Action operads comments to fix

1. Introduction

- add references to works that cite the original preprints: Google Scholar link 1, Google Scholar link 2
- add reference to Ed's thesis
- should we reference the original preprints as well?

2. Action operads

- The comments below can be got rid of if '0-ary multiplication' $\mu\colon \Lambda(0) \to \Lambda(0)$ satisfies $\mu(g;) = g$. This is what is done in Ed's thesis (Lemma 1.13) but seems odd and I can't find any other places this is done. (It doesn't help that nearly everywhere uses reduced operads where $P(0) = *$.)
- I put in G0abel (2.3.9) to prove: need to show that $g \oplus h = \mu(e_2; g, h) = gh$ (suffices to show that $g \oplus e_0 = \mu(e_2; g, e_0) = g$). Thought it would be a straightforward Eckmann-Hilton argument but it gets stuck. I'm quite sure this doesn't work, as in block sum does *not* agree with the group operation. It does in many of the examples we use, since the $\Lambda(0)$ is often trivial. But it doesn't seem to work in general. When it *does*, then it is the case that $\Lambda(0)$ is abelian (and it's always true that block sum on its own for $\Lambda(0)$ is commutative because of a later result about $E\Lambda$-algebras being spacial - noticed that this also uses the claim that $\alpha(e_0; -) = id_I$)
- The previous comment has a big knock-on effect for results at the start of Section 7. Especially Prop 7.5.4 and what follows really just requires that $e_0 \otimes g = g$. Many of the results here go on to assume that $\Lambda(0)$ is trivial, which could mean that we can just make this an assumption from the off and forget about all of this.
- If making the $\Lambda(0) = *$ assumption, then should change Convention 7.6.1 to say this.
- Proof of 2.3.4: just needs checking - there's probably a simpler proof which doesn't use induction. (I usually avoid induction, if possible, but here it does actually seems to help see what's going on.) Ed's thesis has a short proof but I think it doesn't quite cover everything.
- I have been changing tensor product to block sum for a lot of things, we need to go through and decide how to do that consistently
- Ex 2.2.6: Action operad formed by an abelian group A: $A^\bullet$. How does this multiplication work with $A^\bullet(0)$ which is the trivial group? E.g., do we treat the single element of $A^\bullet(0)$ as the empty list? So it has no effect in the operad multiplication:

$$\begin{aligned}
\mu(e_2; e_0, (a_1, \ldots, a_m)) &= \mu((e, e); (), (a_1, \ldots, a_m)) \\
&= (e + a_1, \ldots, e + a_m) \\
&= (a_1, \ldots, a_m).
\end{aligned}$$

3. Operads in the category of categories

- Prop 3.3.11-4 The proofs need filling out: Seems to correspond to stuff in Yau's book around Theorem 18.3.1 and Chapter 19. (Possibly worth some remarks still but may be easier to just reference Yau here.)
- I'd like to have an example around here though, such as how the hexagon identities for symmetric/braided monoidal categories pop out of these generic algebra axioms - there's an example of the symmetry axiom in Ed's thesis. This is kind of covered in the Borel construction section when talking about clubs, but not quite as explicitly.

4. Monoidal structures and multicategories

- Intro
- Lemma 4.3.2: Needs rewording. Is the *underlying set of the free monoid*?
- Re: What is an action morphism? Added a remark (4.3.5 or near) to give some reference. (Not happy with this remark. Really needs clarifying. Think it relates to the things called $g^{\otimes}$ in Section 3?)
- Related: Notation 3.4.2 - ''we write $g^{\otimes}$ for the image of the map $(!; id, \dots, id)$ in $E\Lambda(n)_{\Lambda(n)}X^n$'. Is the map in there or is the image of the map in there?
- Also related: Definition 3.4.3
- Do we want another notation to emphasise the underlying monoid? (Think we settled on $\Lambda^{\oplus}$?)
- Defn of $\mathbf{\Lambda}$-multicategory: check all the specifics. Lots of notation was inconsistent round here.

5. Invertible objects

- The notation in the very first sentence needs to be explained somewhere!
- Rewrite intro: Need to explain that the goal is to understand some group actions
- New notation: added earlier (line 905, search beta_to_oplus), just need to implement, search for action maps or superscript tensors
- Fix weakly invertible section
- Lemma 5.3.10ish: Needs sorting
- Corollary 5.3.13ish: What is it actually saying?

Leftover fixes that I'm not sure about:

- Move comment (QQQ)
- Fix paragraph; make clear we are determining composition
- Explain M strategy, include forward refs

6. Invertibility and group actions

- I want to write $\Lambda^{\oplus}$ for the underlying monoid maybe??
- **why? This one involves real math**
- not happy with last section

7. Computing automorphisms of the unit

- Lemma 7.1.1: Mentions the morphism $g^{\otimes}$ in the statement but the morphisms in the proof are all just labelled g.

- 4.1.3/7.1.2 check 2.3.10: need to make sure this is in an earlier section, and ref'ed (group action actually seems to rely on the lemma previous to the one referenced: 'G0abel' (AC is stuck on this), not 'calclem' (which gives the group hom.))
- Do we actually need $\Lambda(0) \times \Lambda(m) \to \Lambda(m)$ to be a group action? If not, then we don't need to know that $g \oplus h = g \cdot h$ for elements $g, h \in \Lambda(0)$. I.e., Lemma 2.3.9.
- No examples seen have non-trivial $\Lambda(0)$ - we could keep the results around here and remark that 'if' it can be shown (Lemma 2.3.9) that $g \oplus h = g \cdot h$, then the proofs will still go through.
- 7.1.2: Switches between $\oplus$ and $\otimes$ a few times. Is this fine?
- Is the induced homomorphism $\Lambda(0) \to \Lambda(m)$ given by $- \oplus e_m$?
- explain purpose
- improve proof 4.2.3
- check commutative Square
- redo 4.4
- insert diagram
- consistent text after 4.5.3
- move something to earlier
- highlight that star means the inverse under tensor product for morphisms
- check the note

8. A FULL DESCRIPTION OF L_n

- Think about n vs 2n in AGndef
- check reference
- rewrite calculation
- check universal property
- insert for a simple example

9. EXAMPLES

- Actually read this section, fix anything

Comments addressed

10. Invertible objects

- Include notation for η as the unit here
- Change to equalizers
- Change to $(LX)_{inv} = LX$
- Fix ()s
- Include triangle NO
- Uniform gp superscripts
- Remove actually
- Ref η
- Replace with is, remove parts
- Remove proof
- Fix ab superscripts, same as gp
- qi
- Under red line: move? make remark? delete some?
- Where do we say this?
- Need 2-adjunction: this should follow from Thm 8.6 in the enriched_sketches paper I saved
- include forward ref to where we use crefepi: I can't find it
- Get better Eckmann-Hilton ref: don't care anymore

11. Invertibility and group actions

- Forward ref
- definition env
- little wording fixes
- change G to Lambda
- S vs Sigma for symmetric groups: I picked Sigma
- Think about free monoid lem again
- Fix triangle
- lots of notation issues (e, G, length bars)
- why splitting
- missing ref?
- splits by construction: hmm
- ref?
- for v, v' not delta of something
- inverses for morphisms under comp vs tensor
- more G's (x2)
- another missing ref
- another G
- include corollary?
- forward refs
- practical?

12. Computing automorphisms of the unit

- in the next two results

- 4.1.2 two boxes
- the above following square
- insert =
- check 4n or 2n (it is correct in 7.2.1)
- mentioned Delta, I
- fixed proof 4.3.2
- remove functor
- isomorphism symbol
- clarify this
- make sure length and size notation is introduced earlier
- bad line break at the beginning of 4.5
- change prove to shows
- bad line break
- insert the proof from Ed's email
- put a short proof
- change express to describe
- isomorphism symbol
- change make sure to ensures
- remove calculation
- change we want to do

13. A full description of L_n

- bad line break
- remove exposition
- fix fancy G
- change G to lambda
- isomorphism symbol
- tensor product given component wise
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14. Examples

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