$$A \times \coprod P(n) \times_{G(n)} B^n \cong \coprod A \times \left(P(n) \times_{G(n)} B^n\right)$$
$$\longrightarrow \coprod A^n \times \left(P(n) \times_{G(n)} B^n\right)$$
$$\longrightarrow \coprod P(n) \times_{G(n)} (A \times B)^n \cong \underline{P}(A \times B)$$

$$t : A \times P(n) \times_{G(n)} B^n \longrightarrow A^n \times P(n) \times_{G(n)} B^n \longrightarrow P(n) \times_{G(n)} (A \times B)^n$$

$$t^* : P(n) \times_{G(n)} A^n \times B \longrightarrow P(n) \times_{G(n)} A^n \times B^n \longrightarrow P(n) \times_{G(n)} (A \times B)^n$$

$$(a_1\, b_1)\,(a_1\, b_2) \cdots (a_1\, b_n)\,(a_2\, b_1) \cdots (a_2\, b_n) \cdots (a_m\, b_n)$$

$$(a_1\, b_1)\,(a_2\, b_1) \cdots (a_m\, b_1)\,(a_1\, b_2) \cdots (a_m\, b_2) \cdots (a_m\, b_n)$$

$p \in P(m)$, $q \in P(n)$

$\mu(p, q, \ldots, q) \in P(nm)$ $\quad$ Iso in $P(mn) \times_{G(mn)} (A \times B)^{mn}$

$\mu(q, p, \ldots, p) \in P(mn)$

---

the operad for braided strict mon. cats is

- $G(n) = Br_n$, $\pi : G(n) \to \Sigma_n$
- $P(n) = E\, Br_n$

$$A \times B \xrightarrow{\cong} B \times A \rightrightarrows \begin{matrix} B \\ A \end{matrix}$$

Why is the free strict monoidal cat 2-monad not ps-commutative?
t, t^* both given by diagonals
Gives a strength for any operad
$\gamma_{A,B}$ is just the permutation $(A \times B)^{mn} \xrightarrow{\cong} (A \times B)^{mn}$

We think it <u>is</u> ps-commutative.

<u>Thm #1</u>: $P(n) = EG(n)$, $\underline{P}$ is ps-comm

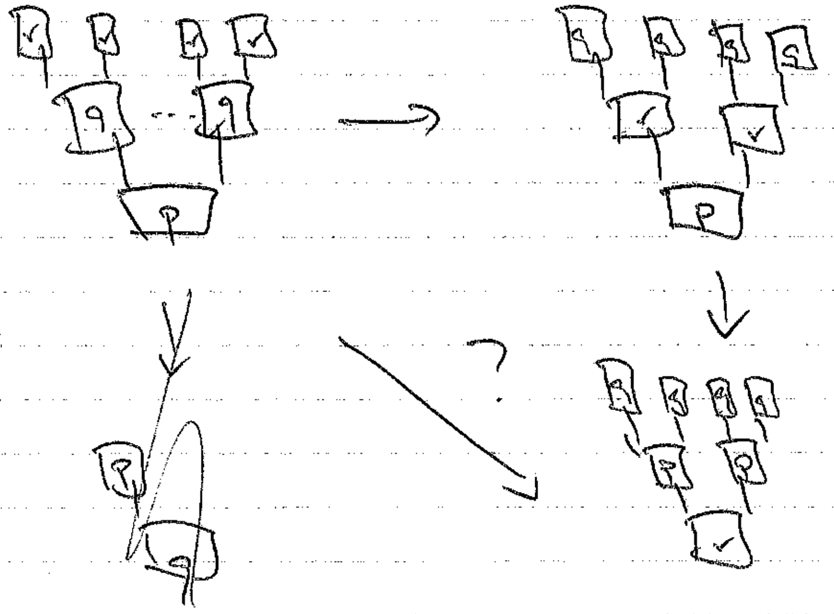
$[p, f] = [q, g]$
<u>Pf</u>: Read Hyland-Power better.

<u>Cor</u>: Free strict monoidal cat 2-monad & Free braided " " 2-monad are ps-comm.

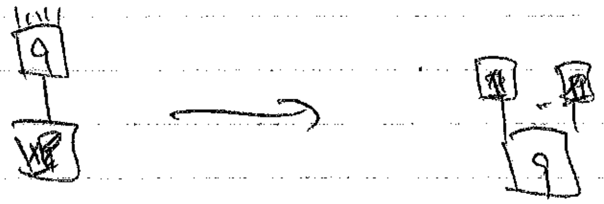
Aside: $A_0 = 1$ $\quad G(n) = A_n$
$A_1 = 1$ $\quad P(n) = A_n$
$A_2 = 1$ $\quad$ P-alg $X \mapsto X^n/A_n \to X$
$A_3 = \mathbb{Z}/3$ $\quad abc = bca = cab$
A_4 = semidirect of $\mathbb{Z}/3$ and $\mathbb{Z}/2 \times \mathbb{Z}/2$?
$abcd = bcad$
$= bade$

<u>Thm #2</u>: $P(n)$ contractible $\Rightarrow$ $\underline{P}$ ps-comm.
This implies Thm #1 (obviously).

3.3



(1) - (3)



= id

(4) - (5)

Strength:

$$t_{AB} : A \times \underline{P} B \longrightarrow \underline{P}(A \times B)$$

$$A \times \coprod_{n \geq 0} P(n) \times_{G(n)} B^n \longrightarrow \coprod_{n \geq 0} P(n) \times_{G(n)} (A \times B)^n$$

$$(a, [\rho, \underline{b_i}]) \qquad [\rho, (\underline{a_i, b_i})]$$

$$(a, [\rho, \underline{b_i}]) \longmapsto [\rho, (a, \underline{b_i})]$$

$$(a, [\rho, b_1, \ldots, b_n]) \longmapsto [\rho, (a, b_1), \ldots, (a, b_n)]$$

$$(a, [\rho \cdot g, \underline{b_i}]) \longmapsto [\rho \cdot g, (a, \underline{b_i})]$$

$$= [\rho, \pi_g (a, \underline{b_i})] = [\rho, (a, \pi_g \underline{b_i})]$$

$$(a, [\rho, \pi_g \underline{b_i}]) \longmapsto [\rho, (a, \pi_g \underline{b_i})]$$

well-defined.

$$\begin{tikzcd} 1 \times \underline{P}A \arrow[r, "t_{1A}"] \arrow[dr, "\lambda_{\underline{P}A}"'] & \underline{P}(1 \times A) \arrow[d, "\underline{P}(\lambda_A)"] \\ & \underline{P}A \end{tikzcd}$$

$$\begin{tikzcd} 1 \times \coprod P(n) \times_{G(n)} A^n \arrow[r] \arrow[dr] & \coprod P(n) \times_{G(n)} (1 \times A)^n \arrow[d] \\ & \coprod P(n) \times_{G(n)} A^n \end{tikzcd}$$

$$\begin{tikzcd} (*, [\rho, \underline{a_i}]) \arrow[r, mapsto] \arrow[dr, mapsto] & {[\rho, (*, \underline{a_i})]} \arrow[d, mapsto] \\ & {[\rho, \underline{a_i}]} \end{tikzcd}$$

$$(a,[\varphi,\underline{b_i}]) \longmapsto [\varphi,(a,\underline{b_i})]$$

$$\begin{tikzcd}
A\times B \arrow[r, "1\times\eta_B"] \arrow[dr, "\eta_{A\times B}"'] & A\times \underline{P}B \arrow[d, "t_{A,B}"] \\
& \underline{P}(A\times B)
\end{tikzcd}
\qquad
\begin{tikzcd}
A\times B \arrow[r, "1\times\eta_B"] \arrow[dr, "\eta_{A\times B}"'] & A\times \coprod P(n)\times_{G(n)} B^n \arrow[d, "t_{AB}"] \\
& \coprod P(n)\times_{G(n)} (A\times B)^n
\end{tikzcd}$$

$$\begin{tikzcd}
(a,b) \arrow[r, mapsto] \arrow[dr, mapsto] & (a,[\mathbb{1},b]) \arrow[d, mapsto] \\
& [\mathbb{1},(a,b)]
\end{tikzcd}$$

$\cancel{(A\times B)\times \underline{P}C \longrightarrow \underline{P}((A\times B)\times C)}$

$$\begin{tikzcd}
(A\times B)\times \underline{P}C \arrow[rr, "t_{A\times B,C}"] \arrow[d, "\alpha"] & & \underline{P}((A\times B)\times C) \arrow[d, equal] \\
A\times(B\times \underline{P}C) \arrow[r, "1\times t_{BC}"'] & A\times \underline{P}(B\times C) \arrow[r, "t_{A,B\times C}"'] & \underline{P}(A\times(B\times C))
\end{tikzcd}$$

$$\begin{tikzcd}
((a,b),[\varphi,\underline{c_i}]) \arrow[rr, mapsto] \arrow[d, mapsto] & & {[\varphi,((a,b),c_i)]} \arrow[d, mapsto] \\
(a,(b,[\varphi,\underline{c_i}])) \arrow[r, mapsto] & (a,[\varphi,(b,\underline{c_i})]) \arrow[r, mapsto] & {[\varphi,(a,(b,\underline{c_i}))]}
\end{tikzcd}$$

$$\begin{tikzcd}
A \times \underline{P}^2 B \arrow[r, "t_{A,PB}"] \arrow[d, "1 \times \mu_B"'] & \underline{P}(A \times \underline{P}B) \arrow[r, "Pt_{A,B}"] & \underline{P}^2(A \times B) \arrow[d, "\mu_{A\times B}"] \\
A \times \underline{P}B \arrow[rr, "t_{AB}"'] & & \underline{P}(A \times B)
\end{tikzcd}$$

$$\begin{tikzcd}
(a, [q, [v_1, \underline{b}_1], \ldots, [v_n, \underline{b}_n]]) \arrow[r, mapsto] \arrow[d, mapsto] & [q, (a, \underline{[v_i, \underline{b}_i]})] \arrow[r, mapsto] & [q, \underline{[v_i, (a, \underline{b}_i)]}] \arrow[d, mapsto] \\
(a, [q(v_1, \ldots, v_n), \underline{b}]) \arrow[rr, mapsto] & & {[q(v_1, \ldots, v_n), \underline{(a, \underline{b})}]} \\
& & {[q(v_1, \ldots, v_n), (a, \underline{b})]} \arrow[u, equal]
\end{tikzcd}$$

Costrength: $\underline{P}A \times B \xrightarrow{t^*_{AB}} \underline{P}(A \times B)$

$(\coprod P(n) \times_{\Sigma(n)} A^n) \times B$

$([p, \underline{a}_i], b) \longmapsto [p, (\underline{a}_i, b)]$

$([p \cdot g, \underline{a}_i], b) \longmapsto [p \cdot g, (\underline{a}_i, b)] = [p, \pi_g(\underline{a}_i, b)] = [p, (\pi_g \underline{a}_i, b)] = t^*_{AB}([p, \pi_g \underline{a}_i], b)$.

Pseudo-commutativity:

$$\begin{tikzcd}
t_{A\times 1}(\coprod P(n) \times_{\Sigma(n)} A^n) \times (\coprod P(n) \times_{\Sigma(n)} B^n) \arrow[r] \arrow[d] & \coprod P(n) \times_{\Sigma(n)} (A \times (\coprod P(n) \times B^n))^n \arrow[d] \\
& \coprod \coprod P(n) \times_{\Sigma(n)} P(k_1) \times_{\Sigma(k_1)} \times (A \times B)^{k_1} \times \ldots \arrow[d] \\
\coprod P(n) \times_{\Sigma(n)} ((\coprod P(n) \times A^n) \times B)^n \arrow[r] & \coprod \coprod P(n) \times_{\Sigma(n)} P(k_1) \times_{\Sigma(k_1)} \cdots \arrow[r] & \coprod P(n) \times_{\Sigma(n)} (A \times B)^n
\end{tikzcd}$$

$$\cancel{[p,a_i],[q,b_i] \longmapsto [p,(a_i,[q,b_i])] \longmapsto [p,[q,(a_i,b_i)]]}$$

$$\cancel{([p,a_i],[q,b_i]) \longmapsto}$$

$$\underline{P}A \times \underline{P}B$$

$$\cancel{[q,([p,a_i],b_i)] \longmapsto [q,[p,(a,b)]]}$$

$$([p,\underline{a}],[q,\underline{b}]) \longmapsto [p,(a_1,[q,\underline{b}]),\ldots,(a_n,[q,\underline{b}])] \longmapsto [p,[q,(a_1,\underline{b})],\ldots,[q,(a_n,\underline{b})]]$$

$p \in P(n)$, $q \in P(m)$,
$\underline{a} = a_1,\ldots,a_n$, $\underline{b} = b_1,\ldots,b_m$.

$$\downarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$$

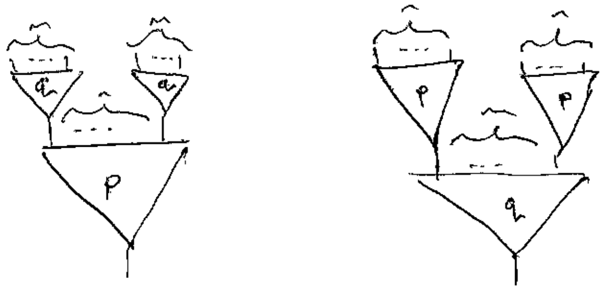
$$[p(q,\ldots,q),(a_i,\underline{b})]$$

$$[q,([p,\underline{a}],b_1),\ldots,([p,\underline{a}],b_m)] \longmapsto [q,[p,(\underline{a},b_1)],\ldots,[p,(\underline{a},b_m)]] \longmapsto [q(p,\ldots,p),(\underline{a},b_i)]$$

Need isomorphisms

$$[q(p,\ldots,p),(\underline{a},b_1),\ldots,(\underline{a},b_m)] \cong [p(q,\ldots,q),(a_1,\underline{b}),\ldots,(a_n,\underline{b})].$$

Do $p(q,\ldots,q)$ and $q(p,\ldots,p)$ make sense? Yes, 'n' q's from $[q,(a_i,\underline{b})]$'s. & vice versa.



$$\underline{P}(A \times B)$$