Stochasticity

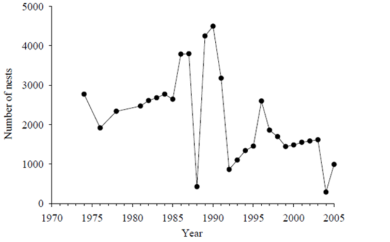


Figure 1. The population trend of Pelagic cormorants at Middleton Island, Alaska from 1974 to 2005 (Hatch et al., unpublished data).

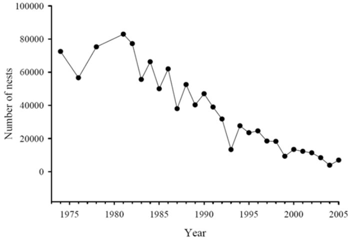


Figure 7. The population trend of black-legged kittiwakes on Middleton Island, Alaska from 1974 to 2005 (Hatch et al., unpublished data).

Learning objectives

1. Introduction
2. Geometric Growth
3. Logistic growth

A random variable is a variable whose value can't be predicted with certainty.

A random variable is a variable whose value can't be predicted with certainty.

Examples?

- Weather
- Our own behavior
- Population size

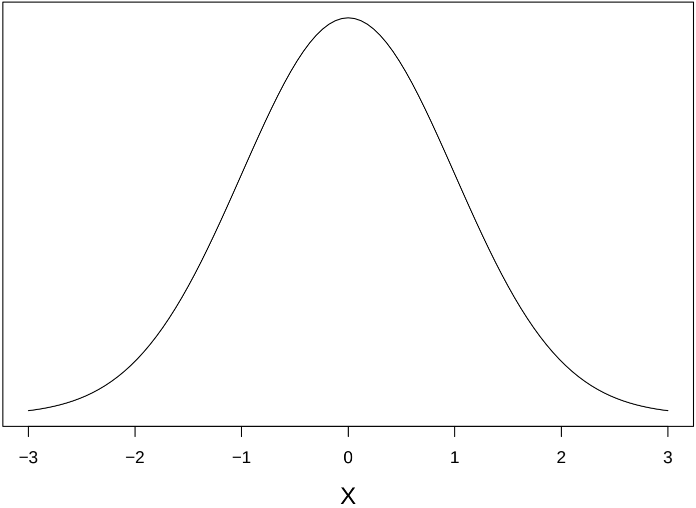
A random variable (X) can be described by a probability distribution.

There are many types of probability distributions

- Normal (or Gaussian)
- Poisson
- Binomial
- Multinomial
- etc. . .

Normal (Gaussian) Distribution

$$X \sim \mathsf{Normal}(\mu = 0, \sigma^2 = 1)$$



Normal (Gaussian) Distribution

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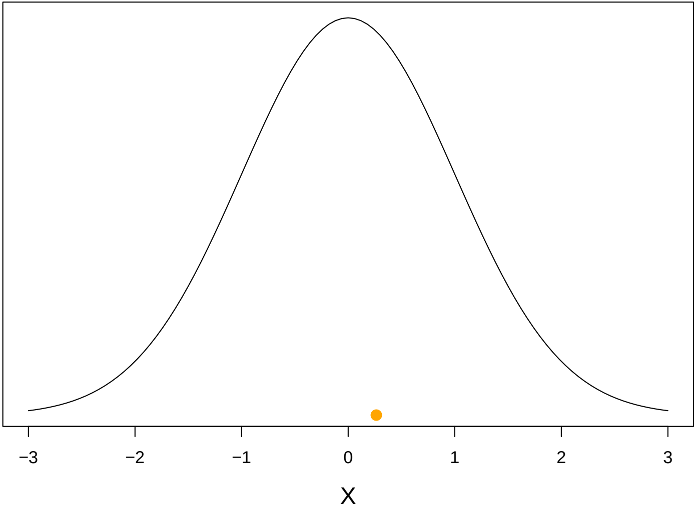

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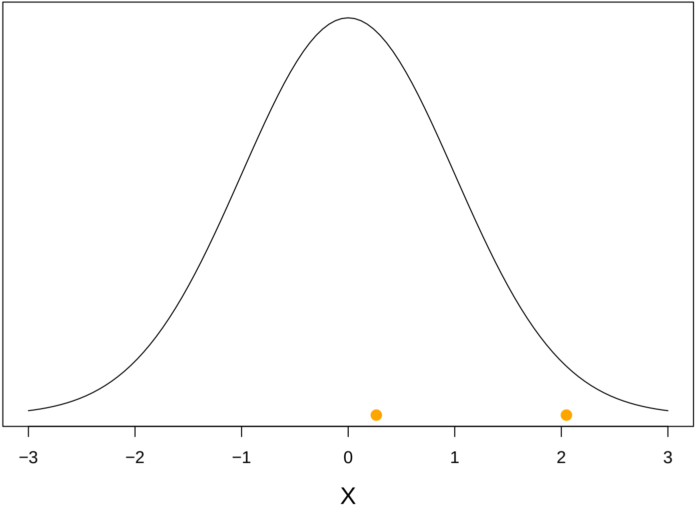

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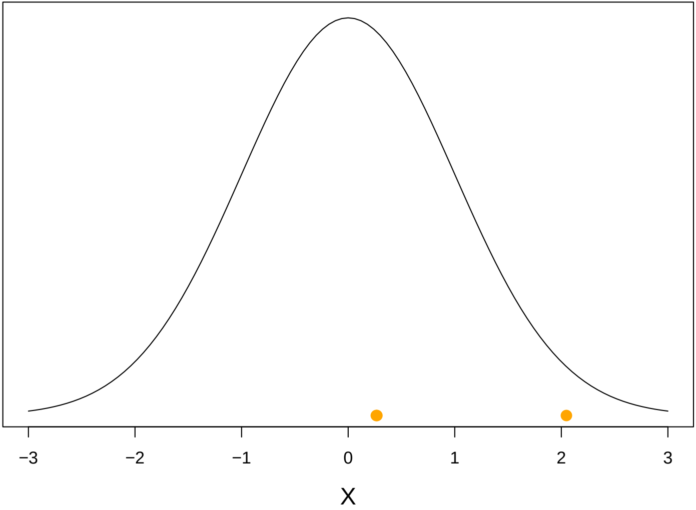

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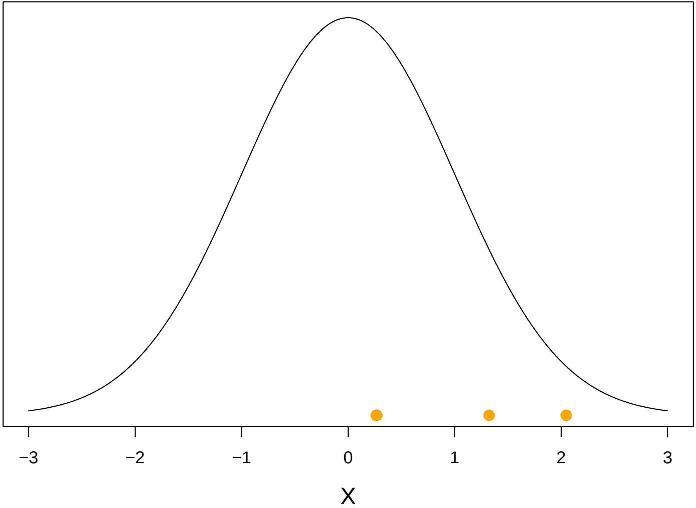

Normal (Gaussian) Distribution

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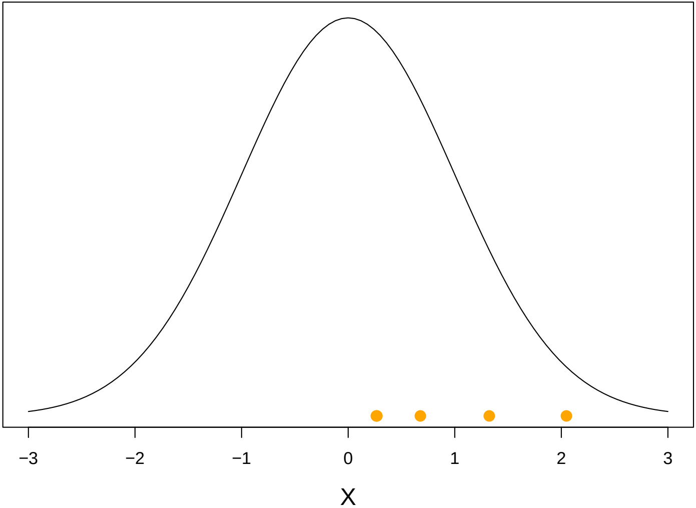

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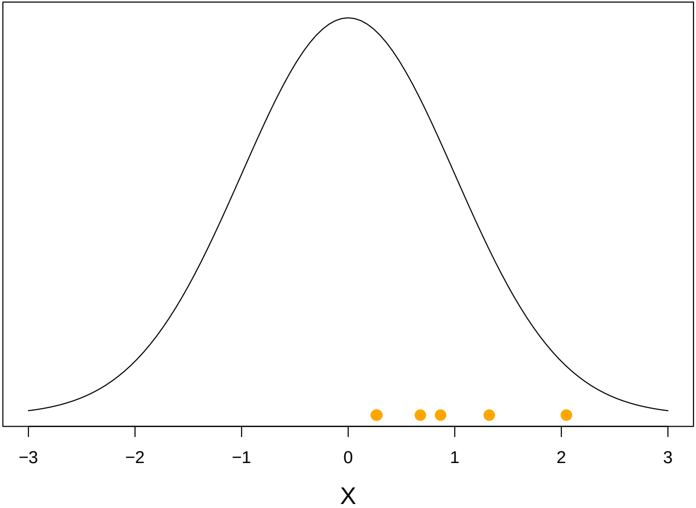

Normal (Gaussian) Distribution

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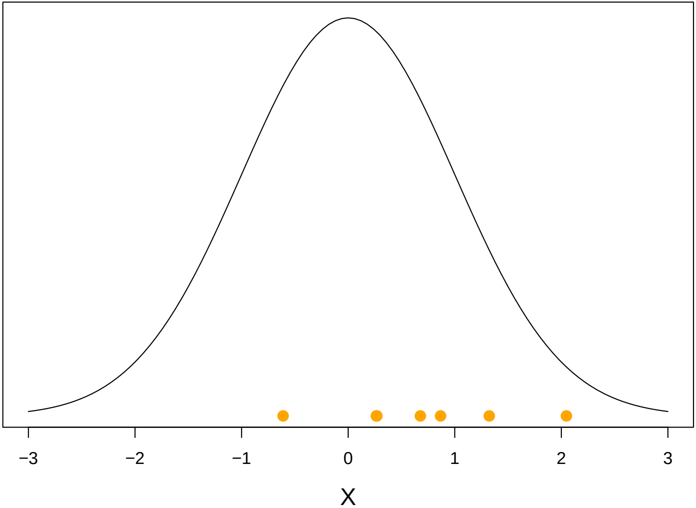

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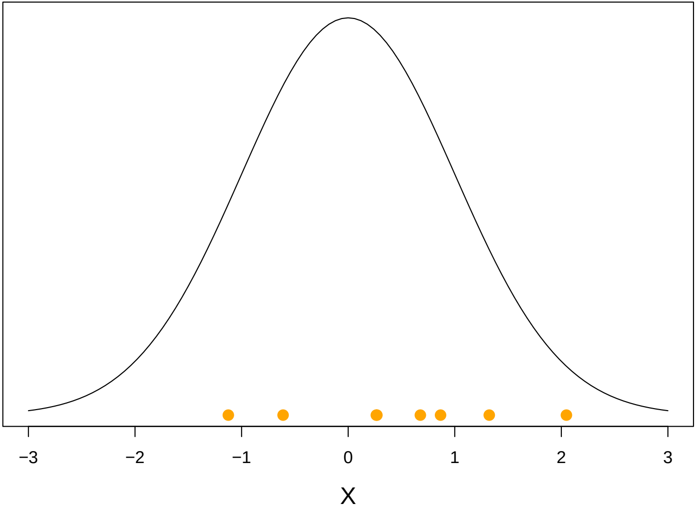

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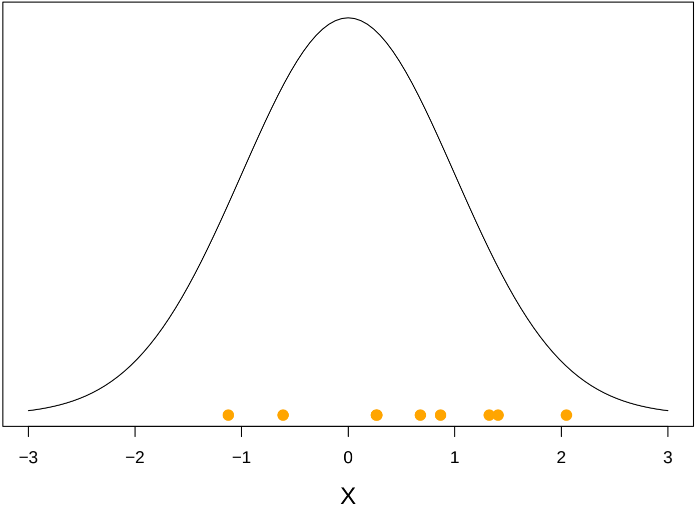

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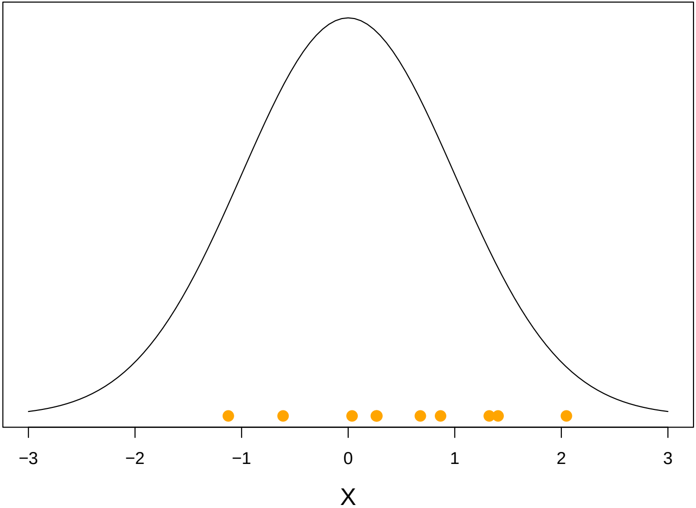
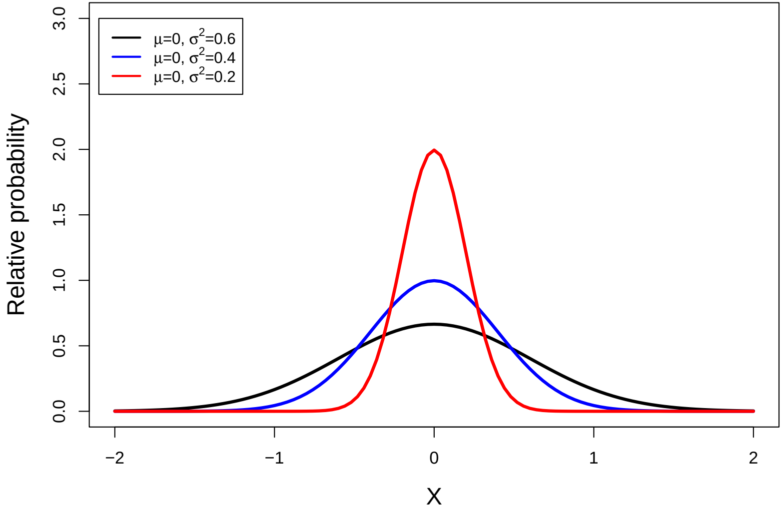
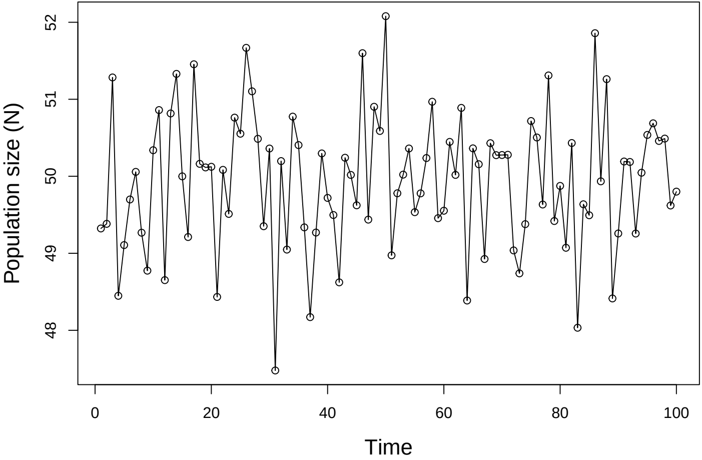


Normal (Gaussian) distribution



$$N_t \sim \mathsf{Normal}(\mu = 50, \sigma^2 = 1)$$



Environmental stochasticity

- Random variation in weather, habitat, etc. . . among years

Environmental stochasticity

- Random variation in weather, habitat, etc... among years

Demographic stochasticity

- Random variation in the number of births and deaths among years

Geometric growth with environmental stochasticity

$$N_{t+1} = N_t + N_t r + X_t$$

where

$$X_t \sim \mathsf{Normal}(0, \sigma_e^2)$$

Geometric growth with environmental stochasticity

$$N_{t+1} = N_t + N_t r + X_t$$

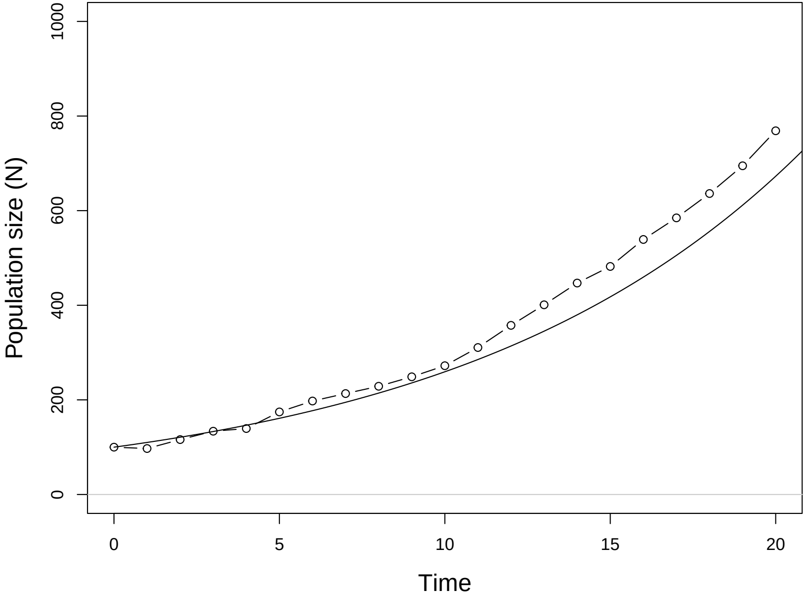
where

$$X_t \sim \text{Normal}(0, \sigma_e^2)$$

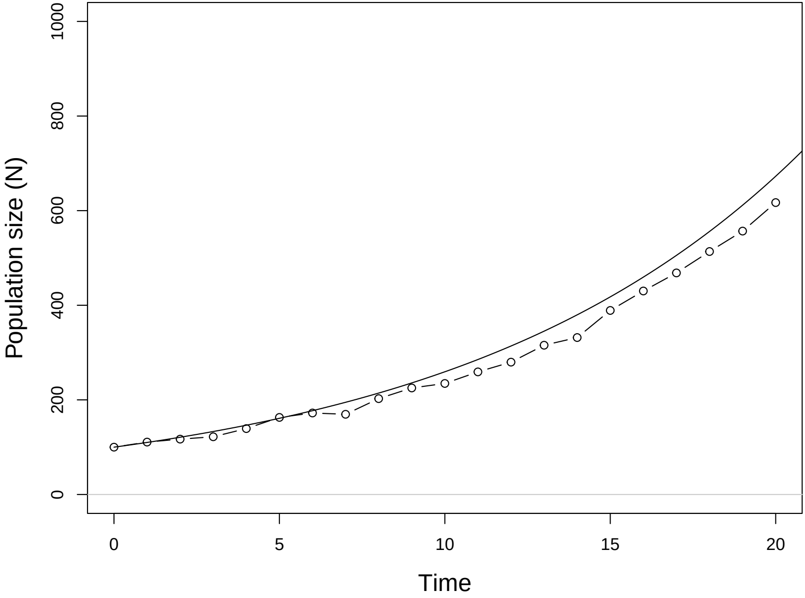
R code

```
nYears <- 20
N <- X <- rep(NA, nYears)  ## Create empty N and X
N[1] <- 100                ## Initial value of N
r <- 0.1                   ## Growth rate
sigma.e <- 10              ## StdDev of enviro variation
for(t in 2:nYears) {
    X[t-1] <- rnorm(n=1, mean=0, sd=sigma.e)
    N[t] <- N[t-1] + N[t-1]*r + X[t-1]
}
```

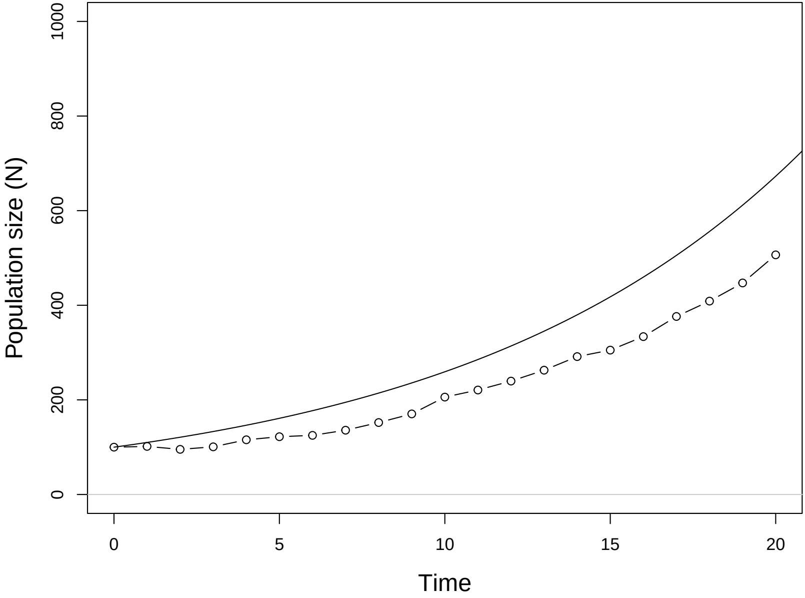
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



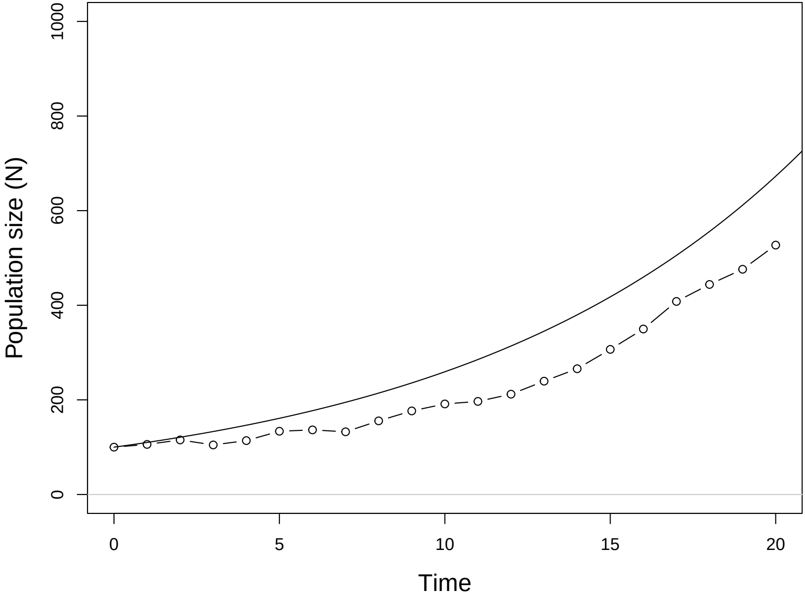
EXAMPLE $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



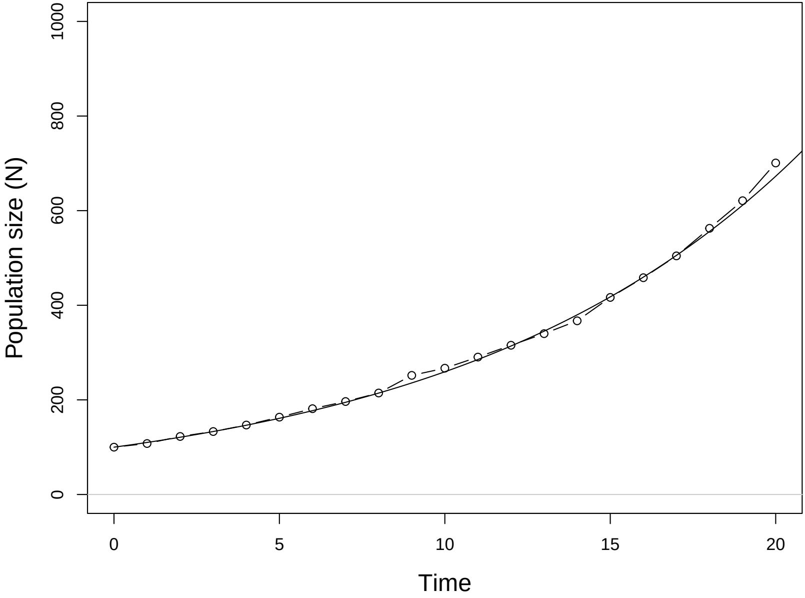
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



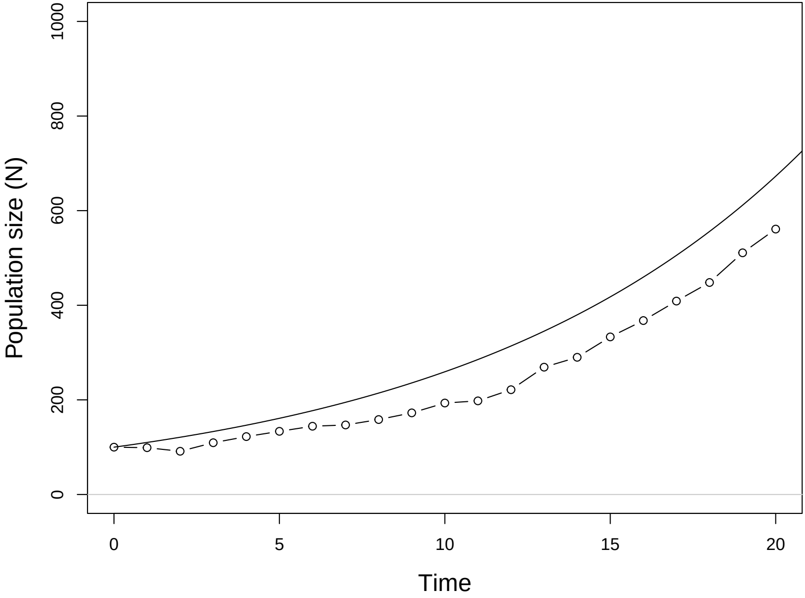
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



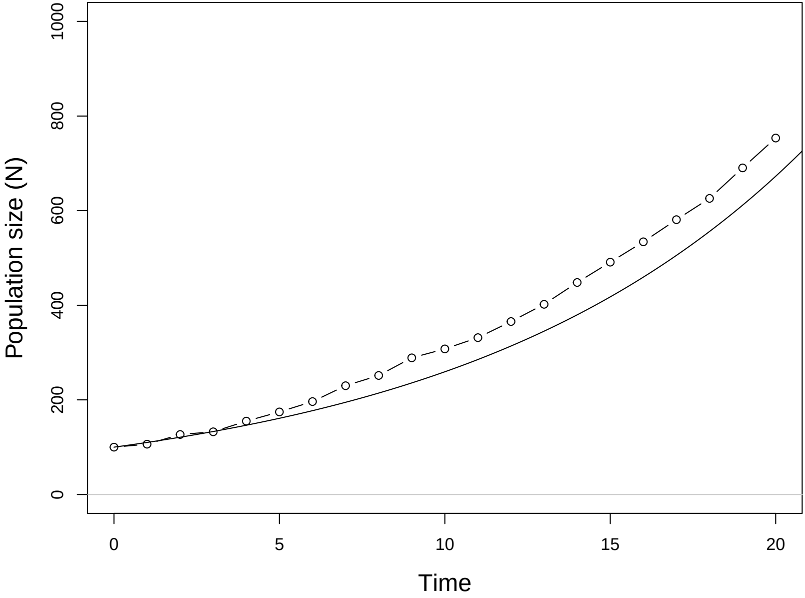
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



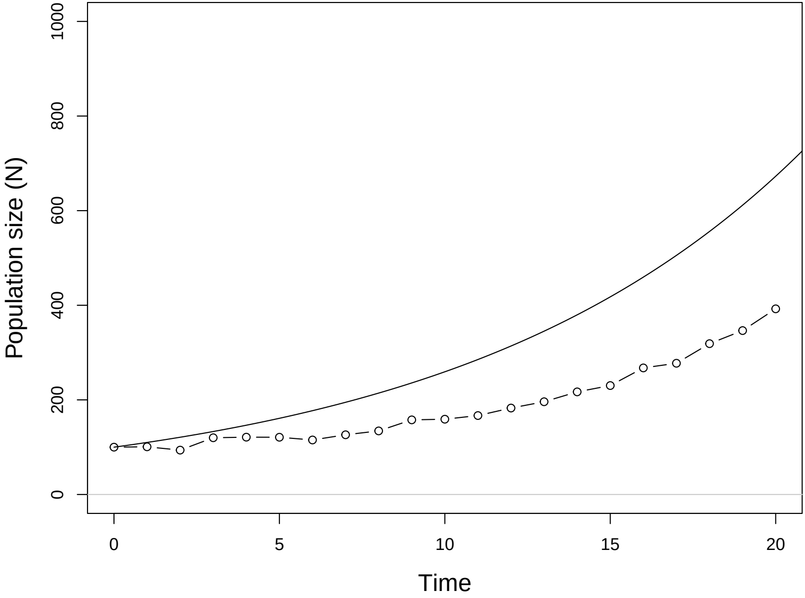
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



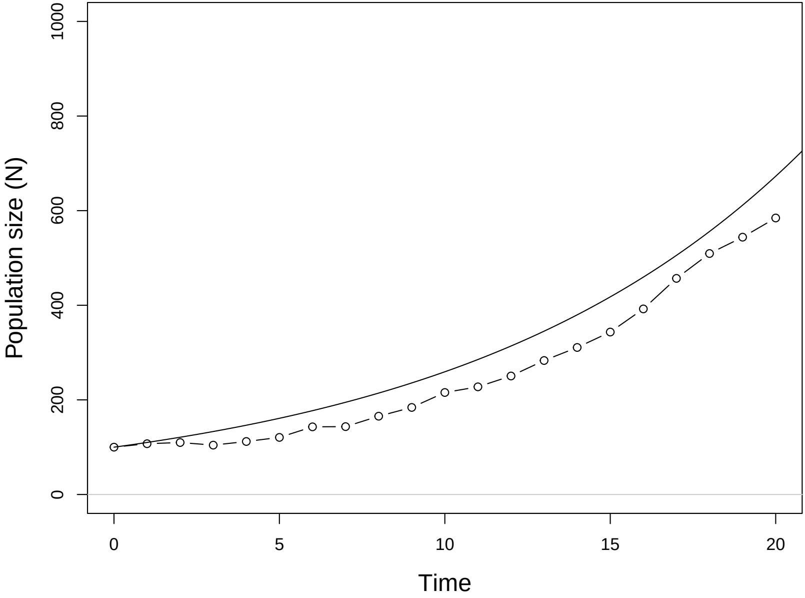
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



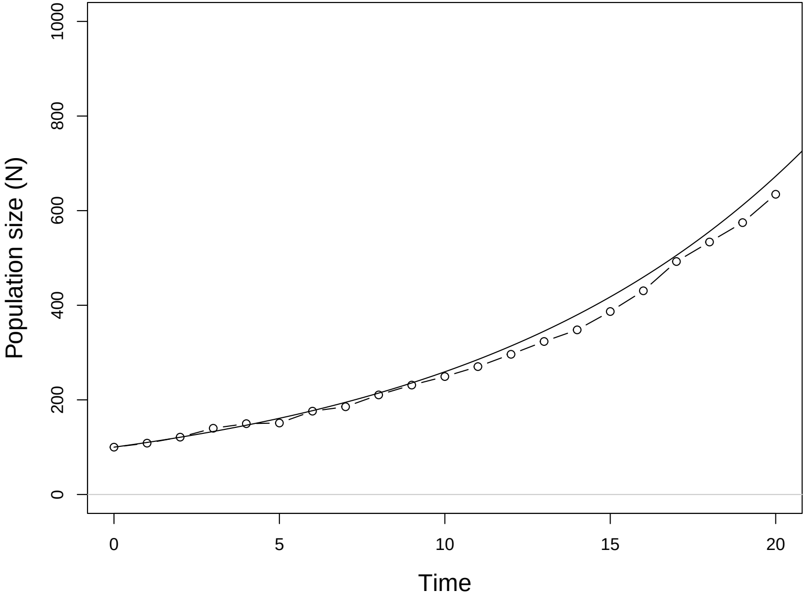
EXAMPLE $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



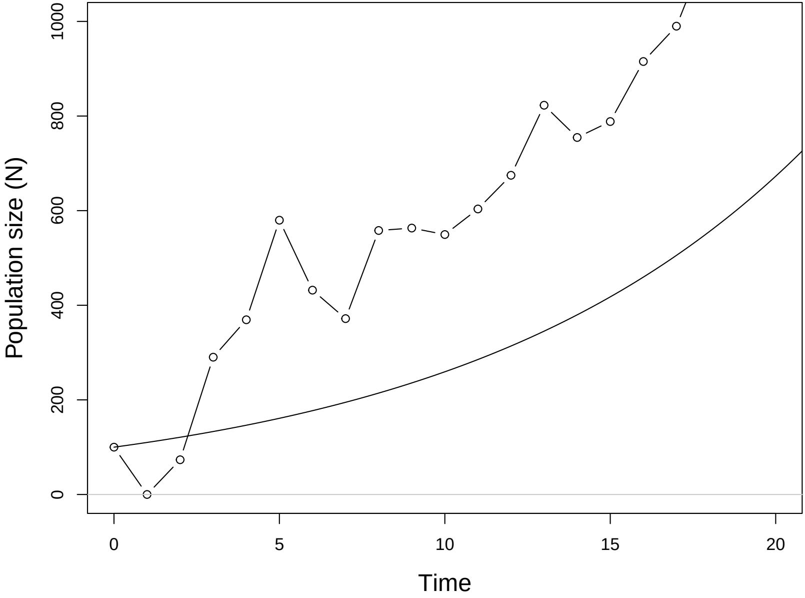
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



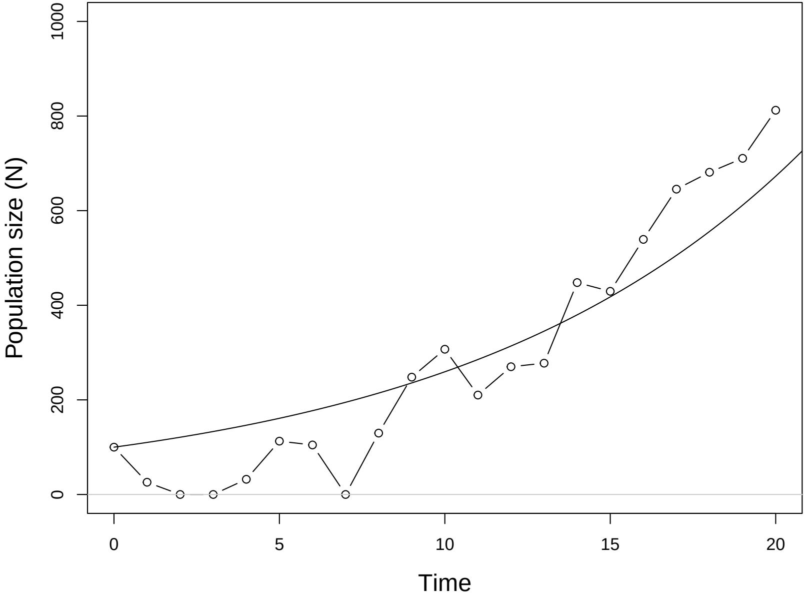
EXAMPLE $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 100$



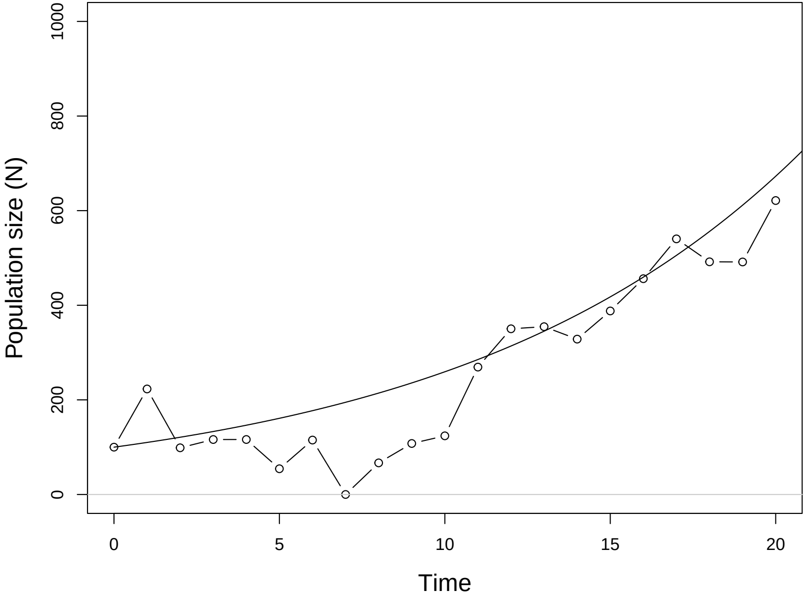
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 10000$



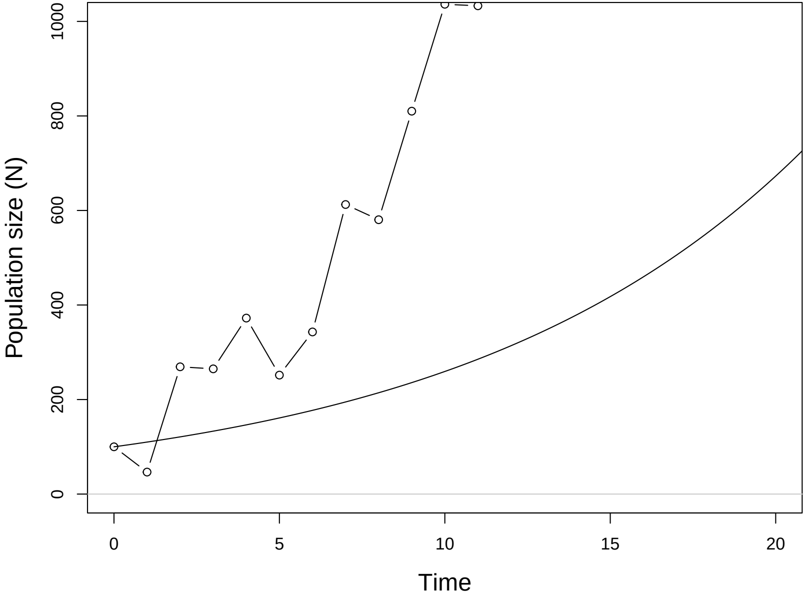
Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 10000$



Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 10000$



Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 10000$



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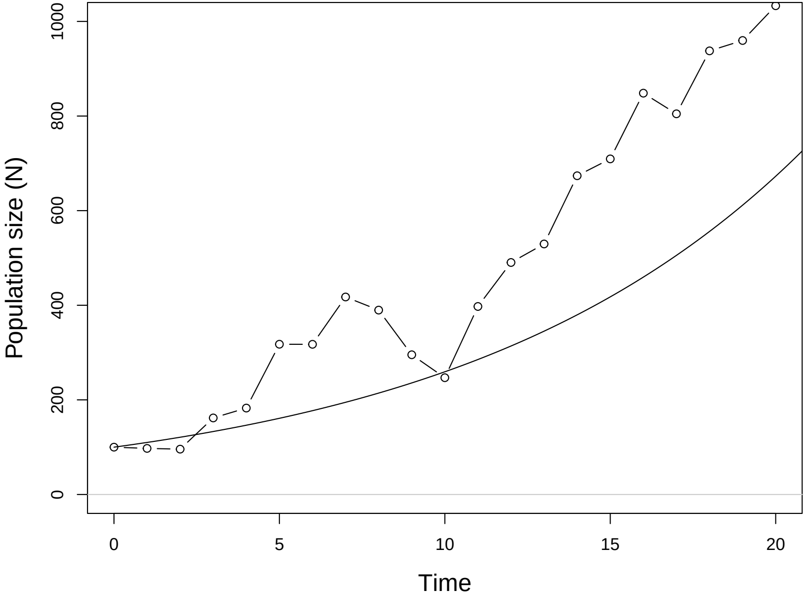

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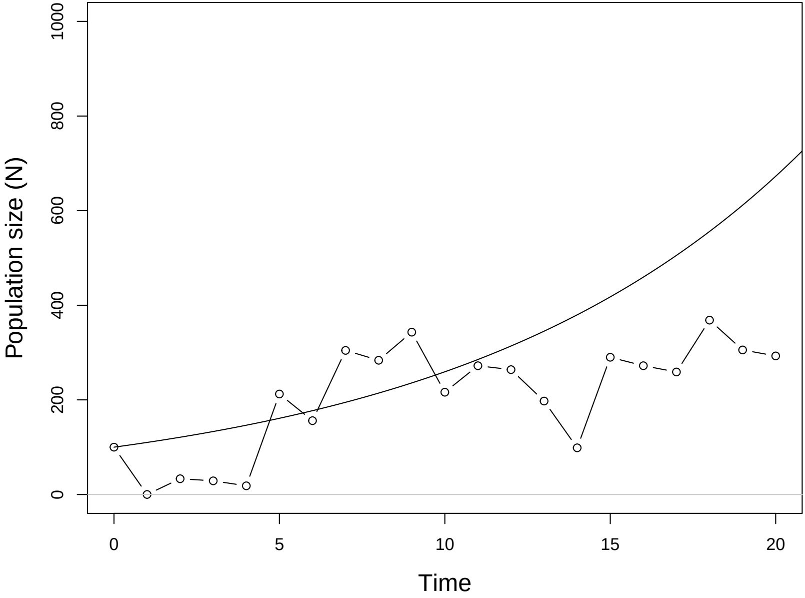
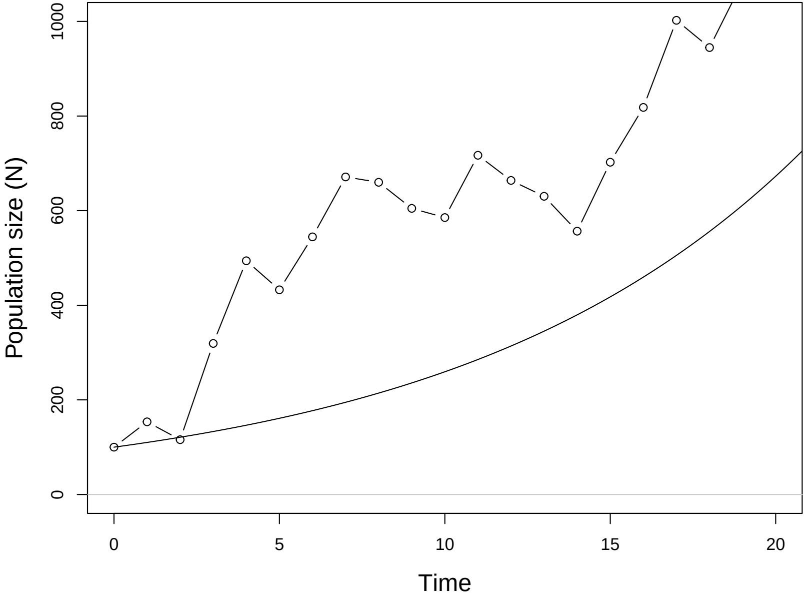


Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 10000$



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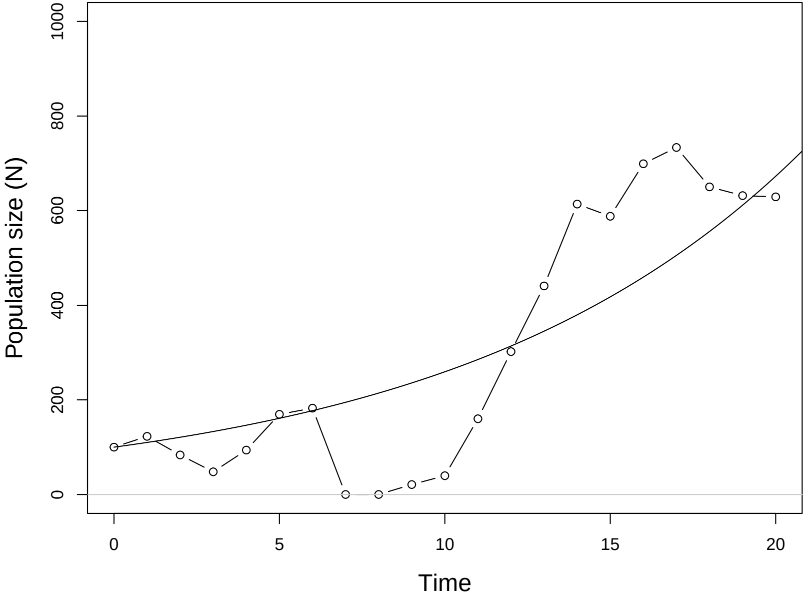

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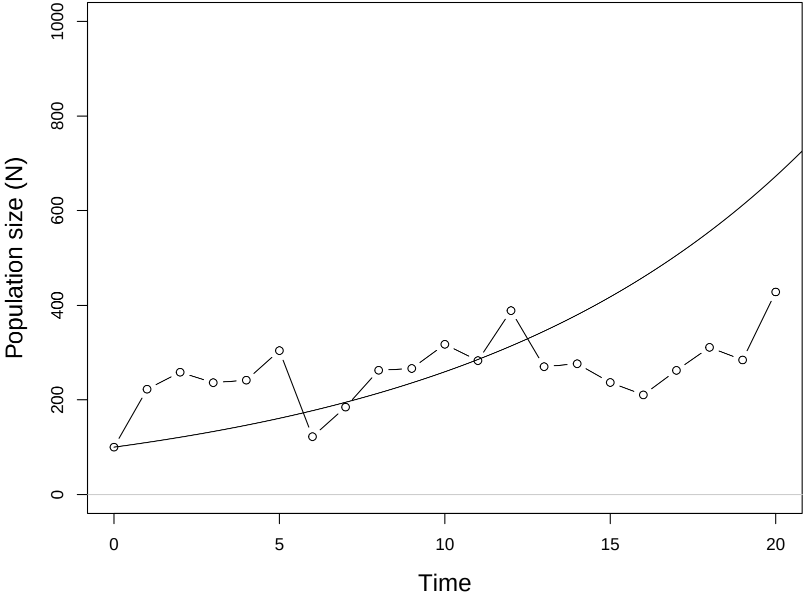
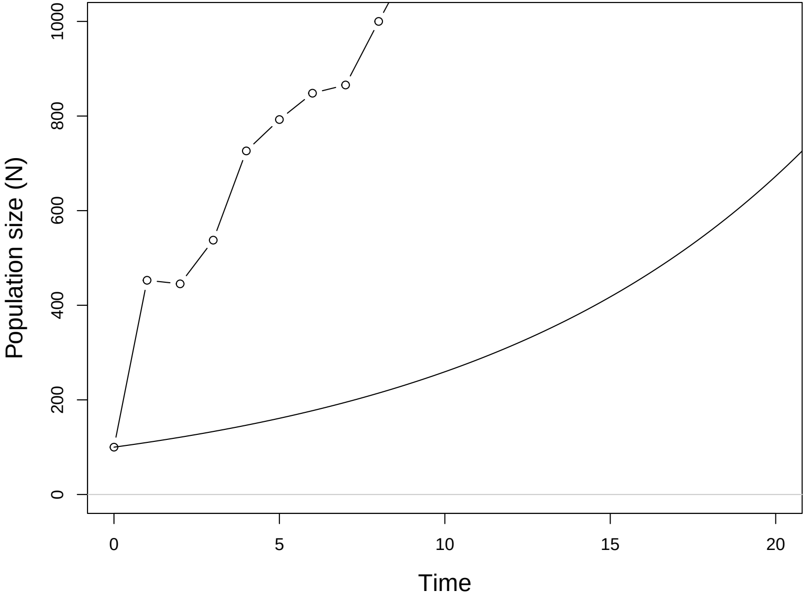


Example $N_0 = 100,\ r = 0.1,\ \mu = 0,\ \sigma_e^2 = 10000$



Geometric growth with demographic stochasticity

$$N_{t+1} = N_t + N_t r_t$$

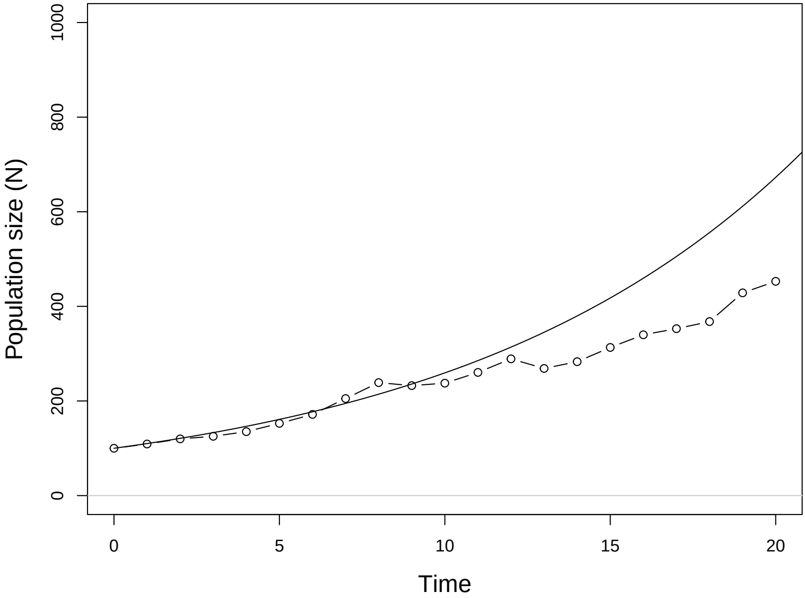
where

$$r_t \sim \text{Normal}(\bar{r}, \sigma_d^2)$$

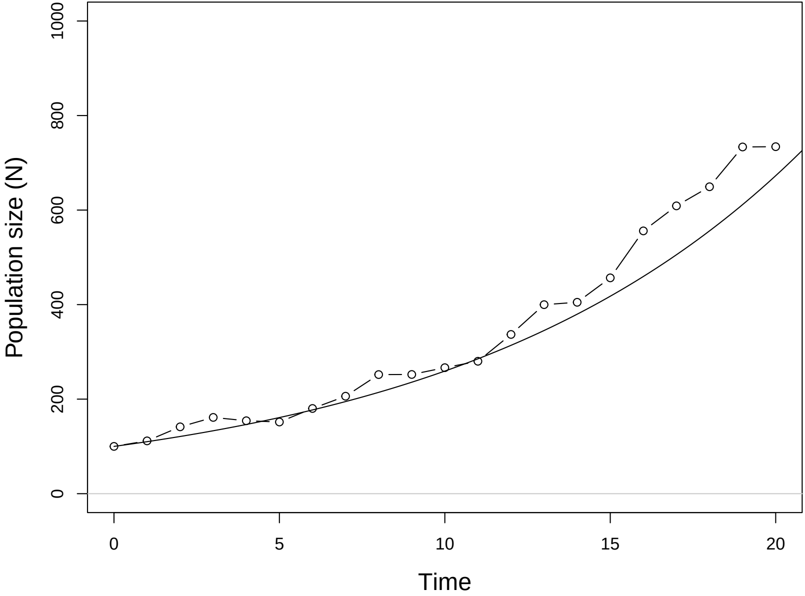
R code

```
nYears <- 20
N <- r <- rep(NA, nYears)  ## Create empty N and r
N[1] <- 100                ## Initial value of N
r.bar <- 0.5               ## Average growth rate
sigma.d <- 0.1             ## StdDev of growth rate
for(t in 2:nYears) {
    r[t-1] <- rnorm(n=1, mean=r.bar, sd=sigma.d)
    N[t] <- N[t-1] + N[t-1]*r[t-1]
}
```

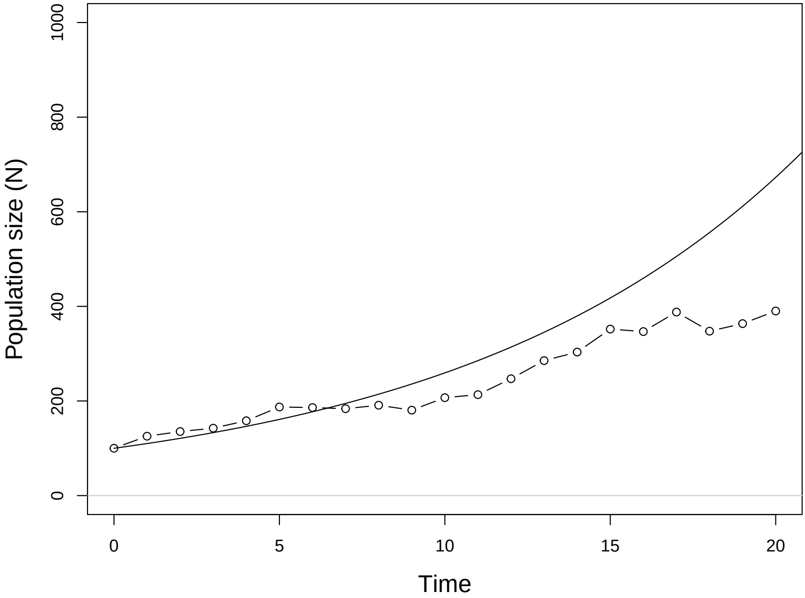
EXAMPLE $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



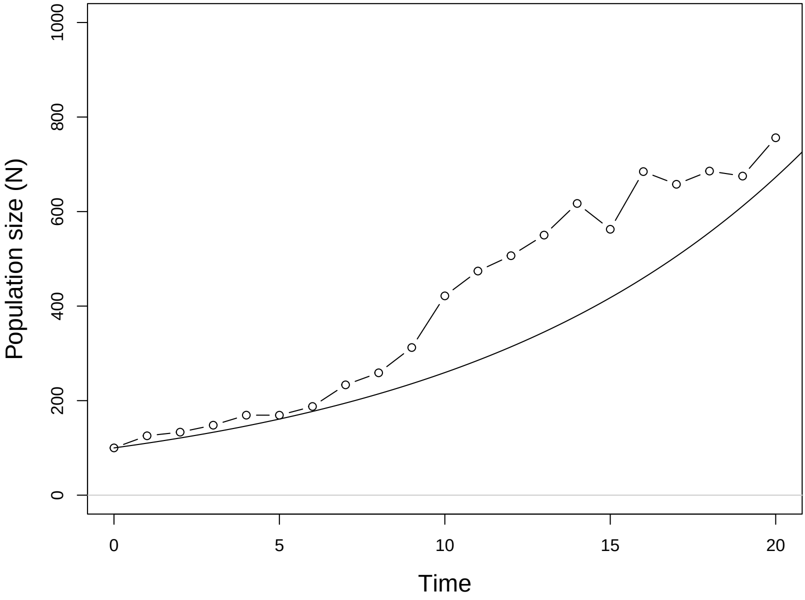
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



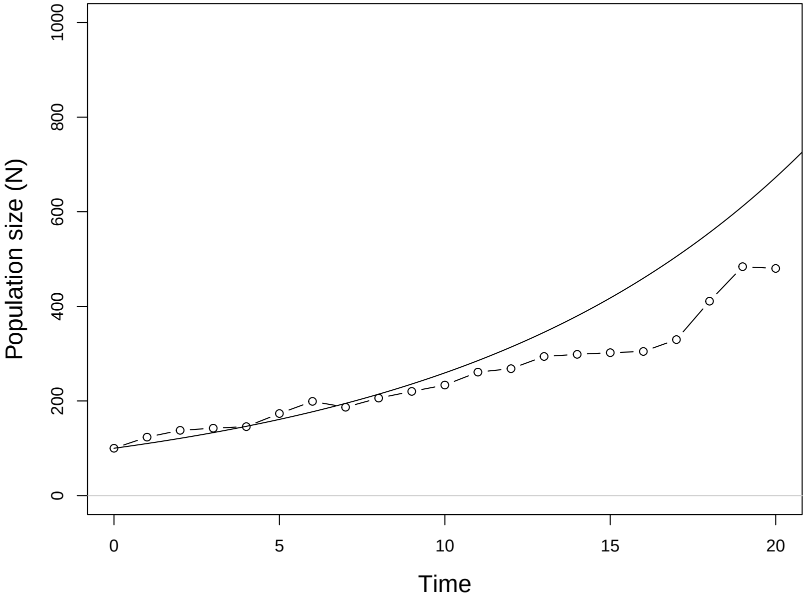
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



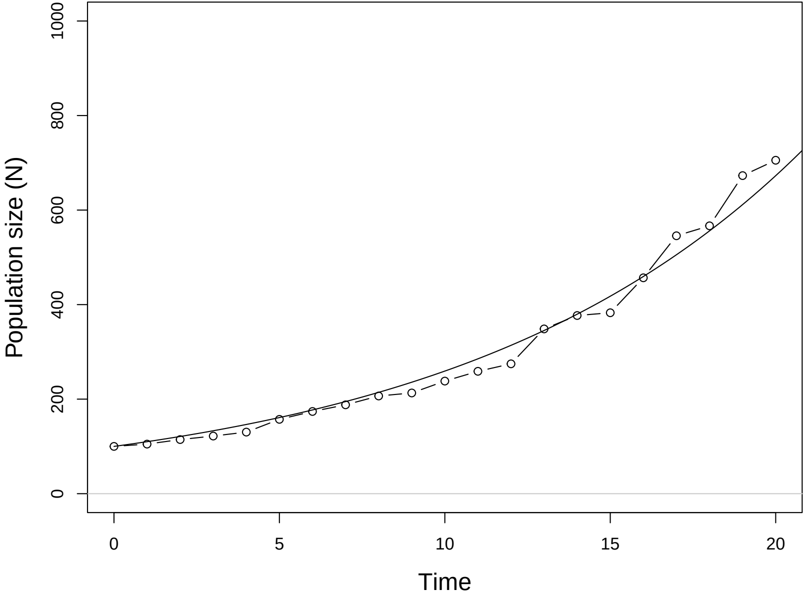
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



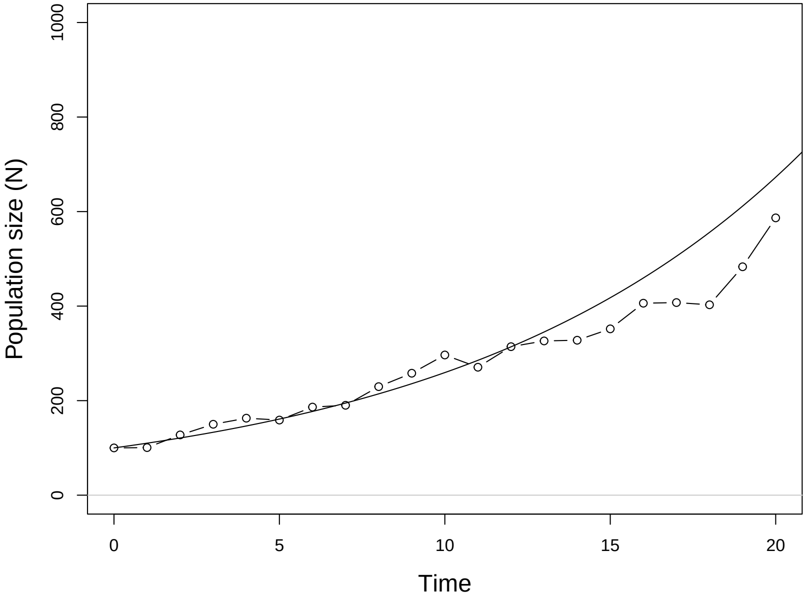
EXAMPLE $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



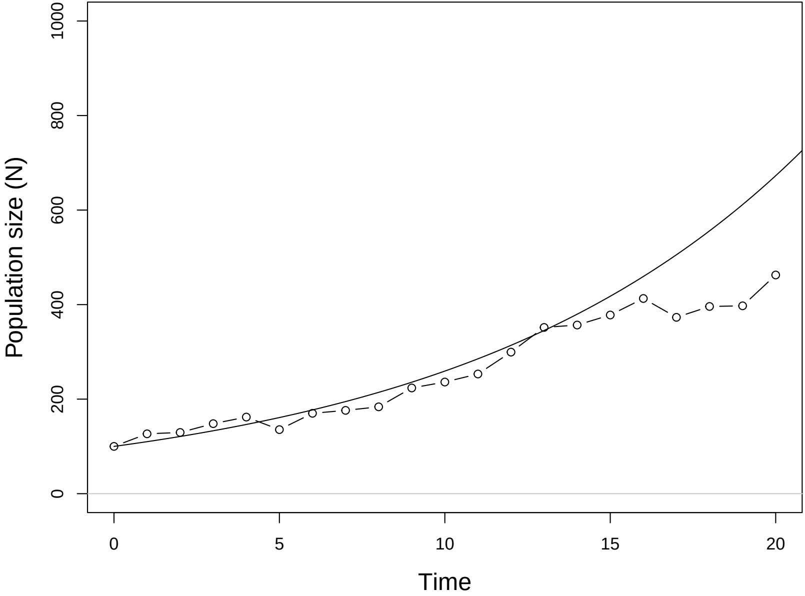
EXAMPLE $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



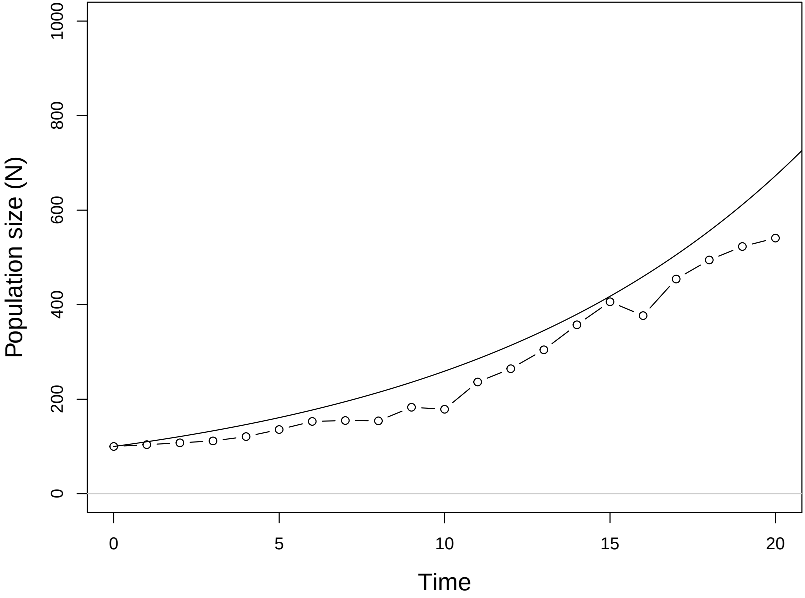
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



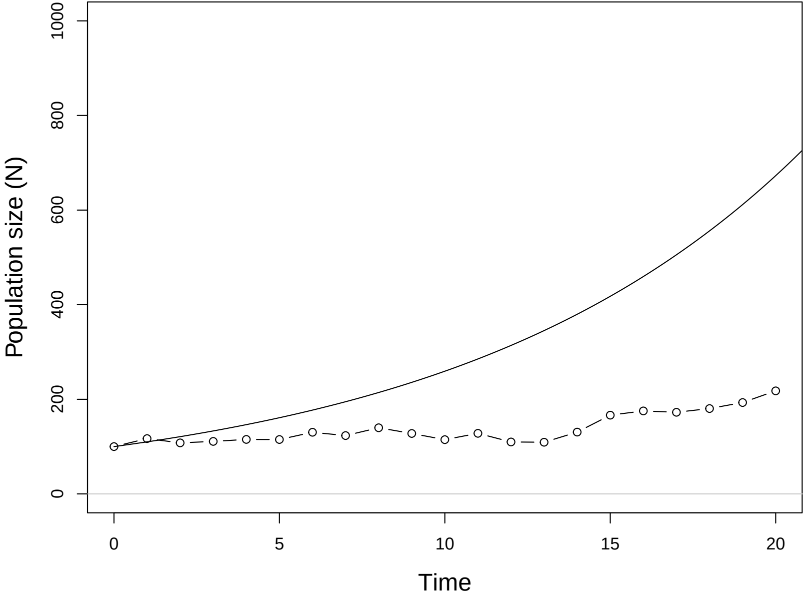
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



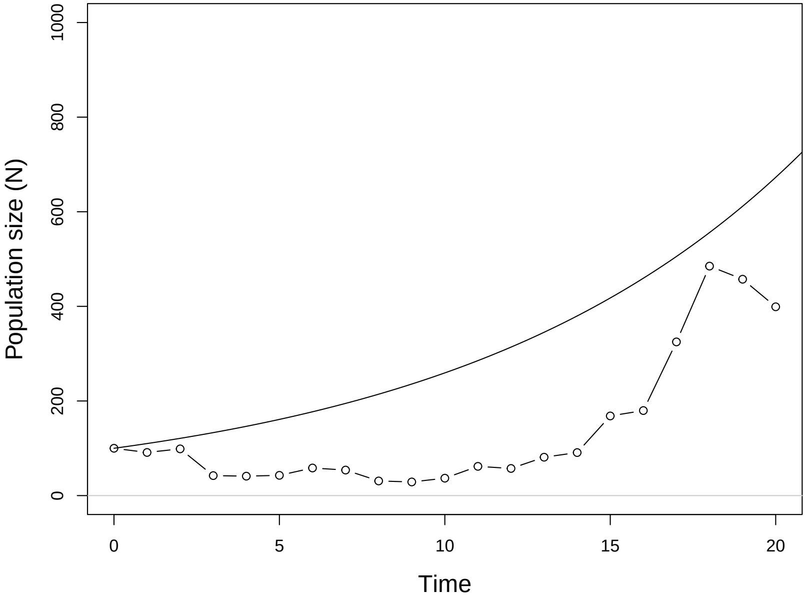
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



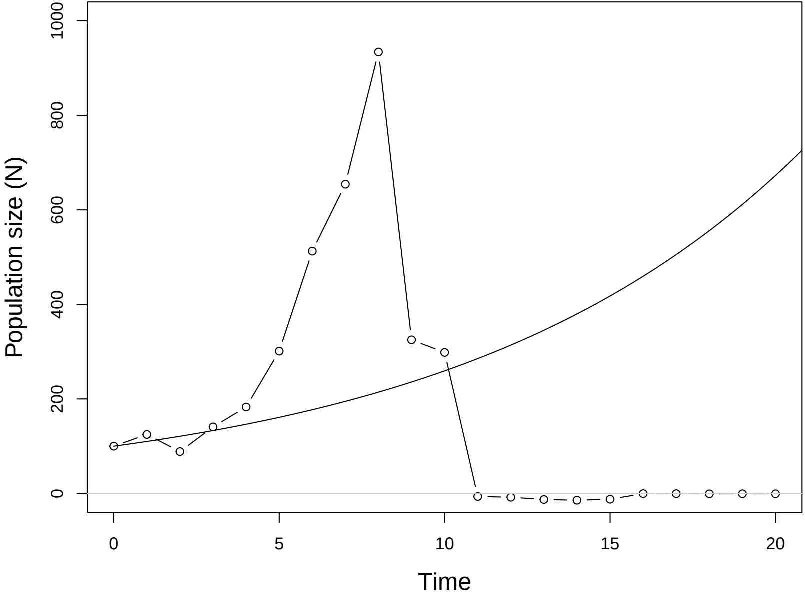
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.01$



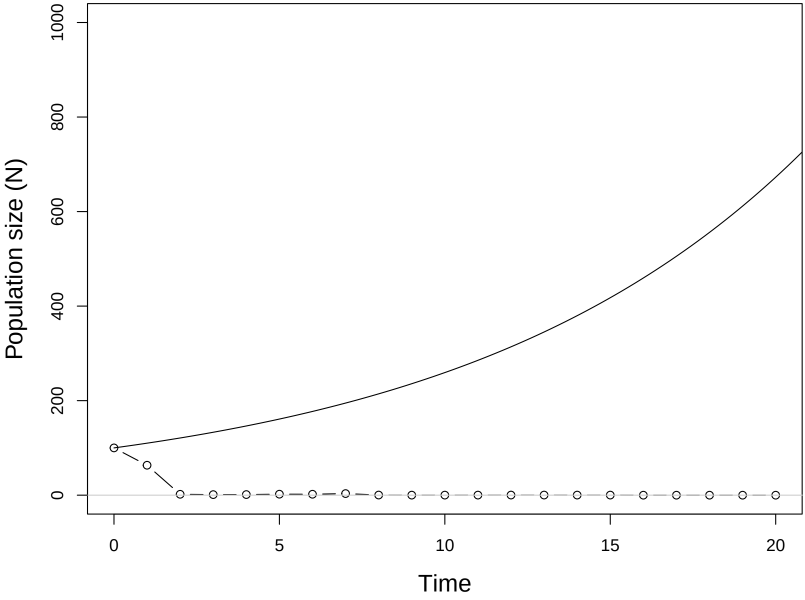
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



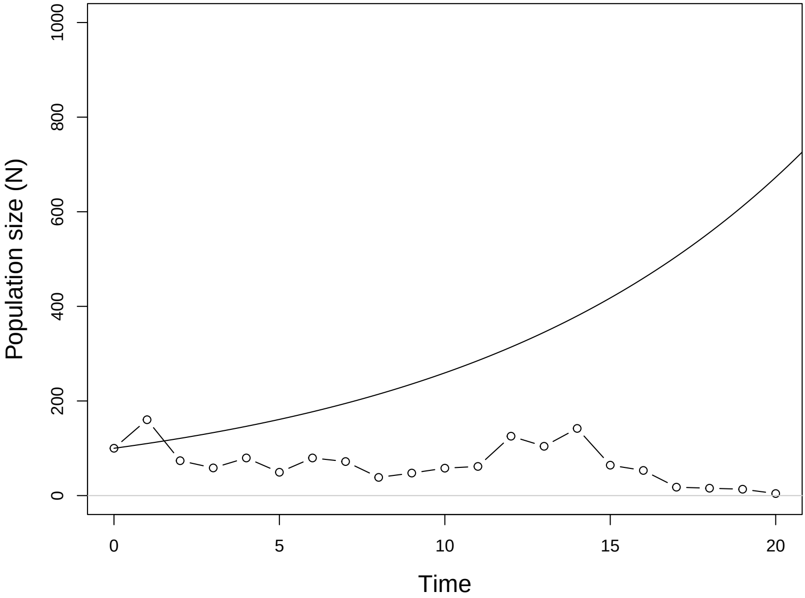
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



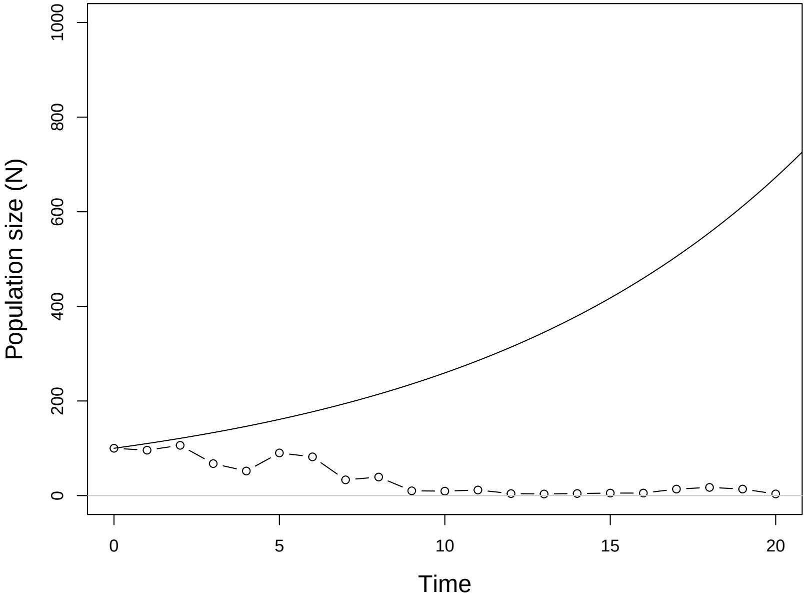
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



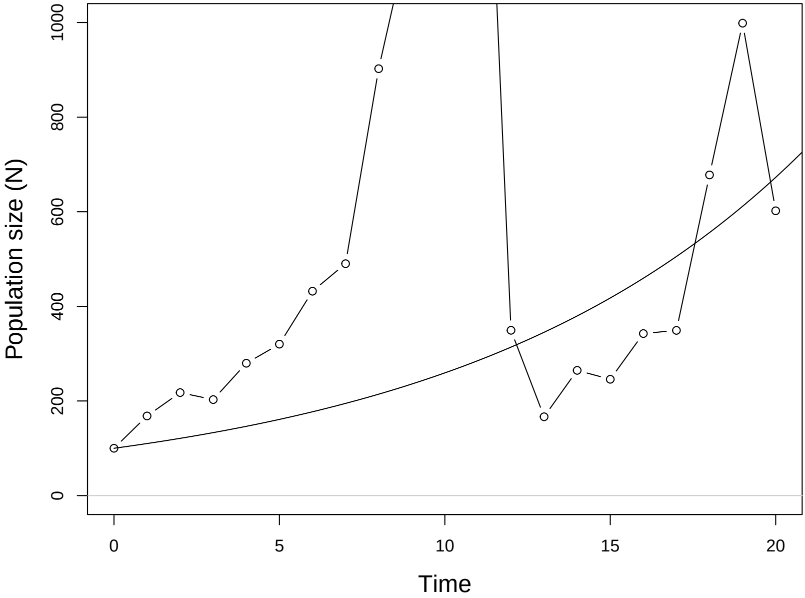
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



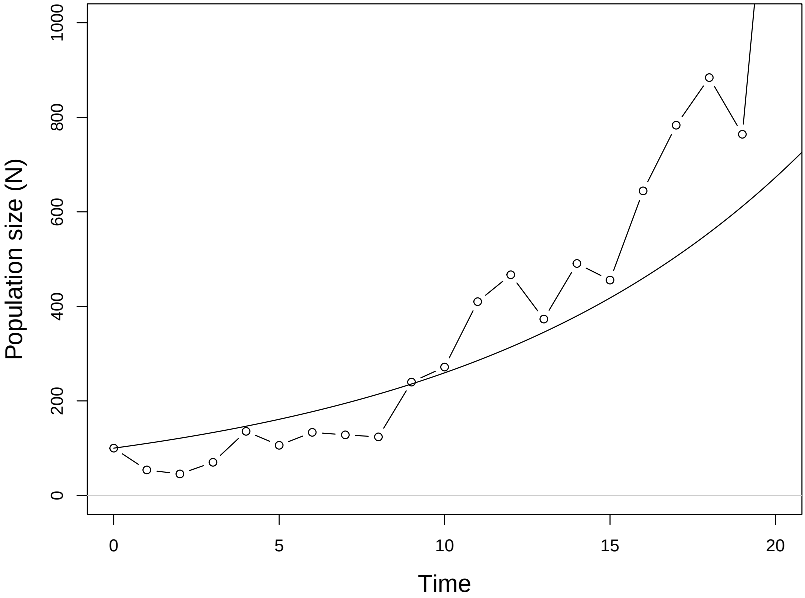
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



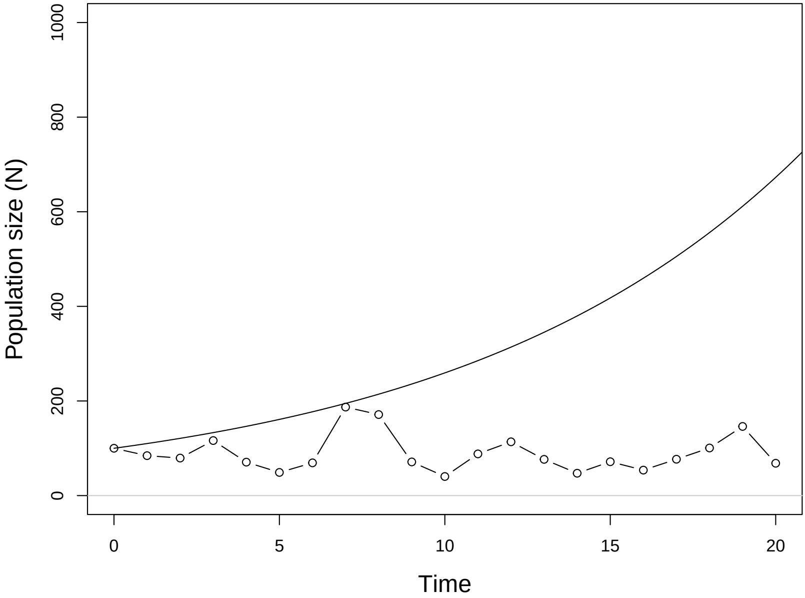
EXAMPLE $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



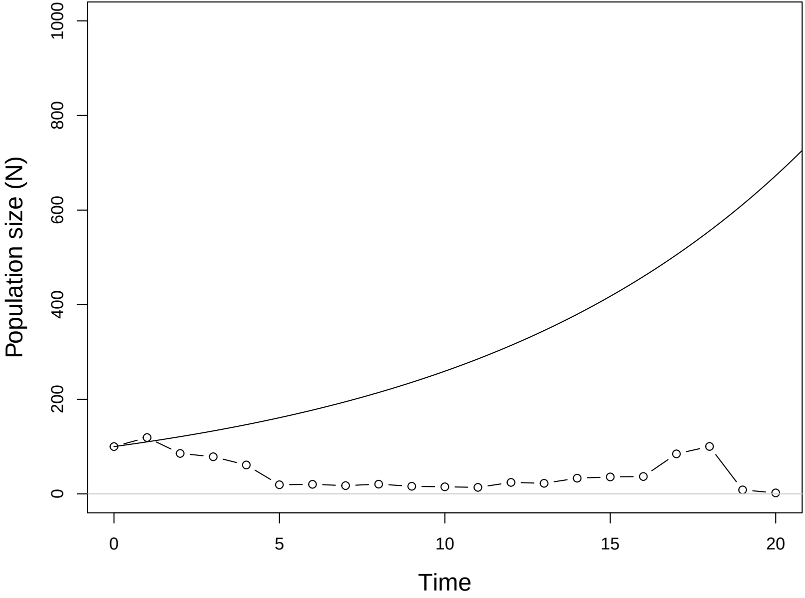
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



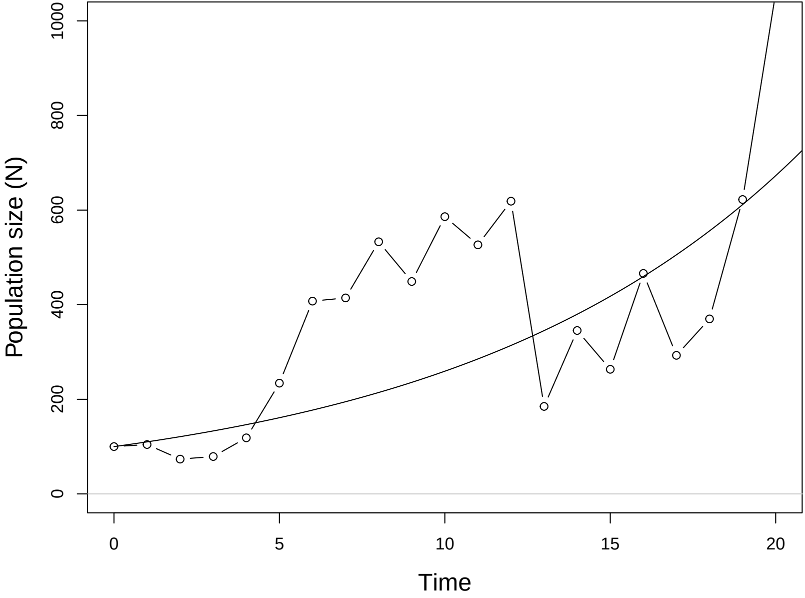
Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



Example $N_0 = 100,\ \bar{r} = 0.5,\ \sigma_d^2 = 0.25$



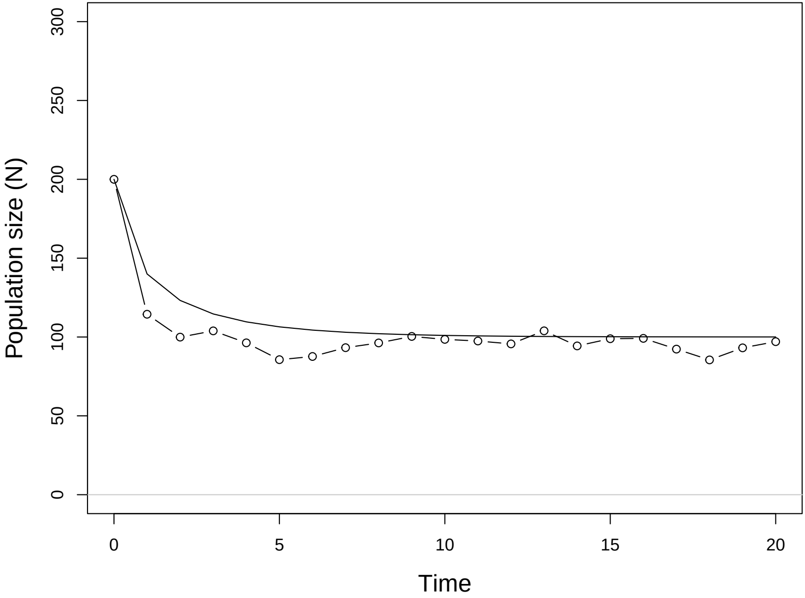
Logistic growth with stochastic carrying capacity

$$N_{t+1} = N_t + N_t r_{max}(1 - N_t/K_t)$$

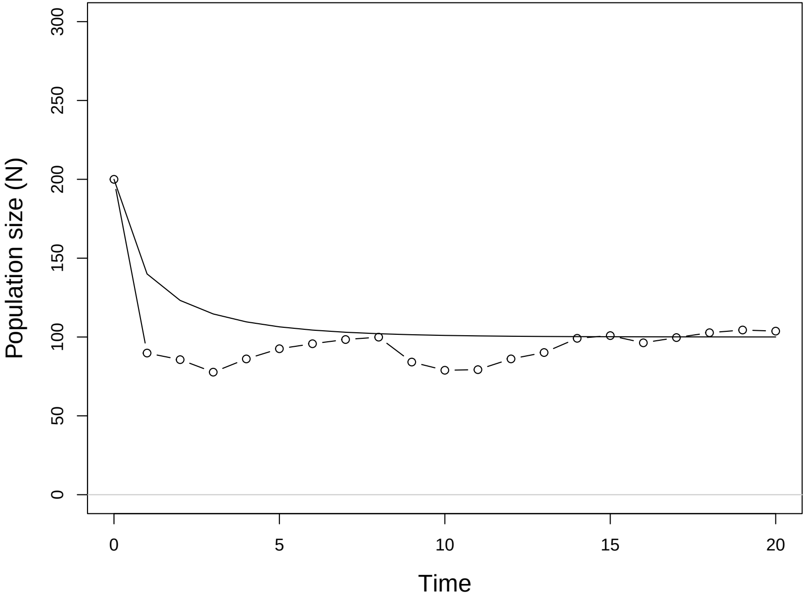
where

$$K_t \sim \mathsf{Normal}(\bar{K}, \sigma_e^2)$$

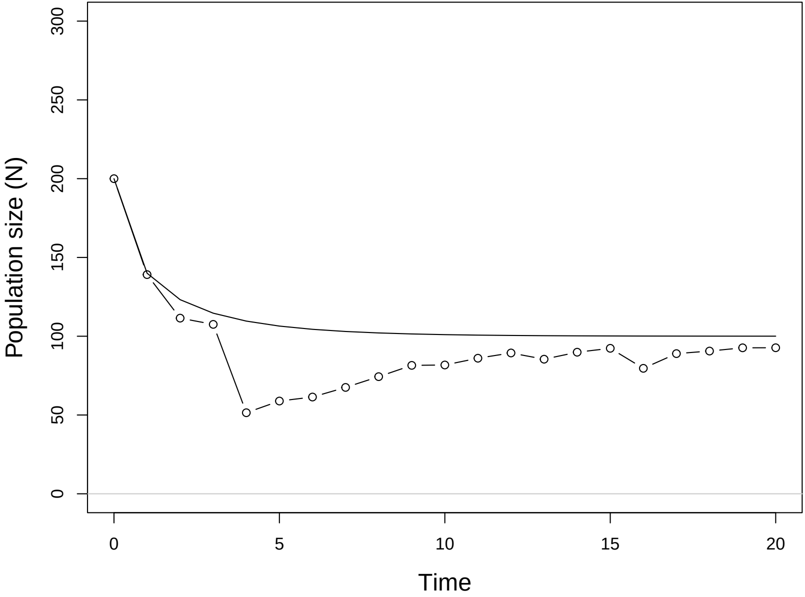
LOGISTIC EXAMPLE, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$



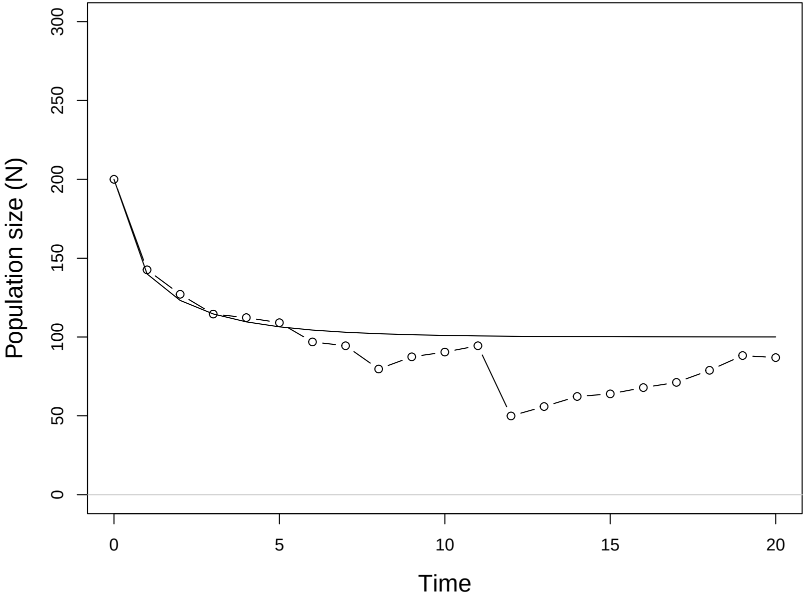
LOGISTIC EXAMPLE, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$

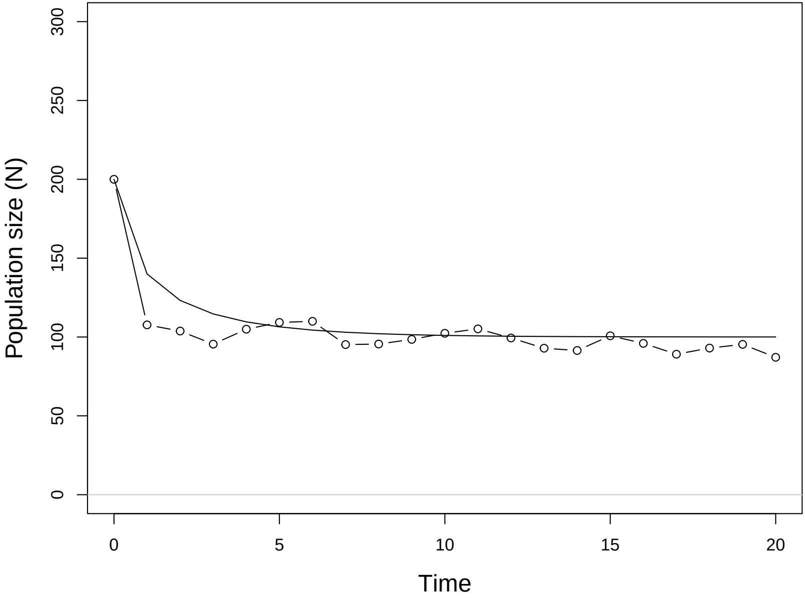


Logistic example, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$

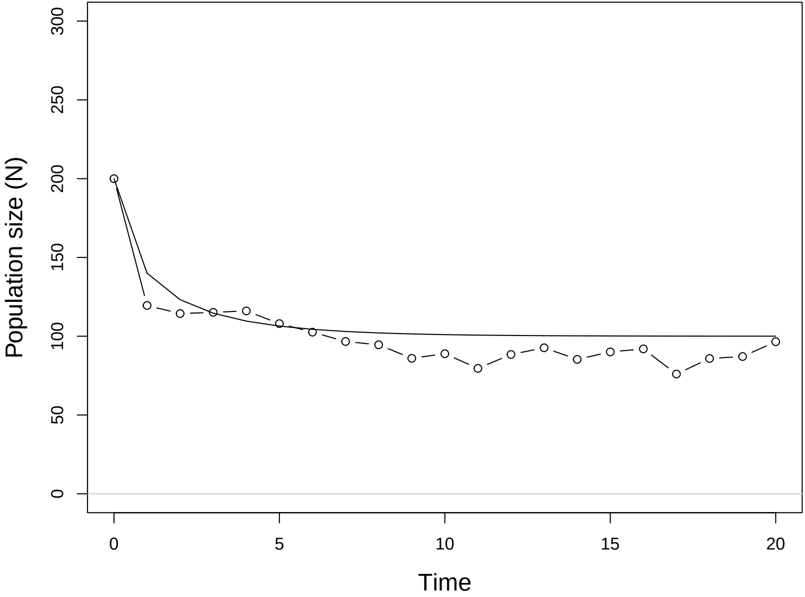


Logistic example, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$

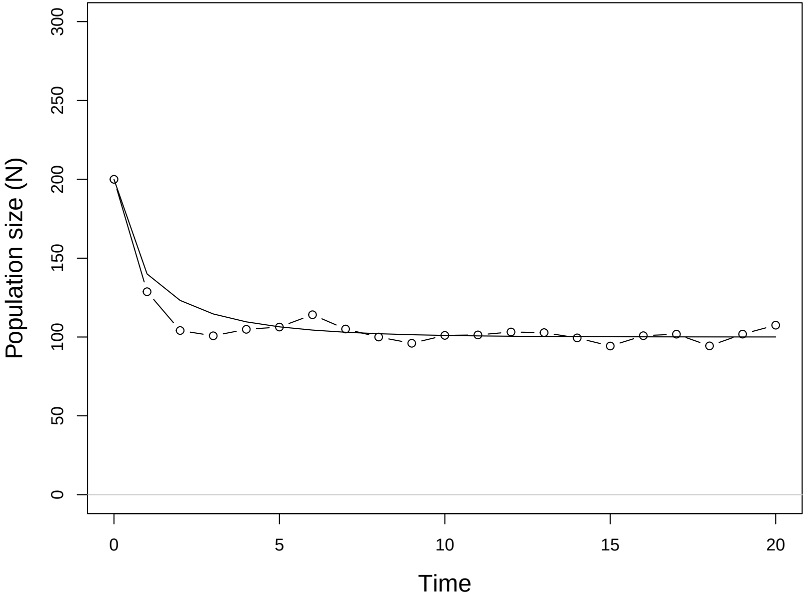



Population size (N)
0
50
100
150
200
250
300
Time
0
5
10
15
20

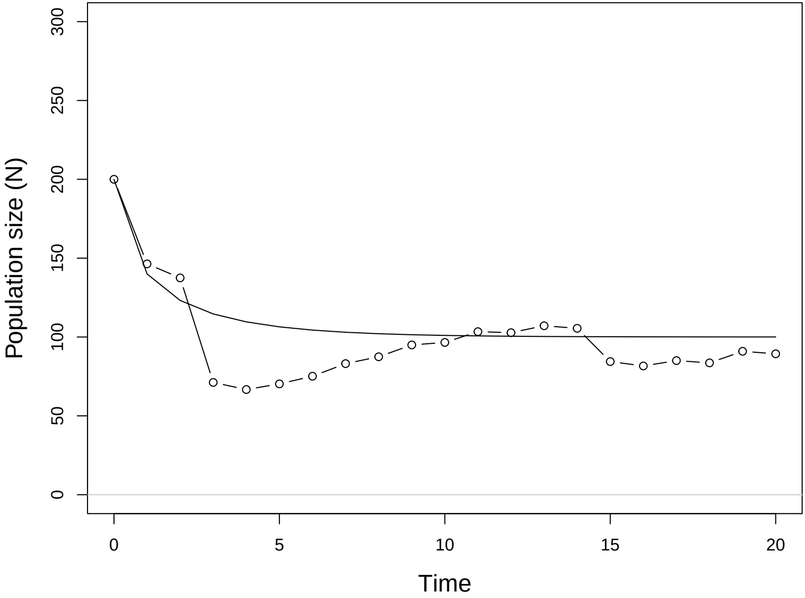
Logistic example, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$



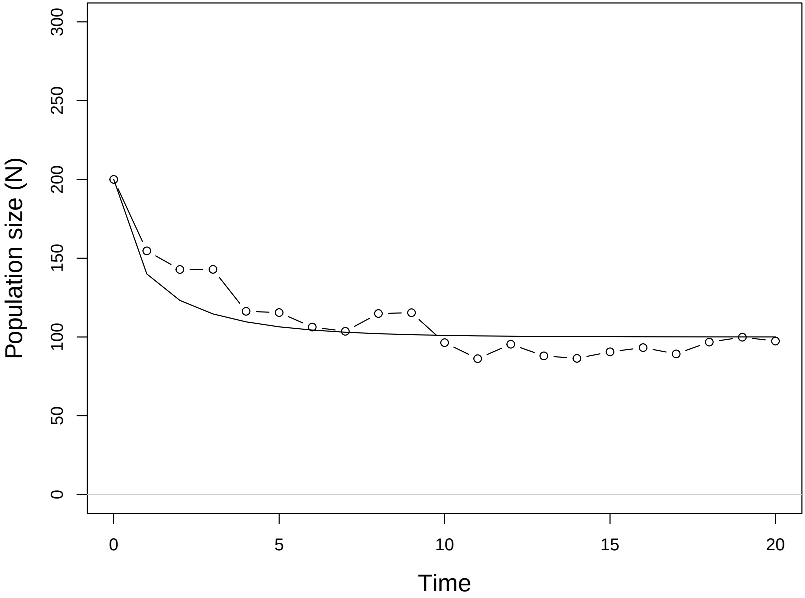
LOGISTIC EXAMPLE, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$



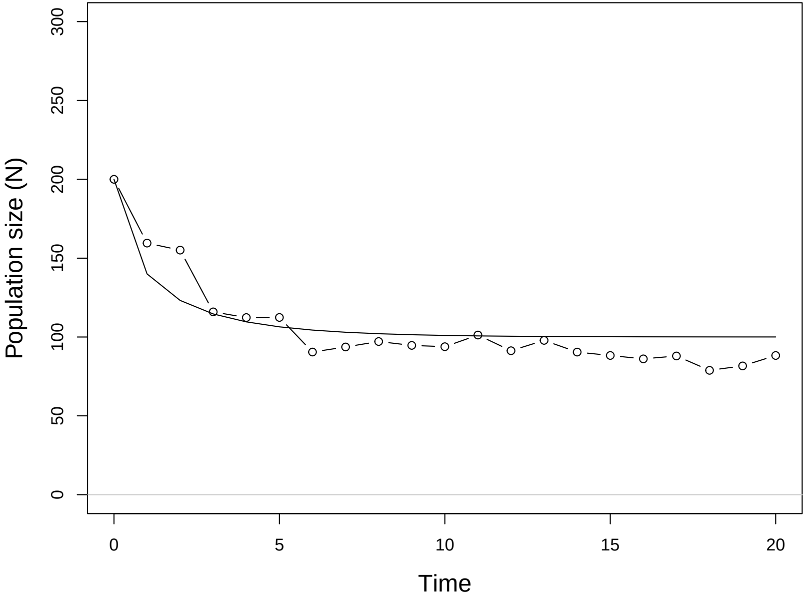
Logistic example, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$



Logistic example, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$



LOGISTIC EXAMPLE, $r_{max} = 0.2,\ \bar{K} = 100,\ \sigma_e^2 = 400$



Purely deterministic models are too rigid.

Purely stochastic models don't describe population processes.

The goal is to develop a mechanistic model that represents our biological understanding while allowing for stochasticity.