

$$(l_1, m | n_1) \stackrel{?}{=} (l_2, m | n_2) \text{ s.t. } l_1 n_1 = l_2 n_2$$

\uparrow \uparrow
 h', h'' k', k''

Assume $\mu(l; h'') \mu(h'; 1) = \mu(l; k'') \mu(k'; 1)$.

Consider the preimages of $\{1, \dots, l_n\}$.

$\mu(-; 1)(\{1, \dots, l_n\}) = \{1, \dots, l_n\}$, so the preimage is given by the preimages of

$$F = \mu(h'; 1) \quad \text{and} \quad G = \mu(k'; 1)$$

using $\{1, \dots, l_n\}$.

$$F^{-1}(\{1, \dots, l_n\}) = \{l_1 \circ \pi(h'')^{-1}(1), -1, -2, \dots, -(l_1-1)$$

$$l_1 \circ \pi(h'')^{-1}(n_1), -1, -2, \dots, -(l_1-1)\}$$

$$G^{-1}(\{1, \dots, l_n\}) = \{l_2 \circ \pi(k'')^{-1}(1), -1, \dots, -(l_2-1)$$

$$l_2 \circ \pi(k'')^{-1}(n_2), -1, \dots, -(l_2-1)\}$$

n_1 blocks of l_1 adjacent #s in F^{-1}
 n_2 blocks of l_2 adjacent #s in G^{-1}

If $l_1 = l_2$, done.

If $l_1 \neq l_2$, $\{1, \dots, l_1\} \neq \{1, \dots, l_2\}$

Need $\{1, \dots, d, l_1\} = \{1, \dots, d, l_2\}$

Claim: $F(\{1, \dots, d, l_1\}) = \{1, \dots, d, l_1\}$

Idea: use "overlapping" blocks from $F^{-1}G$ until we get to $d, l_1 = d, l_2$

Claim #2: If $F(\{1, \dots, k\}) = \{1, \dots, k\}$ then $k=1$ or $k=\text{end}$

Still need d, l_1 is not the end: true b/c $d, l_1 \leq l_1 n_1 < l_2 n_2$ if $n_1 > 1$.