PHY202 Portfolio 1

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September 23, 2016

1 Vector Arithmetic

The addition of two vectors, $\vec{v}$ and $\vec{u}$ can done by decomposing the two into their x and y (and z if in 3D) forms and adding them.

$$\begin{aligned}\vec{v}+\vec{u} &= (v_x+u_x)\hat{\imath}+(v_y+u_y)\hat{\jmath}+(v_z+u_z)\hat{k} \\ &= \langle u_x+v_x, u_y+v_y, u_z+v_z\rangle\end{aligned}$$

When a vector is given in magnitude and angle, trigonometry can be used to find the x and y components.

$$\vec{v}=\langle r,\theta\rangle$$

$$v_x = r\cos\theta, v_y = r\sin\theta$$

Similarly, to convert a vector back into magnitude angle notation:

$$\vec{v}=\left\langle \|\vec{v}\|, \tan^{-1}\frac{v_y}{v_x}\right\rangle$$

1.1 Coulomb's Law

According to Coulomb's Law, the force between two static electrically charged particles can be given by:

$$F_E = k\frac{q_1 q_2}{r^2}$$

F_E is parallel to a line connecting the center points of q_1 and q_2. Coulomb's k constant and ϵ_0 are defined as:

$$k=\frac{1}{4\pi\epsilon_0}$$

$$k\approx 9\cdot 10^9\frac{N\ m^2}{C^2}$$

$$\epsilon_0\approx 8.85\cdot 10^{-12}\frac{C^2}{N\ m^2}$$

The net force on an object is the sum of each individual force

$$F_{net}=\Sigma_{i=1}^{n}F_i$$

When calculating net force from multiple charges, break up the forces into x and y components and then add together. Convert the force back into a magnitude and angle if necessary.

1.2 Electric Fields from Point Charges

Electric fields from point charge extend radially outwards from the origin point. Positive charges radiate outwards, while negative charges radiate inwards. The general definition of electric field is as follows:

$$\vec{E}\equiv\frac{F}{q}{}^{N}/_{C}$$

We can combine the electric field formula with Coulomb's law to obtain the electric field from a point charge.

$$\vec{E}=\frac{F}{q}$$

$$\vec{E}=\frac{\frac{kqQ}{r^2}}{q}$$

$$\vec{E}=\frac{kQ}{r^2}$$

Finding the net field from multiple point charges is similar to finding the net force from multiple point charges. Break up the fields into x and y components and add together. Remember that negative charges have negative fields and positive charges have positive fields.

2 Continuous Charge Distributions

2.1 Electric Fields

We cannot use $E = \frac{kQ}{r^2}$ for non point charges. To find the field from shaped charges, we must integrate the electric field with respect to the charge.

$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$

However, to know the amount of charge we first must find the charge density. The charge density is defined as follows, depending on the dimensionality of the object.

$$\mathbb{R}^1 : \lambda = \frac{q}{l}$$
$$\mathbb{R}^2 : \sigma = \frac{q}{A}$$
$$\mathbb{R}^3 : \rho = \frac{q}{V}$$

Where l, A, and V are length, area, and volume, respectively.

Once the charge density is determined, dq can be replaced with either λdl, σdA, or ρdV. r is the distance from the charge to the point you are measuring the field from. Note the distance should change depending on the "slice" of the charge you are integrating. It is very important to draw a free body diagram to be able to determine the sign of your charges.

2.2 Electric Potential

To find the total voltage of a point charge, we can use the equation

$$V = k\frac{q}{r}$$

however, if we want to find the total voltage of a shaped charge, we must integrate over the whole charge.

$$V = k \int \frac{dq}{r}$$

2.3 Problem Solving Strategy

Continuous charge distribution problems can be solved using the following 4 step strategy:

1. Determine the value of r
   (a) r is equal to the distance from the reference point to the charge "slice".
   (b) Should always be a positive value.
2. Determine dq
   (a) Depends on the dimensionality of charge object
   (b) λdl for one dimension
      i. If integrating around a circle, $dl = rd\theta$
   (c) σdA for two dimensions
   (d) ρdV for three dimensions
3. Determine the limits of integration (where is the charge located?)
4. Check to make sure the integration makes sense
   (a) Follow the units and make sure they are correct
   (b) Is the sign of the integrated value correct?

3 Gauss' Law

Gauss' law is defined as "the total electric flux out of a closed surface is equal to the charge enclosed devided by the permittivity". In mathematical terms, it is defined as:

$$\Phi_E = \frac{q_{enc}}{\epsilon_0}$$

We can put this together with the other definition of flux that we have:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

To get the relationship:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \ = \frac{q_{enc}}{\epsilon_0}$$

Electric flux is the measure of how much electric field is flowing out of an area. Positive flux means electric field is flowing out, while negative flux means electric field is flowing in.

3.1 Gaussian Surfaces

To find the electric field or flux in an area, we can create a theoretical surface to apply Gauss' law. Use the following problem-solving strategy:

1. Draw charge area and imaginary gaussian surface around it
   (a) Area vector should be either perpendicular or parallel to electric field to simplify dot product
      i. If area is parallel, then $\vec{E} \cdot d\vec{A}$ integrates to $\vec{E}\vec{A}$
      ii. If area is perpendicular, then $\vec{E} \cdot d\vec{A}$ equates to 0 and the integral becomes 0
   (b) Field should be uniform to simplify integral
2. Integrate over the surface of the Gaussian surface
   (a) $\oint \vec{E} \cdot d\vec{A}$, where A is the surface area of the gaussian surface
   (b) If everything is constant, you end up with $\vec{E}\vec{A}$
3. Calculate charge enclosed by surface
   (a) Find charge enclosed using $\frac{q_{enc}}{\epsilon_0}$
      i. If constant charge density, use one of the charge density formulas
         A. $q = \lambda l$
         B. $q = \sigma A$
         C. $q = \rho V$
      ii. If nonuniform charge density, integrate it to find total charge
4. Substitute values and solve for electric field, E
   (a) If the field is constant, this should be equal to $E = \frac{q_{enc}}{\epsilon_0 \vec{A}}$

4 Electric PE and Work Energy

4.1 Electric Potential Energy

Recall the generic equation for potential energy:

$$\Delta U = -\int \vec{F} \cdot ds$$

We can relate this to electric field by substituting in $F \equiv \vec{E}q$ for force to obtain

$$\Delta U = -\int \vec{E}q \cdot ds$$

$$\Delta U = -q \int_a^b \vec{E} \cdot ds$$

Note that the integral only depends on the end points to determine change in potential energy. The path that is taken is not important. If we substitute in the E for a point charge, we obtain the following equation for change in PE from point charges

$$\Delta U = -q_1 \int_a^b \frac{kq_2}{r^2} dr$$

$$\Delta U = -kq_1q_2 \int_a^b \frac{1}{r^2} dr$$

$$\Delta U = -kq_1q_2 \left[-\frac{1}{r}\right]_a^b$$

$$\Delta U = kq_1q_2 \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

If the initial reference point is determined to be infinitely (∞) far away, the $\frac{1}{r_i}$ component can be eliminated and we obtain our formula for change in potential energy.

$$\Delta U = \frac{kq_1q_2}{r}$$

If we look closely, we can see that this contains the equation for change in voltage due to a point charge, so change in potential energy can be rewritten as:

$$\Delta U = q\Delta V$$

4.2 Work Energy Theorem

Work is equal to the integral of force over a given distance

$$W = \int_a^b \vec{F} \cdot dx$$

If we replace $\vec{F}$ with Coulomb's law for force from point charges and solve, we obtain the following equation

$$W = \int_a^b k\frac{q_1q_2}{r^2} dr$$

$$W = kq_1q_2 \int_a^b \frac{1}{r^2} dr$$

$$W = kq_1q_2 \left[-\frac{1}{r}\right]_a^b$$

$$W = kq_1q_2 \left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

We can relate this to change in potential. Since the work is nonconservative, it is equal to the sum of the change in potential energy and change in kinetic energy.

$$\Sigma W_{NC} = \Delta U + \Delta K$$

5 Capacitor Networks

A capacitor is an electric component used to store electric charge, the most common of which is the parallel plate capacitor. The capacitance of a parallel plate capacitor is given by the following relationship:

$$C = \frac{Q}{V}$$

According to Gauss' law, the electric field between the two plates is given by

$$E = \frac{Qd}{\epsilon_0 A}$$

This can be rewritten to relate the capacitance to surface area and plate distance

$$C = \frac{\epsilon_0 A}{d}$$

5.1 Capacitors In Series

In a circuit, connecting capacitors in series is equivalent to creating one capacitor whose d value is the sum of each individual d value. The total capacitance value is given by the inverse sum of each inverse capacitance.

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

The total voltage, V, of the capacitors is given by the sum of each individual capacitor's change in potential.

$$V_{total} = V_1 + V_2 + V_3 + \ldots + V_n$$

The charge q remains constant for each capacitor. It is the same as the total charge of the system.

5.2 Capacitors In Parallel

Connecting capacitors in parallel is equivalent to creating one capacitor whose surface area is the sum of each individual surface area. The total capacitance is the sum of each capacitance.

$$C_{total} = C_1 + C_2 + C_3 + \ldots + C_n$$

The total charge of the system is the sum of each capacitor's charge.

$$Q_{total} + Q_1 + Q_2 + Q_3 + \ldots + Q_n$$

The voltage of each capacitor remains constant. It is the same as the total voltage of the system.

5.3 Solving Capacitor Networks

To solve a network of capacitors, group similar capacitors to create equivalent capacitances. Keep solving until the total capacitance of the network is reached, then work backwards to find individual charges and voltages. Use the equation

$$Q = C\Delta V$$

PHY202 Portfolio 2

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October 21, 2016

1 Resistor Networks

A resistor is an electric component used to store impede current flow. The voltage, current, and resistance of any given resistor is given by the following proportion:

$$R = \frac{V}{I} \quad V = IR \quad I = \frac{V}{R}$$

Any device that follows Ohm's law is considered to be ohmic. If the current vs voltage graph of the device is graphed, it would be linear, revealing a linear relationship between resistance, current, and voltage.

1.1 Resistivity

The resistance of a resistor is dependent on a property called *resistivity*, ρ, and also its length and cross sectional area.

$$R = \rho \frac{l}{A}$$

$$l = \text{length} \quad A = \text{cross-sectional area}$$

1.1.1 Temperature

The resistivity of resistors is often dependent on its temperature. It is given by the following expression:

$$\rho = \rho_0 \left[1 + \alpha(T - T_0)\right]$$

Where ρ_0 is the reference resistivity at temperature T_0, which is often either 20°C or 0°C. α is the *temperature coefficient of resistivity*, it is a constant number.

1.2 Resistors In Series

Connecting resistors in series with one another is equivalent to having one resistor whose resistance is the sum of each individual one. So, the equivalent resistance is given by:

$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$$

The total voltage of the system is found by adding each voltage.

$$V_{eq} = V_1 + V_2 + V_3 + \ldots + V_n$$

The current of each resistor remains constant. It is the same as the current of the system.

1.3 Resistors In Parallel

Multiple resistors in series can be grouped together as one resistor. The equivalent resistance of this can be found by adding the inverse sums.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

The total current of the system is the sum of each resistor's charge.

$$I_{eq} = I_1 + I_2 + I_3 + \ldots + I_n$$

The voltage of each resistor remains constant. It is the same as the total voltage of the system.

1.4 Solving Resistor Networks

1. Draw out the complete circuit and label resistors and batteries
2. Group together resistors in series or parallel and find their equivalent resistance
   (a) Pick a small branch of the circuit
   (b) The resistors are usually right next to each other
3. Redraw the new circuit each time you simplify a branch
4. Repeat until you have the whole circuit as one equivalent resistor and one voltage source
5. Since the total voltage of the circuit is equivalent to the power source, work backwards to find currents and voltages
   (a) When resistors are in parallel, they have equal voltages
   (b) When resistors are in series, they have equal currents
   (c) Remember $V = IR$

1.5 Drift Velocity

Drift velocity of electrons in a current-carrying wire can be found using the following formula:

$$I_{avg} = nqv_dA$$

Where n is the number of particles, q is the charge of each particle, v_d is the drift velocity, and A is the cross sectional area of the wire.

1. Find the average current of the wire
   (a) If current density is constant, then use it
   (b) If current density isn't constant, then integrate to find the current
2. Plug in all values and then solve for v_d

2 Kirchhoff's Rules

When starting a Kirchhoff's laws problem, number and label the direction of all currents in the circuit. Direction is arbitrary, but must be kept consistent. Whenever a current hits a junction, split it into different currents.

2.1 Junction Rule

At any point junction point, the sum of the currents going in is equal to the sum of currents going out.

$$\Sigma I_{in} = \Sigma I_{out}$$

For each junction point, we write an equation that relates the currents we have defined as going in as equal to the currents defined as going out.

2.2 Loop Rule

The sum of changes along any loop of wire is always zero. Going from negative to positive terminal of a battery will increase the voltage by the battery's voltage. Going across a resistor in the direction of your current will cause a voltage drop, while going across a resistor against the current will cause a voltage "gain". This is written as an equation that will equal zero.

$$\Sigma \Delta V_n = 0$$

2.3 Solving as a Matrix

To solve a Kirchhoff's laws problem, we will need to create a matrix and perform a gaussian elimination on it to get an answer into *reduced row echelon form.* The size of the matrix depends on how many unknowns currents we have. If we have x currents, then the size of the matrix should be x rows $\times$ $(x+1)$ columns.
If we have y junctions in our problem, then we use $y-1$ of our junction rule equations, and fill the rest of the matrix with loop rule equations.
Here is an example matrix:

$$\left[\begin{array}{cccc|c} a_1 & b_1 & c_1 & d_1 & 0 \\ a_2 & b_2 & c_2 & d_2 & 0 \\ a_3 & b_3 & c_3 & d_3 & 2 \\ a_4 & b_4 & c_4 & d_4 & 6 \end{array}\right]$$

The first column represents the coefficient of our first current in each equation, the second represents the coefficient of current 2, and so on. The last column represents any constant value that the equation is equal to. *i.e.*, the last row in the example matrix represents $a_4 + b_4 + c_4 + d_4 = 6$.
This matrix can be solved using a `rref()` function on any calculator or CAS. The resultant matrix will look similar to an identity matrix with an added column on the right. This last column will contain the values of each current in order from I_1 to I_n, n being the number of currents being solved for.

3 RC Circuits

Any given circuit that contains both resistors and capacitors will have something called a time constant, τ,

$$\tau = RC$$

where R is the total resistance of the circuit and C is the total capacitance of the circuit. The time constant is the amount of time that it takes the capacitor to reach 63% of its maximum voltage, it is measured in seconds. When the capacitor is charging, we can use these formulas to determine the amount of voltage or charge given $t =$ time.

$$Q(t) = Q_0(1 - e^{-\frac{t}{\tau}})$$

$$V(t) = V_{battery}(1 - e^{-\frac{t}{\tau}})$$

When the capacitor is discharging, these formulas are used to determine the voltage or charge.

$$Q(t) = Q_0 e \frac{-t}{\tau}$$

$$V(t) = V_0 e \frac{-t}{\tau}$$

The current flowing through the circuit can be given by

$$I(t) = \frac{V}{R} e \frac{-t}{\tau}$$

it does not depend on whether or not the capacitor is charging or discharging.

3.1 Problem Solving

1. Find the equivalent resistance of the circuit
   (a) Series: $R_{eq} = R_1 + R_2 + R_3 \dots$
   (b) Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$
2. Find the equivalent capacitance of the circuit
   (a) Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots$
   (b) Parallel: $C_{eq} = C_1 + C_2 + C_3 \dots$
3. Solve $\tau = RC$

3.2 Multi Loop Circuits

If the capacitor is fully charged in a multi-loop circuit, then that branch can be ignored because no current is flowing through it. If possible, turn it into a single loop by finding the equivalent resistance and capacitance.

4 Magnetic Force Problems

4.1 Force on a Wire

The magnetic force on a wire from a field is:

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

Where I is the current of the wire, $\vec{L}$ is a vector pointing in the direction of the current, whose magnitude is the length of the wire. $\vec{B}$ is the direction of the magnetic field. When using the right hand rule, your pointer finger is the $I\vec{L}$, your middle finger is the $\vec{B}$ field, and your thumb is the resultant $\vec{F}_b$.

4.2 Force on a Charged Particle from Field

The magnetic force on a particle from a field:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

Where q is the charge of the particle, $\vec{v}$ is the initial velocity of the vector, and $\vec{B}$ is the direction of the magnetic field.

4.2.1 Circular Path of a Particle

When a particle enters a field, its radius to some fixed point will stay constant, as well as the magnitude of its velocity. This is given by the equation:

$$r = \frac{mv}{qB}$$

The period of rotation for the particle is:

$$T = \frac{2\pi m}{qB}$$

4.3 Force between two Wires

The force from two parallel wires on each other is given by:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

l is the length of both wires (assumed to be the same), I_1 and I_2 are the currents of wire 1 and 2, respectively. a is the distance between the wires. If the current on the wires are both travelling in the same direction, the force will be attractive. If the current is opposite, the force will be repulsive.

4.4 Torque and Magnetic Moment

Torque of a coil of wire is defined as:

$$\tau = nI\vec{A} \times \vec{B}$$

Where n is the number of loops, I is the current, and A is the cross-sectional area, perpendicular to the coil of wire. $nI\vec{A}$ can be reduced to just μ, the *magnetic dipole moment* of the loop.

$$\mu = nI\vec{A}$$

So, torque can be related to μ and $\vec{B}$:

$$\tau = \mu \times \vec{B}$$

If finding torque of a circuit without any loops of wire, $n = 1$.

5 Biot-Savart Law

The Biot-Savart law can be used to find the magnetic field from a current-carrying wire at some point. It is given by the following expression:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{s} \times \hat{r}}{r^2}$$

μ_0 is a constant called the *permeability of free space.* $d\vec{s}$ is each piece of wire, and $\hat{r}$ is a unit vector pointing from $d\vec{s}$ to the point P, where we are measuring the field.

5.1 Problem Solving

1. Draw your free body diagram
   (a) Determine the $\hat{r}$ vector. It points from $d\vec{s}$ to P, the point you are determining the field at. It is a unit vector with magnitude of 1.
   (b) Draw and label P, $d\vec{s}$, and the direction of your currents.
2. Find r
   (a) It is the distance from each portion of $d\vec{s}$ to P.
   (b) If integrating an infinite wire, set it equal in terms of θ to integrate more easily.
3. Find $d\vec{s}$
   (a) $d\vec{s}$ is the derivative of the arc length of the wire
4. Determine limits of integration
5. Evaluate the integral

5.2 Ready-Made Equations

The magnetic field from an infinite wire and from a solenoid are given to us and listed on the formulae sheet.

Infinite Wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

r is the distance from the wire to some point you are measuring the field at.

Center of a Solenoid:

$$B = \mu_0 \frac{N}{l} I$$

N is the total number of loops in the solenoid, l is the length of the solenoid. $\frac{N}{l}$ can be replaced with just n, the nubmer of loops per unit length.

6 Ampere's Law

Ampere's law is the magnetic field analogue to Gauss's law for electricity. We define a 1-dimensional closed loop that magnetic field flows through. The integral of the loop dotted with the magnetic field is equal to the current enclosed multiplied with μ_0. In mathematical terms, this is equal to:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

6.1 Solving an Ampere's Law Problem

1. Create a 1D shape that the magnetic field flows through
2. Integrate $\oint \vec{B} \cdot d\vec{s}$, this should leave you with $\vec{B} \cdot S$, where S is the arc length/circumference of the shape.
3. Find the amount of current enclosed.
   (a) If the current is constant, use that.
   (b) If a current density is given, integrate that to find the current enclosed.
4. Solve for $\vec{B}$ or whatever you are trying to find.

PHY202 Portfolio 3

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November 18, 2016

1 Lenz's Law

1.1 Magnetic Flux

The amount of electric field passing through an area can be calculated by the equation for magnetic flux.

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

The area must be an open surface in 3D. If it is closed then there will be net magnetic field of 0.

1.1.1 Problem Solving

To calculate magnetic flux, first identify the magnetic field and the area you are getting flux in. In most cases, $\vec{B}$ will be constant with respect to area so it can be pulled out of the integral.

$$\Phi_B = \vec{B} \cdot \int d\vec{A}$$

Identify which direction the area is pointing in. If a direction is not explicitly given, assume it points along a positive axis. Integrate $d\vec{A}$, which in most cases $\int d\vec{A} = A$, so just find the area of the surface.

$$\Phi_B = \vec{B} \cdot \vec{A}$$

$$\Phi_B = \left\|\vec{B}\right\|\left\|\vec{A}\right\| \cos\theta$$

At this point, plug in the given values of B and A and solve for flux.

1.2 Lenz's Law

In 1834, Russian physicist Heinrich Lenz said:

> The direction of current induced in a conductor by a changing magnetic field due to Faraday's law of induction will be such that it will create a field that opposes the change that produced it.

This is shown as the negative sign in Faraday's Law:

$$\varepsilon = -\frac{\partial \Phi}{\partial t}$$

In problem solving, the current will always be opposite of the direction of the flux derivative.

2 Faraday's Law

Faraday's law of induction states that the induced electromotive force in a closed circuit is equal to the negative rate of change of the magnetic flux enclosed in the circuit.

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Where N is the total number of loops in the circuit, $d\Phi_B$ is the change in flux, and dt is the change in time. This can be simplified into the equation:

$$|\varepsilon| = Blv$$

Where B is the magnitude of the magnetic field, l is the length of the object in the magnetic field, and v is the velocity of the object going into the field. To find the generated EMF in an AC generator, the following equation is used:

$$\varepsilon = NBA\omega \sin(\omega t)$$

ω is the angular frequency in radians per second of the generator.

2.1 Self-Inductance

An inductor can create opposite EMF when it is faced with a change in EMF. This is given by the equation:

$$\varepsilon = -L\frac{dI}{dt}$$

Where L is the value of self-inductance, dI is the change in current and dt is the change in time. To calculate the self-inductance of an inductor, the following two formulas can be used:

$$L = \frac{N\Phi_B}{I}$$

$$L = \frac{\mu_0 N^2 A}{l}$$

N is the total number of loops in the circuit, I is the current flowing through the circuit, Φ_B is the magnetic flux. For the second equation, A is the cross-sectional area and l is the length.

3 LR and LC Circuits

3.1 LR Circuits

In an LR Circuit, the equation for an energizing inductor is given as:

$$I = \frac{\epsilon}{R}\left(1 - e^{-t/\tau}\right)$$

$\frac{\epsilon}{R}$ is the current in the circuit. τ is the LR time constant, it is equal to $\frac{L}{R}$. The equation for a de-energizing inductor is

$$I = \frac{\epsilon}{R}e^{-t/\tau}$$

To find the potential energy of an inductor, use:

$$U = \frac{1}{2}LI^2$$

Where L is the value of inductance and I is the current.

3.1.1 Problem Solving with a Switch

When an LR circuit has a switch that has been closed for a long time, the inductor will act as a wire because there is no change in current. When the switch is just closed, there will be no current because the inductor is opposing all of the change.

When an LR circuit has a switch that has just been open, the current will not change. If the switch has been open for a long time there will be no current because any source of potential will be cut off.

For any time between that, you will have to use the energizing or de-energizing inductor formulas with the values of t. Find the value of τ using $\tau = L/R$.

3.2 LC Circuits

When an inductor and capacitor are in series in a circuit, the current will "bounce" between them and create an alternating current. The charge is described as a cosine wave, where the capacitor will first be filled at full charge.

$$Q(t) = Q_{max}\cos(\omega t)$$

The current is 90 degrees ahead of the charge.

$$I(t) = -I_{max}\sin(\omega t)$$

$$I(t) = -\omega Q_{max}(\omega t)$$

To find the angular speed in radians per second, ω_0 use the following equation

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Where L is the inductance of the circuit and C is the capacitance of the circuit. To get the frequency in hertz, divide by 2π.

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

4 Driven RLC Circuits

In an RLC circuit, the charge is given by the following sinusoidal formula:

$$Q(t) = Q_{max}e^{-Rt/2L}\cos(\omega_d t)$$

To find ω_d, the angular speed of a driven RLC circuit:

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}}$$

4.1 Average Voltage

Since the average of a sine wave is zero, to find the average voltage something called the *root-mean-square* is used. To find V_{RMS}, divide the voltage by $\sqrt{2}$.

$$V_{RMS} = \frac{V}{\sqrt{2}}$$

4.2 Reactance

Reactance in an inductor and capacitor is analogous to the resistor of a circuit. To find the reactance of an inductor:

$$X_L = 2\pi f L = \omega L$$

To find the reactance of a capacitor:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

The impedance of a circuit is the effective resistance in an AC circuit, which is combined from the two reactances and the resistance, units in ohms.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

4.3 Phase Angle

The phase angle is the angle that the voltage leads the current by. It is calculated by:

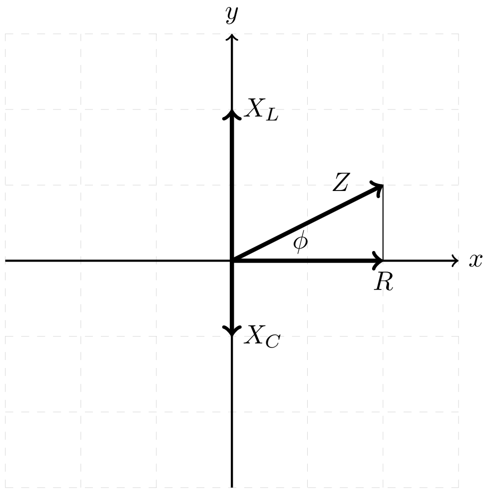
$$\tan\phi = \frac{X_L - X_C}{R}$$

4.4 Drawing Phasors

A phasor diagram represents voltage or current across a cdevice in an AC circuit. It is a graph that rotates at angular speed ω. The projection of the vector onto the y-axis gives the magnitude at that point in time.

4.4.1 Reactance Phasors

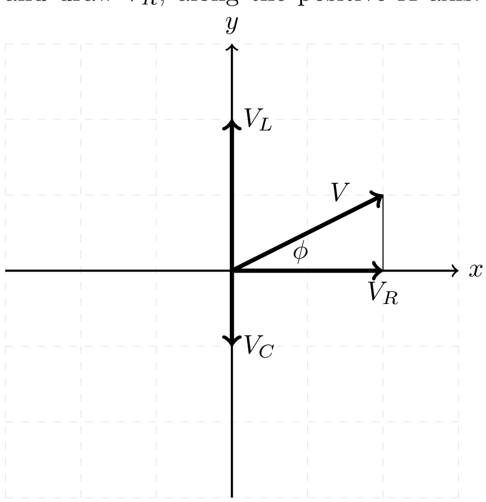
To draw a reactance phasor diagram, draw X_L along the positive Y axis, draw X_C along the negative Y axis, and draw the resistance, R, along the positive X axis. Then, perform vector addition to get the impedance.



In this case, ϕ is the angle that the impedance leads the resistance.

4.4.2 Voltage Phasors

To draw a voltage phasor diagram, draw V_L along the positive Y axis, draw V_C along the negative Y axis, and draw V_R, along the positive X axis. Then, perform vector addition to get the voltage of the circuit.



In this case, ϕ is the angle that the voltage leads the current.

4.4.3 ELI the ICE man

A mnemonic to remember the sign of the phase angle in an inductor or capacitor is *ELI the ICE man.* ε leads I in an inductor (L), I leads ε in a capacitor (C).

5 EM Waves

Maxwell stated that changing electromagnetic fields should produce changing magnetic fields and vice-versa. The equations for $\vec{E}$ field and $\vec{B}$ field in a wave are:

$$\vec{E} = E\sin(kx - \omega t)\hat{\jmath}$$

$$\vec{B} = B\sin(kx - \omega t)\hat{k}$$

The wave number, k is equal to 2π divided by the wavelength. It is the number of cycles/radians per unit distance.

$$k = \frac{2\pi}{\lambda}$$

The ratio of the electric field to the magnetic field is equal to the speed of light.

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E}{B} = \lambda f$$

The intensity and direction of an EM wave can be found by calculating the *Poynting* vector, $\vec{S}$.

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

To find just the intensity of an EM wave, this simplified formula can be used:

$$I = \frac{EB}{2\mu_0}$$

The direction of the Poynting vector determines the direction of the propagating wave, while the magnitude determines the average intensity of the wave.

PHY202 Final Exam Portfolio

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December 12, 2016

1 Polarization of Light

When unpolarized light passes through a polarizer, the average intensity of the light will be half of the original intensity.

$$I_1 = \frac{I_0}{2}$$

This light will now be polarized. If it passes through another polarizer, the new intensity is dependent on the relative angle of the two polarizers.

$$I_2 = I_1 \cos^2 \theta$$

This equation can be applied as many times as there are extra polarizers, just remember that θ is the angle between polarizer n and $n-1$, not the angle that is made to the horizontal.

2 Snell's Law and Total Internal Reflection

2.1 Law of Reflection

When light is reflected off of a mirror, the angle of incidence and the angle of reflection are equal to each other.

$$\theta_i = \theta_r$$

In this case, the angles are measured relative to the surface normal.

2.2 Speed of Light

As light enters different mediums, it will speed up or slow down based on the medium's index of refraction. This new speed of light, v, can be found using the following proportionality:

$$v = \frac{c}{n}$$

Where c is the speed of light in a vacuum, and n is the medium's index of refraction.

2.3 Snell's Law

To find the refraction angle when light enters a new medium, Snell's law can be used.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Where n_1 is the first medium's index of refraction, θ_1 is the angle of incidence relative to the normal, n_2 is the second medium's index, and θ_2 is the angle of refraction relative to the normal.

2.4 Critical Angle

The critical angle is the maximum incidence angle that will still result in refraction. To find this, set the refraction angle to be $\frac{\pi}{2}$, since anything larger will be outside of the medium, then solve for the incidence angle.

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \left(\frac{\pi}{2}\right) \\ \sin \theta_1 &= \frac{n_2}{n_1} \\ \theta_1 &= \sin^{-1} \left(\frac{n_2}{n_1}\right) \end{aligned}$$

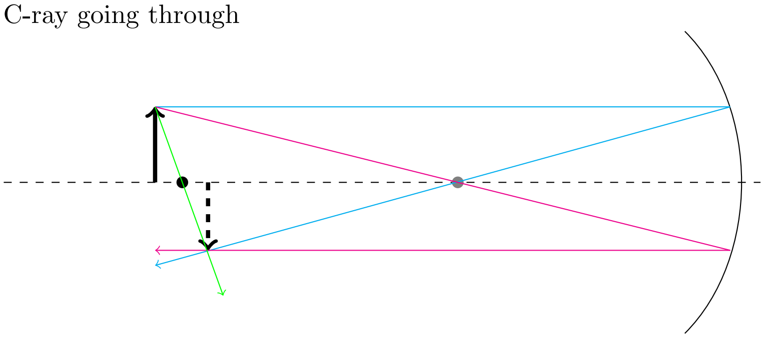
Any angle of incidence larger than this critical angle will result in *total internal reflection*.

3 Ray Diagrams for Lenses and Mirrors

The first step in drawing a ray diagram is to identify what type of mirror or lens needs to be drawn. If it is a mirror, see if it curves towards or away the object. If it curves towards, it is convex, if it curves away, it is concave. If drawing a diagram for a lens, identify if it is converging or diverging. Thin on top and bottom and thick in middle is converging, while thin in the middle and thick on top and bottom is diverging. Draw the ray diagram according to what kind of lens or mirror it is. The point where all of the rays intersect is the image position.

3.1 Concave/Converging Mirrors

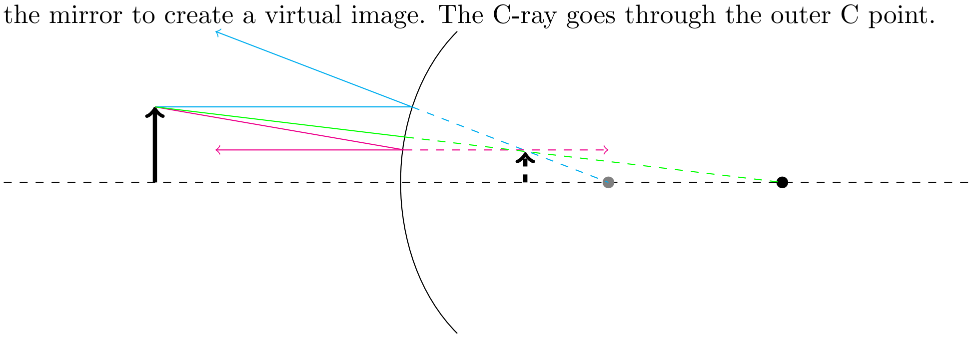
Draw the object, mirror, focal points, and the inner center of radius point (twice the focal point). Starting from the top of the object, draw a ray parallel to the horizontal then reflecting through the focal point (P-Ray). Draw a ray parallel to the focal point and reflecting parallel to the horizontal (F-Ray). Draw the C-ray going through



In this image, the P-ray is blue, the F-ray is magenta, and the C-ray is green.

3.2 Convex/Diverging Mirrors

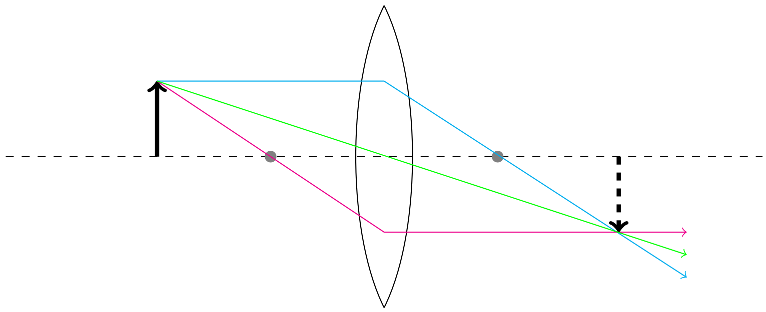
Draw the object, mirror, and focal point, this time the focal point is on the opposite side of the image. Start from the top of the image and draw the F, P, and C-rays as before, except this time they also extend past the mirror to create a virtual image. The C-ray goes through the outer C point.



In this image, the P-ray is blue, the F-ray is magenta, and the C-ray is green.

3.3 Converging Lenses

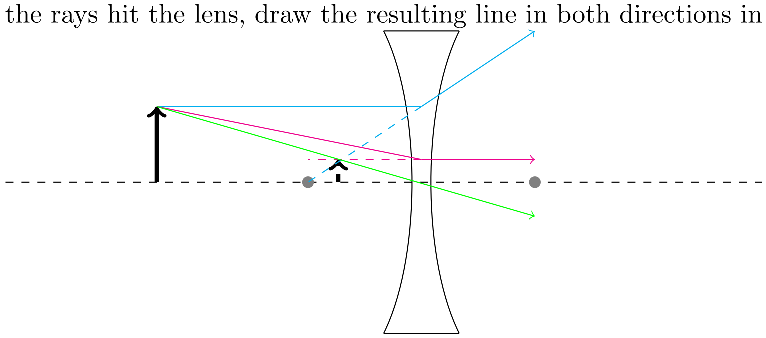
Draw the object, lens, and both focal points, which will be equidistant to the lens on both sides. Draw the P-Ray parallel to the horizontal and then towards the outer focal point. Draw the F-Ray towards the inner focal point then parallel to the horizontal when it reaches the lens. Draw the C-Ray going throught the middle of the lens.



In this image, the P-ray is blue, the F-ray is magenta, and the C-ray is green.

3.4 Diverging Lenses

Draw the object, lens, and focal points. Draw the P-Ray parallel, then towards the inner focal point. Draw the F-ray towards the outer focal point, then parallel. Draw the C-Ray towards the center of the lens. When the rays hit the lens, draw the resulting line in both directions in order to find the image.



In this image, the P-ray is blue, the F-ray is magenta, and the C-ray is green.

4 Calculations for Single Lenses and Mirrors

Given the radius of curvature of a lens or mirror, the focal point can be found by dividing it by two.

$$f = \frac{R}{2}$$

For a lens that has two different radii on its sides, the lens maker's equation can be used:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

The magnification of a lens or mirror, M, is given by the following equation:

$$M = \frac{h'}{h} = -\frac{q}{p}$$

Where h' is the image height, h is the object height, q is the image distance, and p is the object distance. This equation can be used to find one of the distances when the other three are known.
The general equation for a lens or mirror is given by:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Where f is the focal point, p is object distance, and q is the image distance.

4.1 Sign Conventions

Refer to this handy table for sign conventions:

Quantity	**Positive**	**Negative**
Focal Point	Converging	Diverging
Image Distance	Real	Virtual
Image Height	Upright	Inverted

5 Calculations for Double Lens Systems

When dealing with double lens systems, first solve the first lens for the image height and distance. This image becomes the object for the second distance. To get the value of p for the second lens, subtract the image distance from the distance between the two lenses.

$$p_2 = d - q_1$$

The object height will be the same as the first image height because it is measured relative to the same horizontal axis.

6 Interference Problems

In an interference problem, it is important to remember the following variables:

- d, the distance between the slits in a double-slit
- D, the size of the slit in a single slit
- y, the distance from the center of the projection
- L, the distance from the light projector to the screen
- θ, which is equal to $\tan^{-1}\left(\frac{y}{L}\right)$
- ϕ, the angle at which the waves of light are out of phase by in a double-slit at angle θ

6.1 Double Slit

To find the bright spots in a double-slit interference problem, use the following equation:

$$d \sin\theta = m\lambda$$

Where m is an integer greater than or equal to zero. To find the dark spots in a double-slit, use this equation:

$$d \sin\theta = (m + {}^{1}\!/_{2})\lambda$$

To find the phase angle of the two light waves coming out of the slits at angle θ, use this equation to find ϕ:

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$

6.2 Single Slit

To find the dark spots in a single-slit interference problem, use this equation:

$$D \sin\theta = m\lambda$$

Where m is an integer greater than or equal to one.

6.3 Diffraction Grating

If given the slit distance as a density, convert to meters and take the inverse to get d, the slit distance.

$$d = \frac{1}{\rho}$$

To find the bright spots, use the previously defined bright spot equation.

$$d \sin \theta = m\lambda$$