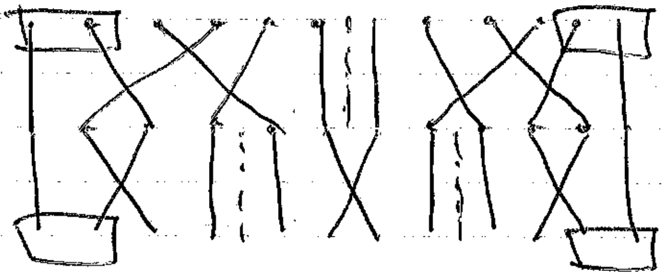
8.1

Think about $\mu(1;h'')\mu(h;1) = \mu(1;k'')\mu(k;1)$
by using $\mu(1;k'')^{-1}\mu(1;h'')\mu(h;1) = \mu(k;1)$

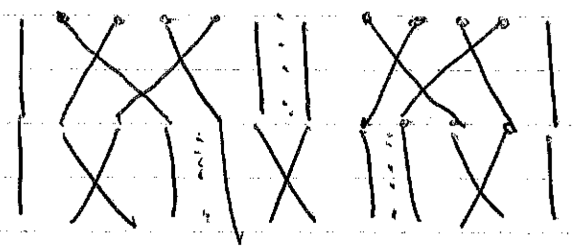
Possible strategy: $\mu(k;1)$ moves elements in l_2-sized blocks.
Try to show that $\mu(1;k'')^{-1}\mu(1;h'')\mu(h;1)$ cannot do that.
① Claim: $\pi(k'')$ (or $\pi(h'')$) : $\nexists\, \alpha$ s.t. $\pi(k'')(\alpha+1) = \pi(k'')(\alpha)+1$
We agree this is true. Moving on.



(3,2)

$(2,2)^{-1} = (2,2)$

Different blocks ⇒ not adj



$(i,j) = \cancel{((i-1), j+1)}$ $\quad i=1 \quad (1,j) = j$

$(2,j) = ?$

$\underline{(i,j)} = (i-1)m + j$

$\underline{(i,j)} = (j-1)n + i$

$\pi((i-1)m + j) = (j-1)n + i$

$\pi((k-1)mn + (i-1)m + j) = (k-1)mn + (j-1)n + i$

$(i,j),\ (i,j+1) \longrightarrow (j-1)n+i$ vs $jn+i$

$(i,m),\ (i+1,1) \longrightarrow (m-1)n+i$ vs $i+1$

$(i,j,k) = (k-1)mn + (i-1)m + j$

$\pi(i,j,k) = (k-1)mn + (j-1)n + i$

Write $(k-1)mn + (j-1)n + i = (k'-1)m'n' + (j'-1)n' + i'$

$\pi'(i',j',k') = (k'-1)(m'n') + (i'-1)m' + j'$

① (i,j,k) vs $(i,j+1,k)$

② (i,m,k) vs $(i+1,1,k)$

③ (n,m,k) vs $(1,1,k+1)$

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① $\pi'(\pi(i,j,k)) = (k'-1)m'n' + (i'-1)m' + j'$

$\pi'(\pi(i,j+1,k)) = \pi'((k-1)mn + jn + i)$

$= \pi'((k-1)mn + (j-1)n + n + i)$

$= \pi'((k'-1)m'n' + (j'-1)n' + i' + n)$

Write $n = \alpha m'n' + \beta n' + \gamma$

$= \pi'((\alpha + k'-1)m'n' + (\beta + j' - 1)n' + i' + \gamma)$

One case: $(\alpha + k' - 1)m'n' + (i' + \gamma - 1)m' + \beta + j'$

$\beta = 1,\ \alpha = \gamma = 0 \Rightarrow n = n'$

$\alpha > 0$ or $\gamma > 0 \Rightarrow$ not possible

② $\pi'(\pi(i,m,k)) = \pi'((k-1)mn + (m-1)n + i)$

$= (k'-1)m'n' + (i'-1)m' + j'$

If $n = n'$: $(k'-1)m'n' + (i-1)m' + m$

$\pi'(\pi(i+1,1,k)) = \pi'((k-1)mn + i + 1)$

$\pi'((k'-1)m'n' + i + 1)$

$= (k'-1)m'n' + \begin{cases} im' \\ +2 \end{cases}$

③ $\pi'(\pi(n,m,k)) = \pi'((k-1)mn + (m-1)n + n)$

$= \pi'(kmn)$

$\pi'(\pi(1,1,k+1)) = \pi'(kmn + 1)$

$\Rightarrow$ ~~$m = m'$ or $(k-1)m + m = 2$~~

$\Rightarrow m = 2,\ i = 1$