

## Action operads comments to fix

- Structure might need tweaking. Some comments in relevant sections below.
- The ‘Free  $\Lambda$ -monoidal categories’ section is tiny now, since we decided to appeal to Yau’s book for characterising  $\Lambda$ -monoidal categories. Some possibly useful remarks and lemmas in here, but might need stripping down and chucking elsewhere.

### NOTATIONAL STUFF

- Use  $e_n$  for identities in action operads
- Use  $\text{id}$  for the identity *of* a  $\Lambda$ -operad (command is ‘backslash id’)
- Underline monads arising from operads. So if  $P$  is the operad, then  $\underline{P}$  is the associated monad. Check later on for consistency.
- Try using  $B\Lambda$  for  $\not\leq$ .
- Remove bold-face  $\Lambda$ ’s (think this might be the command ‘backslash ML’) and just add a remark about the context being obvious when an action operad is being considered as just an operad or ‘with’ its actions
- Check the  $\beta/\delta$  stuff.
  - Check notation
  - Maybe introduce these earlier
  - Use  $p$ ’s and  $q$ ’s instead of Greek letters ( $\tau$ ,  $\sigma$ , etc.) when using the  $\beta/\delta$  notation.
  - Look at Example 2.6
- Dual use of  $A \times_B C$  for pullbacks and coequalizers. Think about this to avoid ambiguity.
- Forward- and back-reference notation and conventions to and from where they are used.

### 1. INTRODUCTION

- add references to works that cite the original preprints: Google Scholar link 1, Google Scholar link 2
- add reference to Ed’s thesis
- should we reference the original preprints as well?
- I’ve added the original intros from the preprints with some other minor comments. Should hopefully be able to tidy this up into a coherent introduction now.

### 2. ACTION OPERADS

- Think we could split this into ‘Action operads’ and ‘Operads with general group actions’? Probably after ‘Presentations of action operads’.
- Proof of 2.3.4: just needs checking - there’s probably a simpler proof which doesn’t use induction. (I often avoid induction since it can hide what’s going on, but here it does actually seem to help see what’s going on.) Ed’s thesis has a short proof but I think it doesn’t quite cover everything.
- I have been changing tensor product to block sum for a lot of things, we need to go through and decide how to do that consistently (Alex: Need to

a general read through for consistency. I've tried to match a few things up along the way, but it's all been a bit ad-hoc so far.)

### 3. OPERADS IN THE CATEGORY OF CATEGORIES

- 'Operads in the category of categories' could be split into a section with that title, followed by one about the 'Borel construction'.
- Remark below was from ages ago. I think we might make it explicitly clear that Conventions 3.2 is in play.
- Definition of pseudomorphism and the remark following it discuss other's alternatives which include an equivariance axiom. I need to think this through to see whether our definitions actually differ or whether they're the same because of the equivariance from the coequalizer.
  - Thinking about it, we do need the equivariance axiom in our definition. Since we're using the non-equivariant maps as the basis of the definition, in order to be able to induce the equivariant maps from them we need the  $\alpha_n$  to coequalize the actions.

### 4. MONOIDAL STRUCTURES AND MULTICATEGORIES

- Intro needs filling out.
- Theorem 4.4.5 about pseudo-commutativity - shift these requirements into a definition of a pseudo-commutative operad, then restate the theorem in simpler terms (like the Guillou paper)
  - This is mostly done - but there's still the thing with the equivariance axiom. I still can't tell where it gets used, or if it's just another case of something like Conventions 3.2 meaning that it's all fine and with those conventions we don't need these extra equivariance axioms that have been hanging around. In which case, we can just mention the conventions and be done, possibly describing what the axiom would be if we *were* to include it.
  - It's currently axiom 2 in the definition, so the proof would need rewording if we take this out.
- Defn of  $\Lambda$ -multicategory: check all the specifics. Lots of notation was inconsistent round here after merging with the other stuff.