利用R程式驗證幾何分配的遺失記憶性

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幾何分配

不斷重複伯努力實驗，直到第一次成功事件發生為止的機率分配其機率分配函數為：

$$P(x)=\begin{cases} p(1-p)^{x-1} & x=1,2,3\ldots \\ 0 & otherwise \end{cases}$$

則稱X服從幾何分配，記做 $X\sim Geometric(p)$

幾何分配的遺失記憶性

幾何分配具有遺失記憶性的特質，不管前面失敗了幾次，還需要到實驗成功的次數與前面沒實驗過時所需的次數相同。其式為:

$$P(X>a+b|X>a)=P(X>b)$$
$$a,b=1,2,3\ldots$$

證明:

$$P(X>a+b|X>a)=\frac{P(X>a+b,X>b)}{P(X>a)}$$
$$=\frac{P(X>a+b)}{P(X>a)}=\frac{(1-p)^{a+b}}{(1-p)^{a}}$$
$$=(1-p)^{b}=P(X>b)$$

範例程式

令p=0.2 b=1,2,3,4,5 a固定為2，則可以編寫以下R的程式碼並產生結果如右圖

```r
a1 <- 1-pgeom(1,0.2);b1 <- 1-pgeom(6,0.2);c1 <- 1-pgeom(4,0.2);
d1 <- b1/c1
a2 <- 1-pgeom(2,0.2);b2 <- 1-pgeom(7,0.2);c2 <- 1-pgeom(4,0.2);
d2 <- b2/c2
a3 <- 1-pgeom(3,0.2);b3 <- 1-pgeom(8,0.2);c3 <- 1-pgeom(4,0.2);
d3 <- b3/c3
a4 <- 1-pgeom(4,0.2);b4 <- 1-pgeom(9,0.2);c4 <- 1-pgeom(4,0.2);
d4 <- b4/c4
a5 <- 1-pgeom(5,0.2);b5 <- 1-pgeom(10,0.2);c5 <- 1-pgeom(4,0.2);
d5 <- b5/c5
t1 <- cbind(a1,a2,a3,a4,a5)
colnames(t1) <- 1:5
rownames(t1) <- c("P(x>b)")
t2 <- cbind(b1,b2,b3,b4,b5)
colnames(t2) <- 6:10
rownames(t2) <- c("P(x>a+b)")
t3 <- cbind(c1,c2,c3,c4,c5)
colnames(t3) <- rep(4,5)
rownames(t3) <- c("P(x>a)")
t4 <- cbind(d1,d2,d3,d4,d5)
colnames(t4) <- 6:10
rownames(t4) <- c("P(x>a+b|x>a)")

par(mfrow=c(2,2))
barplot(t1,ylim=c(0,0.7),col="red",xlab="b",ylab="p",main="P(X>b)")
barplot(t2,ylim=c(0,0.7),col="green",xlab="a+b",ylab="p",main="P(X>a+b)")
barplot(t3,ylim=c(0,0.7),col="blue",xlab="a",ylab="p",main="P(X>a)")
barplot(t4,ylim=c(0,0.7),col="red",xlab="a+b",ylab="p",main="P(X>a+b|x>a)")
```

圖形

從下圖可以看出，當以$X>a$為條件時，則$P(X>a+b|X>a)$的圖形會與$P(X>b)$結果相同，證明了幾何分配如果前面實驗失敗了 a 次，再實驗 b 次仍皆失敗的機率，與重新實驗b次皆失敗的機率相同