Decomposing the Government-Private School Performance Differential: Village Ethnic Politics and School Sorting

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Abstract

The emergence of rural, secular, affordable private schools across South Asia is one of the most promising developments in the education sector in decades. Yet the question of whether private schools are actually superior to government schools remains unsettled. Observational studies consistently show that private school students outperform government school students, but it is unclear whether this is because (a) private schools provide students with a better education, or (b) more talented students attend private schools. This paper leverages a source of variation in school choice – in caste-homogeneous villages, school choice is driven by perceptions of academic potential, while in caste-heterogeneous villages school choice is motivated by a desire to segregate students by caste – to shed new light on this question. It finds that the government-private school test gap is two times larger when school choice is driven by academic potential (even after controlling for a substantial number of observable and unobservable factors), suggesting at least 50% of the difference between government and private school test scores can be explained by differences in student composition, not teaching quality.

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1 Introduction

The rapid rise of affordable and apparently high quality private schools in South Asian rural communities is one of the most exciting developments in the education sector in decades. Private schools account for an ever rising share of children attending these schools – in 2005, 33% of Pakistani primary school students and 20-24% of Indian rural primary school students attended a private school, and the students in these private schools consistently outperform their government school counterparts, even when controlling for observable student characteristics (Jimenez et al., 1991; Jimenez and Lockheed, 1995; Pratham, 2005; Andrabi et al., 2011; Desai et al., 2009; Tooley and Dixon, 2003; Alderman et al., 2003, 2001). This has given rise to the hope that private schools may someday circumvent reform-resistant government schools and finally deliver quality educations to the hundreds of millions of children in the region.

Despite the promise of these developments, however, the true significance of the rapid emergence of private schools hinges critically on the question of whether these private schools are actually delivering superior educations, or whether they just attract students who are more academically inclined or come from families that prioritize educational attainment.

This paper leverages variation across villages in the factors that drive school choice to provide new insight into this question. In particular, it takes advantage of the fact that school choice in rural Pakistan is motivated by different factors in caste-homogeneous and caste-heterogeneous villages. This paper shows that in caste-diverse villages, school choice is driven by social considerations – high-status families send their children to private schools to keep them in homogeneous social settings. In caste-homogeneous villages where caste sorting is unnecessary, by contrast, school choice is driven by parental perceptions of academic potential – families send their more academically gifted children to private schools.

Because school choice is primarily based on academic potential in homogeneous villages and not in heterogeneous villages, differences in the government-private test score gap across these types of villages can be interpreted as a lower-bound on the contribution of student sorting to the test score gap. Using this empirical framework, this paper finds that at least 1/2 of the test score gap between government and private schools that persists after controlling for both observable and many unobservable student characteristics can be attributed to student sorting, suggesting that while private schools may still be outperforming government schools, the magnitude of this differential is likely grossly overstated in non-experimental work.

It is difficult to overstate the potential importance of the answer to this question for education policy in the developing world. Not only do private schools constitute a substantial portion of current enrollments in South Asia, but enrollment is also growing explosively. From 2000 to 2005 in rural Pakistan, for example, "the number of private schools in Pakistan increased from 32,000 to 47,000." (Andrabi et al., 2007, p. vi), and evidence suggests this growth continues today. Moreover, private schools deliver educations at a fraction of the cost of government schools by

¹There is likely still *some* sorting on potential in heterogeneous villages, thus this result can only be interpreted as a lower-bound.

²Of course efforts have been made to address the possibility of student "sorting." Randomizing school assignments is untenable, but in two major cases private school vouchers have been randomly assigned. Even these randomizations have proven problematic, however. Angrist et al. (2002) examines a voucher lottery system in Colombia and finds a small positive effect of vouchers, but inference is clouded by the fact that voucher students who performed poorly were at risk of losing their vouchers, making it impossible to separate this incentive effect from the private school effect. And several studies have been conduced of a voucher system in Chile, but as Bellei (2008) notes, the slight private-school advantage these studies show may be down to the fact that private school admissions are selective and poorly performing students can be expelled from private schools, making it difficult to disentangle selectivity from school effects.

not requiring formal teacher training and by hiring local secondary-educated women as teachers rather than college-educated teachers who have to move to the villages where they teach (Andrabi et al., 2007). Thus the question of whether these lower-cost educations are of similar quality to government school educations also speaks to the importance of different teacher training and employment practices.

The findings of this paper are unlikely to put to rest the debate over whether private schools are superior to government schools. Indeed, private school students continue to outperform government school students even in the most homogeneous villages, just by a dramatically smaller margin. But even after using some of the most sophisticated econometric methods available, this paper still establishes (as a "lower bound") that sorting explains a very large portion of the public-private test score gap. This should give analysts pause when examining other empirical results that claim to fully control for sorting.

This paper is organized out as follows: Section 2 provides an overview of the rural Pakistan context from which data for this analysis is drawn. Section 3 details how the determinants of school choice vary with village caste heterogeneity. Section 4 then shows how the government-private test score gap varies with caste heterogeneity, and presents evidence this is due to differences in student sorting. Section 5 then examines and rules out a number of alternative possible explanations for this empirical regularity. And finally, Section 7 discusses the strengths and weaknesses of these findings and their interpretation.

2 Study Context

The focus of this paper is the rural, secular, primary school educational ecosystem of 112 randomly selected villages (*mauzas*) in the Punjab districts of Attock, Faisalabad, and Rahim Yar Khan in Pakistan. These 112 villages were the subject of the Learning and Educational Attainment in Punjab Schools (LEAPS) panel survey. LEAPS villages were selected through probability proportional to population sampling (stratified at the district level) from the universe of all villages in these districts with at least one private school. The survey ran from to 2003-2007, and includes detailed surveys of households, students, teachers, and both government and private schools in these villages. In addition, some data used in this analysis comes from a listing census conducted in sample villages prior to the start of the LEAPS survey that include basic demographic information on all households, allowing for the computation of accurate village level statistics.

The LEAPS survey is organized around two panels of students – one (initiated in 2003 with an initial population of 12,110 children) which followed students for four years, and one (initiated in 2005 with an initial population of 11,852 students) which followed students for two years. Each panel represents the universe of enrolled students in sample villages in Class 3 in both government and private schools. Both students and their teachers in both panels were administered annual exams in English, math, and Urdu by the LEAPS team, and these tests were subsequently standardized using Item Response Theory (IRT) methods.

2.1 Caste in Punjab

This paper focuses on how village-level caste politics influence the selection of students into private and government schools. Before directly examining this dynamic, however, some background on the nature of caste in Pakistan is warranted.

Caste – known variously as *biraderi* or *zaat* – is a central aspect of rural social identity in Pakistan, especially in Punjab. While *biraderi* is a somewhat distinct concept from the idea of "caste" in India, "it retains a very important feature of the [Indian subcaste] – that of an inherent, inbuilt hierarchy that governs social interactions. Society is hierarchically ordered with the Syeds at the top, followed by the landowning castes, then by the service castes or kammis, and finally by the Musallis, who occupy the lowest rung of the social ladder. This ordering dictates much of the social life in a Punjabi village and is most profound in the notions of community cooperation, where solidarity is strongest within a biraderi."(Gazdar and Mohmand, 2007, p. 29).

Biraderi is correlated with wealth, land holdings, and education, but it is not synonymous with economic class. As a related report observes, "while economic power is required to reinforce biraderi-based dominance, membership of a dominant biraderi can help mitigate some of the effects of being economically poor. As one respondent put it, 'the poorest Jatt is still better off than the richest kammi." (Gazdar and Mohmand, 2007, p. 13) While the centrality of caste politics is relatively universal across villages in Punjab, however, there is significant variation in village *biraderi* composition. Figure 1 below shows the density plot of villages of different levels of caste fractionalization, as measured using a simple herfindahl index.³ As the figure shows, there is significant variation in the degree of fractionalization both within and across the three districts of the LEAPS survey.

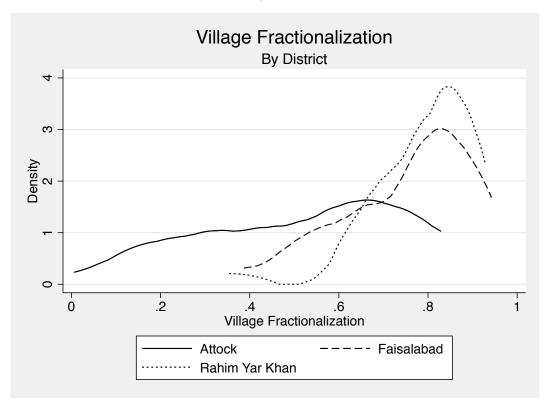


Figure 1:

As shown in Table 1, this variation in heterogeneity is not clearly related to village wealth, land inequality, or adult education. There is some relationship to primary school enrollment rates and

³The herfindahl is a common measure of fractionalization equal to the probability that any two randomly selected individuals belong to the same group. Village fractionalization is computed using data from a village census conducted in 2002 to facilitate household sampling for the LEAPS survey which includes data on the *biraderi* of all households in LEAPS villages.

village size, but on the whole fractionalization appears to be relatively independent of other compositional characteristics of villages.

(4) (1) (2) (3) (5) (6) Median Adult Enrollment Land Schools Log Num HH Wealth Literacy Gini Pct per HH -16.0** Fractionalization -262.5 -7.640.073 -0.00170.68** (-0.44)(-1.36)(1.35)(-2.43)(-0.18)(2.09)District FE Yes Yes Yes Yes Yes Yes

112

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Table 1: Village Characteristics and Fractionalization

Observations

3 Caste Politics and School Sorting

112

The key to determining whether the government-private test score gap is caused by private schools delivering quality educations or by more academically-inclined students attending private schools is understanding what motivates parents to pick one type of school over the other. This Section provides an overview of how households make these choices, and how those choices vary by village caste composition.

3.1 Selection in Homogeneous Villages

As shown in Table 2 below, which regresses the choice to send a child to a private school on a number of characteristics, parental perceptions of child intelligence are an extremely strong predictor of whether a parent will send their child to a private school. Most notably in this table, this pattern holds even *within individual households*. As shown in Column 2, which includes household fixed effects, many parents send the child they perceive to be more intelligent to private school and the child they perceive to be less intelligent to government schools.

Indeed, this pattern also extends beyond school choice into other domains, such as household expenditure on educational materials and the amount of time parents spend helping their children with school work. In the words of the original LEAPS survey authors, "through their choices of whether to enroll a child, through the choice of school ([government] or private) and finally through the amount they chose to spend, households pick "winners" and try to carry them through." (Andrabi et al., 2007, p. 103)

The implications of this behavioral tendency for understanding the government-private school test gap is clear: if parents are choosing to send their more academically-inclined children to private schools, and if parents have more information about student quality than researchers are able to measure in surveys and control for statistically, then standard analyses are likely to systematically overstate the quality of private school educations.

t statistics in parentheses

^{*} p;0.10, ** p;0.05, *** p;0.01

Table 2: School Choice and Child Intelligence

	(1)	(2)
Mom Reports Child Above Average Intelligence	0.058**	0.041*
	(2.82)	(1.99)
Mom Has Some Schooling	0.080	-0.032
	(1.51)	(-0.27)
Mom Has Some Schooling	0.084**	0.084
	(3.23)	(0.71)
Log Month Expenditure	0.043	-0.038
	(1.78)	(-1.05)
Age	-0.021***	-0.017**
	(-3.76)	(-3.26)
Age Squared	0.00025	0.00017
	(1.78)	(1.64)
Female	0.029	-0.0012
	(1.27)	(-0.04)
Constant	-0.24	0.35
	(-1.13)	(1.85)
Village Fixed Effects	Yes	No
Household Fixed Effects	No	Yes
Observations	3426	3426

t statistics in parentheses

^{*} p < 0.05, ** p < 0.01, *** p < 0.001

3.2 Selection in Heterogeneous Villages

While the tendency for parents to invest in "winners" rather than distribute resources is a general tendency in the LEAPS data, it is not the only factor that shapes school choice. In more caste-heterogeneous villages, the *social* composition of schools becomes increasing salient and washes out much of this "sorting on intelligence." Instead, *all* children from "high status" *biraderis* – regardless of perceived academic potential – are sent to private schools to isolate them from lower-status families. And as a result, children from "low status" *biraderis* become concentrated in government schools. School choice, in other words, ends up being driven by social rather than academic considerations.

3.2.1 Evidence of Segregation

This segregation of schools can be seen in a number of ways. Figure 2 below plots the number of biraderis represented in each school for high, medium, and low fractionalized villages. As the figure shows, despite large differences in the fractionalization of the villages in which these schools operate, a remarkably similar numbers biraderis a present among their student bodies.

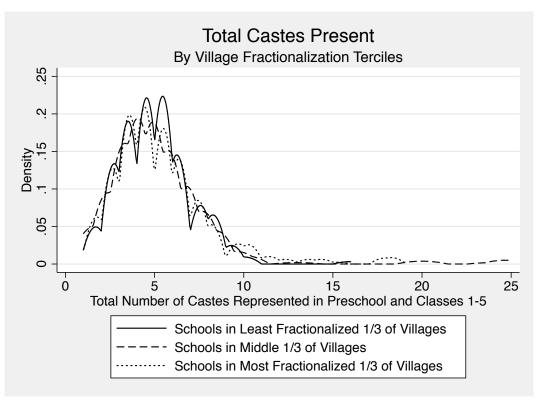


Figure 2:

This figure is easily interpreted, but does not take into account the relative size of the population of each *biraderi* within the student body. Figure 3 plots the population share of the two largest biraderis among students in the village at large and in each school level respectively. Again, this figure shows that while in more fractionalized villages the share of students from the top two castes in the average school is indeed slightly lower, it does not come close to keeping pace with changes in village demographics.

Village Caste Concentration **School Caste Concentration** By Village Fractionalization Terciles By Village Fractionalization Terciles .03 8 Density 2 3 Density 5 20 40 60 oc Percentage of Students in Largest Two Castes Ó 100 Percentage of Village in Largest Two Castes Least Fraction zed 1/3 of Villages Schools in Least Fraction lized 1/3 of Villages - - Middle 1/3 of Villages - Schools in Middle 1/3 of Villages · · · · · Most Fractionalized 1/3 of Villag · · · · Schools in Most Fractionalized 1/3 of Vi

Figure 3: Village and School Fractionalization

Finally, if schools were unsegregated, then we would expected herfindahl indices computed within each school to track closely with herfindahl indices computed at the village level. Yet as shown in Figure 4, this is far from the case. Almost all schools are below the 45 degree line that would indicate school and village diversity moving one for one, and many are well below.

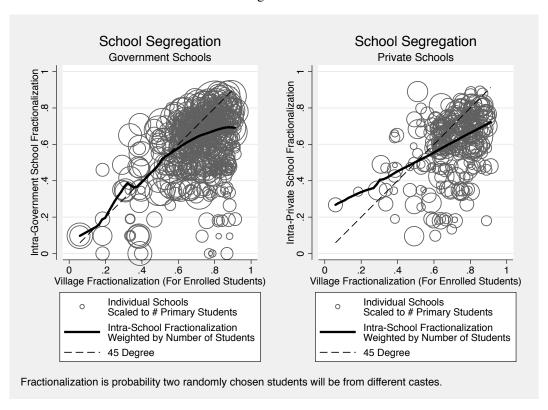


Figure 4:

3.2.2 Evidence of High Caste Private School Attendance

This data shows a clear pattern of segregation, but it does not provide an entirely clear picture of which groups are attending which schools. Grouping *biraderis* into "high" and "low" social

status groupings allows for a better understanding of segregation patterns. The crudeness of these categorizations is unfortunate, but necessary – although *biraderis* are associated with strict hierarchies within villages, there does not exist an explicit global hierarchy of *biraderis* in Pakistan as in with the more familiar *varna* caste designations India. As a result, these hierarchies may vary somewhat from village to village, and as noted previously, this variation may not perfectly follow economic position. Fortunately, interviews with numerous Pakistanis has provided evidence that these categorizations are sufficient consistent at this level of generality and in the region of the LEAPS survey for these rankings to be useful.⁴

As shown in Table 3, in villages with higher caste fractionalization, a larger share of private school students come from higher status *biraderis* and a larger share of government school students come from low *biraderis*. Private schools, in other words, become reservoirs of the social elite.

Table 3: Student Social Status by School Type

	(1)	(2)
	Pct of Students High Status	Pct of Students High Status
Private School	-0.11**	-0.13**
	(-2.30)	(-2.13)
Biraderi Fractionalization	-0.047*	-0.19***
	(-1.85)	(-14.78)
Fractionalization * Private	0.18**	0.21**
	(2.34)	(2.15)
Median Village Expenditure	0.0000014	
	(0.87)	
Village: Pct Adults Literate	0.00022	
	(1.22)	
Log Village Size	0.00074	
	(0.16)	
Village: Pct High Status	1.01***	
	(62.12)	
Constant	-0.0039	1.00***
	(-0.10)	(83.16)
District Fixed Effects	Yes	No
Village Fixed Effects	No	Yes
Observations	782	782

t statistics in parentheses

Demand for caste segregation is also manifest in the dramatically higher prices charged by segregated private schools. As shown in Table 4 below, moving from a perfectly non-fractionalized village to a perfectly fractionalized village is associated with a 600 Rupees increase in annual school fees. Given that the average annual fee for all private schools in the LEAPS survey is 1191 Rupees, this is a very significant amount.⁵

Further, as shown in Table 5, none of these changes are driven by a change in the share of students

^{*} p;0.10, ** p;0.05, *** p;0.01

⁴Some interviewees have also provided threefold divisions, which generate consistent results with those presented here. This work has avoided the Jacoby and Mansuri (2011) methodology – where castes are ranked on the basis of their land holding – due to input from numerous sources that social standing and land holding are not equivalent, and

Table 4: Annual Private School Fees

	(1)	(2)	(3)
	Weighted by School	Weighted by School	Weighted by Primary Students
Biraderi	504.7**	527.9**	608.6**
Fractionalization	(2.33)	(2.50)	(2.37)
Village: Median		61.6	20.8
Expenditures		(1.25)	(0.44)
Expenditure Gini		-49.9	45.5
		(-0.24)	(0.20)
District Fixed Effects	Yes	Yes	Yes
Observations	287	287	285

t statistics in parentheses

in private schools. The percentage of students in private schools in almost perfectly stable, even when controlling for numerous village characteristics.

Table 5: Share of Enrolled Students in Private Schools

	(1) Pct Class 3 in Private	(2) Pct Class 3 in Private
Biraderi Fractionalization	0.056	0.063
	(0.63)	(0.70)
Median Village Expenditure		0.000029**
		(1.99)
Village Land Gini		0.033
		(0.21)
Village: Pct Adults Literate		0.0019
		(1.27)
Log Village Size		0.019
		(0.71)
District Fixed Effects	Yes	Yes
Observations	112	112

t statistics in parentheses

4 School Sorting and Test Scores

Having established that the determinants of school choice vary dramatically between caste-homogeneous and caste-heterogeneous villages in Section 3, this section now turns to an analysis of how differences in sorting impact the test score gap between government and private schools. If private

^{*} p;0.10, ** p;0.05, *** p;0.01

^{*} p;0.10, ** p;0.05, *** p;0.01

in this exercise social-status is of substantially more importance than socio-economic status.

 $^{^5}$ Fees above the 95th percentile – 1900 Rupees – were adjusted down to 1900 Rupees. Without this adjustment, the coefficient on village fractionalization is approximately 950 Rupees with a t-stat of 2.09

schools outperform government schools primarily due to differences in the quality of instruction, then the government-private test score gap should be relatively stable across villages with different caste compositions. If, however, private schools outperform government schools primarily due to differences in the composition of their students, then villages that are subject to different sorting processes should also see differences in the government-private differential.

4.1 Measuring Learning

To measure learning, this analysis employs a lagged-value-added model. Lagged-value-added models have increasingly become the norm in the education research (Gordon et al., 2006; McCaffrey et al., 2003; Hanushek, 2003) due to their potential to take into account not only observational differences between students, but also the potential to control for some unobserved differences, a subject discussed in more detail below.⁶

The lagged-value-added model incorporates the assumption that current knowledge is an additive function of all current and past inputs and an i.i.d. stochastic error term, and can be expressed as:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \epsilon_{i,t} \tag{1}$$

where $Y_{i,t}$ is child *i*'s test scores at time t and $X_{t,i}$ is a vector of child, school, and village controls at time t (a full discussion of the lagged-value-added model and its assumptions can be found in Appendix A).

Note that while the inclusion of a lagged dependent variable effectively controls for unobserved differences that affect differences in test *levels*, it cannot control for unobserved heterogeneity that affects learning *rates*. It is for this reason that while superior to other available methods, value-added analyses can not fully overcome selection issues.⁷

This lagged-value-added model of child learning can be leveraged to measure the contribution of various factors to learning. In this analysis, example, village-school type dummies (i.e. one dummy for village 1, private schools, one dummy for village 1, government schools, etc.) are added to these child-year-level regressions to capture the "value-added" to children's education by schools of a given type in each village. The difference between the private school and government school dummies in a village constitute a measure of the government-private school test score gap. In addition, fixed-effects for each teacher can also be used to estimate the "value-added" of each teacher.

4.2 Convergence in Government-Private Test Scores

Table 6 presents lagged-valued-added estimates of learning as a function of various demographic controls and village fractionalization. It shows that the effect of caste fractionalization on the government-private test score gap is negative and significant for English and Urdu, and negative

⁶Lagged value-added models also account for the fact that learning is not entirely persistent (things learned in the past are often forgotten). This flexibility is discussed in more detail in Appendix ??.

⁷Some analysis have turned to second-differencing the data and focusing on students who change schools (Andrabi et al., 2011), but these analyses have their own limitations, among them limited sample sizes (given that changes between types of school are relatively infrequent in most surveys) and the assumption that school changes are not the result of some unobserved shock (i.e. that school switches are not accompanied by contemporaneous with other changes – a potentially problematic assumption given the relative infrequency with which students change schools).

(albeit insignificant) for math. Further, as shown in columns (2), (5), and (8) of Table 6, the inclusion of various demographic controls such as a child wealth index and dummies for parental education along with the village fixed effects has no significant effect on the results.

To aid in interpretation, Figure 5 plots the government-private test score gap as a function of caste fractionalization (these plots correspond to columns (2), (5), and (8) respectively). In all three cases, the rise in fractionalization is associated with a near 50% decline in the private school premium, although this is by far most striking in the case of English – which is generally considered the path to upward mobility in Pakistan, and is often the focus of private schools in Punjab.

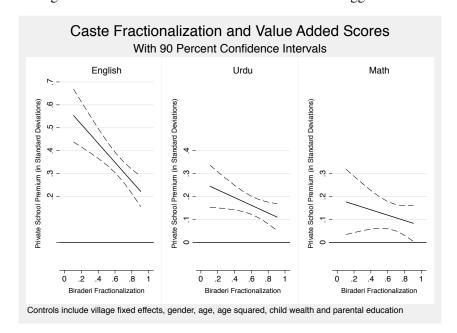


Figure 5: Private School Test Score Premium with Lagged Scores

4.3 Decomposition of Convergence

Further evidence that the convergence in government-private school test scores is driven differences in student sorting – not differences in actual learning outcomes – comes from the fact that while the government-private school test gap decreases, overall learning remain relatively unchanged. As shown in Table 7, overall test scores are essentially flat across all villages – English scores are slightly higher in more fractionalized villages in Column 1, but the magnitude of this difference is relatively small, and once more demographic controls are added in Column 2 this effect disappears. No relationship exists for other subjects. Government scores increase and private scores decline with fractionalization, in other words, but those changes are almost perfectly offsetting. Indeed, this is also illustrated in Column 3 of Table 6, where village fixed effects are replaced with district fixed effects, allowing for a comparison of test scores levels (rather than just the government-private gap) across villages. In the case of English the convergence appears to be driven in equal parts by improvements in government schools and a decline in private schools.

The fact overall educational attainment remains constant is further evidence that as the determinants of school choice changes, it is the *distribution* of academically inclined students that changes, not the performance of the schools themselves.

Table 6: Child Test Scores

		English			Urdu			Math	
	(1)	(2)	(3)	(4)	(5)	(9)	(7)	(8)	(6)
Private School	0.63***	***09.0	0.59***	0.27***	0.26***	0.20***	0.19*	0.19*	0.12
	(8.35)	(7.28)	(7.31)	(4.16)	(4.01)	(2.93)	(1.67)	(1.88)	(1.25)
Fractionalization * Private	-0.43***	-0.41***	-0.42***	-0.17*	-0.17*	-0.094	-0.082	-0.12	-0.052
	(-3.77)	(-3.46)	(-3.68)	(-1.71)	(-1.74)	(-0.98)	(-0.53)	(-0.83)	(-0.39)
Lagged English Scores	0.37	0.36***	0.39***	0.15***	0.14***	0.14***	0.16***	0.15***	0.16***
	(21.16)	(20.03)	(20.61)	(13.05)	(11.37)	(11.69)	(10.78)	(10.06)	(986)
Lagged Math Scores	0.069***	0.072***	0.071***	0.12***	0.12***	0.12***	0.37	0.38***	0.40***
	(8.55)	(8.91)	(8.49)	(14.10)	(13.51)	(13.96)	(29.56)	(27.37)	(28.76)
Lagged Urdu Scores	0.15***	0.15***	0.15***	0.38***	0.38	0.40**	0.23***	0.22***	0.22***
	(14.04)	(13.28)	(12.71)	(34.36)	(32.65)	(31.74)	(17.67)	(17.01)	(16.84)
Child's Wealth Index		0.017***	0.015***		0.0073**	0.0068**		0.014***	0.016***
		(5.19)	(4.29)		(2.46)	(2.28)		(3.52)	(3.73)
Educated Parent		0.058***	0.053***		0.052***	0.049***		0.046***	0.043***
		(4.75)	(4.07)		(4.77)	(4.40)		(3.35)	(3.08)
Biraderi Fractionalization			0.21**			0.095			0.14
			(2.56)			(1.40)			(1.38)
Village: Pct Adults Literate			0.00017			-0.00054			0.00036
			(0.18)			(-0.62)			(0.27)
Log Village Size			0.019			0.014			0.0088
			(1.47)			(1.01)			(0.50)
Village Land Gini			0.053			0.061			-0.25*
			(0.47)			(0.63)			(-1.88)
Constant	0.25	0.40*	0.57**	0.56**	***69.0	0.78***	0.12	0.31	*09.0
	(0.99)	(1.78)	(2.28)	(2.58)	(2.78)	(2.80)	(0.37)	(0.99)	(1.74)
Village Fixed Effects	Yes	Yes	$ m N_0$	Yes	Yes	$_{ m o}^{ m N}$	Yes	Yes	$_{ m o}^{ m N}$
District Fixed Effects	No	No	Yes	No	No	Yes	No	No	Yes
Observations	37147	26141	26141	37147	26141	26141	37147	26141	26141

Controls for age, age squared, gender, and class omitted from table. Standard errors clustered at village level. *pi0.10, **pi0.05, ***pi0.01

Table 7: Child Test Scores and Fractionalization

	Eng	English	U	Urdu		Math	ıth	
	(1)	(2)	(3)	(4)	(5)	(9)	(7)	(8)
Private School	0.31***	0.29***	0.31***	0.29***	0.14***	0.14***	0.11***	0.087**
	(10.98)	(10.42)	(10.98)	(10.42)	(5.74)	(5.65)	(3.17)	(2.58)
Biraderi Fractionalization	0.13*	960.0	0.13*	0.096	0.085	0.069	0.13	0.13
	(1.70)	(1.33)	(1.70)	(1.33)	(1.26)	(1.08)	(1.34)	(1.46)
Lagged English Scores	0.40***	0.39***	0.40**	0.39***	0.16***	0.14***	0.17***	0.16***
)	(22.32)	(21.10)	(22.32)	(21.10)	(13.39)	(11.90)	(10.27)	(9.92)
Lagged Math Scores	0.067***	0.070***	0.067***	0.070***	0.12***	0.12***	0.39***	0.40***
	(8.18)	(8.37)	(8.18)	(8.37)	(14.51)	(14.00)	(30.92)	(28.87)
Lagged Urdu Scores	0.15***	0.15***	0.15***	0.15***	0.39***	0.40***	0.23	0.22
	(13.26)	(12.80)	(13.26)	(12.80)	(34.01)	(31.76)	(17.96)	(16.87)
Village: Pct Adults Literate	0.00067	0.00022	0.00067	0.00022	-0.00018	-0.00053	0.00043	0.00036
	(0.68)	(0.23)	(0.68)	(0.23)	(-0.21)	(-0.61)	(0.31)	(0.27)
Log Village Size	0.017	0.019	0.017	0.019	0.011	0.014	0.0070	0.0087
	(1.33)	(1.39)	(1.33)	(1.39)	(0.73)	(1.01)	(0.35)	(0.50)
Village Land Gini	0.0097	0.045	0.0097	0.045	0.013	0.059	-0.28**	-0.25*
	(0.08)	(0.39)	(0.08)	(0.39)	(0.13)	(0.61)	(-2.22)	(-1.90)
Child's Wealth Index		0.015***		0.015***		**8900.0		0.016***
		(4.22)		(4.22)		(2.27)		(3.73)
Educated Parent		0.053***		0.053***		0.049***		0.043***
		(4.13)		(4.13)		(4.42)		(3.08)
Constant	0.44	0.63**	0.44	0.63**	0.67	0.80	0.32	0.61*
	(1.57)	(2.52)	(1.57)	(2.52)	(2.65)	(2.83)	(0.87)	(1.76)
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	37147	26141	37147	26141	37147	26141	37147	26141

Observations 37147 26141 37147 26141 37147 Controls for age, age squared, gender, and class omitted from table. Standard errors clustered at village level. * p;0.10, ** p;0.05, *** p;0.01

It is worth noting here that up to this point, an implicit assumption has underlay the arguments made in this paper. For it to be the case that sorting by caste reduces the degree to which private schools enroll disproportionately academically inclined students, it must be the case that residual academic potential – potential that cannot be explained by things like parental education and wealth – must be equally distributed across different castes (or be distributed slightly in favor of lower status *biraderis*). If not, and even the least talented "high status" students were more talented than the most talented "low status" students, then the concentration of "high status" students in private schools would result in *divergence*, rather than *convergence*, of test scores. As shown in Table 8, however, there is no evidence that those from higher social status *biraderis* have higher residual talent than those from low status *biraderis*. As evident from the top row of coefficients, after controlling for other observational factors, student caste does not appear to have any consistent effect on test scores.

5 Alternate Explanations

Observational studies pointing to the superior performance of private schools are not the only reason that education reform advocates have voiced excitement about the potential of private school growth; private schools also operate in fundamentally different ways from government schools. For that reason, it is important to examine not only whether the government-private test score gap varies with caste heterogeneity, but also whether other factors some suspect explain school differences can be ruled out as alternate explanations.

5.1 Differences in Teacher Incentives

Advocates of private schools argue that not only are observational studies able to control for many factors, but there is also evidence to explain *why* private schools outperform government schools. In particular, they point out that private schools appear to address the biggest problem in government schools: low effort. High absenteeism and low accountability in government schools has been well documented (Muralidharan and Kremer, 2008; Chaudhury et al., 2006), but appears less prevalent in private schools. The reason, many argue, is that in private schools good teachers are better paid, and poor teachers are let go. This line of reasoning is also buoyed by a growing body of literature that suggests that what matters for success is not the availability of educational "inputs" (like qualified, well paid teachers or good facilities), but incentive schemes that reward effort on the behalf of teachers.(Hanushek, 1997, 2003; Banerjee et al., 2007).

The conclusion of the LEAPS survey authors (full disclosure: this author was a research assistant on the LEAPS project, though not an author) is that this is the case in Pakistan. Private schools deliver better educational outcomes despite hiring only secondary-educated local women with no training and providing them with relatively low wages because private schools incentize good teaching by paying good teachers more, improving effort. This differentiates them from government schools, which offer salaries which are higher but unresponsive to performance. As a result, the authors argue, government school teachers exert less effort. This is illustrated in Figure 6 which appears in the original LEAPS report (Andrabi et al., 2007). It shows that while frequently absent private school teachers are paid less, absent government school teachers are actually paid *more*. And where private school teachers with high score students are paid more, no such relationship exists for government teachers.

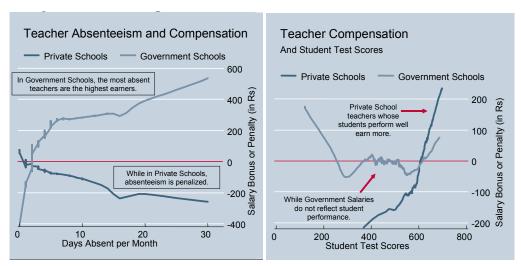
Table 8: Child Social Status and Residual Talent

		English			Urdu			Math	
	(1)	(2)	(3)	(4)	(5)	(9)	(7)	(8)	(6)
High Status Biraderi	-0.035	-0.062	-0.11**	-0.044	-0.11	-0.081	0.0056	-0.019	0.034
	(-0.79)	(-0.93)	(-2.41)	(-0.75)	(-1.55)	(-1.61)	(0.00)	(-0.22)	(0.50)
Private School	0.30**	0.20	0.18	0.22*	0.18	0.026	-0.10	-0.13	-0.24
	(2.53)	(1.50)	(1.46)	(1.92)	(1.37)	(0.21)	(-0.96)	(96.0-)	(-1.55)
Fractionalization * Private	0.0010	0.12	0.100	-0.097	-0.057	0.11	0.35**	0.42*	0.47**
	(0.01)	(0.60)	(0.54)	(-0.60)	(-0.29)	(0.63)	(2.02)	(1.97)	(2.13)
Lagged English Scores	0.32***	0.31	0.40***	0.15***	0.15***	0.15***	0.17***	0.17***	0.16***
	(7.79)	(6.95)	(10.37)	(4.80)	(4.19)	(5.12)	(3.81)	(3.74)	(3.93)
Lagged Math Scores	**990.0	0.061*	0.050	0.12***	0.094**	0.10**	0.31	0.29	0.38***
	(2.21)	(1.68)	(1.51)	(3.04)	(2.23)	(2.57)	(6.38)	(5.93)	(8.33)
Lagged Urdu Scores	0.17***	0.18***	0.16***	0.34***	0.35***	0.40***	0.25***	0.25***	0.25
	(4.72)	(4.11)	(4.04)	(8.85)	(7.57)	(8.74)	(5.51)	(4.96)	(4.94)
Child's Wealth Index		-0.0055	-0.0080		0900.0	0.0046		-0.0094	-0.0062
		(-0.43)	(-0.69)		(0.57)	(0.52)		(-0.57)	(-0.44)
Educated Parent		0.12***	0.13		0.14***	0.14***		0.15***	0.17***
		(2.95)	(3.52)		(3.25)	(4.01)		(2.85)	(3.68)
Biraderi Fractionalization			-0.0096			-0.076			-0.024
			(-0.09)			(-0.70)			(-0.16)
Village: Pct Adults Literate			0.00000016			-0.0030**			-0.0022
			(0.00)			(-2.03)			(-0.84)
Log Village Size			0.011			0.016			0.026
			(0.33)			(0.52)			(0.44)
Village Land Gini			-0.031			0.054			-0.11
			(-0.17)			(0.28)			(-0.37)
Constant	-0.62	0.20	1.00*	-0.15	2.16***	2.28***	-0.71	2.98***	2.59***
	(-1.25)	(0.29)	(1.71)	(-0.26)	(3.07)	(3.46)	(-1.07)	(3.44)	(2.90)
Village Fixed Effects	Yes	Yes	$ m N_{o}$	Yes	Yes	No	Yes	Yes	$^{ m No}$
District Fixed Effects	No	No	Yes	No	No	Yes	No	No	Yes
Observations	1859	1381	1381	1859	1381	1381	1859	1381	1381

Controls for age, age squared, gender, and class omitted from table. Standard errors clustered at village level. *pi0.10, **pi0.05, ***pi0.01

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Figure 6:



Source: Andrabi et al. (2007), pages 72, 74. Salary bonus or penalty is the degree to which salary differences from predicted values independent of absenteeism or test scores.

Table 9 below recreates the analyses underlying the two plots in Figure 6 with two adjustments: first, rather than measuring performance using average test scores (in levels), more rigorous teacher "value-added" scores (Specification 4) are employed to measure teacher contributions to learning; second, a village fractionalization measure is included in all regressions as an interaction term. If it is the case that differences in incentive schemes are driving test score convergence, then we should see (a) the private-school salary penalty for absenteeism decline in fractionalization, and (b) the compensation premium (compensation above what is otherwise predicted) for performance decline. As shown in the table, however, the absenteeism penalty actually *increases* in fractionalization (which under the incentive story should result in *increased* private school scores), and no relationship exists between performance and compensation. In sum, there is no relationship between fractionalization and how government teachers are compensated.

5.2 Differences in School Inputs

A second potential explanation for variation in the government-private school test score gap is that the availability of school resources varies with village caste heterogeneity. To test this, this paper examines the relationship between village fragmentation and different school inputs.

Table 10 presents a series of school-level regressions in which measures of school- and teacher-quality in private schools are regressed against village fractionalization and a number of controls. The table shows that there is little or no evidence that – at least in terms of visible inputs – differences in inputs can explain the decline in private school performance in fractionalized villages. If anything, private school teachers appear to be slightly *better* educated in fractionalized villages.

Table 11 repeats this exercise for government schools. It should be noted that government schools in Pakistan are administered at the state level, and are thus relatively insulated from village politics, making any such differences unlikely. Nevertheless, results are presented for thoroughness. These results show no difference in school inputs across villages.

Table 9: Village Fractionalization and Teacher Compensation

	Private '	Teachers	Governme	nt Teachers
	(1) Log Salary	(2) Log Salary	(3) Log Salary	(4) Log Salary
Days Absent Last Month	0.041**	-0.0068	0.0017	0.0041***
	(2.00)	(-0.85)	(0.40)	(2.79)
Biraderi Fractionalization	0.24	0.21	-0.085	-0.050
	(1.18)	(0.78)	(-1.47)	(-0.74)
Days Absent * Fractionalization	-0.063**		0.0050	
	(-2.05)		(0.77)	
Gender	-0.32***	-0.27**	-0.012	0.0095
	(-3.78)	(-2.17)	(-0.72)	(0.54)
Age of teacher	0.0053	0.023**	0.021***	0.018***
	(1.21)	(2.54)	(12.18)	(14.48)
Average Value Added Score		0.22		-0.033
		(0.48)		(-0.48)
Value-Added * Fractionalization		-0.47		-0.020
		(-0.67)		(-0.22)
Constant	7.07***	8.02***	7.51***	7.60***
	(29.09)	(20.15)	(47.83)	(61.24)
District Fixed Effects	Yes	Yes	Yes	Yes
Observations	619	154	1302	618

Controls for Experience and Teacher Education excluded from table.

Robust t-statistics clustered at the village level in parenthesis

^{*} p;0.10, ** p;0.05, *** p;0.01

Table 10: Private Teacher Characteristics and Village Fractionalization

	(1)	(2)	(3)	(4)	(5)	(9)
	Days Absent	Female	From Village	Teacher English Exam Score	More than Grade School Education	Basic School Facility Index
Biraderi Fractionalization	-0.91	-0.052	0.31*	0.19	0.20*	-0.023
	(-1.32)	(-0.60)	(1.71)	(1.06)	(1.68)	(-0.07)
Median Village Expenditures	0.000054	0.00000057	0.0000030	0.000016	0.000013	0.000049
•	(0.64)	(0.46)	(0.12)	(0.50)	(0.68)	(1.13)
Log Number of Households	-0.29	-0.030*	-0.036	0.045	0.0072	-0.13
	(-1.47)	(-1.67)	(-0.74)	(0.86)	(0.29)	(-1.43)
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1494	1494	1494	892	1494	493

All results clustered at the village level.
All regressions weighted by number of students.
Robust t-statistics presented in parenthesis.
* p₁0.10, ** p₁0.05, *** p₁0.01

Table 11: Government Teacher Characteristics and Village Fractionalization

	(1)	(2)	(3)	(4)	(5)	(9)
	Days Absent	Female	From Village	Teacher English Exam Score	More than Grade School Education	Basic School Facility Index
Biraderi Fractionalization	-0.33	0.099	0.26	0.093	-0.10	0.32
	(-0.34)	(0.72)	(1.20)	(0.36)	(-0.84)	(0.63)
Median Village Expenditures	-0.00018	0.000026	-0.0000068	0.000052*	0.000021	0.000033
•	(-0.99)	(0.99)	(-0.23)	(1.75)	(1.23)	(0.53)
Log Number of Households	-0.33	-0.040*	-0.040	0.034	0.000076	-0.094
	(-1.41)	(-1.80)	(-0.59)	(0.71)	(0.00)	(-0.85)
District Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	886	886	886	477	886	291

All results clustered at the village level.
All regressions weighted by number of students.
Robust t-statistics presented in parenthesis.
* p;0.10, ** p;0.05, *** p;0.01

6 Discussion and Policy Implications

7 Discussion and Policy Implications

Private schools represent a radical departure from the educational status quo in nearly every way. Where government schools hire teachers with college educations, emphasize teacher training, and pay high wages, private schools hire secondary-educated women from the local community for a fraction of the cost, invest nothing in teacher training, and pay them in proportion to their performance. If government schools were to emulate their business model, they would save millions. But would educational outcomes improve?

The results presented here cannot conclusively answer this question. Even in the most heterogeneous villages, private schools still outperform government schools (just by dramatically less than in homogeneous villages). But these results do suggest at least two major reasons for caution on the part of policy-makers thinking about the implications of private school growth, and whether recent calls for the widespread distribution of private school vouchers (e.g. Chakrabarti and Peterson (2008), Kelkar (2006), and Panagariya (2008)) are good policy.

First, this analysis shows that even with a lagged-value-added specification, parental education and wealth controls, and panel data, it is still easy to vastly over-estimate the contributions of private schools to learning using observational data. As such, this study illustrates the need for researchers and policy-makers to maintain a health degree of skepticism when reading observational studies that claim to separate sorting from school quality.

Second, while this analysis only shows that 50% of the private school premium can be explained by sorting, it is important to remember that this represents a lower-bound. The difference between homogeneous and heterogeneous villages is not "sorting on intelligence" and "no sorting on intelligence," but rather "sorting on intelligence" and "less sorting on intelligence." As such, sorting is still likely contributing to the private school premium in heterogeneous villages. Indeed, as shown in Table ??, perceived intelligence remains an important determinant of school choice even in highly fractionalized villages.

In light of persistent poor performance among government schools, the hope that private schools will reform the South Asian education sector is understandable, and may yet prove to be well founded. But as shown here the superiority of private school is not self-evident, and the government would do well to gather more evidence before embracing private schools as a substitute for government schools.

References

- Alderman, H., Kim, J., and Orazem, P. F. (2003). Design, Evaluation, and Sustainability of Private Schools for the Poor: The Pakistan Urban and Rural Fellowship School Experiments. *Economics of Education Review*, 22(3):265–274.
- Alderman, H., Orazem, P. F., and Paterno, E. M. (2001). School Quality, School Cost, and the Public/Private School Choices of Low-Income Households in Pakistan. *The Journal of Human Resources*, 36(2):304–326.
- Andrabi, T., Das, J., Ijaz Khwaja, A., and Zajonc, T. (2011). Do Value-Added Estimates Add Value? Accounting for Learning Dynamics. *American Economic Journal: Applied Economics*, 3(3):29–54.
- Andrabi, T., Das, J., Khwaja, A., Vishwanath, T., and Zajonc, T. (2007). *Pakistan: Learnings and Educational Achievements in Punjab Schools*. The World Bank.
- Angrist, J., Bettinger, E., Bloom, E., King, E., and Kremer, M. (2002). Vouchers for Private Schooling in Colombia: Evidence from a Randomized Natural Experiment. *The American Economic Review*, 92(5):1535–1558.
- Banerjee, A. V., Cole, S., Duflo, E., and Linden, L. (2007). Remedying Education: Evidence from Two Randomized Experiments in India. *The Quarterly Journal of Economics*, 122(3):1235–1264.
- Bellei, C. (2008). The Private-Public School Controverersy: The Case of Chile. In Chakrabarti, R. and Peterson, P. E., editors, *School Choice International: Exploring Public-Private Partner-ships.*, page 272. The MIT Press, Cambridge, Massachusetts.
- Chakrabarti, R. and Peterson, P. E. (2008). *School Choice International: Exploring Public-Private Partnerships*. MIT Press, Cambridge, Massachusetts.
- Chaudhury, N., Hammer, J., Kremer, M., Muralidharan, K., and Rogers, F. H. (2006). Missing in Action: Teacher and Health Worker Absence in Developing Countries. *The Journal of Economic Perspectives*, 20(1):91–116.
- Chay, K. Y., McEwan, P. J., and Urquiola, M. (2005). The Central Role of Noise in Evaluating Interventions That Use Test Scores to Rank Schools. *The American Economic Review*, 95(4):1237–1258.
- Currie, J. and Thomas, D. (1995). Does Head Start Make a Difference? *The American Economic Review*, 85(3):341–364.
- Desai, S., Dubey, A., Vanneman, R., and Banerji, R. (2009). Private schooling in india: A new educational landscape. *India Policy Forum*.
- Fuller, B., editor (2002). *Inside Charter Schools: The Paradox of Radical Decentralization*. Harvard University Press.
- Gazdar, H. and Mohmand, S. K. (2007). Social Structure in Rural Pakistan.
- Glewwe, P., Ilias, N., and Kremer, M. (2010). Teacher Incentives. *American Economic Journal: Applied Economics*, 2(3):205–227.

- Gordon, R., Kane, T. J., and Staiger, D. O. (2006). Identifying Effective Teachers Using Performance on the Job. The Hamilton Project Policy Brief No. 2006-01. *Brookings Institution*.
- Hanushek, E. A. (1997). Assessing the effects of school resources on student performance: An update. *Educational evaluation and policy analysis*, 19(2):141–164.
- Hanushek, E. A. (2003). The Failure of Input-based Schooling Policies. *The Economic Journal*, 113(485):F64–F98.
- Jacoby, H. and Mansuri, G. (2011). Crossing boundaries: gender, caste and schooling in rural Pakistan. *World Bank Policy Research Working Paper No. 5710*.
- Jimenez, E. and Lockheed, M. E. (1995). Public and Private Secondary Education in Developing Countries: A Comparative Study. *World Bank Discussion Papers*.
- Jimenez, E., Lockheed, M. E., and Paqueo, V. (1991). The Relative Efficiency of Private and Public Schools in Developing Countries. *The World Bank Research Observer*, 6(2):205–218.
- Kane, T. J. and Staiger, D. O. (2002). The Promise and Pitfalls of Using Imprecise School Accountability Measures. The Journal of Economic Perspectives, 16(4):91–114.
- Kelkar, V. (2006). Let Every Parent be a Consumer. *India Today*.
- McCaffrey, D. F., Lockwood, J. R., Koretz, D., and Hamilton, L. (2003). *Evaluating value-added models for teacher accountability*. Rand Corp.
- Muralidharan, K. and Kremer, M. (2008). Public and Private Schools in Rural India. In Chakrabarti, R. and Peterson, P. E., editors, *School Choice International: Exploring Public-Private Partnerships*. MIT Press.
- Panagariya, A. (2008). India: The Mergeing Giant. Oxford University Press, New York.
- Pratham (2005). Annual Status of Education Report. Pratham Documentation Center, New Delhi.
- Rothstein, J. (2010). Teacher Quality in Educational Production: Tracking, Decay, and Student Achievement. *The Quarterly Journal of Economics*, 125(1):175–214.
- Tooley, J. and Dixon, P. (2003). *Private schools for the poor: A case study from India*. Centre for British Teachers, Reading, UK.

A Value-Added Test Scores

The lagged-value-added model incorporates the assumption that current knowledge is an additive function of all current and past inputs and an i.i.d. stochastic error term. This can be written formally as:

$$Y_{i,t} = \alpha_t X_{i,t} + \alpha_{t-1} X_{i,t-1} + \dots + \alpha_1 X_{i,1} + \epsilon_{i,t}$$
 (2)

where $Y_{i,t}$ is child i's test scores at time t and $X_{t,i}$ is a vector of child, school, and village controls at time t. As data on past inputs are usually unavailable, however, they are generally subsumed

into a lagged dependent variable included as a control. In this case the lagged-value-added model can be re-written as:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \epsilon_{i,t} \tag{3}$$

Where $Y_{i,t-1}$ is assumed to capture all past inputs and unobservable heterogeneity across students. The specification given by Equation 3 is the model primarily employed in this paper. As the interest of this analysis is on the difference between government and private school students, primary interest is on a dummy for school type included in the vector of controls $X_{i,t}$.

Three aspects of this specification are worth emphasizing. First, while the inclusion of a lagged dependent variable effectively controls for unobserved differences that affect differences in test *levels*, it cannot control for unobserved heterogeneity that affects learning *rates*. It is for this reason that while superior to other available methods, value-added analyses can not fully overcome selection issues.⁸

Second, the β term can be interpreted as the "persistence parameter" in that it estimates the degree to which past learning may carry forward. A value of one is equivalent to assuming that children do not forget past lessons, while a value of zero corresponds to students forgetting all past lessons each year. While imposing a persistence parameter of one may seem reasonable – it amounts to regressing the difference in test scores between time t and time t-1 on controls – a growing literature has shown that the test score gains of short term interventions often "die out" over time, suggesting that not all that is learned is retained (Banerjee et al., 2007; Glewwe et al., 2010; Currie and Thomas, 1995; Rothstein, 2010). Andrabi et al. (2011) shows that imposing the restriction that $\beta = 1$ biases learning estimates.

Finally, lagged test scores are generally measured with error, which leads to an often significant attenuation bias in the estimate of β and biased estimates of other coefficients (Kane and Staiger, 2002; Chay et al., 2005; Andrabi et al., 2011). Thus keeping with best practices, lagged test scores from all three subjects – English, Urdu, and math – are included in all specifications to instruments for the primary lagged test score of interest.

Note further that all child-level results are presented using heteroskedastic-robust standard errors clustered at the level of the village.

A.1 Village-Level Estimates

In addition to estimating learning differences at the level of the child, the primary analysis of this paper is also estimated at the level of the village. To do so, Specification 3 is augmented with with fixed effects for each village-school type combination, as below:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \mathbb{I}_{i,j,type,t}\gamma_{j,type} + \epsilon_{i,t}$$

⁸Some analysis have turned to second-differencing the data and focusing on students who change schools (Andrabi et al., 2011), but these analyses have their own limitations, among them limited sample sizes (given that changes between types of school are relatively infrequent in most surveys) and the assumption that school changes are not the result of some unobserved shock (i.e. that school switches are not accompanied by contemporaneous with other changes – a potentially problematic assumption given the relative infrequency with which students change schools).

Where $\mathbb{I}_{j,type}$ is a vector of dummies for the village j of child i in school type $type \in \{private, government\}$ at time t. The difference between the village private-school dummy coefficients and the village government-school dummy coefficients are then extracted as a village-level estimate of the government-private test score gap. These village-level gaps are then regressed against a series of village-level controls, include village fractionalization, wealth, size, land fractionalization, and adult literacy variables Z_i reported at the level of the village j, as below:

$$Gap_i = Z_i\delta + \eta_i$$

A.1.1 Teacher-Value Added Estimates

A similar method is employed when estimating the contribution of individual teachers to child learning. Specification 3 is again employed with the addition of fixed effects for each individual teacher, as below:

$$Y_{i,t} = X_{i,t}\alpha + Y_{i,t-1}\beta + \mathbb{I}_{i,k,t}\zeta_k + \epsilon_{i,t}$$

where \mathbb{I}_k is a vector of dummies for whether student i was taught by teacher k in year t. The fixed effect coefficients ζ_k are then extracted as an estimate of teacher k's contribution to student learning in various regressions. Teacher regressions are all weighted by the number of students taught by a given teacher, both because our interest is in the experience of the average child and because value-added estimates for teachers with small classes are extremely imprecise and are otherwise prone to skew results.