

This is a section-by-section list of all the major results, with their dependencies. Throughout: track down specific refs, ie Theorem 2.7 in [3], not just [3]

1. INTRODUCTION

Nothing to say here, I think.

2. ACTION OPERADS

Definition. symmetric operad

Definition. non-symmetric operad

Definition. braided operad

Definition. operad map

Definition. action operad

Definition. map of action operads

Definition/Example. ribbon braids, and their (action) operad

Result (1). π is a map of operads. **Dependency:** defs

Result (2). Operads internal to groups are action operads. **Dependency:** (1)

Result (3). The kernel of an action operad is an action operad. **Dependency:** (1,2)

Result (4). The image, in Σ of an action operad is an action operad. **Dependency:** (1)

Result (5). A kernel/image short exact sequence. **Dependency:** (3,4)

Result (6). Some calculations with e'_i s. **Dependency:** defs

Result (7). Some calculations with $\Lambda(0)$. **Dependency:** (6)

Result (8). The big β, δ theorem. **Dependency:** (1)

Result (9). π is zero or surjective. **Dependency:** (8)

Examples. Cyclic, reflexive, hyperoctahedral, alternating. **Dependency:** (8)

Definition. lfp stuff

Result (10). The category of action operads is lfp. **Dependency:** defs, external

Result (11). $U : \mathbf{AOp} \rightarrow \mathbf{Sets}/\mathcal{S}$ preserves limits and filtered colimits. **Dependency:** defs **Note:** seriously check proof

Result (12). $F : \mathbf{Sets}/\mathcal{S} \rightarrow \mathbf{AOp}$ left adjoint to U . **Dependency:** external

Definition. presentations for action operads **Dependency:** (12)

3. OPERADS WITH EQUIVARIANCE

Definition. Λ -operad

Definition. map of Λ -operads

Definition. category of Λ -operads

Result (13). Λ is a Λ -operad. **Dependency:** defs

Definition. algebra over a non-symmetric operad **Note:** delete?

Definition. algebra over a Λ -operad

Definition. category of algebras over a Λ -operad

Result (14). Endomorphism operad is a Λ -operad. **Dependency:** defs **Note:** should have independent endomorphism operad def beforehand, maybe rework all this stuff

Result (15). Change-of-operad functor. **Dependency:** defs

Result (16). Algebras are operad maps into endomorphisms operad. **Dependency:** (14)

Definition. monad associated to a Λ -operad

Result (17). Monad algebra category is operad algebra category. **Dependency:** defs

Result (18). Λ -algebras, as a Λ -operad, are monoids. **Dependency:** defs, maybe (16) **Note:** unclear hypotheses, should say in sets I think

Result (19). Three-part theorem about the adjunction between Λ - and Σ -operads and their categories of

algebras. **Dependency:** defs **Note:** check proof

Definition. monad map **Note:** some text after that needs to be in an environment

Definition. cocomplete SMC **Note:** no emph in def, is wrong

Result (19). Lax symmetric monoidal functors transport operads, with a comparison monad map. **Dependency:** FUTURE! **Note:** eep in general! where did we define the tensor product over a group notation?

Result (20). Operad maps induce monad maps. **Dependency:** stuff that isn't in an environment above **Note:** continued eep

Result (21). Combining to get an adjunction. **Dependency:** (19, 20) **Note:** continued eep

Definition. collections, maps, the category thereof

Definition. substitution product of collections

Result (22). Substitution product gives monoidal structure, and monoids are operads. **Dependency:** (19, 20)

Result (23). $B\Lambda$ is a strict monoidal category. **Dependency:** FUTURE! also (6)

Result (24). n -fold Day convolution is a functor $B\Lambda \rightarrow \mathbf{Sets}$. **Dependency:** (23)

Result (25). Substitution product as coend using Day convolution. **Dependency:** ?? **Note:** seriously check proof

Proof of (22). **Dependency:** (23,24,25) **Note:** seriously check proof

4. OPERADS IN THE CATEGORY OF CATEGORIES

Note: worth revisiting introductory material, maybe some of it needs environments

Definition. pseudoalgebras

Definition. strict algebras **Dependency:** previous defn

Definition. pseudomorphisms

Definition. strict morphisms **Dependency:** previous defn

Definition. algebra transformations

Definition. P -alg, strict and strong

Definition. 2-monads versions of the above

Result (26). 2-monad and operad algebra 2-categories agree, strict and strong. **Dependency:** definitions here

Result (27). 2-monad from an operad is finitary. **Dependency:** definitions here **Note:** check proof

Result (28). 2-monad from an operad preserves bijective-on-objects functors. **Dependency:** definitions here

Result (29). Pseudoalgebras equivalent to strict ones. **Dependency:** definitions here **Note:** worth explaining how this strictifies unbiased monoidal categories to strict ones, but not biased ones

Definition. 2-cartesian 2-monad **Note:** we seem to need some definitions here

Result (30). Coequalizer of actions is sometimes the quotient. **Dependency:** none **Note:** this looks like it could be improved, many aspects unclear

Result (31). Unit for \underline{P} is cartesian for any symmetric operad P . **Dependency:** definitions here

Result (32). The 2-monad \underline{P} preserves pullbacks iff group action is free. **Dependency:** (30)

Result (33). Multiplication for \underline{P} is cartesian if all group actions are free. **Dependency:** (30) **Note:** has some suspect proof-by-example looking text

Result (34). \underline{P} is 2-cartesian if and only if all group actions are free (symmetric case). **Dependency:** (31, 32, 33; 30)

Result (35). \underline{P} is 2-cartesian if and only if all group actions are free (symmetric case). **Dependency:** (31, 32, 33; 30)

Result (36). Lemma about free Σ -actions on categories with Λ -action. **Dependency:** defs **Note:** needs to be fixed up a bit in the whole groups actions on categories rework

Result (37). \underline{P} is 2-cartesian if and only if all group actions are free (Λ case). **Dependency:** (35, 36; 30, 31, 32, 33, 34)

5. THE BOREL CONSTRUCTION FOR ACTION OPERADS

Definition. EG, BG

Definition. isofibration

Result (38). $p : EU \Rightarrow B$ pointwise isofibration. [Dependency: definitions here](#) [Note: needs ref](#)

Result (39). E right adjoint to set of objects functor, symmetric monoidal wrt cartesian products. [Dependency: definitions here](#)

Definition. the category $B\Lambda$ [Note: this was used back in \(23\)](#)

Result (40). Lax symmetric monoidal functors induce functors between categories of Λ -operads [Dependency: \(22; 23, 24, 25\)](#)

Result (41). $E\Lambda$ is an action operad. [Dependency: \(13, 39, 40; 22, 23, 24, 25\)](#)

Definition. Λ -monoidal categories

Result (42). Formula for morphisms in $E\Lambda(n) \times_{\Lambda(n)} X^n$. [Dependency: \(30?\)](#)

Result (43). $E\Lambda$ is finitary and 2-cartesian. [Dependency: \(30?\)](#) [Note: currently just in free text](#)

[Delete \$\Lambda_\infty\$ stuff??](#)

Definition. cartesian monad, collections for those, and operads for those. [Note: should unify with the 2-cartesian definitions??](#)

Definition. clubs

[I think the free text explicitly breaking down the definition of a club has some errors, double check](#)

Result (44). $B\Lambda$ is a club. [Dependency: defs here, \(30, 42, 43\)](#)

Result (45). Characterization of which clubs are action operads. [Dependency: defs here, \(8\)](#)

Result (46). Presentations of clubs, for action operads. [Dependency: defs here, external?](#)

Result (47). Presentations of strict monoidal structures arising from action operads, via clubs. [Dependency: \(46\)](#)

[I think this is where NG puts the presentation for strict symmetric monoidal cats](#)

6. MONOIDAL STRUCTURES AND MULTICATEGORIES

[Lots of examples here, unify them; should go in a section with presentation stuff](#)

Definition. coboundary category

Definition. coboundary functors

Definition. 2-category of coboundary stuff

Result (48). Strictification of coboundary cats. [Dependency: defs here](#)

Definition. stuff to define operad: disjoint, contains, $s_{p,q}$'s, J_n 's

Result (49). Action operad structure on J 's. [Dependency: \(8\), defs here](#) [Note: issue in proof](#)

[Conflict: \$C\$ vs \$J\$ notation; I understand now - \$C\$ is the monad, \$J\$'s give the club](#)

Result (50). The 2-monad C for strict coboundary cats is a club. [Dependency: \(47\)](#)

Result (51). $C1 \cong BJ$. [Dependency: \(49\)](#)

Result (52). $C \cong EJ$. [Dependency: \(45, 50, 51\)](#)

Definition. strength stuff [Note: change to left/right](#)

Definition. pseudo-commutative 2-monad

[AC should redo all our straight line string diagrams to look pretty like his curvy, colored one](#)

Definition. pseudo-commutative operad

Result (53). Pseudo-commutative operads give pseudo-commutative 2-monads. [Dependency: defs here](#)

Result (54). Non-symmetric operads never give pseudo-commutative 2-monads. [Dependency: defs here](#)
[Note: last sentence of proof should probably be tightened up](#)

Definition. symmetric pseudo-commutative 2-monad

Result (55). From operad to symmetric p-c 2-monad. [Dependency: \(53\)](#)

Definition. contractible operad

Result (56). P contractible and has t 's, then p-c. [Dependency: \(53, 55\)](#)

Result (57). Contractible symmetric operads have symm p-c. **Dependency:** (56; 53, 55)

Remark about symmetrization not preserving contractibility doesn't read well

Result (58). The 2-monad for strict braided monoidal cats has two non-symmetric p-c structures.

Result (59). The 2-monad for strict braided monoidal cats is EB. **Dependency:** external

Definition. positive braids

Definition. minimal braids **Note:** combine defs?

Result (60). Bijection between positive minimal braids and permutations. **Dependency:** external

Proof of (58). **Dependency:** defs here, (53 ,60) **Note:** seriously check proof