Goal thm:

$\Sigma_n : K_n \to L_n$ is an equiv of Λ-mon cats
= equiv of categories
= equiv in Λ-MonCat$_{ps}$

(1) Σ_n is a strict Λ-mon functor b/c every invertible obj is weakly invertible

K_n = free Λ-mon cat on n weakly invertible
L_n = " " strictly "

(2) Prop: (1) $\Sigma_n = \underbrace{\Sigma_1 \amalg \cdots \amalg \Sigma_1}_{n}$ ✓

(2) Σ_1 equiv $\Rightarrow$ Σ_n equiv

- true in Cat
- think I proved this before
- think it follows from: $\amalg$ in Λ-MonCat$_s$ is a bicategorical coproduct in Λ-MonCat$_{ps}$

(3) Prove Λ-MonCat$_{ps}(L_1, X) \cong \Lambda$-MonCat$_s(K_1, X)$

- Directly by computing L_1
  Claim: $L_1 = \mathbb{Z} \times B\Lambda^{ab}_\infty$
  $\Lambda^{ab}_\infty = \operatorname{colim} \Lambda(n)^{ab}$
- Is this even true? Maybe $\simeq$ instead...

(4) "(3) $\Leftrightarrow$ K_1 is the ps-morphism classifier for L"
Need (3) $\mathbb{Z}$-nat in X

(5) $\mathbb{Z}$-monads coherence for $E\Lambda$ $\Rightarrow$
$K_1 \to L_1$ is an equiv

Other idea:

$\Lambda\text{-MonCat}_s$ with endomorphism L

- L is a 2-monad

(Prop.?) $\Rightarrow$ $L\text{-Alg}_s$ = 2cat of
- Λ-mon cats w/ strictly inv obj's
- all 1-, 2-cells

Q: $\text{Ps-}L\text{-Alg}_{ps}$?

Want: pseudo-L-alg's = Λ-monoid w/ weakly inv obj's

L-ps-alg's:

$LX \xrightarrow{w} X$

$x \in X \rightsquigarrow x^{-1} \in LX \rightsquigarrow w(x^{-1}) \in X$

$$\begin{tikzcd} L^2X \arrow[r, "Lw"] \arrow[d, "\mu"'] \arrow[dr, phantom, "\cong"] & LX \arrow[d, "w"] \\ LX \arrow[r, "w"'] & X \end{tikzcd}$$

$x, w(x^{-1}) \longmapsto x, w(x^{-1}) \longmapsto w(x, w(x^{-1}))$

?