B401 - HW 8

1)

(a) $L \subseteq \{a,b\}^*$

Start → S

accept → A

$S \xrightarrow{\varepsilon} P$ $\quad P \xrightarrow{a} P$ $\quad P \xrightarrow{\varepsilon} Y$ $\quad P \xrightarrow{a} P_0$

$P_0 \xrightarrow{b} P_1$ $\quad P_1 \xrightarrow{\varepsilon} K$ $\quad K \xrightarrow{\varepsilon} Z$ $\quad Z \xrightarrow{\varepsilon} A$ ✓

(b) $L \subseteq \{a,b\}^*$

Suppose we have K, a language that is a subset of L. if we have $P = a^k b^k a^k b^k$, and $P \in L$; and another subset z that is equal to $U \cdot Y \cdot V$ and $z \leq |k|$, $P' = Y$ because we clip off UV. Because of this, and that the length of $|z| \leq k$, we are not able to obtain a CFG that correctly generates the correct language.

(c) $L \subseteq \{a,b\}^*$

Start → S

accept → A

$S \xrightarrow{\varepsilon} P$ $\quad P \xrightarrow{a} P$ $\quad P \xrightarrow{\varepsilon} P_0$ $\quad P_0 \xrightarrow{a} P$

$P \xrightarrow{\varepsilon} K$ $\quad K \xrightarrow{a} K_0$ $\quad K_0 \xrightarrow{\varepsilon} P$ $\quad P \xrightarrow{b} P$

$P \xrightarrow{\varepsilon} A$

2) (a) Construct PDA for $L = \{a^{p+q} b^q c^p \mid p,q \geq 0\}$

Start → S

accept → A

(b)

$S \xrightarrow{\varepsilon} P$ $\quad (s, \underline{z}, aaabcc)$

$P \xrightarrow{a} P$ $\quad \Rightarrow (P, >\underline{z}, aabcc)$

$P \xrightarrow{\varepsilon} f$ $\quad \Rightarrow (P, >>\underline{z}, abcc)$

$f \xrightarrow{b} f_0$ $\quad \Rightarrow (f, >>>\underline{z}, bcc)$

$f_0 \xrightarrow{\varepsilon} k$ $\quad \Rightarrow (f_0, >>>>\underline{z}, cc)$

$k \xrightarrow{c} k_0$ $\quad \Rightarrow (k, >>>>>\underline{z}, c)$

$k_0 \xrightarrow{\varepsilon} k_1$ $\quad \Rightarrow (k, >>>>\underline{\varepsilon}, \omega)$

$k_1 \xrightarrow{\varepsilon} A$ $\quad \Rightarrow (A, >>>>\underline{z}, \omega)$

2 (c)

CFG

$S \rightarrow aSb \mid K$

$K \rightarrow aKb \mid \varepsilon$

PDA

$S \xrightarrow{\varepsilon} P$

$P \xrightarrow{S} P$

$P \xrightarrow{(S \rightarrow K)} P$

$P \xrightarrow{(K \rightarrow aKb)} P$

$P \xrightarrow{(a \rightarrow \varepsilon)} P$

$P \xrightarrow{(a \rightarrow \varepsilon)} P$

$P \xrightarrow{(\varepsilon \rightarrow A)} P \rightarrow A$

3

$\Sigma = \{0, 1\}$

$c : \Sigma^* \rightarrow \Sigma^*$ (outputs even positions)

$c(160101) = 011$

$c(1) = \varepsilon$

Suppose we have L, a subset of Σ. If we were to make a PDA recognizing L, then we'd have a different one recognizing the transitions that are made to skip the odd numbered inputs. And saying this would allow us to conclude that for the accepting states of these 2 PDA's (lets call them f and f'), the output would be equal to ε. So $L(f) = c(L)$.