unobserved differences that affect differences in test *levels*, it cannot control for unobserved heterogeneity that affects learning *rates*. It is for this reason that while superior to other available methods, value-added analyses can not fully overcome selection issues.[7]

In the lagged-value-added model, coefficients on independent variables are interpreted as the contribution of each variable to learning. In this analysis, village-school-type dummies are added to measure the performance of each type of school in each village. (In other words, results include one dummy for private schools in village 1, one dummy government schools in village 1, one

[6]Lagged value-added models also account for the fact that learning is not entirely persistent (things learned in the past are often forgotten). This flexibility is discussed in more detail in Appendix A.

[7]Some analysis have turned to second-differencing the data and focusing on students who change schools (Andrabi et al., 2011), but these analyses have their own limitations, among them limited sample sizes (given that changes between types of school are relatively infrequent in most surveys) and the assumption that school changes are not the result of some unobserved shock (i.e. that school switches are not accompanied by contemporaneous with other changes – a potentially problematic assumption given the relative infrequency with which students change schools).

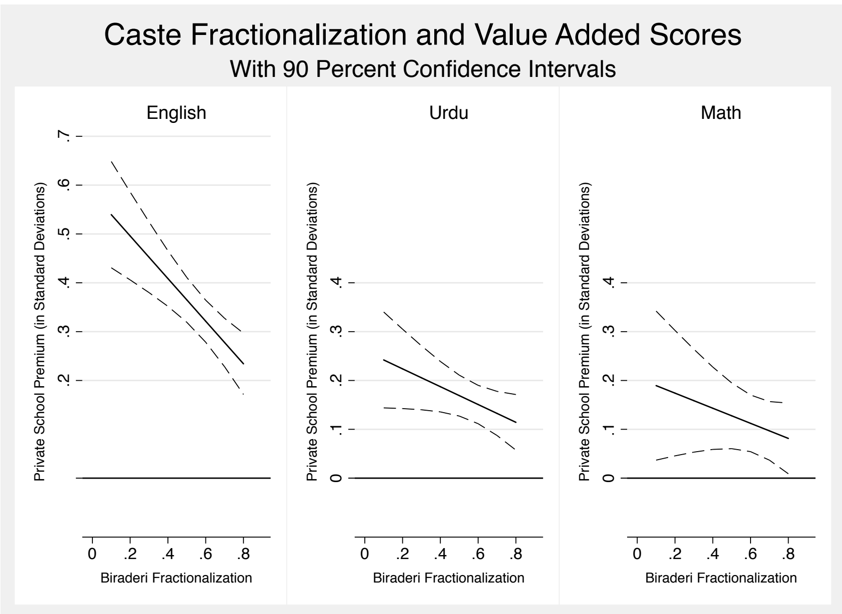
dummy for private schools in village 2, etc.). The difference between the private school and government school dummy coefficients within a village is the village-level measure of government-private performance differential.

4.2 Convergence in Government-Private Test Scores

Table 6 presents lagged-valued-added estimates of learning as a function of various demographic controls and village fractionalization. It shows that the effect of caste fractionalization on the government-private performance differential is negative and significant for English and Urdu, and negative (albeit insignificant) for math. Further, as shown in columns (2), (5), and (8) of Table 6, the inclusion of various demographic controls such as a child wealth index and dummies for parental education along with the village fixed effects has no significant effect on the results.

To aid in interpretation, Figure 4 plots the government-private performance differential as a function of caste fractionalization (these plots correspond to columns (2), (5), and (8) respectively). In all three cases, the rise in fractionalization is associated with a near 50% decline in the private school premium, although this is by far most striking in the case of English – which is generally considered the path to upward mobility in Pakistan, and is often the focus of private schools in Punjab.

Figure 4: Private School Test Score Premium with Lagged Scores



4.3 Decomposition of Convergence

Further evidence that the convergence in government-private school test scores is driven differences in student sorting – not differences in actual learning outcomes – comes from the fact that while the government-private school test gap decreases, overall learning remains relatively unchanged. As shown in Table 7, overall test scores are essentially flat across all villages – English scores are slightly higher in more fractionalized villages in Column 1, but the magnitude of this

Table 6: Child Test Scores

	English			Urdu			Math		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Private School	0.62***	0.58***	0.57***	0.27***	0.26***	0.21***	0.21*	0.20*	0.16
	(0.072)	(0.078)	(0.078)	(0.073)	(0.070)	(0.073)	(0.12)	(0.11)	(0.11)
Fractionalization * Private	-0.46***	-0.44***	-0.44***	-0.18	-0.18	-0.11	-0.12	-0.15	-0.11
	(0.12)	(0.13)	(0.12)	(0.12)	(0.11)	(0.11)	(0.18)	(0.16)	(0.15)
Lagged English Scores	0.37***	0.36***	0.39***	0.15***	0.14***	0.14***	0.16***	0.15***	0.16***
	(0.018)	(0.018)	(0.018)	(0.012)	(0.012)	(0.012)	(0.015)	(0.015)	(0.016)
Lagged Math Scores	0.069***	0.072***	0.071***	0.12***	0.12***	0.12***	0.37***	0.38***	0.40***
	(0.0080)	(0.0080)	(0.0083)	(0.0083)	(0.0086)	(0.0086)	(0.013)	(0.014)	(0.014)
Lagged Urdu Scores	0.15***	0.15***	0.15***	0.38***	0.38***	0.40***	0.23***	0.22***	0.22***
	(0.011)	(0.012)	(0.012)	(0.011)	(0.012)	(0.012)	(0.013)	(0.013)	(0.013)
Child's Wealth Index		0.017***	0.015***		0.0073**	0.0068**		0.014***	0.016***
		(0.0033)	(0.0035)		(0.0030)	(0.0030)		(0.0040)	(0.0042)
Educated Parent		0.057***	0.052***		0.052***	0.049***		0.045***	0.043***
		(0.012)	(0.013)		(0.011)	(0.011)		(0.014)	(0.014)
Biraderi Fractionalization			0.23***			0.090			0.15
			(0.084)			(0.068)			(0.11)
Village: Pct Adults Literate			0.00014			-0.00061			0.00024
			(0.00097)			(0.00086)			(0.0013)
Log Number of Households			0.020			0.015			0.011
			(0.013)			(0.014)			(0.018)
Village Land Gini			0.049			0.060			-0.25*
			(0.11)			(0.100)			(0.13)
Constant	0.25	0.40*	0.57**	0.55**	0.69***	0.79***	0.12	0.31	0.61*
	(0.26)	(0.22)	(0.25)	(0.22)	(0.25)	(0.28)	(0.34)	(0.32)	(0.35)
Village Fixed Effects	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
District Fixed Effects	No	No	Yes	No	No	Yes	No	No	Yes
Observations	37147	26141	26141	37147	26141	26141	37147	26141	26141

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

difference is relatively small, and once more demographic controls are added in Column 2 this effect disappears. No relationship exists for other subjects. Government scores increase and private scores decline with fractionalization, in other words, but those changes are almost perfectly offsetting. Indeed, this is also illustrated in Column 3 of Table 6, where village fixed effects are replaced with district fixed effects, allowing for a comparison of test scores levels (rather than just the government-private gap) across villages. In the case of English the convergence appears to be driven in equal parts by improvements in government schools and a decline in private schools.

The fact overall educational attainment remains constant is further evidence that as the determinants of school choice changes, it is the *distribution* of academically inclined students that changes, not the performance of the schools themselves.

4.4 Caste and Residual Academic Potential

For it to be the case that sorting by caste reduces the degree to which private schools enroll disproportionately academically inclined students, it must be the case that residual academic potential – potential that cannot be explained by things like parental education and wealth – must be equally distributed across different castes (or be distributed slightly in favor of lower status *biraderis*). If not, and even the least talented "high status" students were more talented than the most talented "low status" students, then the concentration of "high status" students in private schools would result in *divergence*, rather than *convergence*, of test scores. As shown in Table 8, however, there is no evidence that those from higher social status *biraderis* have higher residual talent than those from low status *biraderis*. As evident from the top row of coefficients, after controlling for other observational factors, student caste does not appear to have any consistent effect on test scores.

5 Alternate Explanations

Observational studies pointing to the superior performance of private schools are not the only reason that education reform advocates have voiced excitement about the potential of private school growth; private schools also operate in fundamentally different ways from government schools. For that reason, it is important to examine not only whether the government-private performance differential varies with caste heterogeneity, but also whether other factors some suspect explain school differences can be ruled out as alternate explanations.

5.1 Differences in Teacher Incentives

Advocates of private schools argue that not only are observational studies able to control for many factors, but there is also evidence to explain *why* private schools outperform government schools. In particular, they point out that private schools appear to address the biggest problem in government schools: low effort. High absenteeism and low accountability in government schools has been well documented (Muralidharan and Kremer, 2008; Chaudhury et al., 2006), but appears less prevalent in private schools. The reason, many argue, is that in private schools good teachers are better paid, and poor teachers are let go. This line of reasoning is also buoyed by a growing body of literature that suggests that what matters for success is not the availability of educational

Table 7: Child Test Scores and Fractionalization

	English		Urdu		Math	
	(1)	(2)	(3)	(4)	(5)	(6)
Private School	0.31***	0.29***	0.14***	0.14***	0.11***	0.088**
	(0.028)	(0.028)	(0.025)	(0.024)	(0.035)	(0.034)
Biraderi Fractionalization	0.14*	0.10	0.080	0.057	0.13	0.12
	(0.075)	(0.073)	(0.066)	(0.064)	(0.094)	(0.089)
Lagged English Scores	0.40***	0.39***	0.16***	0.14***	0.17***	0.16***
	(0.018)	(0.018)	(0.012)	(0.012)	(0.016)	(0.016)
Lagged Math Scores	0.067***	0.070***	0.12***	0.12***	0.39***	0.40***
	(0.0082)	(0.0083)	(0.0083)	(0.0086)	(0.013)	(0.014)
Lagged Urdu Scores	0.15***	0.15***	0.39***	0.40***	0.23***	0.22***
	(0.011)	(0.012)	(0.011)	(0.012)	(0.013)	(0.013)
Village: Pct Adults Literate	0.00058	0.00017	-0.00027	-0.00060	0.00031	0.00025
	(0.00097)	(0.00098)	(0.00086)	(0.00085)	(0.0014)	(0.0013)
Log Number of Households	0.017	0.019	0.012	0.015	0.0083	0.011
	(0.013)	(0.013)	(0.015)	(0.014)	(0.019)	(0.017)
Village Land Gini	-0.0043	0.036	0.0079	0.057	-0.29**	-0.26*
	(0.13)	(0.12)	(0.11)	(0.098)	(0.13)	(0.13)
Child's Wealth Index		0.015***		0.0068**		0.016***
		(0.0035)		(0.0030)		(0.0042)
Educated Parent		0.053***		0.049***		0.043***
		(0.013)		(0.011)		(0.014)
Constant	0.45	0.64**	0.68***	0.81***	0.34	0.62*
	(0.28)	(0.25)	(0.25)	(0.28)	(0.37)	(0.35)
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	37147	26141	37147	26141	37147	26141

Standard errors in parentheses
* p¡0.10, ** p¡0.05, *** p¡0.01

Table 8: Child Social Status and Residual Talent

	English			Urdu			Math		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
High Status Zaat	-0.040	-0.072	-0.11**	-0.037	-0.10	-0.081	0.0014	-0.025	0.037
	(0.045)	(0.070)	(0.046)	(0.058)	(0.071)	(0.052)	(0.066)	(0.085)	(0.067)
Private School	0.32***	0.22*	0.21*	0.28**	0.26*	0.14	-0.037	-0.060	-0.11
	(0.12)	(0.13)	(0.12)	(0.12)	(0.14)	(0.13)	(0.11)	(0.14)	(0.15)
Fractionalization * Private	-0.040	0.081	0.048	-0.21	-0.19	-0.067	0.28	0.33	0.29
	(0.20)	(0.21)	(0.20)	(0.19)	(0.23)	(0.20)	(0.19)	(0.24)	(0.24)
Lagged English Scores	0.32***	0.31***	0.41***	0.16***	0.16***	0.16***	0.18***	0.19***	0.18***
	(0.042)	(0.046)	(0.040)	(0.031)	(0.036)	(0.029)	(0.045)	(0.046)	(0.040)
Lagged Math Scores	0.060**	0.048	0.045	0.11***	0.076*	0.089**	0.30***	0.27***	0.37***
	(0.030)	(0.035)	(0.032)	(0.038)	(0.042)	(0.040)	(0.050)	(0.050)	(0.048)
Lagged Urdu Scores	0.17***	0.19***	0.17***	0.34***	0.35***	0.41***	0.25***	0.26***	0.26***
	(0.038)	(0.046)	(0.042)	(0.039)	(0.045)	(0.046)	(0.046)	(0.053)	(0.052)
Child's Wealth Index		-0.0067	-0.011		0.0020	0.0010		-0.014	-0.0094
		(0.013)	(0.012)		(0.010)	(0.0090)		(0.017)	(0.014)
Educated Parent		0.14***	0.14***		0.14***	0.14***		0.17***	0.18***
		(0.043)	(0.039)		(0.044)	(0.035)		(0.055)	(0.046)
Biraderi Fractionalization			-0.029			-0.063			-0.042
			(0.10)			(0.13)			(0.17)
Village: Pct Adults Literate			0.00037			-0.0032**			-0.0013
			(0.0018)			(0.0016)			(0.0027)
Log Number of Households			0.021			0.025			0.050
			(0.036)			(0.033)			(0.061)
Village Land Gini			0.027			0.11			-0.15
			(0.18)			(0.21)			(0.32)
Constant	0.57	0.71	0.75	1.18*	2.35***	2.22***	1.57**	2.83***	2.19**
	(0.56)	(0.71)	(0.60)	(0.64)	(0.74)	(0.67)	(0.74)	(0.92)	(0.94)
Village Fixed Effects	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
District Fixed Effects	No	No	Yes	No	No	Yes	No	No	Yes
Observations	1790	1323	1323	1790	1323	1323	1790	1323	1323

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

"inputs" (like qualified, well paid teachers or good facilities), but incentive schemes that reward effort on the behalf of teachers (Hanushek, 1997, 2003; Banerjee et al., 2007).

The conclusion of the LEAPS survey authors (full disclosure: this author was a research assistant on the LEAPS project, though not an author) is that this is the case in Pakistan. Private schools deliver better educational outcomes despite hiring only secondary-educated local women with no training and providing them with relatively low wages because private schools incentize good teaching by paying good teachers more, improving effort. This differentiates them from government schools, which offer salaries which are higher but unresponsive to performance. As a result, the authors argue, government school teachers exert less effort. This is illustrated in Figure **??** which appears in the original LEAPS report (Andrabi et al., 2007). It shows that while frequently absent private school teachers are paid less, absent government school teachers are actually paid *more*. And where private school teachers with high score students are paid more, no such relationship exists for government teachers.

DROP FIGURE

Table 9 below recreates the analyses underlying the two plots in Figure **??** with two adjustments: first, rather than measuring performance using average test scores (in levels), more rigorous teacher "value-added" scores (Specification 4) are employed to measure teacher contributions to learning; second, a village fractionalization measure is included in all results as an interaction term. If it is the case that differences in incentive schemes are driving test score convergence, then we should see (a) the private-school salary penalty for absenteeism decline in fractionalization, and (b) the compensation premium (compensation above what is otherwise predicted) for performance decline. As shown in the table, however, the absenteeism penalty actually *increases* in fractionalization (which under the incentive story should result in *increased* private school scores), and no relationship exists between performance and compensation. In sum, there is no relationship between fractionalization and how government teachers are compensated.

Table 9: Village Fractionalization and Teacher Compensation

	Private Teachers		Government Teachers				
	(1)	(2)	(3)	(4)	(5)	(6)	(7
	Log Salary	Log Salary	Log Salary	Log Salary	est5	est6	es
Days Absent Last Month	0.017***	0.014			-0.0060*	0.0094	
	(0.0039)	(0.010)			(0.0033)	(0.012)	
Biraderi Fractionalization		-0.0028		-0.11		0.30	
		(0.13)		(0.094)		(0.20)	
Days Absent * Fractionalization		0.0055				-0.023	
		(0.018)				(0.017)	
Average Value Added Score			0.014	0.23*			0.
			(0.046)	(0.14)			(0.0
Value-Added * Fractionalization				-0.37*			
				(0.20)			
Mauza Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Y
Observations	3685	3685	784	784	4638	4638	37

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

5.2 Differences in School Inputs

A second potential explanation for variation in the government-private school performance differential is that the availability of school resources varies with village caste heterogeneity. To test this, this paper examines the relationship between village fragmentation and different school inputs.

Table 10 presents a series of school-level results in which measures of school- and teacher-quality in private schools are regressed against village fractionalization and a number of controls. The table shows that there is little or no evidence that – at least in terms of visible inputs – differences in inputs can explain the decline in private school performance in fractionalized villages. If anything, private school teachers appear to be slightly *better* educated in fractionalized villages.

Table 11 repeats this exercise for government schools. It should be noted that government schools in Pakistan are administered at the state level, and are thus relatively insulated from village politics, making any such differences unlikely. Nevertheless, results are presented for thoroughness. These results show no difference in school inputs across villages.

6 Discussion and Policy Implications

Private schools represent a radical departure from the educational status quo in nearly every way. Where government schools hire teachers with college educations, emphasize teacher training, and pay high wages, private schools hire secondary-educated women from the local community for a fraction of the cost, invest nothing in teacher training, and pay them in proportion to their performance. If government schools were to emulate their business model, they would save millions. But would educational outcomes improve?

The results presented here cannot conclusively answer this question. Even in the most heterogeneous villages, private schools still outperform government schools (just by dramatically less than in homogeneous villages). But these results do suggest at least two major reasons for caution on the part of policy-makers thinking about the implications of private school growth, and whether recent calls for the widespread distribution of private school vouchers (e.g. Chakrabarti and Peterson (2008), Kelkar (2006), and Panagariya (2008)) are good policy.

First, this analysis shows that even with a lagged-value-added specification, parental education and wealth controls, and panel data, it is still easy to vastly over-estimate the contributions of private schools to learning using observational data. As such, this study illustrates the need for researchers and policy-makers to maintain a health degree of skepticism when reading observational studies that claim to separate sorting from school quality.

Second, while this analysis only shows that 50% of the private school premium can be explained by sorting, it is important to remember that this represents a lower-bound. The difference between homogeneous and heterogeneous villages is not "sorting on intelligence" and "no sorting on intelligence," but rather "sorting on intelligence" and "*less* sorting on intelligence." As such, sorting is still likely contributing to the private school premium in heterogeneous villages. Indeed, perceived intelligence remains an important determinant of school choice even in highly fractionalized villages.

In light of persistent poor performance among government schools, the hope that private schools

Table 10: Private Teacher Characteristics and Village Fractionalization

	(1)	(2)	(3)	(4)	(5)	(6)
	Days Absent Last Month	Female	From Village	Teacher English Exam Score	More than Grade School Education	Basic School Facility Index
Biraderi Fractionalization	-0.81*	-0.029	0.10	0.31*	0.19*	-0.46
	(0.45)	(0.088)	(0.16)	(0.16)	(0.10)	(0.46)
Median Village						
	-0.00013*	0.000011	0.0000025	0.000082***	0.000024	0.000045
Expenditures						
	(0.000066)	(0.000011)	(0.000019)	(0.000025)	(0.000019)	(0.000081)
Log Number						
	-0.070	-0.034**	-0.025	-0.036	0.016	-0.25
of Households						
	(0.13)	(0.017)	(0.041)	(0.050)	(0.023)	(0.15)
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1656	1656	1656	1125	1656	496

Standard errors in parentheses
* p¡0.10, ** p¡0.05, *** p¡0.01

Table 11: Government Teacher Characteristics and Village Fractionalization

	(1)	(2)	(3)	(4)	(5)	(6)
	Days Absent Last Month	Female	From Village	Teacher English Exam Score	More than Grade School Education	Basic School Facility Index
Biraderi Fractionalization	-0.67	0.048	0.13	0.19	0.092	0.45
	(0.56)	(0.13)	(0.19)	(0.19)	(0.11)	(0.68)
Median Village Expenditures	0.000089	0.000031	-0.000016	0.000029	0.000016	-0.000081
	(0.000077)	(0.000027)	(0.000025)	(0.000022)	(0.000015)	(0.000096)
Log Number of Households	-0.068	-0.033	-0.082	-0.015	-0.034**	-0.24*
	(0.096)	(0.024)	(0.050)	(0.036)	(0.015)	(0.13)
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1335	1337	1335	990	1337	295

Standard errors in parentheses

* p¡0.10, ** p¡0.05, *** p¡0.01

will reform the South Asian education sector is understandable, and may yet prove to be well founded. But as shown here the superiority of private school is not self-evident, and the government would do well to gather more evidence before embracing private schools as a substitute for government schools.

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A Value-Added Test Scores

The lagged-value-added model incorporates the assumption that current knowledge is an additive function of all current and past inputs and an i.i.d. stochastic error term. This can be written formally as:

$$Y_{i,t} = \alpha_t X_{i,t} + \alpha_{t-1} X_{i,t-1} + \cdots + \alpha_1 X_{i,1} + \epsilon_{i,t} \tag{2}$$

where $Y_{i,t}$ is child i's test scores at time t and $X_{t,i}$ is a vector of child, school, and village controls at time t. As data on past inputs are usually unavailable, however, they are generally subsumed into a lagged dependent variable included as a control. In this case the lagged-value-added model can be re-written as:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \epsilon_{i,t} \tag{3}$$

Where $Y_{i,t-1}$ is assumed to capture all past inputs and unobservable heterogeneity across students. The specification given by Equation 3 is the model primarily employed in this paper. As the interest of this analysis is on the difference between government and private school students, primary interest is on a dummy for school type included in the vector of controls $X_{i,t}$.

Three aspects of this specification are worth emphasizing. First, while the inclusion of a lagged dependent variable effectively controls for unobserved differences that affect differences in test *levels*, it cannot control for unobserved heterogeneity that affects learning *rates*. It is for this reason that while superior to other available methods, value-added analyses can not fully overcome selection issues.[8]

Second, the β term can be interpreted as the "persistence parameter" in that it estimates the degree to which past learning may carry forward. A value of one is equivalent to assuming that children do not forget past lessons, while a value of zero corresponds to students forgetting all past lessons each year. While imposing a persistence parameter of one may seem reasonable – it amounts to regressing the difference in test scores between time t and time $t-1$ on controls – a growing literature has shown that the test score gains of short term interventions often "die out" over time, suggesting that not all that is learned is retained (Banerjee et al., 2007; Glewwe et al., 2010; Currie and Thomas, 1995; Rothstein, 2010). Andrabi et al. (2011) shows that imposing the restriction that $\beta = 1$ biases learning estimates.

Finally, lagged test scores are generally measured with error, which leads to an often significant attenuation bias in the estimate of β and biased estimates of other coefficients (Kane and Staiger, 2002; Chay et al., 2005; Andrabi et al., 2011). Thus keeping with best practices, lagged test scores from all three subjects – English, Urdu, and math – are included in all specifications to instruments for the primary lagged test score of interest.

Note further that all child-level results are presented using heteroskedastic-robust standard errors clustered at the level of the village.

A.1 Village-Level Estimates

In addition to estimating learning differences at the level of the child, the primary analysis of this paper is also estimated at the level of the village. To do so, Specification 1 is augmented with with fixed effects for each village-school type combination, as below:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \mathbb{I}_{i,j,type,t}\gamma_{j,type} + \epsilon_{i,t}$$

Where $\mathbb{I}_{j,type}$ is a vector of dummies for the village j of child i in school type $type \in \{private, government\}$ at time t. The difference between the village private-school dummy coefficients and the village government-school dummy coefficients are then extracted as a village-level estimate of the government-private test score gap. These village-level gaps are then regressed against a series of village-level controls, include village fractionalization, wealth, size, land fractionalization, and adult literacy variables Z_j reported at the level of the village j, as below:

[8] Some analysis have turned to second-differencing the data and focusing on students who change schools (Andrabi et al., 2011), but these analyses have their own limitations, among them limited sample sizes (given that changes between types of school are relatively infrequent in most surveys) and the assumption that school changes are not the result of some unobserved shock (i.e. that school switches are not accompanied by contemporaneous with other changes – a potentially problematic assumption given the relative infrequency with which students change schools).

$$Gap_j = Z_j\delta + \eta_j$$

A.1.1 Teacher-Value Added Estimates

A similar method is employed when estimating the contribution of individual teachers to child learning. Specification 1 is again employed with the addition of fixed effects for each individual teacher, as below:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \mathbb{I}_{i,k,t}\zeta_k + \epsilon_{i,t}$$

where $\mathbb{I}_k$ is a vector of dummies for whether student i was taught by teacher k in year t. The fixed effect coefficients ζ_k are then extracted as an estimate of teacher k's contribution to student learning in various results. Teacher results are all weighted by the number of students taught by a given teacher, both because our interest is in the experience of the average child and because value-added estimates for teachers with small classes are extremely imprecise and are otherwise prone to skew results.

B Biraderi Classification

make better