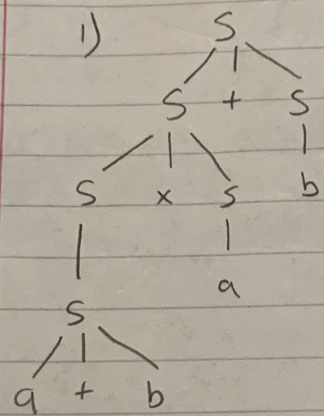
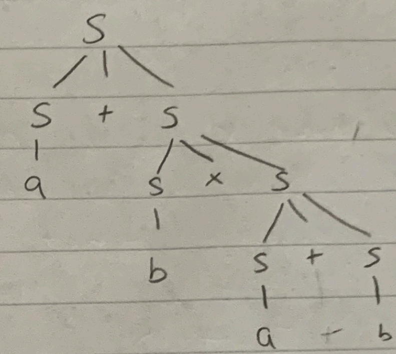
B401 - HW 7

1. CFG G over $\{a, b, +, \times\}$ with initial non-terminal S,
Production rules $S \rightarrow S+S \mid S \times S \mid a \mid b$
give 2 parse trees of G for string $a + b \times a + b$.

1)



2)



2 (a) $S \rightarrow aSb \mid B$
$B \rightarrow Bb \mid b$

$\Rightarrow aSb$
$\Rightarrow aBb$
$\Rightarrow aBbb$
$\Rightarrow abbb$ ✓

(b) $S \rightarrow XY$
$X \rightarrow aabB \mid \varepsilon$
$Y \rightarrow aB \mid \varepsilon$

$\Rightarrow XY$
$\Rightarrow aa\varepsilon bY$
$\Rightarrow aabaY$
$\Rightarrow aaba\varepsilon$
$\Rightarrow aaba$ ✓

(c) $S \rightarrow aSd \mid X \mid Y$
$X \rightarrow bXd \mid n$
$Y \rightarrow aYc \mid A$
$A \rightarrow bAc \mid n$

$\Rightarrow aSd$
$\Rightarrow aYd$
$\Rightarrow aaYcd$
$\Rightarrow aabAccd\varepsilon$
$\Rightarrow aabcAcd$
$\Rightarrow aabccd$ ✓

(b) $S \to AaA \mid X \mid Y \mid Z$
$X \to bXb \mid Y \mid Z \mid \varepsilon$
$Y \to aYb \mid A \mid Z$
$Z \to bZa \mid A \mid Y \mid Z \mid \varepsilon$

$\Rightarrow AaA$
$\Rightarrow YaA$
$\Rightarrow aYbaA$
$\Rightarrow abaX$
$\Rightarrow ababXb$
$\Rightarrow ababb$ ✓

3. Suppose you have X as a context free Grammar that generates Y. Start w/ nonterminal node, S. With the rules, you could conclude that some non-terminal node Z would make

$Z \to S$
$\Rightarrow SZ \mid \varepsilon$
$\Rightarrow SZa \mid a$

4. $L = \mathcal{L}(G)$ where $G = CFG$, $L^R = \{w^R \mid w \in L\}$
So to generate L^R
$S \to w \mid w \in L$
$S \to w^R \mid w \in L,$

5 (a) suppose $L = a^k b^{2k} a^k$ is an element of L, and it's length is $> k$. This means there exists a subset that is $abc \subseteq L$, with $aL \neq \varepsilon$. If we clip with the clipping thm, we get w' which is $\subseteq L$. So it is in L

(b) $\{a^i b^3 a^i \mid i \geq 3\}$
suppose $L = a^k b^3 a^k$ is an element of L, and it's length is $> k$. There exists a subset that is composed of XYZ also in L, with $XL \neq \varepsilon$. With the clipping thm, we can prove that L is CF