MANHATTAN COLLEGE DEPARTMENT OF MATHEMATICS

Project Proposal:
A Markov Chain Analysis
of Strategy Realized in
A Single Hand in Blackjack

Nicholas Italiano

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1 INTRODUCTION

Have you ever seen the movie *21(2008*? It is a pretty good Hollywood movie adaptation! The premise of the 2002 book *Bringing Down the House* by Ben Mezrich is as follows: a team of six MIT students under the mentorship of Professor Mickey Rosa (Kevin Spacey)– who Dr. Gerhardt may have known– make serious money by implementing "Advantage Player

Gambling (card counting)" at the Blackjack tables. Albeit dramatized, the movie features the actions of an advantage player– gambling as a team sport (card counting is legal if there is a- no communication between players and b- card counting does not violate the particular casino's rules)– which can also be used for other casino games[2].

What both the book and movie affirm, but fail to explain to the appropriate audience, are the mathematical reasons why the statement " card counting strategies on top of regular strategy give a player advantage of over a half-percent (0.05%) edge over the casino" holds true[2]. Instead the focus is on briefly explaining one of many card counting strategies: the "Hi-Lo" system. The "Hi-Lo" system also referred to as the "Complete Point System" as was first formalized in the 1966 book *Beat the Dealer* by Mathematician Edward O. Thorpe[4]. What does the statement about edge mean quantitatively? How do strategies (regular, card counting, shuffle tracking, etc.) increase or decrease the relative odds, or more formally defined, the expected loss/gain per hand? How do we apply probabilistic methods correctly in order to answer the aforementioned inquires? One answer: Markov Chains (MC's)

To answer my above questions of interest, we will be doing a MC analysis of strategy realized in a single hand of Blackjack. By the phrase "strategy realized in a single hand " I mean to apply MC's to preexisting mathematical methods derived by prior analyses. Nothing new, however, we will be required to reproduce results from 1) quantitatively determine what strategy is better to implement in an a single hand (restricting strategies to "regular" as and "card counting") by applying MC's, 2) computing the expected loss/gain that a player/dealer edge of a single and specific to that chosen strategy, and 3) demonstrating the power of MC's by simulating a single hand, and making correct analytical and graphical inferences.
This proposal assumes the reader has familiarity with advantage playing in gambling (not presuming the latter is true, nor judging in any case, and that this statement was not made to be offensive in any way ;-)!).

2 Motivation

Anticipating a final project, where not only do we get to apply math to a topic of our choosing, but also one that requires a great deal of knowledge amassed from study of probability models (second time is the charm!) I set out to answer the two questions (which also were, many times, the fundamental thesis outlined in cited research papers)[4,5]. Being familiar with literature about the game I needed to i) research how MC's were applied mathematically, and ii) how they were simulated in analyses. Luckily, I discovered that MATLAB has user-built simulations with references published on Mathworks.com– and since I'm only taking two grad courses this spring– I was inspired to learn more.

Over the last month or so I learned three main takeaways from research and coding efforts to help begin our analysis: 1) coding a GUI- responsive blackjack program in MATLAB is more difficult than expected (thus complexity of simulation and/or coding in analysis will be time pending); 2) user developed strategies like the Complete Point System do make a difference, but not in the long run due to Gamblers ruin 2) and 3) the innate structure of Blackjack ends itself to Discrete- time Markov Chain Analysis as does sports betting, e-commerce and even web development (more on that in the paper)[4,2,3].

3 Background

The game of blackjack can be modeled as a MC. Specifically, we can model the game as a Discrete-time MC (DTMC) which is defined as a family of random variables $X_1, i \in \mathbb{N}$, Explicitly,

$$\{X_n : n > 0\}.$$

Here, the probability of transitioning from state i to state $i+1$ is defined by the matrix $\underline{\mathbf{P}}$ which is called the one-step transition matrix[1]. $\underline{\mathbf{P}}$ can also be expressed as the Markov property (will be outlined and formally defined in paper) which is the probability distribution of the value of each state that depends on the value of the state preceding it, and is memoryless with respect to how the system arrived in the preceding state[2,3]:

$$\underline{\mathbf{P}} = (P^{(1)}_{i,j}),$$

where the Markov property is easily recognized when expressed equivalently as the one-step transition matrix[1]. Explicitly,

$$(P^{(1)}_{i,j}) \equiv \{X_{n+1} = j \mid X_{n+1} = i\}$$

is the for $n > 0$.

In Blackjack, state space can be modeled as all possible hands that a player may have: hard 4 through 21, soft 12 through 21, and bust (22 or more)[3]. Each time indexed by i represents the player drawing another card: X_1 is the two-card hand the player is originally dealt, X_2 is his hand after one hit, and so on. The chain stops at X_N for some finite N when the player either chooses to stand or goes bust. For $m > N$, define X_m to equal X_N. It is clear from the rules of the game that it is irrelevant how a player arrived at his current hand[2].

In the preceding paragraph we outlined a players hand using MC's. Now we must describe the dealer state space, transitional probabilities (similar to the players but we must consider the fact the dealer has one card facedown and stops upon hitting combination of cards that yield greater than a 17), how we plan to calculate the player advantage and expected return. Consider the state space in which the dealers hand can transition to to be H, the state of the dealers cards to be Y_i with one-step transition matrix $\underline{\mathbf{Q}} = (Q^{(1)}_{i,j})$. Since the dealer has absorbing states between a hard 17 or bust 22+, like state 0 and N in the Gambler's Ruin model, the matrix $(Q^{(\infty)}_{i,j})$ will include a set $A \subset H$ of essential absorbing states). The finite number transitions into absorbing states allowing us to solve for the steady state vector π_i.[2,3] Explicitly,

$$\pi_0^T * Q^{(\infty)} = \pi^T$$

where $*$ indicates right matrix multiplication, π_0^T is the probability the dealer is holding hand i, and π^T determines the probability the dealer finishes with hand i (all matrices include the given information from the dealers face up card[3].

With the following information about the state space for how the dealer and player interact, the appropriate DTMC models defined, expressions for steady state vectors pi_i^T (and referencing two papers with explicit methods to a full analysis using both problem setups) we can follow either of two directions: 1) We can do reproduce what Wakin and Rozell achieved in "A Markov Chain Analysis of BlackjackStrategy" and focus on only the "complete point system" strategy (will be outlined in the paper if not familiar) and use the remaining state of cards in the shoe (called the *high low index* to analytically calculate the time spent in each state, determining overall advantage for a single hand[3]. Or 2) reproduce what Coltin achieved by simply ignoring all other strategies but regular play in determining a strategy matrix $C = 10x|H|$, allowing us to compute the expected value of a players winnings in normal play $E[winnings]$. Explicitly,

$$E[winnings] = \mu\ \pi_0^T$$

where mu is computed from C (will be explained in-depth in paper) and π_0^T is a vector containing the of probabilities of starting with each hand.

Unfortunately, it is unclear at this time which method (or arrangements there of) we will commit to in order to answer my (two) non-simulation based questions of interest (again, 1) establishing which strategy is better in the long term by producing models for a single hand of play, and 2) determining the overall player advantage, or "edge", in a single hand using any strategy (obtained from determining the expected loss/gain)[3]. Based of reading and MATLAB code I will most likely chose 2) in favor of simpler math (no linear algebra) and because Coltin (and in a unpublished paper from Duarte) outlined the steps to compute each matrix in pseudocode.

4 Plan of Attack

To properly conduct an a DTMC analysis of a single hand in Blackjack, both by implementing technology to produce a probabilistic simulation and produce deliverables that tells a nice story, we must do the following:

Assumptions Make assumptions regarding the shuffling method, rules, mode of play (single hand, unit bet, no double downs, etc.), and allowable strategies (restricted to "regular" as described in above url's and "Complete Point System" that will be outlined in detail, in the report to follow).

Model Formulate expressions for all transitional probabilities, state spaces, and steady state vectors (noting the appropriate properties inherent to all transitional matrices, e.g., absorbing/recurrent/transient classes).

Determine Conduct by-hand, or use of technology, methods to determine the required strategy matrices (that govern the distribution of particular decisions such as "Splitting", "Double Down?, etc.), player advantages (a value specified in detail in the following report) and the expected value of winnings (loss/gain) in a single hand—per strategy if time requires!.

Calculate Tabulate the above data to make both correct analytical and graphical inferences for "Results and Discussion" section of report, noting significant findings in strategy-specific matrices and expected loss/gain vectors.

Generate Produce graphics and figures that support MC Analysis of a single hand.

Simulate Write executable MATLAB code which simulates the transient and long term behavior of MC's (talk about limitations and parallel programming in "Swinging for the Fences" section).

5 Bibliography

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