The Intrinsic Manifolds of Radiological Images and their Role in Deep Learning

Nicholas Konz[1] Hanxue Gu[1] Haoyu Dong[2] Maciej Mazurowski[1,2,3,4]

[1]Department of Electrical and Computer Engineering, [2]Department of Radiology, [3]Department of Computer Science, [4]Department of Biostatistics & Bioinformatics, Duke University, Durham, North Carolina, USA

Duke

Introduction

- **The Manifold Hypothesis (MH)**: *High dimensional data can be well described by a much smaller number of* **intrinsic dimensions**.
  - Neural networks can learn to convert raw data to abstract, informative features that are *intrinsic* to the dataset [1].

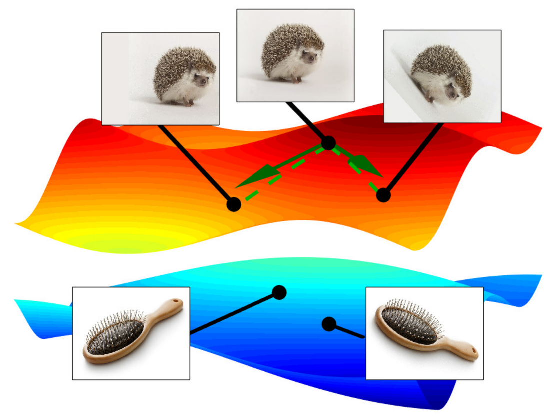

Figure 1: Visualization of intrinsic low-dimensional image manifolds from [2].

- **Why study the intrinsic dimension of medical images?**
  - Medical vs. natural images: different relevant semantics,
  - Due to the MH, understanding the intrinsic structure of medical image datasets is key to analyzing how networks learn from them.

Objectives

1. Estimate the intrinsic dimensions of common radiology datasets, and compare to natural image datasets.
2. Evaluate the relationship of dataset intrinsic dimension with network generalization ability; comparing within and between the domains of radiological and natural images.

Estimating the Intrinsic Dimension of Image Manifolds

- By the MH: our d-dimensional data lies on a manifold $\mathcal{M} \subseteq \mathbb{R}^d$ such that $\dim \mathcal{M} = m \ll d$.
- We can estimate m via **maximum likelihood**:
  - Assume that volume of $\mathcal{M}$ scales exponentially with m as we move away from a point; model volume with k-NN distance T_k.
  - Model data sampling with a Poisson Process, and find m via MLE:

$$\hat{m} = \left[\frac{1}{N(k-1)}\sum_{i=1}^{N}\sum_{j=1}^{k-1}\log\frac{T_k(x_i)}{T_j(x_i)}\right]^{-1}$$

Datasets

- 7 common radiology datasets from different modalities:

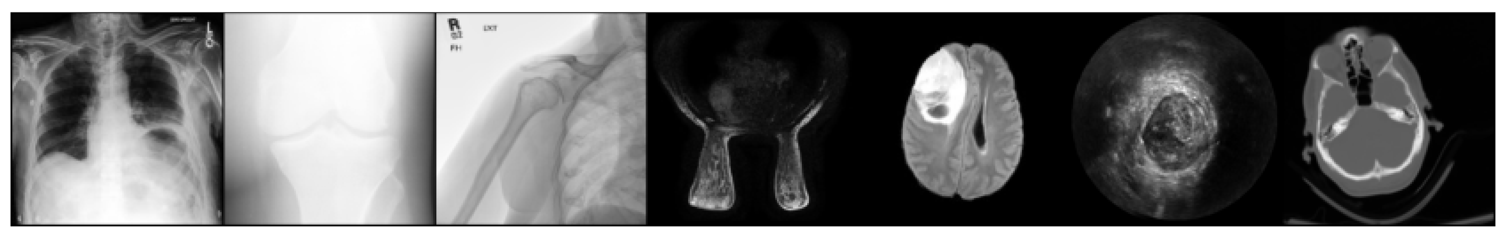

Figure 2: Samples from our seven evaluated datasets.

Finding 1: Radiological vs. Natural Image Intrinsic Dimension

Radiological image datasets tend to have lower intrinsic dimension than natural image datasets:

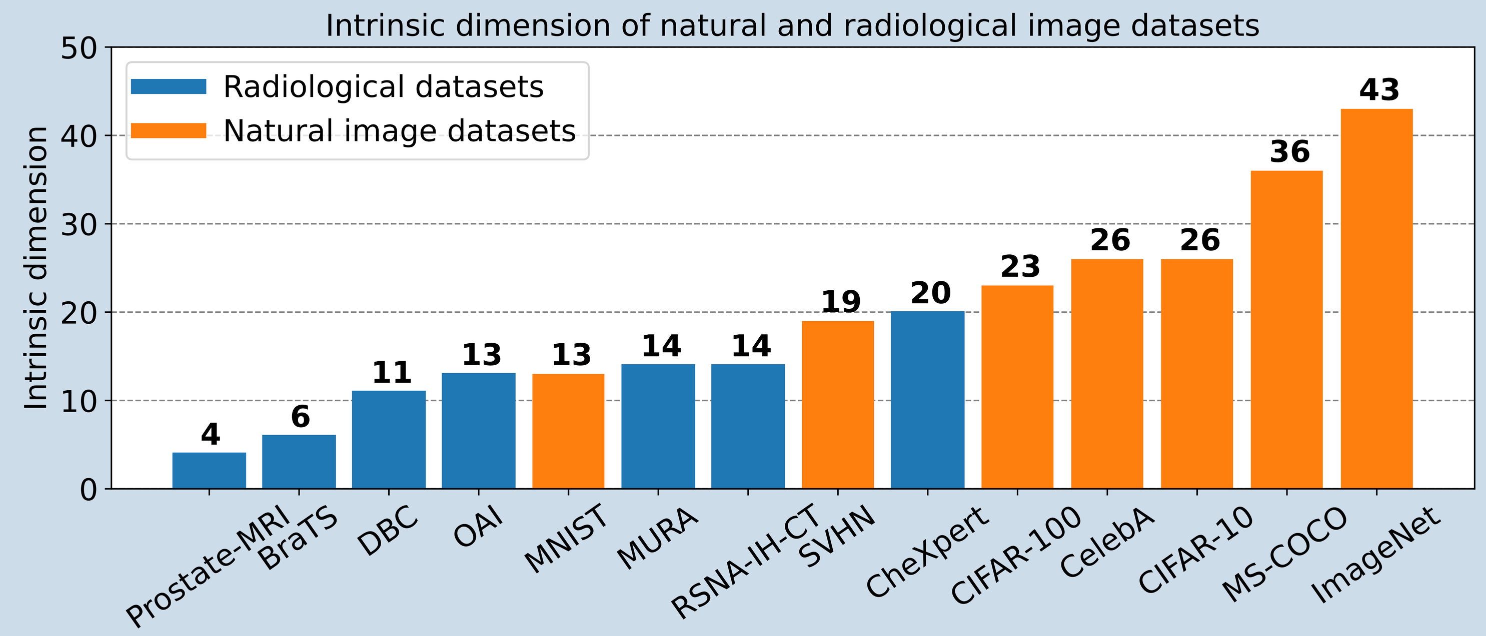

Figure 3: Intrinsic dimension of radiological and natural [3] image datasets.

Finding 2: Intrinsic Dimension and Generalization Ability

- Generalization ability (GA) is sharply linearly correlated with dataset intrinsic dimension (ID) *within* radiological and natural imaging domains, but the steepness of this correlation differs noticeably *between* the two domains.
- The *slope* of this GA vs. ID relationship is practically independent to model choice and/or training set size within an imaging domain.

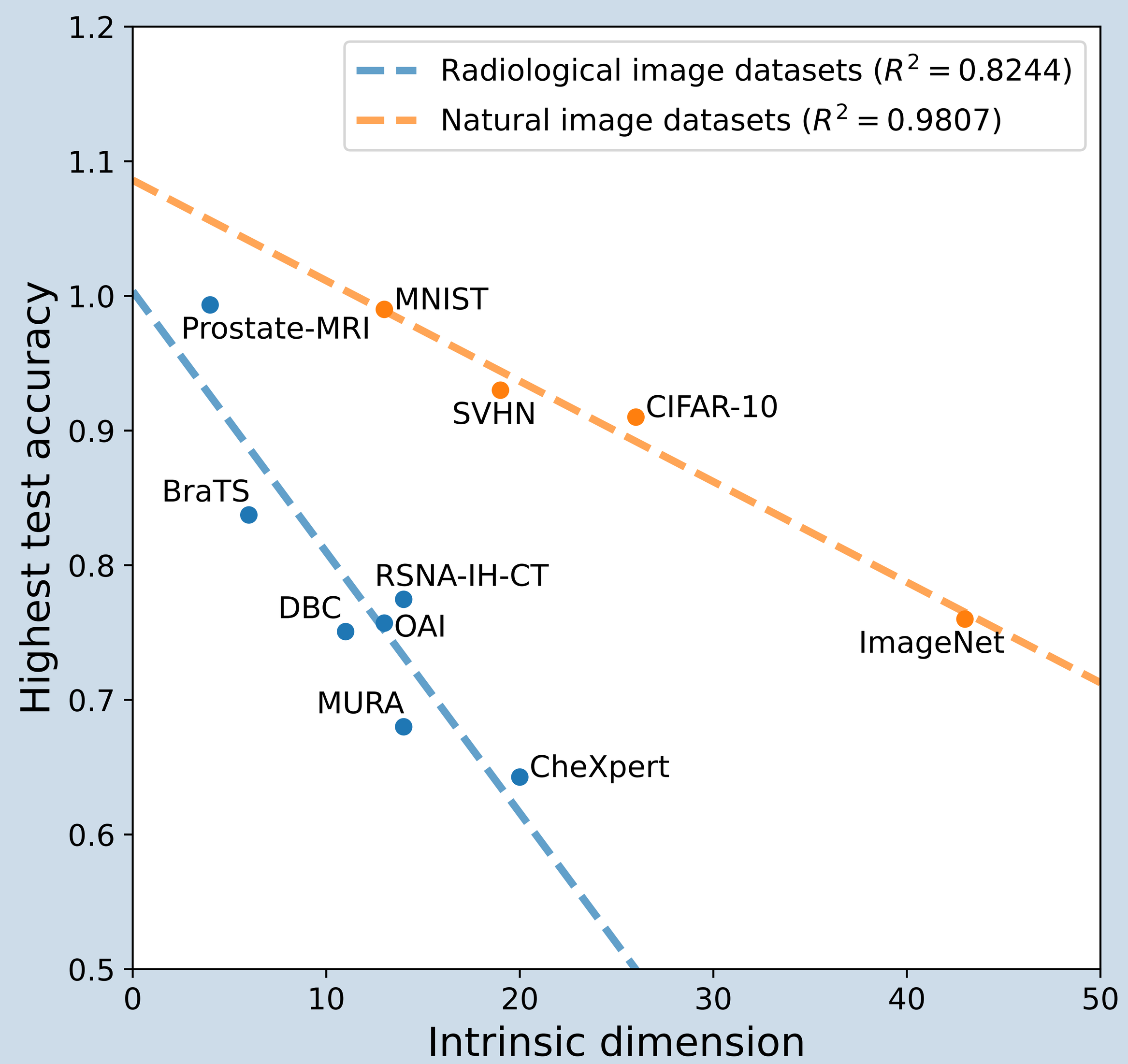

Figure 4: Linearity of model generalization ability with respect to dataset intrinsic dimension, for radiological and natural image datasets ($N_{\text{train}} = 2000$ on ResNet-18).

Additional Findings

- ID $\ll$ extrinsic dimension/ED (number of pixels).
- Intuitively, modifying ED (resizing images) didn't affect ID.

Experimental Settings

- **Radiological vs. natural image IDs (Finding 1):**
  - We estimated the intrinsic dimension of each dataset using 7500 images, evenly class-balanced according to a chosen binary classification task for each.
- **Generalization ability vs. ID (Finding 2):**
  - We trained a network on each dataset for its respective binary classification task, and tested on 750 unseen data points.
  - We evaluated 9 neural network models, each on 7 training set sizes, also performing task choice ablations.

Future Work

- Find theoretical support for the correlation of GA with dataset ID, and explain why the correlation sharpness differs between domains.
- Explore further uses of dataset ID estimation for modeling, experimentation, etc.

Contact Information

- My email: nicholas.konz@duke.edu
- My website: `https://nickk124.github.io/`
- Lab website: `https://sites.duke.edu/mazurowski/`

References

[1] Alessio Ansuini, Alessandro Laio, Jakob H Macke, and Davide Zoccolan.
Intrinsic dimension of data representations in deep neural networks.
Advances in Neural Information Processing Systems, 32, 2019.

[2] Sam Buchanan, Dar Gilboa, and John Wright.
Deep networks and the multiple manifold problem.
In *International Conference on Learning Representations*, 2021.

[3] Phillip Pope, Chen Zhu, Ahmed Abdelkader, Micah Goldblum, and Tom Goldstein.
The intrinsic dimension of images and its impact on learning.
In *9th International Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenReview.net, 2021.