# Meet-in-the-middle Enumeration

The idea of meet in the middle enumeration is that if

$$\sum_{j=1}^{m} L_{i_j} = T \mod N$$

then we can split the indices in half

$$\sum_{j=1}^{m/2} L_{i_j} = x \mod N$$

$$\sum_{j=1}^{m/2} L_{i_{m/2+j}} = T - x \mod N$$

So we only need to enumerate over every possible half-subset, and store in boxes $x$ and $T - x$.

TARGET = 40541043 mod 100000000

NUM_POSSIBLE_HALF_SUBSETS = $\binom{24}{6}$ = 134596

We can filter for actual collisions

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COLLISIONS = {
36528811: [[73132, {0, 1, 5, 6, 9, 13}], [44174, {10, 11, 14, 15, 16, 22}]],
 46707864: [[3038, {0, 1, 5, 6, 9, 10}], [50958, {11, 13, 14, 15, 16, 22}]],
 54947059: [[109448, {0, 1, 4, 18, 19, 21}], [126627, {0, 3, 6, 9, 11, 17}]],
 57383109: [[106620, {0, 1, 5, 6, 9, 22}], [119763, {10, 11, 13, 14, 15, 16}]],
 58586616: [[43225, {0, 1, 5, 6, 9, 16}], [7446, {10, 11, 13, 14, 15, 22}]],
 65511348: [[70191, {0, 1, 5, 6, 9, 15}], [13394, {10, 11, 13, 14, 16, 22}]],
 66419366: [[60485, {0, 1, 5, 6, 9, 11}], [51982, {10, 13, 14, 15, 16, 22}]],
 72876088: [[51446, {0, 1, 5, 6, 10, 11}], [60397, {9, 13, 14, 15, 16, 22}]],
 73664628: [[143, {0, 1, 3, 8, 13, 17}], [133130, {0, 2, 3, 4, 7, 12}]],
 92618571: [[17211, {0, 1, 5, 6, 9, 14}], [130916, {10, 11, 13, 15, 16, 22}]],
 ... (1516 entries omitted) ...
14912628: [[124632, {6, 14, 19, 20, 21, 23}],
            [41703, {7, 10, 14, 17, 18, 20}]],
 42019851: [[20373, {6, 15, 19, 20, 21, 23}], [29935, {7, 10, 15, 17, 18, 20}]],
 48944583: [[94547, {6, 16, 19, 20, 21, 23}], [9101, {7, 10, 16, 17, 18, 20}]],
 50148090: [[130043, {6, 19, 20, 21, 22, 23}],
            [115087, {7, 10, 17, 18, 20, 22}]],
 52289520: [[120636, {6, 14, 15, 19, 21, 23}],
            [126818, {7, 10, 14, 15, 17, 18}]],
 59214252: [[69925, {6, 14, 16, 19, 21, 23}], [27262, {7, 10, 14, 16, 17, 18}]],
 60417759: [[7182, {6, 14, 19, 21, 22, 23}], [114407, {7, 10, 14, 17, 18, 22}]],
 86321475: [[66607, {6, 15, 16, 19, 21, 23}], [45462, {7, 10, 15, 16, 17, 18}]],
 87524982: [[122897, {6, 15, 19, 21, 22, 23}], [6057, {7, 10, 15, 17, 18, 22}]],
 94449714: [[64145, {6, 16, 19, 21, 22, 23}],
            [109308, {7, 10, 16, 17, 18, 22}]]
}
```

And the complexity can be estimated by the first collision, which in this case is:

SMALLEST = [[2477, {1, 8, 13, 17, 18, 20}], [3571, {2, 4, 7, 12, 18, 20}]]

Note that at most we will have to do $\binom{24}{6}$ work, but the expected amount of work is:

$$\text{EXPECTED\_WORK} = \binom{24}{6} / \sqrt{\binom{12}{6}} = 4427.88331674025$$

The above description can be compared with the classic meet-in-the-middle approach [1] who were the first to reduce the complexity from $O(2^n)$ to $O(2^{n/2})$ by splitting the problem in half, generating all subset sums from each half, and searching for complementary pairs. Schroeppel and Shamir [2] achieved the same time complexity with reduced space requirements of $O(2^{n/4})$.

## References


[1] E. Horowitz and S. Sahni, "Computing partitions with applications to the knapsack problem," *Journal of the ACM*, vol. 21, no. 2, pp. 277–292, 1974, doi: 10.1145/321812.321823.

[2] R. Schroeppel and A. Shamir, "$T = O(2^{n/2})$, $S = O(2^{n/4})$ algorithm for certain NP-complete problems," in *SIAM journal on computing*, 1981, pp. 456–464.