Problema 1

S = {el despertador suena a la hora fijada}

T = {el estudiante llega a tiempo}

$$P(T) = P(S)\cdot P(T|S) + P(\bar{S})\cdot P(T|\bar{S}) = 0'7\cdot 0'8 + 0'3\cdot 0'3 = 0'65$$

$$P(\bar{S}|T) = \frac{P(\bar{S})\cdot P(T|\bar{S})}{P(T)} = \frac{0'3\cdot 0'3}{0'65} = 0'1385$$

Problema 2

$$P[(B_1\cap B_2)\cup(R_1\cap R_2)\cup(N_1\cap N_2)]$$

Incompatibles e independientes:

$$\frac{5}{12}\cdot\frac{3}{16}+\frac{4}{12}\cdot\frac{6}{16}+\frac{3}{12}\cdot\frac{7}{16}=\frac{5}{16}$$

Problema 3

Posibles resultados: C, XC, XXC, XXXC, XXXXC, XXXXX

X = {número de veces que se lanza la moneda hasta que salga cara o cinco cruces} = {1, 2, 3, 4, 5}

P(X=1) = P(C) = 1/2
P(X=2) = P(XC) = 1/4
P(X=3) = P(XXC) = 1/8
P(X=4) = P(XXXC) = 1/16
P(X=5) = P(XXXXC) + P(XXXXX) = 1/32 + 1/32 = 1/16

X	1	2	3	4	5
f(x)	1/2	1/4	1/8	1/16	1/16

Problema 4

Dada la función de densidad:

$$f(x,y)=\begin{cases} kx, & (x,y)\in[0,1]\times[0,x] \\ 0, & resto \end{cases}$$

a) Calcula las marginales.
b) Demuestra que no son independientes.

Solución:

$$\int_0^1\int_0^x kx\,dydx = k\frac{1}{3} = 1 \rightarrow k = 3$$

$$f_1(x) = \int_0^x 3xdy = 3x^2, x\in[0,1]$$

Cambio: y<x<1

$$f_2(y) = \int_y^1 3xdx = \frac{3}{2}(1-y^2), y\in[0,1]$$

No indep:

$$3x \neq 3x^2\,\frac{3(1-y^2)}{2}$$

Problema 5

$$\mathrm{E}(X) = \int_0^3 x\cdot\frac{x^2}{9}dx = \left[\frac{x^4}{36}\right]_0^3 = 2'25$$

$$E(Y) = 2E(X) + 30 = 34{,}5$$

$$E(X^2) = \int_0^3 x^2 \frac{x^2}{9} dx = \left[\frac{x^5}{45}\right]_0^3 = 5.4$$
$$Var(X) = 5.4 - 2.25^2 = 0.3375$$
$$Var(Y = 2X + 30) = 2^2 Var(X) = 1.35$$
$$\sigma = 1.1619$$

Problema 6

$(Y \mid X = 11)$	1	2	3	4
$g_2(y \mid 11)$	3/15	2/15	4/15	6/15

$(X \mid Y = 3)$	10	11	12	13	14
$g_1(x \mid 3)$	5/31	4/31	9/31	10/31	3/31

$$E(Y \mid X = 11) = \frac{43}{15}, \quad E(X \mid Y = 3) = \frac{374}{31}$$

Problema 7

X = {nº de accidentes} X ~ P(2) por 1 semana

a) P (X > 0) = 1 − F(0) = 1 – 0.1353 = 0.8647

b) Y ~ P(4) en 2 semanas

P (Y > 2) = 1 – P(Y≤2) = 1 − F(2) = 1 – 0.2381 = 0.7619

c) P(X = 2) · P(X = 2) = [F(2) – F(1)] · [F(2) – F(1)] =

(0.6767-0.4060) · (0.6767-0.4060) = 0.07328

Problema 8

$$X \sim N(36.7, 3.8)$$
$$\bar{X} \sim N\left(36.7, \frac{3.8}{\sqrt{100}} = 0.38\right)$$

a)

$$P(\bar{X} \leq 36.9) = P\left(Z \leq \frac{36.9 - 36.7}{0.38}\right) = P(Z \leq 0.5263 \approx 0.53) = 0.7019$$

b)

$$P(36.5 \leq \bar{X} \leq 37.3) = P\left(\frac{36.5 - 36.7}{0.38} \leq Z \leq \frac{37.3 - 36.7}{0.38}\right) \approx P(-0.53 \leq Z \leq 1.58)$$
$$= \Phi(1.58) - \Phi(-0.53) = \Phi(1.58) - [1 - \Phi(0.53)] = 0.6448$$