Action operads comments to fix

1. Introduction

- add references to work that citing the original papers: Google Scholar link 1, Google Scholar link 2

2. Action operads

- The comments below can be got rid of if '0-ary multiplication' $\mu\colon \Lambda(0) \to \Lambda(0)$ satisfies $\mu(g;) = g$. This is what is done in Ed's thesis (Lemma 1.13) but seems odd and I can't find any other places this is done.
- I put in G0abel (2.3.8) to prove: need to show that $g \oplus h = \mu(e_2; g, h) = gh$ (suffices to show that $g \oplus e_0 = \mu(e_2; g, e_0) = g$). Thought it would be a straightforward Eckmann-Hilton argument but it gets stuck. I'm quite sure this doesn't work, as in block sum does *not* agree with the group operation. It does in many of the examples we use, since the $\Lambda(0)$ is often trivial. But it doesn't seem to work in general. When it *does*, then it is the case that $\Lambda(0)$ is abelian (and it's always true that block sum on its own for $\Lambda(0)$ is commutative because of a later result about $E\Lambda$-algebras being spacial.)
- The previous comment has a big knock-on effect for results at the start of Section 7.
- I have been changing tensor product to block sum for a lot of things, we need to go through and decide how to do that consistently
- Ex 2.2.6: Action operad formed by an abelian group A: $A^\bullet$. How does this multiplication work with $A^\bullet(0)$ which is the trivial group? E.g., do we treat the single element of $A^\bullet(0)$ as the empty list? So it has no effect in the operad multiplication:

$$\begin{aligned}\mu(e_2; e_0, (a_1, \dots, a_m)) &= \mu((e,e); (), (a_1, \dots, a_m)) \\ &= (e + a_1, \dots, e + a_m) \\ &= (a_1, \dots, a_m).\end{aligned}$$

3. Operads in the category of categories

- Prop 3.3.11-4 The proofs need filling out: Seems to correspond to stuff in Yau's book around Theorem 18.3.1 and rest of Section 18/19
- Should we change $E\Lambda(n) \times X^n/\Lambda(n)$ to be $(E\Lambda(n) \times X^n)/\Lambda(n)$?

4. Monoidal structures and multicategories

- Intro
- Use

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  \lmc
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  for lambda monoidal categories
- Lemma 4.3.2: Needs rewording. Is the *underlying set of the free monoid*?
- Re: What is an action morphism? Added a remark (4.3.5 or near) to give some reference. (Not happy with this remark. Really needs clarifying. Think it relates to the things called $g^{\otimes}$ in Section 3?)

- Related: Notation 3.4.2 - ''we write $g^{\otimes}$ for the image of the map $(!; id, \ldots, id)$ in $E\Lambda(n)_{\Lambda(n)}X^n$'. Is the map in there or is the image of the map in there?
- Also related: Definition 3.4.3
- Do we want another notation to emphasise the underlying monoid? (Think we settled on $\Lambda^{\oplus}$?)
- Lemma 4.3.8: Should be a $\Lambda^{\oplus}$, not just Λ?
- Defn of $\mathbf{\Lambda}$-multicategory: check all the specifics. Lots of notation was inconsistent round here.

5. Invertible objects

- The notation in the very first sentence needs to be explained somewhere!
- Rewrite intro: Need to explain that the goal is to understand some group actions
- Decide on ELambda algebras or Lambda monoidal categories throughout (we decided the second!)
- New notation: added earlier (line 905, search beta_to_oplus), just need to implement, search for action maps or superscript tensors
- Fix weakly invertible section
- Lemma 5.3.10ish: Needs sorting

Leftover fixes that I'm not sure about:

- Move comment (QQQ)
- Fix paragraph; make clear we are determining composition
- Explain M strategy, include forward refs

6. Invertibility and group actions

- I want to write $\Lambda^{\oplus}$ for the underlying monoid maybe??
- **why? This one involves real math**
- not happy with last section

7. Computing automorphisms of the unit

- Lemma 7.1.1: Mentions the morphism $g^{\otimes}$ in the statement but the morphisms in the proof are all just labelled g.
- 4.1.3/7.1.2 check 2.3.10: need to make sure this is in an earlier section, and ref'ed (group action actually seems to rely on the lemma previous to the one referenced: 'G0abel' (AC is stuck on this), not 'calclem' (which gives the group hom.))
- Do we actually need $\Lambda(0) \times \Lambda(m) \to \Lambda(m)$ to be a group action? If not, then we don't need to know that $g \oplus h = g \cdot h$ for elements $g, h \in \Lambda(0)$. I.e., Lemma 2.3.9.
- 7.1.2: Switches between $\oplus$ and $\otimes$ a few times. Is this fine?
- Is the induced homomorphism $\Lambda(0) \to \Lambda(m)$ given by $- \oplus e_m$?
- explain purpose
- improve proof 4.2.3
- check commutative Square
- redo 4.4
- insert diagram
- consistent text after 4.5.3

- move something to earlier
- highlight that star means the inverse under tensor product for morphisms
- check the note

8. A full description of L_n

- Think about n vs 2n in AGndef
- check reference
- rewrite calculation
- check universal property
- insert for a simple example

9. Examples

- Actually read this section, fix anything

Comments addressed

10. Invertible objects

- Include notation for η as the unit here
- Change to equalizers
- Change to $(LX)_{inv} = LX$
- Fix ()s
- Include triangle NO
- Uniform gp superscripts
- Remove actually
- Ref η
- Replace with is, remove parts
- Remove proof
- Fix ab superscripts, same as gp
- qi
- Under red line: move? make remark? delete some?
- Where do we say this?
- Need 2-adjunction: this should follow from Thm 8.6 in the enriched_sketches paper I saved
- include forward ref to where we use crefepi: I can't find it
- Get better Eckmann-Hilton ref: don't care anymore

11. Invertibility and group actions

- Forward ref
- definition env
- little wording fixes
- change G to Lambda
- S vs Sigma for symmetric groups: I picked Sigma
- Think about free monoid lem again
- Fix triangle
- lots of notation issues (e, G, length bars)
- why splitting
- missing ref?
- splits by construction: hmm
- ref?
- for v, v' not delta of something
- inverses for morphisms under comp vs tensor
- more G's (x2)
- another missing ref
- another G
- include corollary?
- forward refs
- practical?

12. Computing automorphisms of the unit

- in the next two results

- 4.1.2 two boxes
- the above following square
- insert =
- check 4n or 2n (it is correct in 7.2.1)
- mentioned Delta, I
- fixed proof 4.3.2
- remove functor
- isomorphism symbol
- clarify this
- make sure length and size notation is introduced earlier
- bad line break at the beginning of 4.5
- change prove to shows
- bad line break
- insert the proof from Ed's email
- put a short proof
- change express to describe
- isomorphism symbol
- change make sure to ensures
- remove calculation
- change we want to do

13. A FULL DESCRIPTION OF L_n

- bad line break
- remove exposition
- fix fancy G
- change G to lambda
- isomorphism symbol
- tensor product given component wise
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14. EXAMPLES

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