## Action operads comments to fix

## 1. Introduction

- add references to works that cite the original preprints: Google Scholar link 1, Google Scholar link 2
- add reference to Ed's thesis
- should we reference the original preprints as well?

#### 2. ACTION OPERADS

- Proof of 2.3.4: just needs checking there's probably a simpler proof which doesn't use induction. (I usually avoid induction, if possible, but here it does actually seems to help see what's going on.) Ed's thesis has a short proof but I think it doesn't quite cover everything.
- I have been changing tensor product to block sum for a lot of things, we need to go through and decide how to do that consistently

### 3. Operads in the category of categories

- Definition of pseudomorphism and the remark following it discuss other's alternatives which include an equivariance axiom. I need to think this through to see whether our definitions actually differ or whether they're the same because of the equivariance from the coequalizer.
  - Thinking about it, we do need the equivariance axiom in our definition. Since we're using the non-equivariant maps as the basis of the definition, in order to be able to induce the equivariant maps from them we need the  $\alpha_n$  to coequalize the actions.
- Prop 3.3.11-4 The proofs need filling out: Seems to correspond to stuff in Yau's book around Theorem 18.3.1 and Chapter 19. (Possibly worth some remarks still but may be easier to just reference Yau here.)
  - We decided to just reference Yau's book here since it goes into it in more detail there and this doesn't have much of a bearing on the rest of what we do.
- I'd maybe like to have an example around here though, such as how the
  hexagon identities for symmetric/braided monoidal categories pop out of
  these generic algebra axioms there's an example of the symmetry axiom
  in Ed's thesis. This is kind of covered in the Borel construction section
  when talking about clubs, but not quite as explicitly.

### 4. Monoidal structures and multicategories

- Intro
- Lemma 4.3.2: Needs rewording. Is the underlying set of the free monoid?
- Related: Notation 3.4.2 "we write  $g^{\otimes}$  for the image of the map (!; id, ..., id) in  $E\Lambda(n)_{\Lambda(n)}X^n$ ". Is the map in there or is the image of the map in there?
- Also related: Definition 3.4.3
- Do we want another notation to emphasise the underlying monoid? (Think we settled on  $\Lambda^{\oplus}$ ?)

- Theorem 4.4.5 about pseudo-commutativity shift these requirements into a definition of a pseudo-commutative operad, then restate the theorem in simpler terms (like the Guillou paper)
- $\bullet$  Defn of  $\Lambda\text{-multicategory:}$  check all the specifics. Lots of notation was inconsistent round here.

### 5. Invertible objects

- The notation in the very first sentence needs to be explained somewhere!
- Rewrite intro: Need to explain that the goal is to understand some group actions
- New notation: added earlier (line 905, search beta\_to\_oplus), just need to implement, search for action maps or superscript tensors
- Fix weakly invertible section
- Lemma 5.3.10ish: Needs sorting
- Corollary 5.3.13ish: What is it actually saying?

## Leftover fixes that I'm not sure about:

- Move comment (QQQ)
- Fix paragraph; make clear we are determining composition
- Explain M strategy, include forward refs

## 6. Invertibility and group actions

- I want to write  $\Lambda^{\oplus}$  for the underlying monoid maybe??
- why? This one involves real math
- not happy with last section

### 7. Computing automorphisms of the unit

- Lemma 7.1.1: Mentions the morphism  $g^{\otimes}$  in the statement but the morphisms in the proof are all just labelled g.
- 7.1.2: Switches between  $\oplus$  and  $\otimes$  a few times. Is this fine?
- Is the induced homomorphism  $\Lambda(0) \to \Lambda(m)$  given by  $\oplus e_m$ ?
- explain purpose
- improve proof 4.2.3
- check commutative Square
- redo 4.4
- insert diagram
- consistent text after 4.5.3
- move something to earlier
- highlight that star means the inverse under tensor product for morphisms
- check the note

# 8. A FULL DESCRIPTION OF $L_n$

- Think about n vs 2n in AGndef
- check reference
- rewrite calculation
- check universal property
- insert for a simple example

# 9. Examples

 $\bullet$  Actually read this section, fix anything

### Comments addressed

### 10. Invertible objects

- Include notation for  $\eta$  as the unit here
- Change to equalizers
- Change to  $(LX)_{inv} = LX$
- Fix ()s
- Include triangle NO
- Uniform gp superscripts
- Remove actually
- Ref  $\eta$
- Replace with is, remove parts
- Remove proof
- Fix ab superscripts, same as gp
- q
- Under red line: move? make remark? delete some?
- Where do we say this?
- Need 2-adjunction: this should follow from Thm 8.6 in the enriched\_sketches paper I saved
- include forward ref to where we use crefepi: I can't find it
- Get better Eckmann-Hilton ref: don't care anymore

### 11. Invertibility and group actions

- Forward ref
- definition env
- little wording fixes
- change G to Lambda
- S vs Sigma for symmetric groups: I picked Sigma
- Think about free monoid lem again
- Fix triangle
- lots of notation issues (e, G, length bars)
- why splitting
- missing ref?
- splits by construction: hmm
- ref?
- for v, v' not delta of something
- inverses for morphisms under comp vs tensor
- more G's (x2)
- another missing ref
- another G
- include corollary?
- forward refs
- practical?

## 12. Computing automorphisms of the unit

• in the next two results

- 4.1.2 two boxes
- the above following square
- $\bullet$  insert =
- check 4n or 2n (it is correct in 7.2.1)
- mentioned Delta, I
- fixed proof 4.3.2
- remove functor
- isomorphism symbol
- clarify this
- $\bullet\,$  make sure length and size notation is introduced earlier
- bad line break at the beginning of 4.5
- change prove to shows
- bad line break
- ullet insert the proof from Ed's email
- put a short proof
- $\bullet\,$  change express to describe
- isomorphism symbol
- change make sure to ensures
- remove calculation
- change we want to do

# 13. A FULL DESCRIPTION OF $L_n$

- bad line break
- remove exposition
- fix fancy G
- change G to lambda
- isomorphism symbol
- tensor product given component wise

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### 14. Examples

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