PRP Final Exam

(5:30PM to 6:30PM)

- There are 4 questions
- First question is 4 marks, second question is 3 marks, 3rd and 4th questions are 4 marks each.

① Suppose the covariance matrix of a random vector $[X_1\ X_2\ X_3]^T$ is

$$\begin{bmatrix} 7 & 4 & 0 \\ 4 & 5 & 3 \\ 0 & 3 & 5 \end{bmatrix}.$$

ⓐ Find the correlation coefficient between X_1 and X_2.

ⓑ Find a so that $X_1 - aX_2$ is uncorrelated with X_1

ⓒ Find $Var(X_1 + X_2 + X_3)$.

② Let $X_1, \ldots, X_n$ be independent r.v. with $P(X_i = 1) = P(X_i = -1) = 0.5$

(a) Compute the characteristic function of the following r.v.

$$S_n = \sum_{i=1}^{n} X_i, \quad V_n = \frac{S_n}{\sqrt{n}}.$$

(b) Find the limit of characteristic functions of S_n and V_n as $n \to \infty$.

(3) Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate $\lambda = 0.5$.

(a) Find the probability of no arrivals in time interval $(3,5]$.

(b) Find the probability that there is exactly one arrival in $(0,1]$ <u>and</u> one arrival in $(1,2]$ <u>and</u> one arrival in $(2,3]$ <u>and</u> one arrival in $(3,4]$.

(4) Let $z(t)$ be a white Gaussian noise process of power spectral density $N_0/2$.

(a) Find the probability density of

$$V = \int_0^{\infty} e^{-t} z(t)\, dt.$$

(b) Let $Y(t)$ be the output when i/p $Z(t)$ is passed through a filter with the following response

$$H(\omega) = \begin{cases} 1 & 0 \le |\omega| \le W \\ 0 & \text{otherwise.} \end{cases}$$

Find the joint distribution of $Y(0)$ and $Y\left(\frac{1}{4W}\right)$.