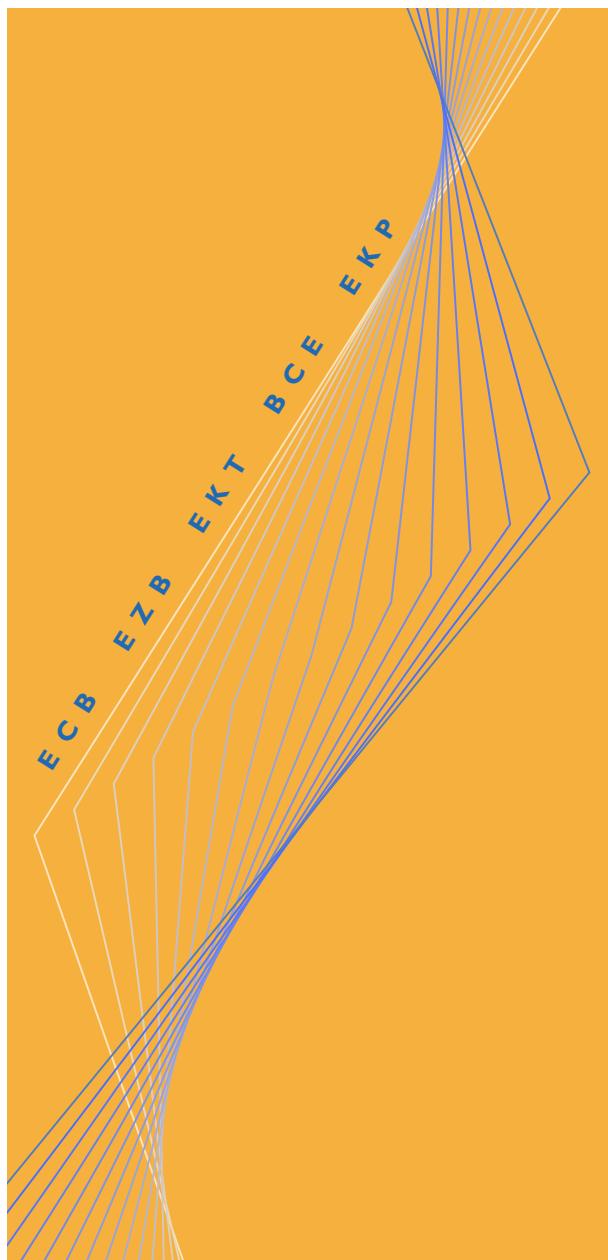


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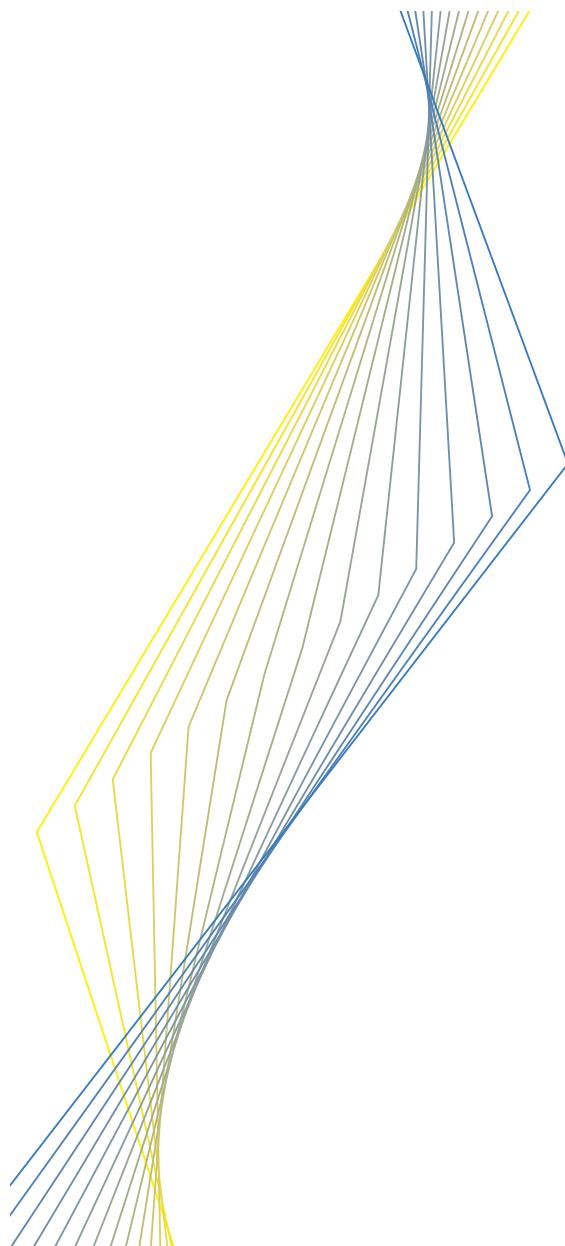
**VALUE AT RISK MODELS
IN FINANCE**

**BY SIMONE MANGANELLI
AND ROBERT F. ENGLE**

August 2001

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Abstract

Value at Risk (VaR) has become the standard measure that financial analysts use to quantify market risk. VaR is defined as the maximum potential change in value of a portfolio of financial instruments with a given probability over a certain horizon. VaR measures can have many applications, such as in risk management, to evaluate the performance of risk takers and for regulatory requirements, and hence it is very important to develop methodologies that provide accurate estimates.

The main objective of this paper is to survey and evaluate the performance of the most popular univariate VaR methodologies, paying particular attention to their underlying assumptions and to their logical flaws. In the process, we show that the Historical Simulation method and its variants can be considered as special cases of the CAViaR framework developed by Engle and Manganelli (1999). We also provide two original methodological contributions. The first one introduces the extreme value theory into the CAViaR model. The second one concerns the estimation of the expected shortfall (the expected loss, given that the return exceeded the VaR) using a simple regression technique.

The performance of the models surveyed in the paper is evaluated using a Monte Carlo simulation. We generate data using GARCH processes with different distributions and compare the estimated quantiles to the true ones. The results show that CAViaR models are the best performers with heavy-tailed DGP.

JEL Classification Codes: C22, G22

NON TECHNICAL SUMMARY

The increased volatility of financial markets during the last decade has induced researchers, practitioners and regulators to design and develop more sophisticated risk management tools. Value at Risk (VaR) has become the standard measure that financial analysts use to quantify market risk. VaR is defined as the maximum potential loss in value of a portfolio due to adverse market movements, for a given probability. The great popularity that this instrument has achieved is essentially due to its conceptual simplicity: VaR reduces the (market) risk associated with any portfolio to just one number, the loss associated to a given probability.

VaR measures can have many applications, and is used both for risk management and for regulatory purposes. In particular, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements imposes to financial institutions such as banks and investment firms to meet capital requirements based on VaR estimates. Providing accurate estimates is of crucial importance. If the underlying risk is not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability or the financial stability of the institutions.

The main objective of this paper is to survey the recent developments in VaR modelling. The focus of the paper is on the underlying assumptions and the logical flaws of the available methodologies and not on their empirical application. In order to facilitate comparison, we restrict our attention to univariate methodologies.

There are also two original theoretical contributions in this paper. The first one introduces the extreme value theory into the Conditional Autoregressive Value at Risk or CAViaR model introduced by Engle and Manganelli (1999). The second one concerns the estimation of the expected shortfall (the expected loss, given that the return exceeded the VaR) using a simple regression technique. In the survey, we show also how the Historical Simulation method is just a special case of the CAViaR models.

The performance of the models surveyed in the paper is evaluated using a Monte Carlo simulation. We generate data using GARCH processes with different distributions and compare the estimated quantiles to the true ones. The results show that CAViaR models are the best performers with heavy-tailed DGP.

1. Introduction

Financial institutions are subject to many sources of risk. Risk can be broadly defined as the degree of uncertainty about future net returns. A common classification reflects the fundamental sources of this uncertainty. Accordingly, the literature distinguishes four main types of risk. *Credit risk* relates to the potential loss due to the inability of a counterpart to meet its obligations. It has three basic components: credit exposure, probability of default and loss in the event of default. *Operational risk* takes into account the errors that can be made in instructing payments or settling transactions, and includes the risk of fraud and regulatory risks. *Liquidity risk* is caused by an unexpected large and stressful negative cash flow over a short period. If a firm has highly illiquid assets and suddenly needs some liquidity, it may be compelled to sell some of its assets at a discount. *Market risk* estimates the uncertainty of future earnings, due to the changes in market conditions.

The most prominent of these risks in trading is market risk, since it reflects the potential economic loss caused by the decrease in the market value of a portfolio. Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk. It is defined as the maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain horizon. In simpler words, it is a number that indicates how much a financial institution can lose with probability θ over a given time horizon. The great popularity that this instrument has achieved among financial practitioners is essentially due to its conceptual simplicity: VaR reduces the (market) risk associated with any portfolio to just one number, that is the loss associated with a given probability.

VaR measures can have many applications, such as in risk management, to evaluate the performance of risk takers and for regulatory requirements. In particular, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements imposes to financial institutions such as banks and investment firms to meet capital requirements based on VaR estimates. Providing accurate estimates is of crucial importance. If the underlying risk is not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability or the financial stability of the institutions.

From a statistical point of view, VaR estimation entails the estimation of a quantile of the distribution of returns. The fact that return distributions are not constant over time poses exceptional challenges in the estimation. The main objective of this paper is to survey the recent developments in VaR modelling. We want to stress that the focus of the paper is on the underlying assumptions and the logical flaws of the available methodologies and not on their empirical application. In order to facilitate comparison, we restrict our attention to univariate methodologies.

There are two original contributions in this paper. The first one introduces the extreme value theory into the Conditional Autoregressive Value at Risk or CAViaR model introduced by Engle and Manganelli (1999). The second one concerns the estimation of the expected shortfall (the expected loss, given that the return exceeded the VaR) using a simple regression technique. In the survey, we show also how the Historical Simulation method is just a special case of the CAViaR models.

The performance of the models surveyed in the paper is evaluated using a Monte Carlo simulation. We generate data using GARCH processes with different distributions and compare the estimated quantiles to the true ones. The results show that CAViaR models are the best performers with heavy-tailed DGP.

The rest of the paper is organised as follows. Section 2 contains the survey. Section 3 discusses several procedures to estimate the expected shortfall. Section 4 describes the implementation and the findings of the Monte Carlo simulation and section 5 briefly concludes.

2. VaR Methodologies

While VaR is a very easy and intuitive concept, its measurement is a very challenging statistical problem. Although the existing models for calculating VaR employ different methodologies, they all follow a common general structure, which can be summarised in three points: 1) Mark-to-market the portfolio, 2) Estimate the distribution of portfolio returns, 3) Compute the VaR of the portfolio.

The main differences among VaR methods are related to point 2, that is the way they address the problem of how to estimate the possible changes in the value of the portfolio. CAViaR models skip the estimation of the distribution issue, as they allow computing directly the quantile of the distribution. We will classify the existing models into three broad categories:¹

- Parametric (RiskMetrics and GARCH)
- Nonparametric (Historical Simulation and the Hybrid model)
- Semiparametric (Extreme Value Theory, CAViaR and quasi-maximum likelihood GARCH)

¹ The number and types of approaches to VaR estimation is growing exponentially and it's impossible to take all of them into account. In particular, Monte Carlo simulation and stress testing are commonly used methods that won't be discussed here. We refer the interested reader to the excellent web site www.gloriamundi.org for a comprehensive listing of VaR contributions.

The results that each of these methods yield can be very different from each other. Beder (1995) applies eight common VaR methodologies² to three hypothetical portfolios. The results show that the differences among these methods can be very large, with VaR estimates varying by more than 14 times for the same portfolio. Hence, in order to decide which methodology to choose, it is necessary to understand the underlying assumptions, as well as the mathematical models and quantitative techniques used. Only after this preliminary step, can the researcher choose the model that she thinks to be closer to her beliefs or objectives.

The basic inspiration of the VaR methodologies to be discussed below usually comes from the characteristics of financial data. The empirical facts about financial markets are very well known, since the pioneering works of Mandelbrot (1963) and Fama (1965). They can be summarised as follows:

1. Financial return distributions are leptokurtotic, that is they have heavier tails and a higher peak than a normal distribution.
2. Equity returns are typically negatively skewed.
3. Squared returns have significant autocorrelation, i.e. volatilities of market factors tend to cluster. This is a very important characteristic of financial returns, since it allows the researcher to consider market volatilities as quasi-stable, changing in the long run, but stable in the short period. Most of the VaR models make use of this quasi-stability to evaluate market risk.

All of the traditional VaR models try to account for some or all of these empirical regularities.

Throughout the rest of the paper we denote the time series of portfolio returns by $\{y_t\}_{t=1}^T$ and the θ -quantile of the return distribution at time t by $q_{t\theta}$. Estimated quantities are denoted by a hat (^).

2.1. Parametric Models

Models such as RiskMetrics (1996) and GARCH propose a specific parameterisation for the behaviour of prices. The family of ARCH models was introduced by Engle (1982) and Bollerslev (1986) and has been successfully applied to financial data. See Bollerslev, Engle and Nelson (1994) for a survey. The simplest GARCH (1,1) can be described as follows:

² EVT and CAViaR are not considered by Beder (1995) as they have been applied to VaR estimation only recently.

$$(1) \quad \begin{aligned} y_t &= \sigma_t \varepsilon_t & \varepsilon_t &\sim i.i.d.(0,1) \\ \sigma_t^2 &= \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}$$

This model has two crucial elements: the particular specification of the variance equation and the assumption that the standardised residuals are i.i.d. The first element was inspired by the characteristics of financial data discussed above. The assumption of i.i.d. standardised residuals, instead, is just a necessary device to estimate the unknown parameters. A further necessary step to implement any GARCH algorithm is the specification of the distribution of the ε_t . The most generally used distribution is the standard normal³. Only after this extra distributional assumption has been imposed, does it become possible to write down a likelihood function and get an estimate of the unknown parameters. Once the time series of estimated variance is computed, the 5% quantile, say, is simply computed as -1.645 (the 5% quantile of the standard normal) times the estimated standard deviation.

Under the RiskMetrics approach the variance is computed using an Exponentially Weighted Moving Average, which correspond to an Integrated GARCH model: $\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda)y_{t-1}^2$, with λ usually set equal to 0.94 or 0.97. RiskMetrics also assumes that standardised residuals are normally distributed (see the Technical Document 1996 for details).

The general finding is that these approaches (both normal GARCH and RiskMetrics) tend to underestimate the Value at Risk, because the normality assumption of the standardised residuals seems not to be consistent with the behaviour of financial returns. The main advantage of these methods is that they allow a complete characterisation of the distribution of returns and there may be space for improving their performance by avoiding the normality assumption. On the other hand, both GARCH and RiskMetrics are subject to three different sources of misspecification: the specification of the variance equation and the distribution chosen to build the log-likelihood may be wrong, and the standardised residuals may not be i.i.d. Whether or not these misspecification issues are relevant for VaR estimation purposes is mainly an empirical issue.

³ This assumption is also motivated by the very important result of Bollerslev and Wooldridge (1992) on quasi-maximum likelihood GARCH. More on this later.

2.2. Nonparametric Methods

One of the most common methods for VaR estimation is the Historical Simulation. This approach drastically simplifies the procedure for computing the Value at Risk, since it doesn't make any distributional assumption about portfolio returns. Historical Simulation is based on the concept of rolling windows. First, one needs to choose a window of observations, that generally ranges from 6 months to two years. Then, portfolio returns within this window are sorted in ascending order and the θ -quantile of interest is given by the return that leaves $\theta\%$ of the observations on its left side and $(1-\theta)\%$ on its right side. If such a number falls between two consecutive returns, then some interpolation rule is applied. To compute the VaR the following day, the whole window is moved forward by one observation and the entire procedure is repeated.

Even if this approach makes no explicit assumptions on the distribution of portfolio returns, an implicit assumption is hidden behind this procedure: the distribution of portfolio returns doesn't change within the window. From this implicit assumption several problems derive.

First, this method is logically inconsistent. If all the returns within the window are assumed to have the same distribution, then the logical consequence must be that all the returns of the time series must have the same distribution: if $y_{t>window}, \dots, y_t$ and $y_{t+1>window}, \dots, y_{t+1}$ are i.i.d., then also y_{t+1} and $y_{t>window}$ must be i.i.d., by the transitive property. Second, the empirical quantile estimator is consistent only if k , the window size, goes to infinity. The third problem concerns the length of the window. This is a very delicate issue, since forecasts of VaR under this approach are meaningful only if the historical data used in the calculations have (roughly) the same distribution. In practice, the volatility clustering period is not easy to identify. The length of the window must satisfy two contradictory properties: it must be large enough, in order to make statistical inference significant, and it must not be too large, to avoid the risk of taking observations outside of the current volatility cluster. Clearly, there is no easy solution to this problem.

Moreover, assume that the market is moving from a period of relatively low volatility to a period of relatively high volatility (or vice versa). In this case, VaR estimates based on the historical simulation methodology will be biased downwards (correspondingly upwards), since it will take some time before the observations from the low volatility period leave the window.

Finally, VaR estimates based on historical simulation may present predictable jumps, due to the discreteness of extreme returns. To see why, assume that we are computing the VaR of a portfolio using a rolling window of 180 days and that today's return is a large negative number. It is easy to predict that the VaR estimate will jump upward, because

of today's observation. The same effect (reversed) will reappear after 180 days, when the large observation will drop out of the window. This is a very undesirable characteristic and it is probably enough to discard the historical simulation method as a reliable one.

An interesting variation of the historical simulation method is the hybrid approach proposed by Boudoukh, Richardson and Whitelaw (1998). The hybrid approach combines RiskMetrics and historical simulation methodologies, by applying exponentially declining weights to past returns of the portfolio. There are three steps to follow. First, to each of the most recent K returns of the portfolio, $y_t, y_{t-1}, \dots, y_{t-K+1}$, is associated a weight, $\frac{1-\lambda}{1-\lambda^K}, \left(\frac{1-\lambda}{1-\lambda^K}\right)\lambda, \dots, \left(\frac{1-\lambda}{1-\lambda^K}\right)\lambda^{K-1}$, respectively⁴. Second, the returns are ordered in ascending order. Third, to find the $\theta\%$ VaR of the portfolio, sum the corresponding weights until $\theta\%$ is reached, starting from the lowest return. The VaR of the portfolio is the return corresponding to the last weight used in the previous sum⁵.

We believe that this approach constitutes a significant improvement over the methodologies discussed so far, since it drastically simplifies the assumptions needed in the parametric models and it incorporates a more flexible specification than the historical simulation approach. To better understand the assumptions behind this method, note that it implies the following:

$$(2) \quad \hat{q}_{t+1,\theta} = \sum_{j=t-K+1}^t y_j I\left(\sum_{i=1}^K f_i(\lambda; K) I(y_{t+1-i} \leq y_j) = \theta\right)$$

where $f_i(\lambda; K)$ are the weights associated with return y_i and $I(\bullet)$ is the indicator function. If we set $f_i(\lambda; K) = 1/K$ we get as a special case the Historical Simulation estimator.⁶ The main difference between these methods is in the specification of the quantile process. In the Historical Simulation method, each return is given the same weight, while in the Hybrid method different returns have different weights, depending on how old the observations are.

Hence, strictly speaking, none of these models is nonparametric, as a parametric specification is proposed for the quantile. This is the same idea behind CAViaR methodology. Boudoukh, Richardson and Whitelaw set λ equal to 0.97

⁴ Note that the role of the term $\frac{1-\lambda}{1-\lambda^K}$ is simply to ensure that the weights sum to 1.

⁵ To achieve exactly the $\theta\%$ of the distribution, adjacent points are linearly interpolated.

⁶ Here, for simplicity of notation but without loss of generality, we assume that no interpolation rule is necessary.

and 0.99, as in their framework no statistical method is available to estimate this unknown parameter. Of course, by recognising that both the Hybrid and Historical Simulation models are just a special case of CAViaR models, one can use the regression quantile framework to estimate the parameter λ and the window length K .⁷

2.3. Semiparametric Models

Recently, alternative methods have been proposed to estimate Value at Risk, such as applications of Extreme Value Theory (see, for example, Danielsson and deVries (1998) or Gouriéroux and Jasak (1998)) and applications of regression quantile technique such as in Chernozhukov and Umantsev (2000) and Engle and Manganelli (1999). Other approaches that can be included under this section are those based on Bollerslev and Wooldridge (1992) quasi-maximum likelihood GARCH, as suggested by Diebold, Schuermann and Stroughair (1999), and independently implemented by McNeil and Frey (2000) and Engle and Manganelli (1999)⁸. A similar idea can be found in Hull and White (1998), although their model is not as robust as those cited above, since the variance is not estimated using QML GARCH.

a) Extreme Value Theory

EVT is a classical topic in probability theory. Many books and surveys are available on the subject, see for example Leadbetter, Lindgren and Rootzen (1983) or Embrechts, Kluppelberg and Mikosch (1997). Here we intend to give some intuition and basic results of EVT, following very closely the approach of Embrechts, Kluppelberg and Mikosch (1997). EVT can be conveniently thought as a complement to the Central Limit Theory: while the latter deals with fluctuations of cumulative sums, the former deals with fluctuations of sample maxima. The main result is due to Fisher and Tippett (1928), who specify the form of the limit distribution for appropriately normalised maxima. Let $M_n = \max\{X_1, \dots, X_n\}$, where $\{X_i\}_{i=1}^n$ is a sequence of i.i.d. random variables. Fisher and Tippett showed that if there exist norming constants $c_n > 0$, $d_n \in \mathbb{R}$ and some non-degenerate distribution function H such that $c_n^{-1}(M_n - d_n) \xrightarrow{d} H$, then

$$(3) \quad H \equiv H_\xi(x) = \exp\left\{-\left(1 + \xi x\right)^{-\frac{1}{\xi}}\right\} \quad 1 + \xi x > 0$$

⁷ Although there might be some theoretical issues, due to the discontinuity of the indicator function.

⁸ See the approach proposed at the end of section 9.

The special case $H_0(x)$ is to be interpreted as $\lim_{\xi \rightarrow 0} H_\xi(x)$. H is called the Generalised Extreme Value (GEV)

distribution and describes the limit distribution of normalised maxima. An important concept for the application of EVT to VaR estimation is Maximum Domain of Attraction: the random variable X belongs to the Maximum Domain of Attraction of the extreme value distribution H (and we write $X \in MDA(H)$) if and only if Fisher-Tippett theorem holds for X with limit distribution H .

ξ is a crucial parameter that determines the shape of the GEV distribution. Distributions that belong to $MDA(H_\xi)$, for $\xi > 0$, are called heavy tailed (examples are Cauchy, Student-t, Pareto, loggamma). Gamma, normal, lognormal and exponential distributions belong to $MDA(H_0)$, while distributions with finite endpoints (such as the uniform and beta) belong to $MDA(H_\vartheta)$, for $\xi < 0$.

There are two alternative ways to implement EVT. The first one is generally built around the Hill estimator. We won't discuss it here and we refer the interested reader to Embrechts, Kluppelberg and Mikosch (1997) and to Danielsson and de Vries (1997).

The second one is based on the concept of exceptions of high thresholds. In this paper we adopt this estimation method because the Hill estimator is designed only for data coming from heavy tailed distributions⁹, while in the Monte Carlo simulation of section 2.4.1 we consider also normal and gamma distributions. Moreover, estimates based on this second approach tend to be more stable than the Hill estimator with respect to the choice of the threshold, as shown by McNeil and Frey (2000).

Suppose X_i are i.i.d. random variables with distribution function $F \in MDA(H_\vartheta)$. Define the excess distribution function as $F_u(y) = P(X_i - u \leq y | X_i > u)$ for $y \geq 0$ and for a given threshold u . Define also the Generalised Pareto Distribution (GPD) as:

$$(4) \quad G_{\xi, \beta}(x) = 1 - (1 + \xi x / \beta)^{-1/\xi} \quad \beta > 0 \quad \text{and} \quad \begin{cases} x \geq 0 & \text{if } \xi > 0 \\ 0 \leq x \leq -\beta / \xi & \text{if } \xi < 0 \end{cases}$$

where again we use the convention $G_{0, \beta}(x) = \lim_{\xi \rightarrow 0} G_{\xi, \beta}(x)$. The following is a crucial property of GPD, due to

Pickands (1975):

⁹ Although extensions that cover non fat-tailed distributions have been proposed; see, for example, Dekkers, Einmahl and de Haan (1989).

$$F \in MDA(H_\xi) \Leftrightarrow \lim_{u \uparrow x_F} \sup_{0 < x < x_F - u} |F_u(x) - G_{\xi, \beta}(x)| = 0$$

where x_F is the right endpoint (which could be infinite) of the distribution F . This result states that if F is in the Maximum Domain of Attraction of a GPD, as the threshold u approaches the endpoint of F , GPD becomes an accurate approximation of the excess distribution function F_u .

The parameters of the GPD can be estimated by maximum likelihood, once the threshold has been chosen. The log-likelihood based on (4) computed on the N upper order statistics is:

$$(5) \quad \begin{aligned} & \max_{\xi, \beta} \left\{ -N \ln \beta - (1 + 1/\xi) \sum_{i=1}^N \ln \left(1 + \frac{\xi}{\beta} (X_{k_i} - u) \right) \right\} \\ & \sum_{i=1}^N \ln \left[\mathbb{I}(\xi > 0) \mathbb{I}(X_{k_i} - u > 0) + \mathbb{I}(\xi < 0) \mathbb{I}(0 \leq X_{k_i} - u \leq -\frac{\beta}{\xi}) \right] \mathbb{I}(\beta > 0) \end{aligned}$$

where $N=N(u)$ denotes the number of observation exceeding the threshold u and $X_{k_1}, X_{k_2}, \dots, X_{k_N}$ are the upper order statistics exceeding this threshold. Note that the second part of the log-likelihood simply reflects the domain constraint of the GPD. Noting that

$$\begin{aligned} 1-F(u+y) &= [1-F(u)]/[1-F_u(y)] \\ &\approx [1-F(u)]/[1-G_{\xi, \beta}(y)] \end{aligned}$$

an estimator for the tail of $1-F(u+y)$ for $y>0$ can be easily derived. The term $1-F(u)$ can be estimated using the empirical distribution function: $[1-F(u)]^\wedge = N/n$.

Substituting the maximum likelihood estimates of ξ and β in the GPD, we get:

$$(6) \quad [1 - F(u + y)]^\wedge = \left(1 + \hat{\xi} \frac{\hat{y}}{\hat{\beta}} \right)^{-\frac{1}{\hat{\xi}}} N/n$$

The \hat{x}_p quantile estimator is now obtained by simply inverting the above formula:

$$(7) \quad \hat{x}_p = u + \left[\left((1-p) \frac{n}{N} \right)^{-\hat{\xi}} - 1 \right] \frac{\hat{\beta}}{\hat{\xi}}$$

Note that this estimator is valid only for very low p , as the approximation is valid only asymptotically.

To summarise, EVT seems to be a very general approach to tail estimation. The main strength is that the use of a GEV distribution to parameterise the tail doesn't seem to be a very restrictive assumption, as it covers most of the commonly used distributions. On the other hand, there are several problems that need to be considered.

First, the assumption of i.i.d. observations seems to be at odds with the characteristics of financial data. Although generalisations to dependent observations have been proposed (see, for example, Leadbetter, Lindgren, and Rootzen (1983) or Embrechts, Kluppelberg and Mikosch (1997)), they either estimate the marginal unconditional distribution or impose conditions that rule out the volatility clustering behaviour typical of financial data. A solution to this problem is suggested in the following sections.

Second, EVT works only for very low probability levels. How low these probability levels must be is hard to tell on a priori ground. A Monte Carlo study might help to shed some light on how fast the performance of the EVT estimators deteriorates as we move away from the tail.

Closely related to this issue is the selection of the cut-off point that determines the number of order statistics to be used in the estimation procedure. The choice of the threshold u presents the same problems encountered in the choice of the number of k upper order statistics that enter the Hill estimator. If the threshold is too high, there are too few exceptions and the result is a high variance estimator. On the other hand, a threshold too low produces a biased estimator, because the asymptotic approximation might become very poor. Unfortunately, as of today there is no statistical method to choose u and one usually relies on simulations and graphical devices. Recently Danielsson and de Vries (1997) and Danielsson, de Haan, Peng and de Vries (1998) have proposed a two step bootstrap method to select the sample fraction on which the Hill estimator is based. Although their result is based on an assumption on the second order property of F that is rarely verifiable in practice, it is certainly an important contribution that can supplement the usual graphical analysis and that can stimulate further research on this very important issue. For example, it would be interesting to know whether their result can be extended also to the GPD approach.

Another issue is that the choice of k based on the minimisation of the mean squared error results in a biased estimator. As the quantile is a non-linear function of the tail estimator, it is important to quantify the size of this bias. Very little attention has been devoted to this problem. Since Value at Risk is the ultimate object of interest, and a bad estimation of it might have very relevant consequences on the profitability of a financial institution, we believe this to be a topic that deserves further exploration.

We will address some of these issues later on.

b) CAViaR

The Conditional Autoregressive Value at Risk, or CAViaR model was introduced by Engle and Manganelli (1999). The basic intuition is to model directly the evolution of the quantile over time, rather than the whole distribution of portfolio returns. They propose the following specification for the Value at Risk:

$$(8) \quad q_{t,\theta} = \beta_0 + \beta_1 q_{t-1,\theta} + l(\beta_2, \dots, \beta_p, y_{t-1}, q_{t-1,\theta})$$

Different models can be estimated by choosing different specifications for the l function. Some of the models considered in the original paper are the Symmetric Absolute Value, $l(\cdot) = \beta_2 |y_{t-1}|$ and the Asymmetric Slope, $l(\cdot) = \beta_2 l(y_{t-1} > 0) - \beta_3 l(y_{t-1} < 0)$. A CAViaR model useful for Monte Carlo simulation is the Indirect GARCH(1,1):

$$(9) \quad q_{t,\theta} = \left(\beta_1 + \beta_2 q_{t-1,\theta}^2 + \beta_3 y_{t-1}^2 \right)^{\frac{1}{2}}$$

A special case of CAViaR is the model suggested by Chernozhukov and Umantsev (2000), who set $\beta_i=0$ and $l(\cdot)=X_t'\beta$, where X_t is a vector containing observable economic variables at time t and β a vector of parameters to be estimated.

The unknown parameters are estimated using non-linear regression quantile techniques.¹⁰ Consider the following model:

¹⁰ Note that the quantile specification proposed by Chernozhukov and Umantsev (2000) is linear in the parameters. Under this assumption, the objective function (11) can be rewritten as a linear programming problem. This guarantees that the quantile regression estimate will be obtained in a finite number of simplex iterations. See, for example, Buchinsky (1998) for more details.

$$(10) \quad y_t = q_{t\theta} + \varepsilon_{t\theta} \quad Quant_\theta(\varepsilon_{t\theta} | \Omega_t) = 0$$

where θ is the probability level associated with the quantile, Ω_t is the information set at time t and $Quant_\theta(\varepsilon_{t\theta} | \Omega_t) = 0$ simply means that the θ -quantile of the error term is 0. White (1994) has shown that the minimisation of the regression quantile objective function introduced by Koenker and Bassett (1978) is able to deliver consistent estimates, under suitable assumptions:

$$(11) \quad \min_{\beta} \frac{1}{T} \left\{ \sum_{t:y_t \geq q_{t,\theta}(\beta)} \theta |y_t - q_{t,\theta}(\beta)| + \sum_{t:y_t < q_{t,\theta}(\beta)} (1-\theta) |y_t - q_{t,\theta}(\beta)| \right\}$$

Engle and Manganelli (1999) show also how to derive the asymptotic distribution of the estimator in order to perform hypothesis testing. We refer to the original paper for further details.

The only assumption required under this framework is that the quantile process is correctly specified. In particular, no assumption on the distribution of the error terms is needed, hence reducing the risk of misspecification. Moreover, even if the quantile process is misspecified, the minimisation of the regression quantile objective function can still be interpreted as the minimisation of the Kullback-Leibler Information Criterion, which measures the discrepancy between the true model and the one under study (for a treatment of quasi-maximum likelihood estimation, see White (1994)).

c) Quasi-Maximum Likelihood GARCH

In our discussion of GARCH models, we stressed how the assumption of normally distributed standardised residuals seemed to be at odds with the fact that financial data tend to exhibit heavy tails. It turns out, however, that the normality assumption might not be as restrictive as one can think. This is due to a very important result by Bollerslev and Wooldridge (1992) who showed that the maximisation of the normal GARCH likelihood is able to deliver consistent estimates, provided that the variance equation is correctly specified, even if the standardised residuals are not normally distributed. We refer to this result as the quasi-maximum likelihood (QML) GARCH. Many papers have exploited this

property. Engle and Gonzalez-Rivera (1991), for example, propose a semiparametric estimation of GARCH models. Engle and Manganelli (1999), at the end of section 9, suggest computing the VaR of a portfolio by first fitting a QML GARCH and then multiplying the empirical quantile of the standardised residuals by the square root of the estimated variance. This estimation method is a mix of a GARCH fitted to portfolio returns and historical simulation applied to the standardised residuals. As a consequence it retains some of the drawbacks of these approaches. First, the assumption that the standardised residuals are i.i.d. is still required. Given this assumption, however, the logical inconsistency of historical simulation is solved, as they use as a window the whole series of standardised residuals. On the other hand, the problem of discreteness of extreme returns remains. Historical simulation will provide very poor and volatile quantile estimates in the tails, for the same reason that kernel methods don't guarantee to provide an adequate description of the tails. A natural alternative seems to use EVT instead of historical simulation. This model was estimated by McNeil and Frey (2000).

Given the very general results of EVT, the QML GARCH augmented by the EVT requires very weak assumptions. The only required assumptions are that the variance is correctly specified and that the standardised residuals are i.i.d. and in the maximum domain of attraction of some extreme value distribution.

d) CAViaR and Extreme Value Theory

Even if more empirical work needs to be done, there is preliminary evidence that CAViaR models work fine at very common probability levels, such as 5% and 1%. However, there might be some problems (essentially due to finite sample size) when one tries to estimate very extreme quantiles, such as 0.1% or even lower. This has been documented in a Monte Carlo study by Engle and Manganelli (1999). A viable alternative could be to incorporate EVT into the CAViaR framework.

The strategy we adopt is the following. We fit a CAViaR model to get an estimate of the quantile $q_{t,\theta}$ with θ large enough (between 5% and 10%, say) so that sensible estimates can be obtained. Then we construct the series of standardised quantile residuals:

$$(12) \quad \hat{\varepsilon}_{t,\theta} / \hat{q}_{t,\theta} = (y_t / \hat{q}_{t,\theta}) - 1$$

Let $g_t(\cdot)$ be the conditional density function of the standardised quantile residuals. We impose the following assumption.

$$A1 - g_t\left(\frac{\varepsilon_{t,\theta}}{q_{t,\theta}}\right) = g\left(\frac{\varepsilon_{t,\theta}}{q_{t,\theta}}\right), \text{ for all } \frac{\varepsilon_{t,\theta}}{q_{t,\theta}} > 0, \text{ for all } t, \text{ provided that } q_{t,\theta} \neq 0 \text{ for all } t.$$

That is, the distribution of the standardised quantile residuals is not time-varying beyond the $(1-\theta)$ -quantile.¹¹

Under this assumption, we can now apply EVT to the standardised quantile residuals to get an estimate of the tail.

$$(13) \quad \Pr(y_t < q_{t,p}) = \Pr(y_t < q_{t,\theta} - (q_{t,\theta} - q_{t,p})) = \Pr\left(\frac{y_t}{q_{t,\theta}} - 1 > z\right) = p, \quad \text{for } p < \theta$$

where $z = \frac{q_{t,p}}{q_{t,\theta}} - 1$ is the $(1-p)$ -quantile of the standardised quantile residuals. Assume that z_p is the EVT estimate of the

$(1-p)$ -quantile of the standardised residuals. The corresponding p -quantile of portfolio returns is then given by $\hat{q}_{t,p} = \hat{q}_{t,\theta}(1 + z_p)$. Note that if $p = \theta$, then $z_p = 0$.

Another nice consequence of this assumption is the possibility to estimate the density function $h_t(\varepsilon_{t,\theta})$ that enters the variance-covariance matrix of regression quantile.¹² Hence, more efficient estimates of the parameters of the CAViaR model can be obtained by weighting the objective function by this density function evaluated at 0:

$$(14) \quad \min_{\beta} \frac{1}{T} \left\{ \sum_{t:y_t \geq q_{t,\theta}(\beta)} \theta |y_t - q_{t,\theta}(\beta)| h_t(0) + \sum_{t:y_t < q_{t,\theta}(\beta)} (1-\theta) |y_t - q_{t,\theta}(\beta)| h_t(0) \right\}$$

¹¹ Here we implicitly assume that $q_{t,\theta}$ is a negative number.

¹² See Engle and Manganelli (1999) for details.

Newey and Powell (1991) showed that by minimising this objective function, the estimator attains the semiparametric efficiency bound, when the quantile function is linear. It is likely that this result extends also to the non-linear case, but no such result exists yet.

Since assumption A1 imposes some restrictions on the error term of CAViaR models, we would like to stress the difference between the set of assumptions required by CAViaR and QML GARCH models. Assumption A1 requires that the standardised quantile residuals are i.i.d. only in the right tail, allowing for any sort of behaviour in the rest of the distribution. QML GARCH, instead, requires that the tails have the same dynamic behaviour as the rest of the distribution. Again, whether the extra restrictions imposed by QML GARCH models are relevant or not, is exclusively an empirical question.

3. Expected Shortfall

Many authors have criticised the adequacy of VaR as a measure of risk. Artzner et al. (1997, 1999) propose, as an alternative measure of risk, the Expected Shortfall, which measures the expected value of portfolio returns given that some threshold (usually the Value at Risk) has been exceeded. Analytically:

$$(15) \quad E_t(y_t | y_t < q_{t,\theta})$$

Since σ_t is a function of variables known at time t , it is possible to rewrite (15) in terms of standardised residuals:

$$(16) \quad E_t(y_t | y_t < q_{t,\theta}) = \sigma_t E_t\left(\frac{y_t}{\sigma_t} \middle| \frac{y_t}{\sigma_t} < \frac{q_{t,\theta}}{\sigma_t}\right)$$

If we assume that the standardised residuals follow an EVT distribution, EVT provides a very simple formula to compute this value. In general, if X has a Generalised Pareto distribution with parameters $\xi < 1$ and β , then for $u < x_F$:

$$(17) \quad E(X - u | X > u) = \frac{\beta + \xi u}{1 - \xi} \quad \beta + u \xi > 0$$

It can be easily shown (see, for example, McNeil and Frey (2000) eq. 16) that for any $x_p > u$,

$$(18) \quad E(X | X > x_p) = x_p \left(\frac{1}{1-\xi} + \frac{\beta - \xi u}{(1-\xi)x_p} \right)$$

Suppose we fit a QML GARCH augmented with EVT and we get estimates for q_t and σ_t in (16). Then, applying the formula in (18) to compute the expectation in (16), we get:

$$(19) \quad \hat{E}_t(y_t | y_t < q_{t,\theta}) = \hat{q}_{t,\theta} \left(\frac{1}{1-\hat{\xi}} - \frac{\hat{\beta} - \hat{\xi} \hat{u}}{1-\hat{\xi}} \frac{\hat{\sigma}_t}{\hat{q}_{t,\theta}} \right)$$

A similar result can be obtained for CAViaR models. Assuming that $q_{t,\theta}$ is a negative number, from (15) we have:

$$(20) \quad E_t(y_t | y_t < q_{t,\theta}) = q_{t,\theta} E_t \left(\frac{y_t}{q_{t,\theta}} \middle| \frac{y_t}{q_{t,\theta}} > 1 \right)$$

Given the assumption that the standardised quantile residuals are i.i.d. beyond a certain threshold, we can again apply EVT to compute the expected shortfall:

$$(21) \quad \hat{E}_t(y_t | y_t < q_{t,\theta}) = \hat{q}_{t,\theta} \left(\frac{1}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi} \hat{u}}{1-\hat{\xi}} \right)$$

Formulae (16) and (20) suggest an alternative way to compute the expected shortfall. Noting that the conditional expectation in both formulae must be constant (given the i.i.d. assumption), one can compute the expected shortfall by simply regressing the returns that exceed the quantile against the corresponding estimated standard deviation (or quantile):

$$(22) \quad y_t = \beta \sigma_t + \eta_t \quad \text{for} \quad y_t < q_t$$

$$(23) \quad y_t = \delta q_t + \eta_t \quad \text{for} \quad y_t > q_t$$

The expected shortfall is now given by, for QML GARCH and CAViaR models respectively:

$$(24) \quad \hat{E}_t(y_t | y_t < q_{t,\theta}) = \hat{\beta} \hat{\sigma}_t$$

$$(25) \quad \hat{E}_t(y_t | y_t > q_{t,\theta}) = \hat{\delta} \hat{q}_t$$

where $\hat{\beta}$ and $\hat{\delta}$ are the regression coefficients of (22) and (23).

We view this second estimation method as complementary to the EVT method. Indeed, this approach can be useful to compute the expected shortfall at relatively high quantiles (such as 5%) where the approximation provided by the EVT may be very poor. On the other hand, the regression method can give very poor estimates at very low quantiles, since only few observations are available. In the extreme case, when the quantile is beyond the boundary of observed data, no observation would be available, and hence the method wouldn't be applicable.

4. Monte Carlo Simulation

4.1. Simulation Study of the Threshold Choice for EVT

As we have already pointed out, a critical aspect for the implementation of EVT is the determination of the threshold beyond which the observations are assumed to follow a Generalised Pareto distribution. To address this issue we have performed a Monte Carlo simulation.

We generated 1000 samples of 2000 observations each, using three different distributions, the normal and the Student-t with 3 and 4 degrees of freedom. For each sample, we estimated the 5%, 1% and 0.05% quantiles using the formula of the extreme value theory provided above and using different threshold values. We chose the threshold values indirectly, by choosing the number of exceptions (k) to be included in the maximum likelihood estimation. We started with $k=10$ and we increased it by 5 until it reached 700. To compare the different estimates, we computed the bias and the mean squared error of the estimators as follows:

$$(26) \quad bias_{\theta,k} = \sum_{j=1}^{1000} \hat{q}_{\theta,k}^{(j)} / 1000 - q_\theta$$

$$(27) \quad MSE_{\theta,k} = \sum_{j=1}^{1000} (\hat{q}_{\theta,k}^{(j)} - q_\theta)^2 / 1000$$

Our goal is to determine how sensitive these estimates are to the choice of the parameter k , to the underlying distribution and to the confidence level of the estimated quantile. The results are reported in Figure 1. We plot the bias and MSE of the EVT estimates against k , together with the empirical quantile estimates, which don't depend on k .

A first result that emerges from these plots is that the 5% and 1% EVT estimates appear to be biased for very low k . This is probably due to the fact that these two confidence levels are not extreme enough for the GPD to be a reasonably good approximation of the tail of the underlying distribution. However, as k increases both the bias and the MSE of the EVT estimates are significantly reduced. In the normal case, the EVT MSE seems to be comparable with the empirical MSE for all values of k greater than 100 for the 5% quantile and for all values of k greater than 30 for the 1% quantile. The 5% and 1% EVT bias, instead, is close to the empirical bias for k between 100 and 350 and for k between 10 and 250, respectively, after which it grows rapidly in the negative direction. The plots look different for the Student-t distributions. In this case, the MSE of the 5% and 1% EVT rapidly deteriorates as k increases, while the bias remains more or less constant until $k = 500$. For these distributions a choice of $k = 100$ or even lower seems to be the most appropriate. Note also that the empirical quantile might be a better estimation method at these confidence levels as it has similar bias, a lower MSE and it does not depend on the choice of any parameter.

The situation changes as we move further out in the tail. For 0.05% quantile estimates, the empirical quantile becomes unreliable, as only few observations are available at these extreme confidence levels. In the Student-t case with 3 degrees of freedom, the empirical quantile has a greater bias than the EVT estimate and a MSE more than 10 times greater. The plots also show the bias-MSE trade-off for the EVT estimates. For the Student-t distributions, the bias is very close to 0 for low values of k and increases gradually as k increases. The MSE, instead, decreases until k equals about 400 and increases afterwards. It is hard to tell what is the optimal k under these circumstances. If one decides to minimise the MSE, a greater bias will result. On the other hand, by choosing a relatively low k , one obtains a more precise estimate on average, but at the cost of a greater variability. It would be helpful to be able to estimate the

bias that results from the minimisation of the MSE, but to our knowledge no such contribution is yet available in the literature.

4.2. Comparison of quantile methods performance

In this section, we compare the performance of the quantile models discussed in section 2 using a Monte Carlo simulation. We generated 1000 samples of 2000 observations for 7 different processes. The first 5 are GARCH processes with parameters [2.5, 0.04, 0.92]. The error terms were generated using random number generators with the following distributions: 1) standard normal, 2) Student-t with 3 degrees of freedom, 3) Student-t with 4 degrees of freedom, 4) Gamma with parameters (2,2), 5) Gamma with parameters (4,2). Note that in order to have error terms with mean zero and variance 1 (as required by GARCH models), the distributions above had to be standardised. For the Student-t distributions, we simply divided the random numbers by $\sqrt{v/(v-2)}$, the standard deviation of a Student-t with v degrees of freedom. For the Gamma (a,b) distributions, instead, we first subtracted the mean, ab , then divided by the standard deviation, $\sqrt{ab^2}$, and finally reversed the sign in order to have an infinite left tail.

We generated also two more processes. The first one is a process with a GARCH variance (with the same parameters as above) but with non-i.i.d. error terms that are alternatively drawn from a Student-t (3) and a Gamma (2,2). The second one is a CAViaR process, generated as follows. We start with an initial value for the quantile, q_0 , and generate the next θ -quantile as $q_{t,\theta} = -\sqrt{\omega + \alpha y_{t-1}^2 + \beta q_{t-1,\theta}^2}$, with parameters [2, 0.08, 0.9]. y_t is then generated from a distribution with θ -quantile equal to $q_{t,\theta}$. This is done by multiplying $q_{t,\theta}$ by a random number drawn from a distribution with $(1-\theta)$ -quantile equal to 1:

$$(28) \quad \theta = \Pr(y_t < q_{t,\theta}) = \Pr(q_{t,\theta} \epsilon_t < q_{t,\theta}) = \Pr(\epsilon_t > 1).$$

In the last term of (28), we reversed the sign of the inequality because $q_{t,\theta}$ is a negative number. The random number was drawn sequentially from a Student-t with 3 and 4 degrees of freedom and from a Gamma (2,2), and then divided by the θ -quantile of these distributions, to ensure that the $(1-\theta)$ -quantile was equal to 1. Note that the distributions were appropriately standardised so that they had mean zero and variance one, and that the θ -quantile of

these standardised distributions is negative for low values of θ . Note also that this is not a GARCH process, and hence the parameters can be consistently estimated using a CAViaR model, but not with a GARCH model.

The performance of the models is evaluated using the equivalents of the bias and MSE for vector estimates:

$$(29) \quad bias_{\theta} = \left\| \sum_{j=1}^{1000} (\hat{q}_{\theta}^{(j)} - q_{\theta}^{(j)}) / 1000 \right\|^2 / 2000$$

$$(30) \quad MSE_{\theta} = \sum_{j=1}^{1000} \left(\left\| \hat{q}_{\theta}^{(j)} - q_{\theta}^{(j)} \right\|^2 / 2000 \right) / 1000$$

where $\|\cdot\|$ denotes the Euclidean norm, $q_{\theta}^{(j)}$ is the (2000×1) vector of true time-varying θ -quantiles and $\hat{q}_{\theta}^{(j)}$ is the (2000×1) vector of estimated θ -quantiles for the sample j .¹³ Note that for each sample we get 2000 quantile estimates and (30) measures the average distance of these estimates from the true quantile values.

We also tested the performance of the models in estimating the expected shortfall. From the true DGP, it is possible to compute the true expected shortfall¹⁴:

$$(31) \quad ES = E(y_t | y_t < VaR_t) = \theta^{-1} \int_{-\infty}^{VaR_t} y_t f_t(y_t) dy$$

where f_t is the pdf of y_t .

For each model, we estimated ES using EVT and/or the regression methods wherever they were applicable and computed the bias and MSE in the same way as shown in (29) and (30).¹⁵

We estimated the quantiles using 9 different methods:

1) *Historical simulation*, 2) *Normal GARCH* (the θ -quantile is computed as the estimated GARCH variance times the θ -quantile of the standard normal distribution), 3) *QML GARCH* (the θ -quantile is computed as the estimated

¹³ Other criteria might be used for the evaluation. For example, we could have used the L-1 norm instead of the L-2 norm, or an asymmetric criterion that gives higher weight to underestimated quantiles (based on the economic justification that the risk of underestimating the VaR is potentially more harmful than the cost of overestimating it). However, there is no clear a priori reason to prefer an L-1 norm to an L-2 norm, and economic intuition doesn't say anything about how to determine the weights in the asymmetric criterion.

¹⁴ The integral was evaluated using the command *quad8* in Matlab, which implements a quadrature algorithm using an adaptive recursive Newton-Cotes 8 panel rule. To compute the integral, we had to modify the pdf of the original Student-t and Gamma distributions using the Jacobian of the transformation, because the random variables were drawn from standardized distributions.

¹⁵ For the regression method, when no observation beyond VaR was available, we set ES equal to infinity and no value is reported in the table.

GARCH variance times the θ empirical quantile of the standardised residuals), 4) *QML GARCH & EVT* (the θ -quantile is computed as the estimated GARCH variance times the θ EVT quantile of the standardised residuals), 5) *EVT*, 6) *CAViaR*, 7) *Efficient CAViaR* (i.e., we use equation (14) as regression quantile objective function), 8) *CAViaR & EVT*, 9) *Efficient CAViaR & EVT*.

To optimise the likelihood of EVT, GARCH and CAViaR models, we used the command *fminsearch* in Matlab, which is an optimisation algorithm based on the simplex search method by Nelder and Meade.¹⁶

We chose a window of 300 observations for the historical simulation method. On the basis of the results of the EVT Monte Carlo study, we decided to estimate the GPD parameters using 120 exceptions, a number that allows us to reduce the MSE at very low quantiles and still keep bias and MSE of higher quantiles within a reasonable range. The CAViaR model augmented with EVT was estimated with a confidence level of 7.5%. Finally, the bandwidth to compute the height of the density function for the estimation of efficient CAViaR models was set equal to 0.1.

The results are shown in Tables 1 and 2. Let's start looking at the quantile results for the first 5 GARCH processes. Not surprisingly, the worst models in terms of both MSE and bias are Historical Simulation, Normal GARCH and EVT, which are the only not correctly specified models for the simulated processes. The only exception is when the underlying process is a Normal GARCH, in which case the Normal GARCH model is correctly specified and is the best performer at all confidence levels.

At extreme confidence levels, such as 0.05%, also the QML GARCH and the plain CAViaR become unreliable. This is due to the fact that when the confidence level is very low, only very few observations exceed the quantile (if any) and CAViaR estimates are imprecise for the same reason that empirical quantile estimates are.

Another interesting feature, although still not surprising, is that by using the efficient CAViaR estimation there is actually a sensible gain in the efficiency of the estimates and a reduction in the bias. Note that for the first 5 DGPs, the assumption that allows the implementation of the efficient CAViaR estimation (i.i.d. standardised quantile residuals) is true.

The best models overall are:

- the normal GARCH in the case of a Normal GARCH DGP,

¹⁶ The initial values were set equal to the true GARCH parameters for GARCH and CAViaR models and equal to [0.15, 0.5] for the EVT. For the efficient CAViaR, the initial values were set equal to the optimizing parameters of the inefficient CAViaR. The termination tolerance was 0.0001 for both the function value and the parameters, the maximum number of evaluations allowed was 1000 and the maximum number of iterations was 500.

- the efficient CAViaR augmented with EVT for GARCH DGP generated using the Student-t with 3 and 4 degrees of freedom,
- the QML GARCH augmented with EVT for the gammas with reversed sign GARCH.

Note that the performance of QML GARCH and QML GARCH augmented with EVT is very similar at 5% and 1% confidence levels. This is consistent with the findings discussed in section 4.1, where we showed that 5% and 1% empirical quantile and EVT estimates are very close for a reasonable choice of the parameter k .

The results are less clear for the non i.i.d. GARCH. Note that in this case, none of the models is correctly specified. At 5% the historical simulation method seems to have the lowest bias, but the CAViaR models have the lowest MSE. At 1% the CAViaR augmented with EVT is the best performer, while at 0.05% none of the models seems to do a good job, although historical simulation and normal GARCH produce the lowest bias and MSE.

For the CAViaR DGP, the best estimates are delivered, not surprisingly, by the CAViaR models. Under this DGP, only the plain CAViaR is correctly specified and also the assumption behind efficient estimation is violated. Despite this, all the CAViaR models produce similar estimates of 5% quantiles. At 1%, instead, the plain CAViaR is clearly superior to the CAViaR augmented with EVT. Finally, at 0.05% the performance of the plain CAViaR is considerably worse, again because of scarcity of data that far out in the tail. The CAViaR model augmented with EVT seems to do a good job at lowering the MSE of the estimate, although plain EVT delivers an estimate with similar MSE and historical simulation produces estimates with much lower bias.

It is interesting to compare the relative efficiency of these estimators, as measured by the ratio of the Mean Square Error (MSE). For 1% quantile estimates, the normal GARCH has a MSE ratio of 0.37 compared to the QML GARCH augmented with EVT, when the true DGP is a normal GARCH. However this ratio quickly deteriorates to 1.13 under a Student-t with 3 degrees of freedom, 2.04 under a Student-t with 4 degrees of freedom.

The MSE ratio of QML GARCH & EVT to efficient CAViaR & EVT is 0.75 under the normal, 4.12 under Student-t with 3 degrees of freedom, 0.45 under the gamma (2,2), 1.08 under the non i.i.d. GARCH and 2.65 under the CAViaR GDP.

Similar results hold for other quantiles. These results show that CAViaR is generally a more robust method for estimating quantiles, when returns have a fat tailed distribution.

Turning our attention to the Expected Shortfall tables, we see analogous results: CAViaR models produce better estimates for heavy-tailed processes and QML GARCH & EVT is better with normal and gamma distributions with an

infinite left endpoint. The results are once again mixed for the last two processes. All the models for the non i.i.d. GARCH are seriously biased and with high MSE, these results being worse at lower confidence levels. Also in the case of CAViaR DGP, all the estimates are biased and with high MSE, although in this case the size of the distortion seems to be considerably lower, especially at very low confidence levels.

A totally new result is the estimation of the Expected Shortfall using the regression technique described in section 3. First of all notice that at 0.05% the regression estimates are not available in most of the cases, because there are no observations beyond the quantile and hence it is not possible to implement the method. At 5% and 1%, instead, the method works reasonably well. The bias of the regression estimate is almost always lower than the bias of the EVT estimate. This feature is particularly striking for the CAViaR models augmented with EVT, in which case the EVT method seems to produce a considerable bias relative to the regression method. The results on MSE are similar: EVT yields a lower MSE when applied to QML GARCH, while the regression method works better when applied to CAViaR models.

5. Conclusion

We discussed the most commonly used methodologies for Value at Risk estimation, paying attention to their underlying assumptions and their weaknesses. The performance of the different models was evaluated through a Monte Carlo experiment. The results show that CAViaR models produce the best estimates with heavy-tailed DGP.

We also introduced a regression-based method for estimating the expected shortfall. This method can be used as an alternative to the extreme value theory approach. The Monte Carlo results indicate that the regression method tends to outperform the EVT approach at very common confidence levels, such as 1% or 5%.

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Appendix A: Tables

Table 1 – Bias and MSE of the quantile estimates.

BIAS - 0.05% quantile	Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Historical Simulation	8.42	275.76	144.42	62.67	42.01	546.09	0.02
Normal GARCH	0	1004.6	470.36	342.78	173.11	621.3	4.12
QML GARCH	1.33	255.34	102.63	8.35	6.3	1220.48	3.28
QML GARCH & EVT	0.26	2.45	2.6	1.28	1.89	1319.13	0.73
EVT	0.3	15.44	2.6	2.14	0.75	1005.84	0.41
CAViaR	3.65	104.76	33.51	22.37	12.12	827.67	0.92
Efficient CAViaR	2.88	141.47	58.77	14.04	8.31	974.02	0.88
CAViaR & EVT	0.06	1.53	1.36	0.89	0.48	1468.89	0.4
Efficient CAViaR & EVT	0.07	1.7	1.51	0.69	0.66	1469.57	0.49

MSE - 0.05% quantile	Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Historical Simulation	25.16	1052.5	462.24	166.96	105.94	1086.08	2.1
Normal GARCH	0.45	1147.3	493.24	351.04	176.58	702.98	5.56
QML GARCH	9.24	3060.5	1939.8	67.01	46.09	2854.82	14.18
QML GARCH & EVT	4.3	497.92	160.96	25.87	27.73	1971.75	2.22
EVT	10.67	875.75	269.61	82.49	44.54	1973.77	1.15
CAViaR	19.07	1430.4	483.97	133.35	75.96	1546.32	7.46
Efficient CAViaR	18.93	1905.4	894.33	137.82	75.43	1991	7.01
CAViaR & EVT	4.51	274.93	109.87	47.63	27.09	2139.6	1.16
Efficient CAViaR & EVT	4.36	275.5	114.41	45.81	30.48	2163.96	1.43

BIAS - 1% quantile	Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Historical Simulation	0.4	8.13	3.84	1.64	1.07	32.02	0.14
Normal GARCH	0	6.14	6.37	56.01	30.08	32.33	1.6
QML GARCH	0	0.29	0.08	0	0	25.77	0.01
QML GARCH & EVT	0	0.14	0.01	0.01	0.09	25.5	0.14
EVT	0.06	0.73	0.31	0.08	0.05	26.11	0.13
CAViaR	0	0.1	0.05	0.24	0.09	25.66	0
Efficient CAViaR	0	0.09	0.04	0.04	0.01	25.66	0
CAViaR & EVT	0	0.03	0.01	0.04	0.02	25.65	0.1
Efficient CAViaR & EVT	0	0.02	0	0.01	0.03	25.65	0.1

MSE - 1% quantile	Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Historical Simulation	7.17	101.31	50.8	32.8	21.62	75.5	1.17
Normal GARCH	0.23	24.81	9.12	57.74	30.96	41.43	2.16
QML GARCH	0.6	23.45	5.13	2.98	1.85	31.91	0.37
QML GARCH & EVT	0.062	21.82	4.46	2.39	3.75	30.44	0.53
EVT	3.62	36.51	19.67	15.36	9.79	42.15	0.4
CAViaR	1.36	15.52	9.74	12.92	7.62	34.38	0.08
Efficient CAViaR	1.27	13.97	8.84	9.59	6.1	34.22	0.08
CAViaR & EVT	0.87	6.34	3.73	6.8	4.37	28.27	0.2
Efficient CAViaR & EVT	0.83	5.29	3.68	5.3	4.69	28.18	0.2

BIAS - 5% quantile	Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Historical Simulation	0.02	0.12	0.07	0.07	0.05	0.59	0.01
Normal GARCH	0	4.08	1.05	5.37	3.32	9.01	2.11
QML GARCH	0	0.02	0	0	0	0.68	0
QML GARCH & EVT	0	0.01	0	0	0	0.7	0
EVT	0.01	0.01	0	0.02	0.01	0.68	0
CAViaR	0	0	0	0	0	0.75	0
Efficient CAViaR	0	0	0	0	0	0.75	0
CAViaR & EVT	0	0	0	0.01	0	0.78	0
Efficient CAViaR & EVT	0	0	0	0.01	0	0.78	0

MSE - 5% quantile	Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Historical Simulation	2.53	11.99	6.81	7.93	5.74	8.42	2
Normal GARCH	0.12	11.01	2.22	5.79	3.59	20.96	3.13
QML GARCH	0.21	3.16	0.87	0.74	0.55	3.04	0.4
QML GARCH & EVT	0.2	3.09	0.83	0.68	0.5	3.04	0.4
EVT	1.7	9.45	5	5.17	3.67	6.2	1.66
CAViaR	0.44	1.35	0.91	1.55	1.13	1	0.1
Efficient CAViaR	0.43	1.16	0.87	1.46	1.07	0.93	0.09
CAViaR & EVT	0.41	1.15	0.79	1.41	1.01	1.02	0.11
Efficient CAViaR & EVT	0.4	1	0.75	1.4	1	0.97	0.09

Table 2 – Bias and MSE of the Expected Shortfall Estimates.

BIAS - .05% quantile		Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Normal GARCH	Normal	0	3375	1291	505	252	1879	12.71
	Reg	-	2221	808	337	168	-	-
QML GARCH	Reg	-	-	-	-	-	-	-
QML GARCH & EVT	EVT	0.75	14.61	12.69	2.55	3.62	34774	10.69
	Reg	-	-	-	-	-	-	-
EVT	EVT	8.57	2.61	20.78	23.07	22.37	18201	10.32
	Reg	-	-	-	-	-	-	-
CAViaR	Reg	-	-	-	-	-	-	-
Efficient CAViaR	Reg	-	-	-	-	-	-	-
CAViaR & EVT	EVT	32.26	102.93	77.21	260	154	4110	11.28
	Reg	-	-	-	-	-	-	-
Efficient CAViaR & EVT	EVT	31.81	101	75.69	265	149	4191	11.62
	Reg	-	-	-	-	-	-	-

MSE - .05% quantile		Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Normal GARCH	Normal	0.53	3712	1344	517	257	2028	14.47
	Reg	-	2662	1092	349	174	-	-
QML GARCH	Reg	-	-	-	-	-	-	-
QML GARCH & EVT	EVT	7.49	2079	613	56.99	48.69	3328914	12.93
	Reg	-	-	-	-	-	-	-
EVT	EVT	21.93	2831	749	140	87.17	1075793	11.43
	Reg	-	-	-	-	-	-	-
CAViaR	Reg	-	-	-	-	-	-	-
Efficient CAViaR	Reg	-	-	-	-	-	-	-
CAViaR & EVT	EVT	41.6	1306	435	392	222	7964	12.61
	Reg	-	-	-	-	-	-	-
Efficient CAViaR & EVT	EVT	41	1317	445	393	225	8911	13.34
	Reg	-	-	-	-	-	-	-

BIAS - 1% quantile		Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Normal GARCH	Normal	0	113.84	63.79	122.98	64.02	107.37	11.81
	Reg	0	12.09	10.74	54.98	28.42	162.22	-
QML GARCH	Reg	0	0.47	0.04	0	0	123.42	10
QML GARCH & EVT	EVT	0.01	0.07	0.11	0.21	0.27	249.62	9.09
	Reg	0	0.28	0.01	0.05	0.11	114.54	14.07
EVT	EVT	8.13	21.1	22.07	33.12	23.94	242.38	6.27
	Reg	0.21	2.86	1.13	0.54	0.36	125.7	13.4
CAViaR	Reg	0.03	0.64	0.21	0.23	0.08	116.99	8.47
Efficient CAViaR	Reg	0.03	0.41	0.24	0.02	0.02	117.28	8.47
CAViaR & EVT	EVT	9.8	13.88	13.17	51.69	32.42	108.24	9.17
	Reg	0	0.02	0.01	0.04	0.01	113.17	14.04
Efficient CAViaR & EVT	EVT	9.61	13.63	12.89	54.16	31.99	110.49	9.17
	Reg	0	0.02	0.01	0.01	0.02	113.07	14.07

MSE - 1% quantile		Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Normal GARCH	Normal	0.3	141.67	68.7	126.25	65.55	121.46	12.68
	Reg	0.69	158.65	23.81	58.02	30.23	502.9	-
QML GARCH	Reg	0.91	180.05	22.56	5.67	3.62	175.76	11.89
QML GARCH & EVT	EVT	0.94	77.31	15.05	5.04	5.24	1566.38	10.17
	Reg	1.07	180.09	23.56	6.02	6.05	163.83	17.16
EVT	EVT	12.77	118.42	63.1	56.49	38.35	1746.66	6.64
	Reg	5.39	133.87	56.26	27.82	17.14	184.85	15.45
CAViaR	Reg	1.9	50.31	25.15	20.83	12.4	153.33	9.67
Efficient CAViaR	Reg	1.8	60.54	26.88	15.59	9.92	157	9.69
CAViaR & EVT	EVT	12.17	49.59	29.68	73.7	45.04	288.97	9.57
	Reg	1.5	55.3	17.32	11.59	7.43	138.6	17.04
Efficient CAViaR & EVT	EVT	11.91	45.43	28.73	72.69	45.23	239.49	9.54
	Reg	1.47	51.68	16.51	10.11	7.63	138.32	16.91

BIAS - 5% quantile		Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Normal GARCH	Normal	0	2.24	2.61	30.74	16.81	20.02	30.7
	Reg	0	9.49	1.72	5.22	2.98	63.01	56.05
QML GARCH	Reg	0	0.08	0.01	0	0	14.47	22.69
QML GARCH & EVT	EVT	0	0.02	0	0.01	0	16.83	23.77
	Reg	0	0.04	0	0.01	0	14.42	22.12
EVT	EVT	9.16	29.88	24.67	33.99	24.97	20.47	9.56
	Reg	0.01	0.27	0.08	0.03	0.03	15.07	24.88
CAViaR	Reg	0	0.02	0.02	0.02	0.01	14.33	21.35
Efficient CAViaR	Reg	0	0.02	0.02	0.02	0.01	14.33	21.41
CAViaR & EVT	EVT	0.47	0.38	0.4	2.18	1.5	15.41	20.67
	Reg	0	0.01	0.01	0.01	0.01	14.33	21.19
Efficient CAViaR & EVT	EVT	0.45	0.35	0.38	2.14	1.47	15.25	20.58
	Reg	0	0.01	0.01	0.02	0.01	14.33	21.15

MSE - 5% quantile		Normal	t(3)	t(4)	Gamma(2,2)	Gamma (4,2)	non-iid GARCH	CAViaR
Normal GARCH	Normal	0.19	11.34	4.35	31.86	17.41	38.27	35.56
	Reg	0.29	59.51	6.74	6.48	3.79	200.98	67.09
QML GARCH	Reg	0.33	15.19	2.94	1.58	1.06	28.7	26.99
QML GARCH & EVT	EVT	0.31	9.71	2.49	1.46	0.98	40.05	27.84
	Reg	0.35	15.17	2.93	1.65	1.08	28.07	26.33
EVT	EVT	11.73	55.16	35.66	43.98	31.56	34.74	9.87
	Reg	2.74	28.96	12.2	10.89	7.22	29.17	25.94
CAViaR	Reg	0.68	8.71	2.63	3.12	2.1	16.94	24.23
Efficient CAViaR	Reg	0.66	10.88	2.51	2.97	1.98	17.13	24.35
CAViaR & EVT	EVT	1.27	4.89	2.88	6.01	3.91	268.12	23.47
	Reg	0.68	10.44	2.5	3.03	1.96	17.1	24.11
Efficient CAViaR & EVT	EVT	1.21	4.44	2.78	5.88	3.88	235.13	23.28
	Reg	0.65	9.54	2.42	2.99	1.95	17.03	23.98

Appendix B: Figures

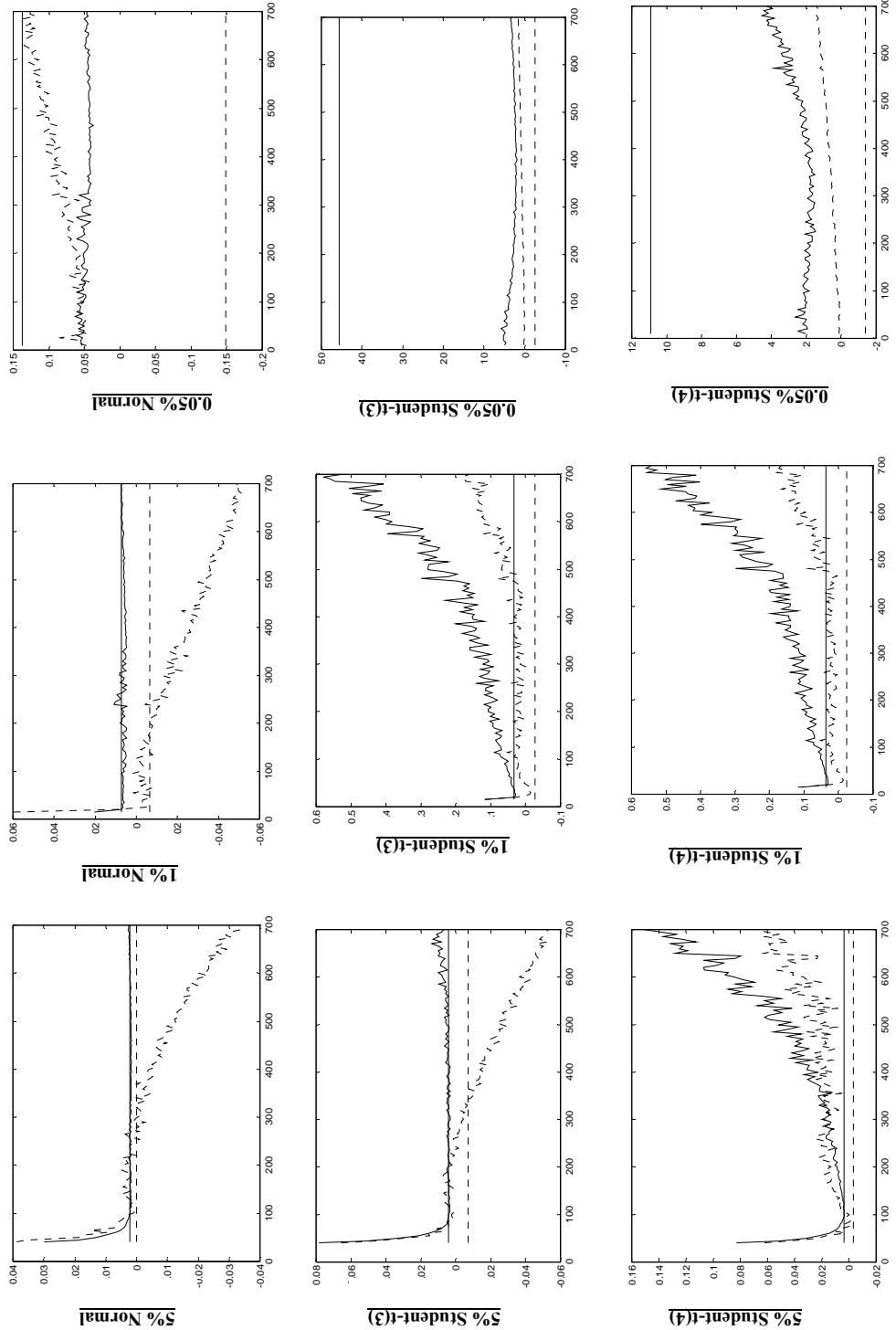


Figure 2.1 - Plot of the Mean Squared Error (solid line) and bias (dashed line) for EVT estimates against the number of exceptions (k) to be included in the maximum likelihood estimation, using 3 different distributions and 3 different confidence levels. The superimposed straight lines represent the Historical Simulation estimates, which do not depend on k . Sample Size = 2000, Replications = 1000.

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