Models of interspecific interactions

Predator-prey dynamics and competition

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- As usual, these models were developed as continuous-time models.
- We will focus on discrete-time versions ($t = 1, 2, \ldots$).
- We will ignore potential extensions with stochasticity, age structure, spatial structure, etc...

How should predator-prey dynamics operate?

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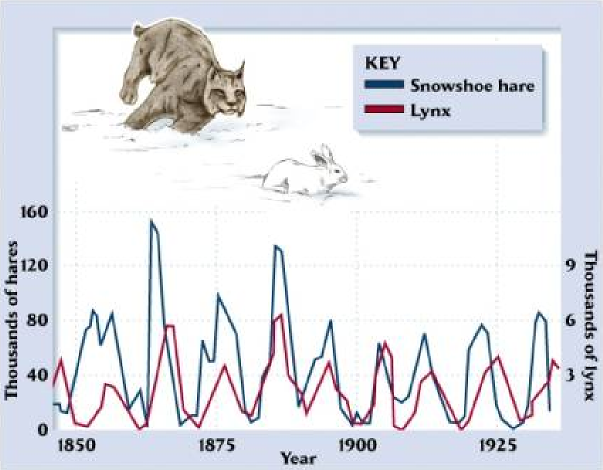
Lynx-hare cycles



Model for prey

$$N_{t+1}^{prey} = N_t^{prey} + N_t^{prey}(r^{prey} - d^{prey} N_t^{pred})$$

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Model for predator

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Model for predator

$$N_{t+1}^{pred} = N_t^{pred} + N_t^{pred}(b^{pred} N_t^{prey} - d^{pred})$$

- Model is based on geometric growth
- r^{prey} is the growth rate of the prey in the absence of predators
- d^{prey} is the predation rate
- b^{pred} is the birth rate of the predators
- d^{pred} is the mortality rate of the predator

Equilibrium for prey occurs when...

$$N^{pred} = \frac{r^{prey}}{d^{prey}}$$

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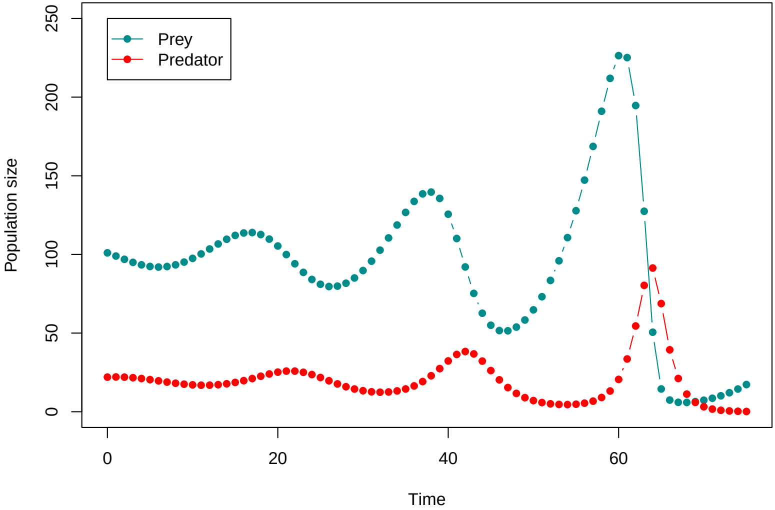
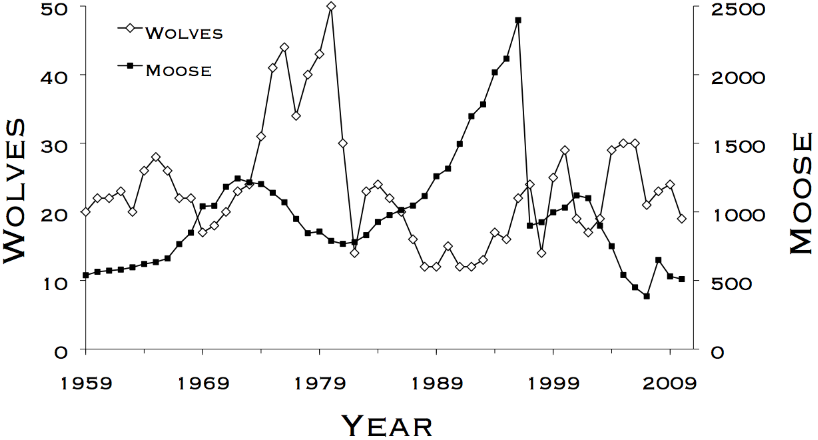
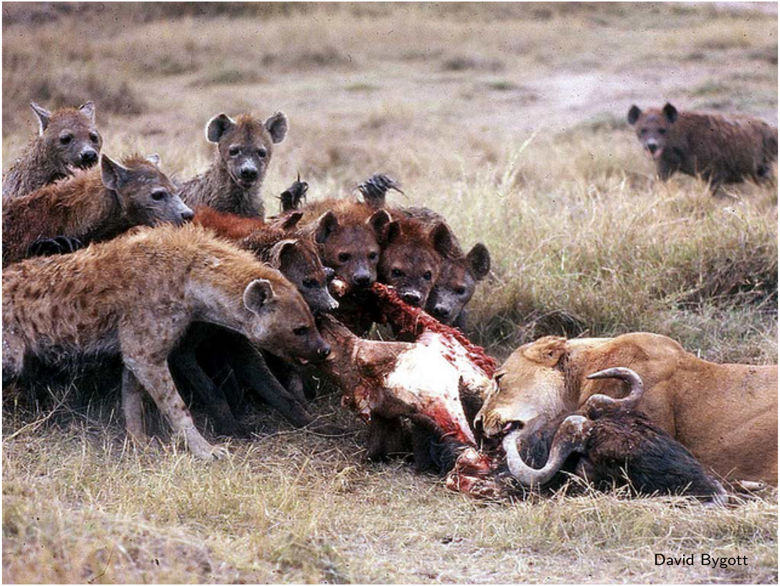
However, it is rare that both equilibrium conditions will be met at the same time, and so the populations will cycle.

Model predicts population cycles





http://www.youtube.com/watch?v=PdwnfPurXcs
https://isleroyalewolf.org/


David Bygott

Model for species A

$$N_{t+1}^{A} = N_t^A + r^A N_t^A (K^A - N_t^A - \alpha^B N_t^B)/K^A$$

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$$N_{t+1}^{A} = N_{t}^{A} + r^{A} N_{t}^{A} (K^{A} - N_{t}^{A} - \alpha^{B} N_{t}^{B}) / K^{A}$$

Model for species B

$$N_{t+1}^{B} = N_{t}^{B} + r^{B} N_{t}^{B} (K^{B} - N_{t}^{B} - \alpha^{A} N_{t}^{A}) / K^{B}$$

Model for species A

$$N_{t+1}^A = N_t^A + r^A N_t^A (K^A - N_t^A - \alpha^B N_t^B)/K^A$$

Model for species B

$$N_{t+1}^B = N_t^B + r^B N_t^B (K^B - N_t^B - \alpha^A N_t^A)/K^B$$

- Model based on logistic growth
- The α parameters are competition coefficients determining how strongly each species affects the other

Equilibrium for species A

$$N^A = \frac{K^A - \alpha^B K^B}{1 - \alpha^A \alpha^B}$$

Equilibrium for species A

$$N^A = \frac{K^A - \alpha^B K^B}{1 - \alpha^A \alpha^B}$$

Equilibrium for species B

$$N^B = \frac{K^B - \alpha^A K^A}{1 - \alpha^A \alpha^B}$$

Outcomes depend on the sign of the numerators

(1) Stable coexistence

(2) Competitive exclusion

(3) Unstable equilibrium

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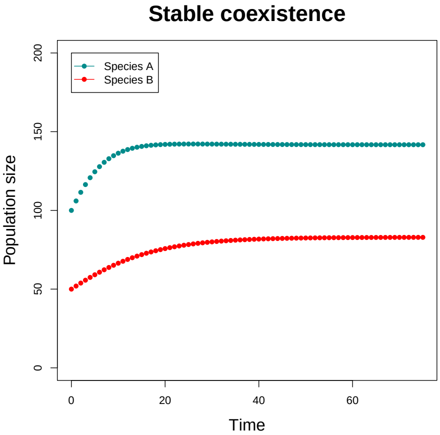
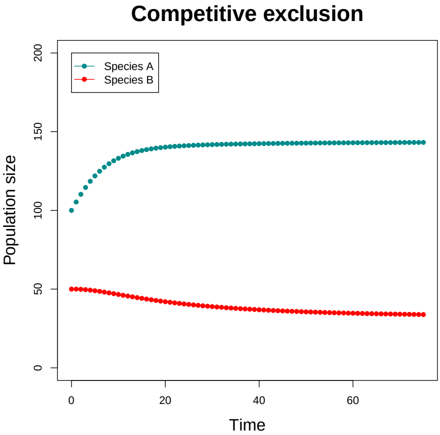
Competitive exclusion principle: Two species with the same niche cannot coexist on the same limiting resource

Outcomes


Stable coexistence
Species A
Species B
Population size
0
50
100
150
200
0
20
40
60
Time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https://youtu.be/KQLPL1qRhn8

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Summary

Predator-prey model is extension of geometric growth

- Predators and prey limit each other's growth potential

Competition model is extension of logistic growth

- Competitors influence each other's density-dependent regulation process

These models could be extended to include:

- More species
- Stochasticity
- Age structure
- Harvest
- Spatial structure
- Additional forms of density dependence