Perturbations from a Inflationary Spectator Field

Nick Konz

Presentation Partners: Autumn Ficker and Roark Habegger

The University of North Carolina
Chapel Hill

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Abstract

Following the observations of inflationary perturbations in recent years, a number of models for single-field inflation have been all but ruled out [3]. However, if we move from single-field theory to considering the presence of an additional "spectator" *curvaton* field during inflation besides the inflaton, a number of inflaton potentials are made viable; this is because with the introduction of an additional field, observed perturbations can be decoupled from the inflaton. I will also consider the presence of a *modulus* spectator field, which generates nonuniform spatial modulations in the decay of the inflaton during reheating [4]. Both of these spectator fields can also modify the observed perturbation power spectrum, which in turn can modify observerables such as the spectral index, tensor-to-scalar ratio, and the non-Gaussianity in observed fluctuations.

1 The Primordial Power Spectrum for Multiple Scalar Fields

In order to approach inflationary perturbations and any non-Gaussianity arising from them, it is advantageous to adopt the so-called δN formalism used in [3] and [4]. This formalism is used to quantify the number of e-folds that one region is ahead of another (due to perturbations), where essentially, δN is analogous to the curvature perturbation (on some uniform-energy-density hypersurface) ζ, and is advantageous when used at sufficiently large scales where any spatial gradient can be neglected. Specifically, ζ can be defined at some time t_f as

$$\zeta(t_f, \vec{x}) = N(t_f, t_i, \vec{x}) - \langle N(t_f, t_i, \vec{x}) \rangle \,, \tag{1}$$

where $N(t_f, t_i, \vec{x}) \equiv \displaystyle\int_{t_*}^{t_f} H(t, \vec{x})\,\mathrm{d}t$ is the number of e-folds, with t_i some initial time, and the angular brackets denote a spatial average (such that the second term in (1) is analagous the the expansion of the unperturbed "background" spacetime) [3][4]. Letting $t_i = t_*$ be the time of horizon crossing, ζ can expanded written in terms of the value(s) of the scalar field(s) (whichever field that may be) at t_*,

$$\zeta(t_f) = N_a \delta\phi_*^a + \frac{1}{2} N_{ab} \delta\phi_*^a \delta\phi_*^b + \mathcal{O}(\delta\phi_*^3), \tag{2}$$

where $\delta\phi_*^a$ is the perturbation of the scalar field ϕ^a at t_*, $\delta\phi_*^a$ is defined similarly with some ϕ^b (here, I only consider the presence of at most two scalar fields), and

$$N_a \equiv \frac{\partial N}{\partial \phi^a}, \quad N_{ab} \equiv \frac{\partial^2 N}{\partial \phi^a \partial \phi^b} \quad [3]. \tag{3}$$

Now, the primordial power spectrum of ζ is defined as

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle \equiv (2\pi)^3 P_\zeta(k_1) \delta(\vec{k}_1 + \vec{k}_2), \tag{4}$$

and taking the first order approximation of (2) (with two scalar fields indexed by a and b with perturbation wavenumbers $\vec{k}_1$ and $\vec{k}_2$, respectively), I find that

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle = \left\langle N_a \delta\phi_*^a N_b \delta\phi_*^b \right\rangle = N_a N_b \left\langle \delta\phi_*^a \delta\phi_*^b \right\rangle. \tag{5}$$

The quantity $\left\langle \delta\phi_*^a \delta\phi_*^b \right\rangle$ can be expressed in terms of the standard power spectrum of any two scalar fields $P^{ab}(k)$,

$$\left\langle \delta\phi_*^a \delta\phi_*^b \right\rangle = (2\pi)^3 P^{ab}(k_1)\delta(\vec{k}_1 + \vec{k}_2), \tag{6}$$

and assuming that the two fields are uncorrelated and have the same amplitude, (6) will only be nonzero for $a = b$, in which case it will follow the standard single light scalar field power spectrum $P(k)$, i.e.

$$P^{ab}(k) = \delta^{ab} P(k) = \delta^{ab} \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi}\right)^2, \tag{7}$$

where $H_* \equiv H(t_*)$. It follows that, plugging (7) into (6) and that into (5),

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle = N_a N_b (2\pi)^3 \delta^{ab} \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi}\right)^2 \delta(\vec{k}_1 + \vec{k}_2) = (2\pi)^3 P_\zeta(k_1)\delta(\vec{k}_1 + \vec{k}_2), \tag{8}$$

so that, following the δ^{ab} and defining the dimension*less* power spectrum $\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2} P_\zeta(k)$,

$$P_\zeta(k) = N_a^2 \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi}\right)^2 \quad \Rightarrow \quad \boxed{\mathcal{P}_\zeta(k) = \left(N_a \frac{H_*}{2\pi}\right)^2}. \tag{9}$$

In the following sections, I will determine the form of N_a for the curvaton and modulating reheating scenarios, and show how that will determine the power spectrum and the observable parameters derived from it.

2 The Curvaton

Imagine that during inflation, there is additional scalar field σ_c known as the *curvaton* (named after the curvature perturbations it produces) present alongside the inflaton, that is subdominant in mass relative to the curvaton. Here, I consider a curvaton with a quadratic potential $U(\sigma) = \frac{1}{2} m_{\sigma_c}^2 \sigma_c^2$, where m_{σ_c} is the mass of the curvaton, following [3]. For the purposes of this paper, I will summarize much of the following intermediate steps that lead to the important results. In order to examine the effect of a curvaton on $\mathcal{P}_\zeta$ and related parameters, it's necessary to consider N_R, which defined as the number of e-folds from some time after the decay of the inflaton, to some time well after the curvaton has decayed [3]. The reason that N_R is of interest is that the curvaton only produces noticeable perturbations once the inflaton has decayed and is no longer the "primary" scalar field at play. After analyzing the properties of the curvaton over the course of its evolution, [3] arrives at the quantity Q, which schematically represents the part of N_R which depends on the curvaton at horizon entry, $\sigma_{c,*}$, and is also a function of the decay rate of the curvaton Γ_{σ_c}, and m_{σ_c}. As such, going back to (9), it follows that $N_a = N_{\sigma_c} = Q_c$ for the curvaton, where $Q_{\sigma_c} \equiv \frac{\partial Q}{\partial \sigma_c}$. For the inflaton with some potential $V(\phi)$, we have that $N_a = N_\phi \equiv \frac{1}{M_{\rm pl}^2} \frac{V}{V_\phi}$, where $V_\phi \equiv \frac{\partial V}{\partial \phi}$, which simply comes from the standard calculation of solo inflaton e-folds [3]. Plugging these different N_a terms into (2) to obtain a ζ (to first order) for each field, the same process of (4) - (9) can be applied (after adding the contribution of each field to the overall power spectrum) to obtain

$$\mathcal{P}_{\zeta,c} = \left(\frac{1}{M_{\rm pl}^4}\frac{V^2}{V_\phi^2} + Q_{\sigma_c}^2\right)\left(\frac{H_*}{2\pi}\right)^2. \tag{10}$$

From (10), the spectral index n_s, its running $n_{\rm run}$, and the tensor-to-scalar ratio r can finally be obtained, which are parameters that can be constrained by observations. By definition, using $k = aH_*$,

$$n_s - 1 \equiv \frac{\mathrm{d}\log\mathcal{P}_{\zeta,c}}{\mathrm{d}\log k} = \frac{k}{\mathcal{P}_{\zeta,c}}\frac{\mathrm{d}\mathcal{P}_{\zeta,c}}{\mathrm{d}k} = \frac{k}{\mathcal{P}_{\zeta,c}}\left\{\frac{1}{M_{\rm pl}^4}\frac{\mathrm{d}}{\mathrm{d}k}\left[\frac{V^2}{V_\phi^2}\left(\frac{H_*}{2\pi}\right)^2\right] + \frac{\mathrm{d}}{\mathrm{d}k}\left[Q_{\sigma_c}^2\left(\frac{H_*}{2\pi}\right)^2\right]\right\}. \tag{11}$$

This is further simplified using chain-rule relations such as $\frac{\mathrm{d}V}{\mathrm{d}k} = \frac{\mathrm{d}\phi}{\mathrm{d}k}V_\phi$, $\frac{\mathrm{d}V_\phi}{\mathrm{d}k} \equiv \frac{\mathrm{d}\phi}{\mathrm{d}k}V_{\phi\phi}$, and $\frac{\mathrm{d}H_*}{\mathrm{d}k} = \frac{1}{a}$. After doing the algebra and defining the first-order slow-roll parameters (SRP) as

$$\epsilon_V \equiv \frac{1}{2}M_{\rm pl}^2\left(\frac{V_\phi}{V}\right)^2 \qquad \eta_V \equiv M_{\rm pl}^2\frac{V_{\phi\phi}}{V}, \tag{12}$$

an expression for the spectral index can be obtained [3] (to first order in SRP) in terms of the SRP,

$$n_s - 1 = -2\epsilon_V - \frac{4\epsilon_V - 2\eta_V}{1 + 2\epsilon_V M_{\rm pl}^2 Q_{\sigma_c}^2}. \tag{13}$$

Similarly, following the same steps, the running of the spectral index can also be obtained as

$$n_{\rm run} = \frac{\mathrm{d}n_s}{\mathrm{d}\log k} = k\frac{\mathrm{d}n_s}{\mathrm{d}k}, \tag{14}$$

where the SRP parameters within n_s can be differentiated with respect to k using their definitions and the chain-rule relations used in (11), to obtain

$$n_{\rm run} = -4\epsilon_V(2\epsilon_V - \eta_V) - \frac{2\left[8\epsilon_V^2 - 6\epsilon_V\eta_V + 2\epsilon_V(2\epsilon_V\eta_V - 2\eta_V^2)M_{\rm pl}^2 Q_{\sigma_c}^2\right]}{\left(1 + 2\epsilon_V M_{\rm pl}^2 Q_{\sigma_c}^2\right)^2} \quad [3]. \tag{15}$$

Now, to find r, note that during inflation, *tensor* perturbation modes are generated following a power spectrum of

$$\mathcal{P}_T = \frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2 \quad [3]. \tag{16}$$

The tensor to scalar ratio is defined to be the ratio between power spectra of tensor and scalar perturbations, so I find that (using the definition for ϵ_V)

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\zeta,c}} = \frac{\frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2}{\left(\frac{1}{M_{\rm pl}^4}\frac{V^2}{V_\phi^2} + Q_{\sigma_c}^2\right)\left(\frac{H_*}{2\pi}\right)^2} = \frac{\frac{8}{M_{\rm pl}^2}}{\frac{1}{2M_{\rm pl}^2\epsilon_V} + Q_{\sigma_c}^2} = \frac{16\epsilon_V}{1 + 2\epsilon_V M_{\rm pl}^2 Q_{\sigma_c}^2}. \tag{17}$$

3 The Modulated Reheating Scenario

The second spectator field scenario that I will consider is known as the *modulated reheating scenario* [4]. In traditional inflation theory, the inflaton decays uniformly during reheating; the crux of this theory is that the inflaton can have a spatially modulating decay rate if an additional so-called *modulus* spectator field, denoted with σ_m, is present (with an energy density much lower than that of the inflaton).

To begin, I'll use the same δN formalism presented in equations (1) - (9) (which can still be applied because I did not specify the spectator field being the curvaton until section 2), and apply (9) to σ_m to obtain

$$\mathcal{P}_{\zeta,m} = \left(\frac{\partial N}{\partial \sigma_m} \frac{H_*}{2\pi} \right)^2, \tag{18}$$

where again, I assume that the modulus field is light, N denotes the number of e-folds, and H_* is the value of H upon horizon entry. To proceed, I will assume that the Universe transitions from inflaton oscillation to radiation dominated instantaneously, i.e. it enters reheating instantly, following [4].

I will next assume that $\sigma_{\text{reh}} = \sigma_*$, i.e. that the modulus field does not change between horizon entry and the beginning of reheating, following [4].

4 Chaotic Inflation

In [3], one of the primary inflaton models discussed is *Chaotic inflation*, which is characterized by the potential

$$V = \lambda M_{\text{pl}}^4 \left(\frac{\phi}{M_{\text{pl}}} \right)^n, \tag{19}$$

where λ is a parameter fixed by observations. Here, I will discuss the case of $n = 4$, which is the primary focus of [3], given that other values such as $n = 2, 6$ are not as affected by the inclusion of a curvaton [3]. Consider $\sigma_* \ll M_{\text{pl}}$ and $p \ll 1$

5 Observational Consequences and Discussion

As shown in [5], multiple field models can provide strong fits to recent observational data. Shown in [5], different multiple field models can be distinguished [3] [4] [1] [5] [2]

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