# **Potential Outcomes**

Nick Eubank

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Why not?

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 $\Rightarrow$  Comparing these two groups would conflate effects of treatment with other unobserved differences.

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- Estimate effect of cholesterol meds on heart attack risk by comparing heart attack rates among those who take cholesterol meds to those that don't?

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ensures that in large enough samples, treated and control subjects will be the same as one another.

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### RAND Study:

- High internal validity (randomized)
- Mixed external validity: study population was average people, but most uninsured today are young, poor, less educated, so may not speak to results of real policies to expand insurance.

Often (though not always) tension between internal and external validity:

- Internal Validity: Maximized by controlling the environment
- External Validity: About "realism" of study

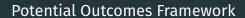
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Which of the following has higher internal validity, which has higher external validity, and why?

- Psychology experiment where undergrads are put in a lab and randomly assigned to solve puzzles, some with emotionally disturbing imagery and some with happy imagery to test effects of emotional stress on problem solving.
- 2. Political scientists interested in how social pressure effects whether people vote mail fliers to a random set of voters that includes data on what elections they've voted in in the past several years so they'll know whether they vote is public. They then see if they turnout at higher rates than a control group.



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- outcome  $y_i$  under treatment t = 1 (denoted  $y_{i,t=1}$ ) to
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· Counter-factual model of causality

... but we can't see both  $y_{i,t=1}$  and  $y_{i,t=0}$ . Each person can only experience one outcome.

So we'll do two things. First, let's move to populations. Ideally we want:

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Called Average Treatment Effect, or ATE

But we still can't actually see ATE. What we can see is:

$$\widehat{ATE} = E(Y_{T=1}|D=1) - E(Y_{T=0}|D=0)$$

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Two concepts:

- $T \in 0, 1$ : Potential states of the world.
- $D \in 0,1$ : Actual assignment of treatment.

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$$\tilde{A}T\tilde{E} = E(Y_{T=1}|D=1) - E(Y_{T=0}|D=0) 
= E(Y_{T=1}|D=1) - E(Y_{T=0}|D=0) + 
E(Y_{T=0}|D=1) - E(Y_{T=0}|D=1)$$

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## **Potential Outcomes**

$$\underbrace{E(Y_{T=1}|D=1)-E(Y_{T=0}|D=1)}_{\text{Treatment on the Treated}} + \underbrace{E(Y_{T=0}|D=1)-E(Y_{T=0}|D=0)}_{\text{Baseline Difference}}$$

Baseline Difference: Absent treatment, would those who actually got treatment have turned out the same as those who hadn't received treatment.

$$\underbrace{E(Y_{T=1}|D=1) - E(Y_{T=0}|D=1)}_{\text{Avg Treatment on the Treated}} + \underbrace{E(Y_{T=0}|D=1) - E(Y_{T=0}|D=0)}_{\text{Selection Effect}}$$

Treatment on the Treated: What we measure. This is equal to Average Treatment effect iff

$$E(Y_{T=1}|D=1) - E(Y_{T=0}|D=1) = E(Y_{T=1}|D=0) - E(Y_{T=0}|D=0)$$
$$= E(Y_{T=1}) - E(Y_{T=0})$$

In other words, ATT = ATE if the response to treatment of people for whom D = 1 is the same as that of those for whom D = 0.

What we estimate is equivalent to  $ATE = E(Y_{T=1}) - E(Y_{T=0})$  if:

- 1. No baseline difference (absent treatment, same outcomes)
- 2. Same treatment response (no difference in how treated and untreated would respond if treated)
- ⇒ Both groups to have same potential outcomes

Suppose we measured the effect of an exercise program on health by just comparing health of people in an exercise class with health of people not in an exercise class. How might these two things have been violated:

- 1. No baseline difference
- 2. Same treatment response

Suppose we measured the effect of advertising on sales by correlating sales with advertising expenditures. How might these two things have been violated:

- 1. No baseline difference
- 2. Same treatment response