PRP Exam
(2:00 PM to 3:00 PM)

- There are 5 questions
- Each question carries 4 marks.

① The joint density of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2} y e^{-xy}, & 0 < x < \infty,\ 0 < y < 2 \\ 0 & \text{o.w.} \end{cases}$$

Calculate $E\left[e^{X/2} \mid Y = 1\right]$.

② Given $f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 & x < 0 \end{cases}$ $f_Y(y) = \begin{cases} e^{-y}, & y \geq 0 \\ 0 & y < 0 \end{cases}$

Find joint pdf $f_{Z,W}(z,w)$ when X & Y are independent and $Z = X+Y$ and $W = \frac{X}{X+Y}$. Conclude whether Z and W are independent.

(3) An urn contains $n+m$ balls, of which n are red and m are black. They are withdrawn from the urn, one at a time and without replacement. Let X be the no. of red balls removed before first black ball is chosen. Express X as sum of indicator r.v.s. Use this to calculate $E(X)$.

(4) Consider the Cauchy density function

$$f_X(x) = \frac{1}{\pi(1+x^2)}, \quad x \in \mathbb{R},$$

What is the mean and the characteristic function associated with this density?
(Hint: Please see last page)

(5) Let Y and Z be independent Gaussian r.v.s. Y has mean 10 and variance 4. Z has mean -2 and variance 9. Let $X = 2Y + 3Z$. Use Chebyshev inequality to find the upper bound on the probability that X differs from mean of X by more than 5.

Hint for Q4:

You can use the following relation

$$\int_0^\infty \frac{\cos(tx)}{b^2+x^2}\,dx = \frac{\pi}{2b}e^{-tb}, \quad t \geq 0.$$