Strategy for the conjecture:

$$(*)\quad h = \mu(\overset{m}{1};\overset{nl}{h''})\,\mu(\overset{mn}{h'};\overset{l}{1}) \qquad h \in G(lmn)$$

Induct on (mn, nl).

$(*)$ implies that h has a particular kind of underlying permutation. If $k \in G(ab)$ s.t. $\pi(k)\big((\underline{a}, b)\big) = (a, \underline{b})$ but k does not have underlying perm as in $(*)$, then $\pi : G(ab) \twoheadrightarrow \Sigma_{ab}$ so choose any h in the preimage of the perm $(\underline{a}, b) \mapsto (a, \underline{b})$.

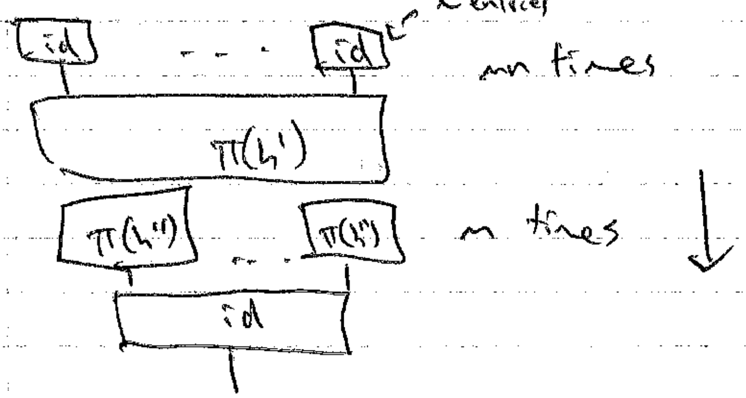
$\Rightarrow$ No canonical ps-comm.

$$\pi(h) = \pi\big(\mu(1;h'')\big)\,\pi\big(\mu(h';1)\big) \qquad \text{SP hom.}$$

$$= \mu\big(1;\pi(h'')\big)\,\mu\big(\pi(h');1\big) \qquad \text{operad map}$$

$$\pi(h'') \in G(nl) \qquad \underline{(a_{i\cdot}, b_k)} \longrightarrow \underline{(a_{i\cdot}, \underline{b_k})}$$

$$\pi(h') \in G(mn) \qquad \underline{(\underline{a_{\cdot j}}, b_k)} \longrightarrow \underline{(a_{\cdot j}, \underline{b_k})}$$



$n=1$: Both sides are id perms ✓
$m=1$: $h = \mu(1; h'')$
If $\mu(1; \pi(h'')) = \mu(1; \pi(k''))$, then
$\pi(h'') = \pi(k'')$ ✓
$\ell = 1$: $h = \mu(h'; 1)$
Same as $m=1$ ✓

$m=n$: $\mu(h'; 1)$ fixes $n\ell$ entries
Eep.

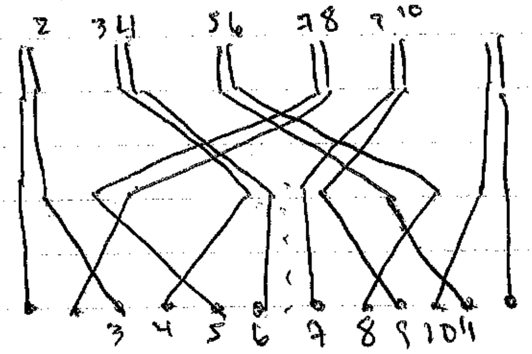
Try to distinguish based on specific entries.

φ/h^2 Write
$\ell+2 \longmapsto \pi(h')(2)\cdot\ell + 2$
Write $\pi(h')(2)\cdot \ell + 2 = \alpha n \ell + \beta$, $0 < \beta \leq n\ell$
$\longmapsto \pi(h'')(\beta) + \alpha n \ell$

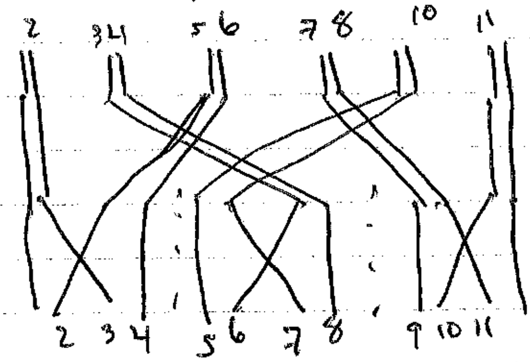
h', h'' $\quad \ell+2 \longmapsto \pi(h'')(\beta) + \alpha n \ell$
k', k'' $\quad \ell+2 \longmapsto \pi(k')(2)\cdot \ell + 2 = \alpha' n' \ell + \beta'$
$\longmapsto \pi(k'')(\beta') + \alpha' n' \ell$
??

$(1, m, n) = (\ell m, 1, n)$

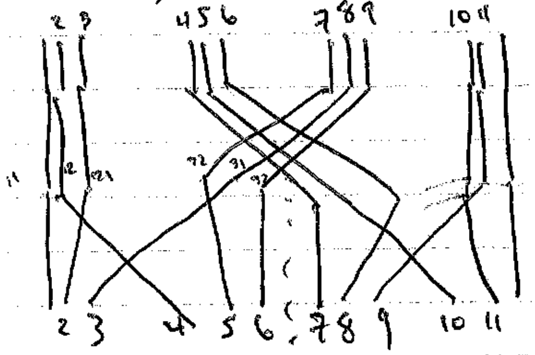
$(2,2,3)$: $(2,2|3)$



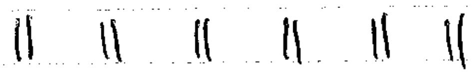
$(2,3,2)$: $(2,3|2)$



$(3,2,2)$: $(3,2|2)$



$(2,6,1)$: $(2,6|1)$



$=$ id perm
so OK

- $(l,n,1)$ = id perm

Claim: if (l,m,n) gives id perm, then $n=1$. Idea: if $\mu(h;1)$ moves something into another "block", then cannot be identity.

- Can reflect through the middle

Nonsense

Let r be the reflection of one group for Σ. If $\Sigma = \sigma$, then

$r\Sigma = \Sigma \Rightarrow r\sigma = \sigma$

$r\sigma_1\sigma_2 = \sigma_1\sigma_2$

$\Rightarrow r\sigma_1 = \sigma_1$

$\Rightarrow m_1 | m_2$ (or $m_2 | m_1$)

Sketch for: if $(l_1, \mu | n_1) = (l_2, \mu | n_2)$ with $h';h''$ and $k';k''$
then $l_1 = l_2$, $n_1 = n_2$

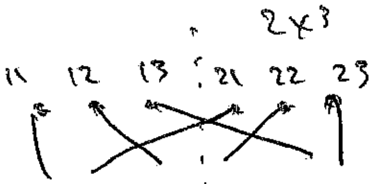
$$\mu(1;h'')\mu(h';1) = \mu(1;k'')\mu(k';1)$$

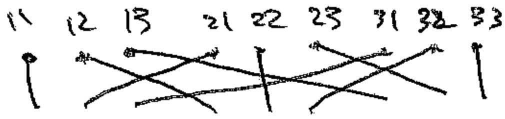
$$\Rightarrow \underbrace{\mu(1;k'')^{-1}\mu(1;h'')}\mu(h';1) = \mu(k';1)$$

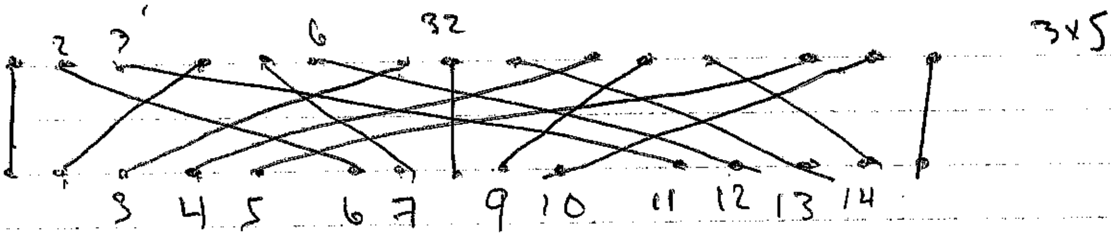
fixes elements 1, ln for each block
Show preimage of ln for LHS is not equal to preimage of ln for RHS

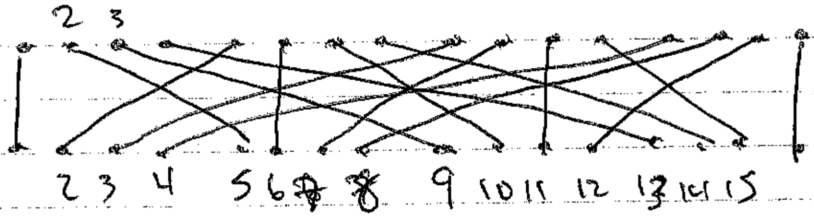
LHS preimage $= \mu(h';1)^{-1}(ln)$
$= \pi(h')^{-1}(n_1)\cdot l_1$
RHS preimage $= \mu(k';1)^{-1}(ln)$
$= \pi(k')^{-1}(n_2)\cdot l_2$

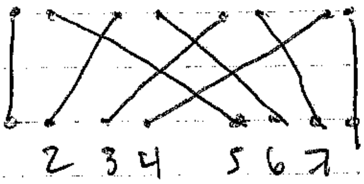
If these are equal, $\pi(h')^{-1}(n_1)l_1 = \pi(k')^{-1}(n_2)l_2$
We know $l_1 n_1 = l_2 n_2$

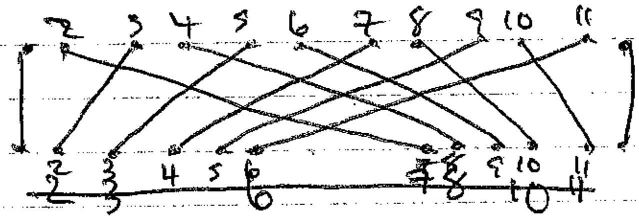


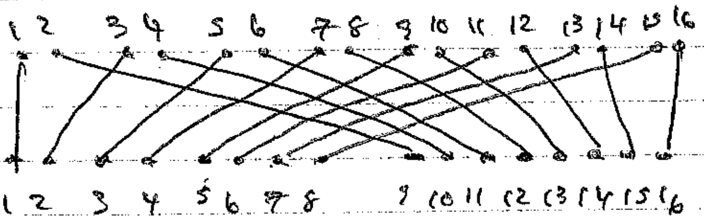












$$\underset{\substack{\uparrow \\ h}}{mn} = \underset{\substack{\uparrow \\ k}}{m'}n'$$

$$\pi(h)(a_{12}) = \pi(k)(a_{12})$$

$2 \mapsto 9 \qquad 2 \mapsto 5$

24/09.1.

~~(1)(2 3 5 2~~ (1)(2 3 5 4)(6) (+1, +2, -1, -2) (2×3)

(3×3) . (2 4)(3 7)(6 8) (+2, -2), (+4, -4), (+2, -2)

(2 3) (+1, -1)

(2 6 12 14 10 4) (+4, +6, +2, -4, -6, -2) (3×5)

(3 11 9 13 5 7) (+8, -2, +4, -8, +2, -4)

(4×6)

(2 5)(3 9)(4 13)(7 10)(8 14)(12 15)

+3 +6 +9 +3 +6 +3

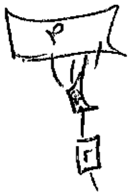
~~(4, 1, 2, 1, 4, 2~~

~~(1, 0, 2, 2, 0, 1)~~

~~(4, 1, 2, 1, 4, 3)~~

(+3), (+6), (+9),

~~(+4) (+7) 12~~



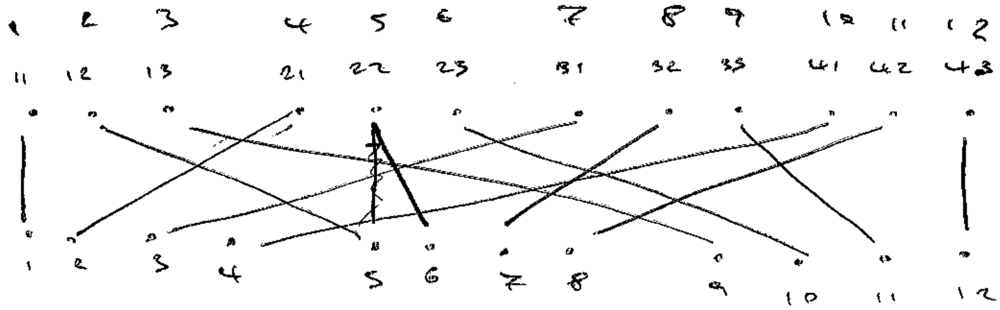
~~(2, 1, 1, 1, 2, 2)~~

~~(3, 3, 4, 2, 2, 1)~~

~~(2, 0, 1, 2, 2, 1)~~

(2×4) ~~(2 5)~~

(2 5 3)(4 6 7)

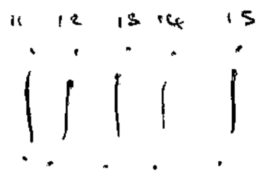


(2×6): ~~(2 7 10 11 6 3)(4 8~~ (2 7 4 8 10 11 6 9 5 3)

(+5, -3, +4, +2, +1, -5, +3, -4, -2, -1)

(3×4): ~~(2 5 6 10 4)(3 9 7)(~~

(2 5 6 10 4)(3 9 11 8 7) (+3, +1, +4, -6, -2)(+6, +2, -3, -1, -4)



(2×8): (2 9 5 3)(4 10 13 7)(6 11)(8 12 14 15)

+7 -4 -2 -1 +6 +3 -6 -3 +5, -5 +4 +2 +1 -7

~~(3 4 6)~~

~~(3 6~~

(2, 2, 3) : (2 3 4 6 8)(5 11 10 9 7)

(2, 3, 2) : (2 3 6 4 8 11 10 7 9 5)

(3, 2, 2) : (2 4 7 5 10 11 9 6 8 3)

(1, 2, 6) : (2 7 4 8 10 11 6 9 5 3)

(2, 1, 6) :