Lab 4 Assignment — Stochasticity and Extinction

Submit your Excel file and R script before Monday. Undergrads only need to do the first exercise in R, but you will get 10 bonus points if you do the second one too.

Random Numbers in Excel

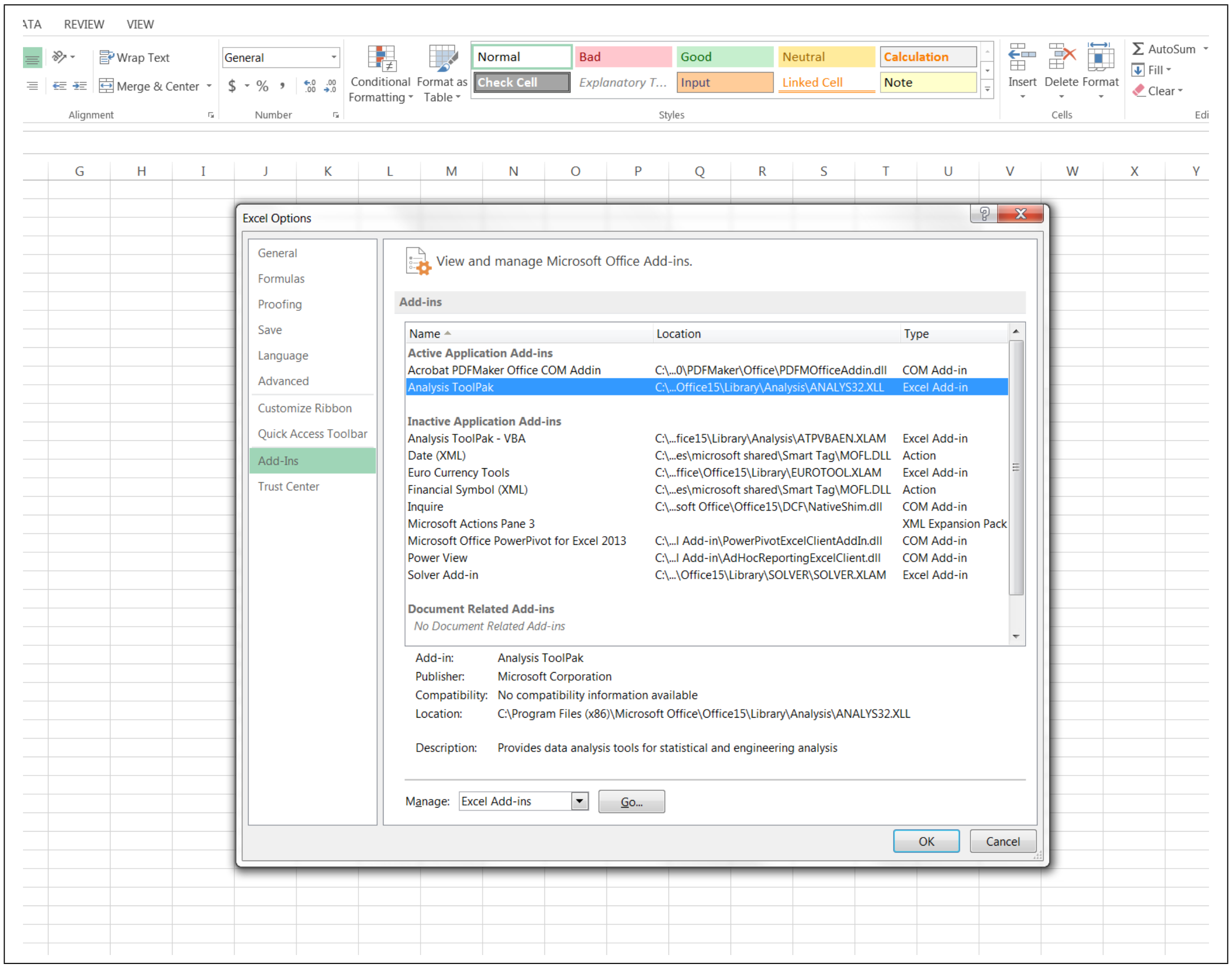


Figure 1: To generate random numbers in Excel, you must load the "Analysis Toolpak" add-in by clicking `File` > `Options` > `Add-Ins` > `Analysis Toolpak`. Then hit "Go" (not "OK") and select "Analysis Tookpak" again.

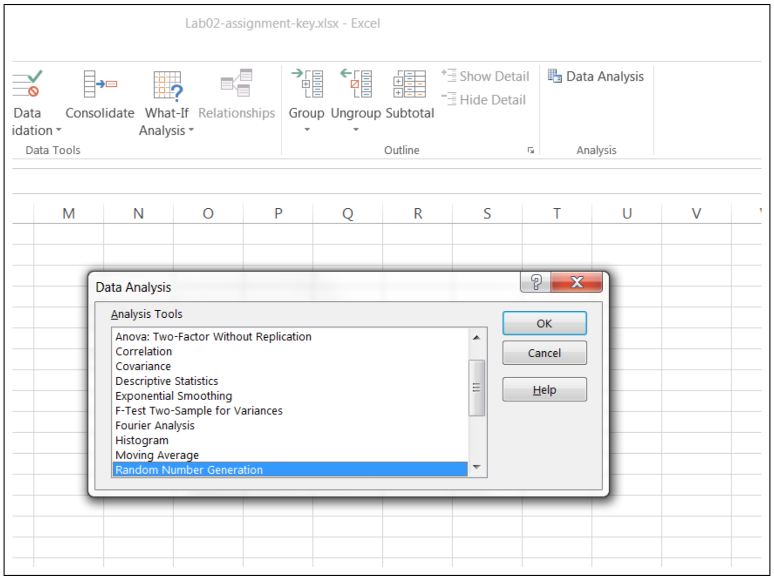


Figure 2: Once the Analysis ToolPak is installed, you can generate random numbers by clicking the "Data Analysis" button in the "Data" tab.

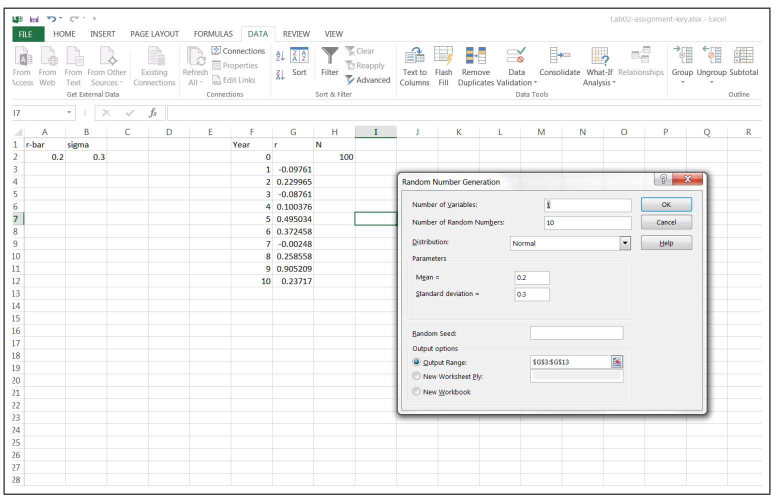


Figure 3: We will use three distributions: Normal, Binomial, and Poisson. The latter two are only used in Exercise II. Note: You can think of the "number of variables" as the number of columns of random variables, and the "number of random numbers" as the number of cells in each column.

Answer each of the following questions and upload your completed Excel file and R script to ELC. Be sure to show your calculations.

Exercise I

1. Suppose a population is growing geometrically with $r = -0.2$ and $N_0 = 500$. There is no random variation, and quasi-extinction occurs when N falls below 20 individuals. What is the time to quasi-extinction (T_e)? In other words, how long will it take for the population to fall below 20 individuals?
2. Now imagine that there is no demographic stochasticity, but environmental stochasticity occurs with $X_t \sim \text{Normal}(\mu = 0, \sigma = 10)$, where μ is the mean and σ is the standard deviation of the random variable X. Generate the random values of X that will be used in the 10 simulations in the next step.
3. Conduct 10 simulations over a 30 year time period, and force the population size to zero after it falls below the threshold. This can be done by multiplying the growth equation by a "test statement", like this: `=(EQUATION)*(CELL>20)`, where `EQUATION` should be replaced with the population growth equation and `CELL` should be replaced with the cell reference for abundance at the previous time step. Plot the projections.
4. What is the average T_e based on these 10 simulations?

Exercise II

In this exercise, you will be exposed to two new probability distributions: the Poisson and the binomial. The Poisson distribution is useful for data that are non-negative integers. It only has a single parameter (often called "lambda" but not to be confused with the finite rate of increase) that describes the expected value of the count data. In stochastic population models, the Poisson distribution can be used to model the number of births (B_t) that occur in a time interval.

The binomial distribution is also useful for data that are non-negative integers, but it has an upper bound. In population models, the upper bound is often population size, and we use the model to describe how many individuals die during some time period.

Assume that a population is geographically closed such that there is no immigration or emigration. The number of individuals born is a Poisson random variable:

$$B_t \sim \text{Poisson}(N_t \times b)$$

and the number that die each year is a binomial random variable:

$$D_t \sim \text{Binomial}(N_t, d)$$

Abundance is just the number that were alive plus the number that were born, minus the number that died:

$$N_{t+1} = N_t + B_t - D_t$$

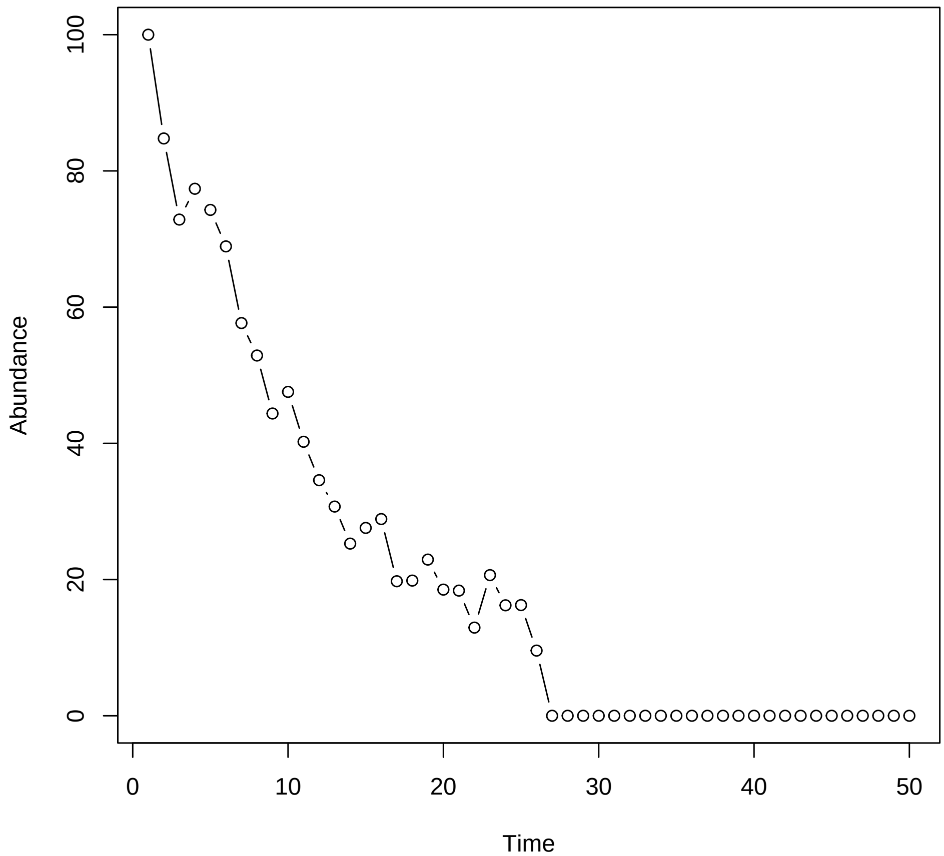
1. Beginning with $N_0 = 300$, conduct one simulation over 5 years, in which $b = 0.3$ and $d = 0.2$. Plot the simulated values of abundance over time. Hint: You have to randomly generate B_t and D_t each year before you can compute N_t.
2. Do you think this population will reach a stochastic equilibrium? Why or why not? (A stochastic equilibrium occurs when a population fluctuates around a long-term average).
3. Can population size ever be less than zero under this model? Why or why not?
4. Do another simulation, but this time make the mortality rate (d) density-dependent according to the model $d_t = 0.2 + 0.001 \times N_t$. Will this population reach a stochastic equilibrium? If so, at what value of abundance does the equilibrium point occur?

Example R code

Geometric growth with environmental stochasticity

```r
r <- -0.1
sigma <- 5
nYears <- 50
extinctionThreshold <- 10

## Repeat the next steps 10 times to do 10 simulations
## Or, do a 'nested for loop' and save each simulation (harder)
N1 <- rep(NA, nYears)  ## Population size
X <- rep(NA, nYears)   ## Random variable for environmental stochasticity
N1[1] <- 100           ## Initial population size
for(t in 2:nYears) {
    X[t-1] <- rnorm(n=1, mean=0, sd=sigma)
    N1[t] <- (N1[t-1] + N1[t-1]*r + X[t-1])*(N1[t-1]>extinctionThreshold)
}
plot(1:nYears, N1, xlab="Time", ylab="Abundance", type="b")
```



Abundance
Time
0
20
40
60
80
100
0
10
20
30
40
50

Poisson-Binomial birth-death model

```
b <- 0.15  ## Birth rate
d <- 0.2   ## Mortality rate
nYears <- 50
N2 <- rep(NA, nYears)  ## Empty vector for population size
B <- rep(NA, nYears)   ## Random variable for nBirths
D <- rep(NA, nYears)   ## Random variable for nDeaths
N2[1] <- 100           ## Initial population size
for(t in 2:nYears) {
    B[t-1] <- rpois(n=1, lambda=N2[t-1]*b)
    D[t-1] <- rbinom(n=1, size=N2[t-1], prob=d)
    N2[t] <- N2[t-1] + B[t-1] - D[t-1]
}
plot(1:nYears, N2, xlab="Time", ylab="Abundance", type="b")
```