Geometric and Exponential Growth

Today's learning objectives

The equations for geometric and exponential growth

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The relationship between geometric growth and the BIDE model

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The difference between continuous and discrete time models of population growth

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The equations for geometric and exponential growth

The relationship between geometric growth and the BIDE model

The difference between continuous and discrete time models of population growth

The definition of density *independent* population growth

The study of spatial and temporal variation in population size and structure

Fundamental Question

How does abundance go from N_t to N_{t+1}?

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Answer: The BIDE Model

$$N_{t+1} = N_t + B_t + I_t - D_t - E_t$$

B=Births, I=Immigrations, D=Deaths, E=Emigrations

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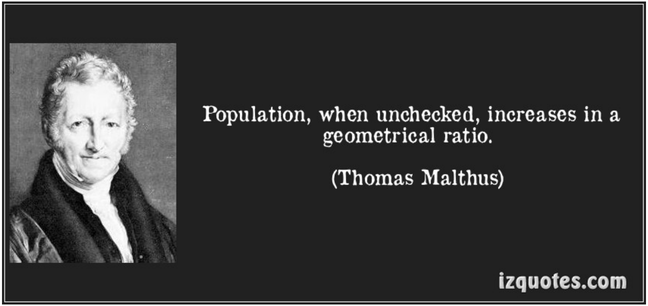
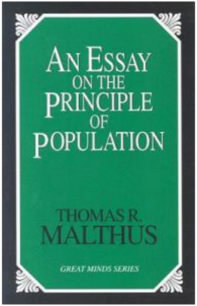
Geometric growth is a simplification of BIDE

Exponential growth is a continuous time version of geometric growth

Reverend Thomas Malthus, 1766–1834





Charles Darwin (*Origin of Species*)

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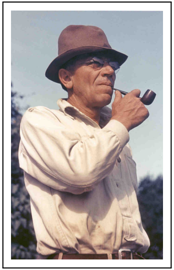
> *"Hence, as more individuals are produced than can possibly survive, there must in every case be a struggle for existence. . . "*

"Every wild species has certain fixed habits which govern the reproductive process, and determine its maximum rate. [...] Thus one pair of quail, if entirely unmolested in an "ideal" environment, would increase at this rate:"

At End of	Young	Adults	Total
1st year	14	2	16
2nd year	(16/2)14=112	16	128
3rd year	(128/2)14=896	128	1024



Aldo Leopold, Game Management 1946

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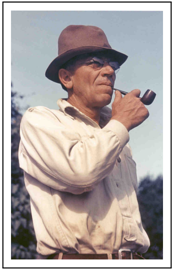
"The maximum rate of increase is of course never attained in nature. Part of it never takes place, part of it is absorbed by natural enemies, and part of it [. . .] is absorbed by hunters."



So What Is Geometric Growth?

Discrete time, $t = 1, 2, \ldots$

$$N_t = N_0(1 + r)^t$$

$r =$ discrete-time version of intrinsic rate of increase

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$$N_t = N_0(1 + r)^t$$

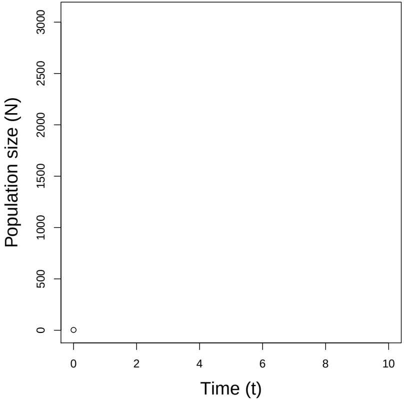
Or, for one time step:

$$N_{t+1} = N_t + N_t r$$

$r =$ discrete-time version of intrinsic rate of increase

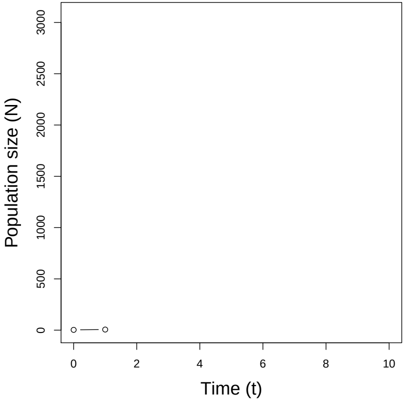
EXAMPLE, $N_{t+1} = N_t + N_t r$

$N_0 = 3$, $r = 1$	
Time (t)	Population size (N_t)
0	3



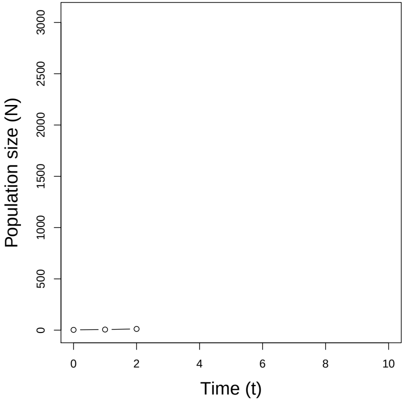
Example, $N_{t+1} = N_t + N_t r$

$N_0 = 3$, $r = 1$	
Time (t)	Population size (N_t)
0	3
1	6



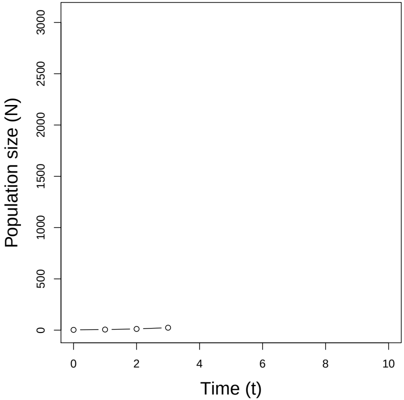
Example, $N_{t+1} = N_t + N_t r$

$N_0 = 3$, $r = 1$	
Time (t)	Population size (N_t)
0	3
1	6
2	12



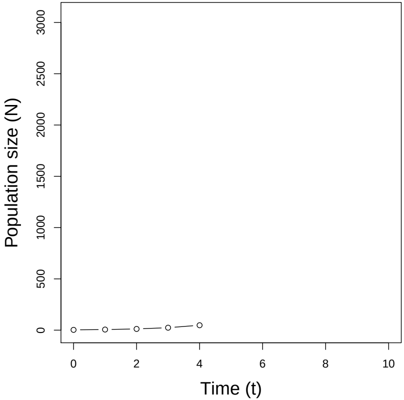
EXAMPLE, $N_{t+1} = N_t + N_t r$

$N_0 = 3$, $r = 1$	
Time (t)	Population size (N_t)
0	3
1	6
2	12
3	24



Example, $N_{t+1} = N_t + N_t r$

$N_0 = 3$, $r = 1$	
Time (t)	Population size (N_t)
0	3
1	6
2	12
3	24
4	48



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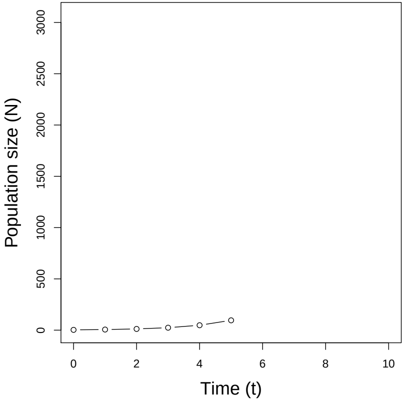
Time (t)	Population size (N_t)
0	3
1	6
2	12
3	24
4	48
5	96



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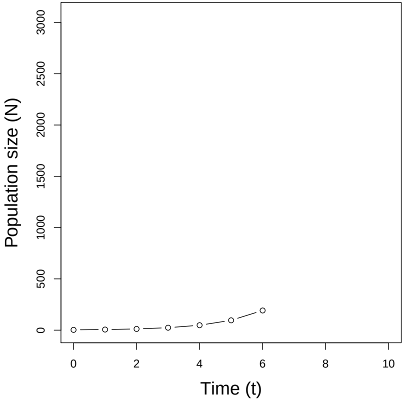
$N_0 = 3$, $r = 1$	
Time (t)	Population size (N_t)
0	3
1	6
2	12
3	24
4	48
5	96
6	192



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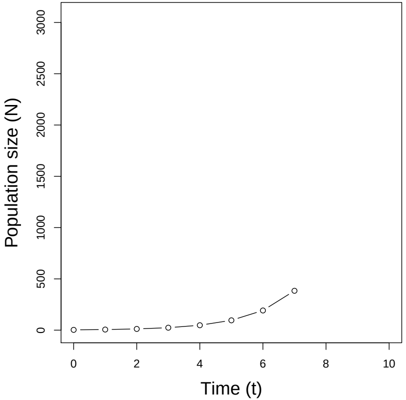
4	48
5	96
6	192
7	384



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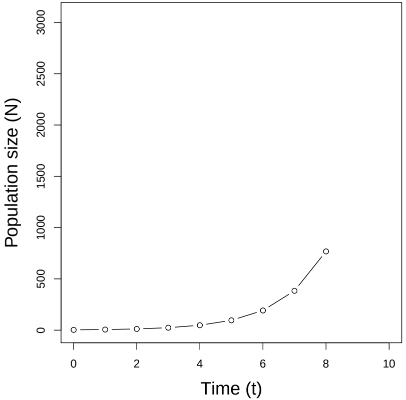
4	48
5	96
6	192
7	384
8	768



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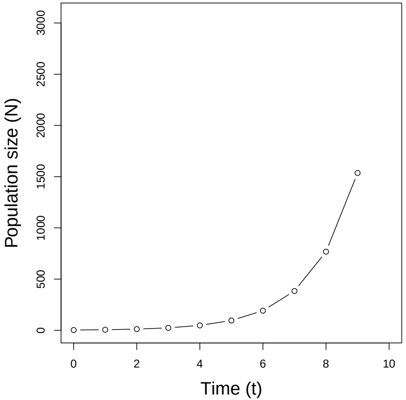

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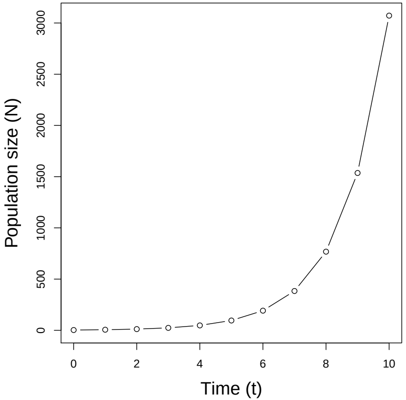
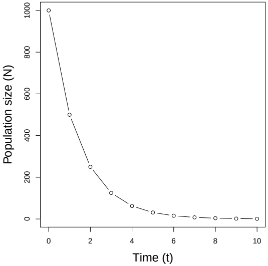
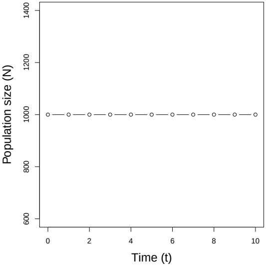
7	384
8	768
9	1536
10	3072



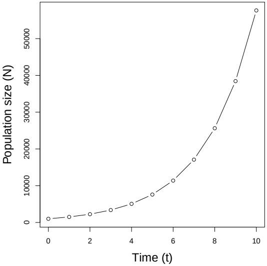
If $-1 \leq r < 0$
population goes extinct



If $r = 0$
population is stable



If $r > 0$
population grows to ∞



r is the discrete growth rate

λ is the finite growth rate

$$\lambda = \frac{N_{t+1}}{N_t}$$

$\lambda = 1 + r$

Fundamental equation of population ecology

$$N_{t+1} = N_t + B_t + I_t - D_t - E_t$$

N_t = Abundance at year t
B = Births
I = Immigrations
D = Deaths
E = Emigrations

Ignore immigration and emigration

$$N_{t+1} = N_t + B_t - D_t$$

$$\begin{aligned} N_t &= \text{Abundance in year } t \\ B &= \text{Births} \\ D &= \text{Deaths} \end{aligned}$$

Step 1: Divide both sides by N_t

$$\frac{N_{t+1}}{N_t} = 1 + \frac{B_t}{N_t} - \frac{D_t}{N_t}$$

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$$\begin{aligned} \frac{N_{t+1}}{N_t} &= 1 + b - d \\ &= 1 + r \\ &= \lambda \end{aligned}$$

Step 3: Geometric growth

$$N_{t+1} = N_t + N_t r$$

So What Is Exponential Growth?

Continuous time version of geometric growth

$$N_t = N_0 e^{rt}$$

$N_0 =$ initial abundance
$r =$ intrinsic rate of increase
$t =$ time (any positive number)

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However, geometric growth models can provide a good approximation of birth flow or birth pulse populations

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Definition: Population growth rate (r) is *not* affected by population size (N).

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Implications: Resources are unlimited and there is no carrying capacity!

Model Assumptions

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- No immigration
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- No age- or stage-structure
- No time lags

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(4) No stochasticity

- No random variation in birth or death
- No random variation in environmental conditions

Can We Apply The Model To Real Data?

All models are wrong, but some are useful. (George Box)

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Is exponential growth a useful model?

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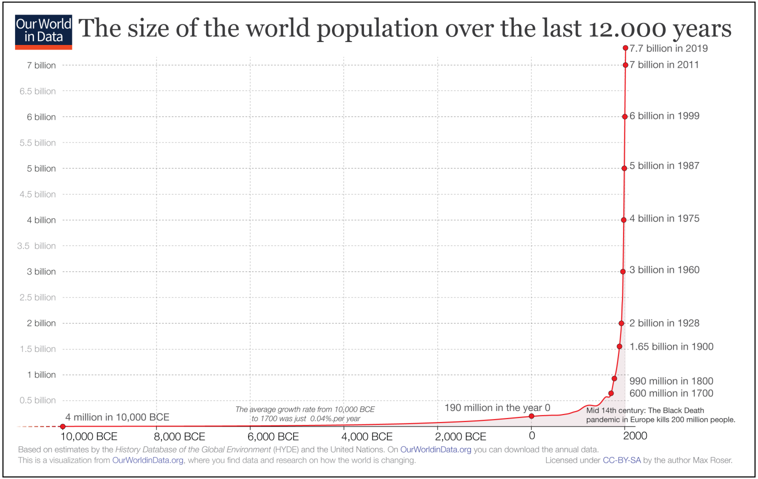
Is exponential growth a useful model?

- Possibly for describing some populations during short time periods, e.g. invasive species
- Also useful as foundation for more realistic models

Is the human population exhibiting exponential growth?



`https://ourworldindata.org/world-population-growth`

Assignment

Read pages 15–19 in Conroy and Carroll

Be prepared for a quiz