PERTURBATIONS FROM AN INFLATIONARY SPECTATOR FIELD

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Honor Pledge: *All work presented here is my own.*

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Abstract

Following the observations of primordial curvature perturbations in recent years, a number of models for single-field inflation have been all but ruled out [2]. However, if a transition is made from single-field theory to considering the presence of an additional "spectator" field, the *curvaton*, alongside the inflaton during inflation, a number of inflaton potentials can be made viable; this is because with the introduction of the curvaton, observed perturbations can be decoupled from the inflaton. I will also consider the presence of a *modulus* spectator field during reheating, which generates nonuniform spatial modulations in the decay of the inflaton during reheating [3]. These spectator fields modify the observed primordial perturbation power spectrum, which in turn can modify observables such as the spectral index and tensor-to-scalar ratio. I also briefly consider how these scenarios can introduce noticeable non-Gaussianity in observed perturbations.

1 The Primordial Power Spectrum of Multiple Scalar Fields

In order to approach inflationary perturbations sourced from more than one primordial scalar field, it is advantageous to adopt the so-called δN formalism used in [2] and [3]. This formalism is used to quantify the number of e-folds that one region of space-time is ahead of another (due to perturbations), where essentially, δN is analogous to the curvature perturbation (on some uniform-energy-density hypersurface) ζ. This paper will be working at sufficiently large scales such that any spatial gradient can be neglected, which is advantageous towards using the δN formalism. With this prescription, ζ can be defined at some time t_f as

$$\zeta(t_f, \vec{x}) = N(t_f, t_i, \vec{x}) - \langle N(t_f, t_i, \vec{x}) \rangle \,, \tag{1}$$

where $N(t_f, t_i, \vec{x}) \equiv \int_{t_*}^{t_f} H(t, \vec{x})\,\mathrm{d}t$ is the number of e-folds, t_i is some initial time, and the angular brackets denote a spatial average (such that the second term in (1) is equivalent to the expansion of the unperturbed "background" space-time) [2][3]. Letting $t_i = t_*$ be the time that the perturbation crosses the instantaneous comoving horizon, ζ can be written in terms of the value of the scalar field (whichever field that may be) at t_*,

$$\zeta(t_f) = N_a \delta\phi_*^a + \mathcal{O}(\delta\phi_*^2), \tag{2}$$

where $\delta\phi_*^a$ is the perturbation of the scalar field ϕ^a at t_*, and $N_a \equiv \partial N / \partial \phi^a$ [2]. Now, the dimension*full* primordial power spectrum of ζ is defined as

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle \equiv (2\pi)^3 P_\zeta(k_1) \delta(\vec{k}_1 + \vec{k}_2), \tag{3}$$

and taking the first order approximation of (2) (with two scalar fields indexed by a and b with perturbation wavenumbers $\vec{k}_1$ and $\vec{k}_2$, respectively), I find that

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle = \left\langle N_a \delta\phi_*^a N_b \delta\phi_*^b \right\rangle = N_a N_b \left\langle \delta\phi_*^a \delta\phi_*^b \right\rangle. \tag{4}$$

The quantity $\left\langle \delta\phi_*^a \delta\phi_*^b \right\rangle$ can be expressed in terms of the standard power spectrum of any two scalar fields $P^{ab}(k)$,

$$\left\langle \delta\phi_*^a \delta\phi_*^b \right\rangle = (2\pi)^3 P^{ab}(k_1)\delta(\vec{k}_1 + \vec{k}_2), \tag{5}$$

and assuming that the two fields are uncorrelated and have the same amplitude, (5) will only be nonzero for $a = b$, in which case it will follow the standard single light scalar field power spectrum $P(k)$, i.e.

$$P^{ab}(k) = \delta^{ab} P(k) = \delta^{ab} \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi}\right)^2, \tag{6}$$

where $H_* \equiv H(t_*)$. It follows that, plugging (6) into (5) and that into (4),

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \right\rangle = N_a N_b (2\pi)^3 \delta^{ab} \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi}\right)^2 \delta(\vec{k}_1 + \vec{k}_2) = (2\pi)^3 P_\zeta(k_1) \delta(\vec{k}_1 + \vec{k}_2), \tag{7}$$

so that, following the δ^{ab} and defining the dimension*less* power spectrum $\mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2} P_\zeta(k)$,

$$P_\zeta(k) = N_a^2 \frac{2\pi^2}{k^3} \left(\frac{H_*}{2\pi}\right)^2 \quad \Rightarrow \quad \boxed{\mathcal{P}_\zeta(k) = \left(N_a \frac{H_*}{2\pi}\right)^2}. \tag{8}$$

Note that N_a is evaluated at horizon entry, t_*. In the following sections, I will determine the form of N_a for the curvaton and modulating reheating scenarios, and show how that will determine the power spectrum and the observable parameters derived from it.

2 The Curvaton

Consider that during inflation, there is additional scalar field σ_c known as the *curvaton* (named after the curvature perturbations it produces) present alongside the inflaton, that is subdominant in mass relative to the curvaton. Here, I consider a curvaton with a quadratic potential $U(\sigma) = \frac{1}{2} m_{\sigma_c}^2 \sigma_c^2$, where m_{σ_c} is the mass of the curvaton, following [2]. For the purposes of this paper, I will summarize the intermediate, technical steps that lead to the important results, as follows.

In order to examine the effect of a curvaton on $\mathcal{P}_\zeta$ and related parameters, it is necessary to consider the quantity N_R, which is defined to be the number of e-folds from some time after the decay of the inflaton, to some time well after the curvaton has decayed [2]. The reason that N_R is of interest is that the curvaton only produces noticeable perturbations once the inflaton has decayed and is no longer the "primary" scalar field at play. After analyzing the properties of the curvaton over the course of its evolution, [2] arrives at the quantity Q, which schematically represents the part of N_R which depends on the curvaton at horizon entry, $\sigma_{c,*}$, and is also a function of the decay rate of the curvaton Γ_{σ_c}, and m_{σ_c}. As such, going back to (8), it follows that $N_a = N_{\sigma_c} = Q_c$ for the curvaton, where $Q_{\sigma_c} \equiv \frac{\partial Q}{\partial \sigma_c}$. For the inflaton with some potential $V(\phi)$, I have that $N_a = N_\phi \equiv \frac{1}{M_{\rm pl}^2} \frac{V}{V_\phi}$, where $V_\phi \equiv \frac{\partial V}{\partial \phi}$, which simply comes from the standard calculation of solo inflaton e-folds [2]. Plugging these different N_a terms into (2) to obtain a ζ (to first order) for each field, the same process of (3) - (8) can be applied (after adding the contribution of each field to the overall power spectrum, and assuming that the curvaton is light) to obtain

$$\boxed{\mathcal{P}_{\zeta,c} = \left(\frac{1}{M_{\rm pl}^4}\frac{V^2}{V_\phi^2} + Q_{\sigma_c}^2\right)\left(\frac{H_*}{2\pi}\right)^2}, \tag{9}$$

the primordial power spectrum for the mixed inflaton-curvaton scenario.

From (9), the spectral index $n_{s,c}$, its running $n_{{\rm run},c}$, and the tensor-to-scalar ratio r_c can finally be obtained, which are parameters that can be constrained by observations. By definition, using $k = aH_*$,

$$n_{s,c} - 1 \equiv \frac{{\rm d}\ln\mathcal{P}_{\zeta,c}}{{\rm d}\ln k} = \frac{k}{\mathcal{P}_{\zeta,c}}\frac{{\rm d}\mathcal{P}_{\zeta,c}}{{\rm d}k} = \frac{k}{\mathcal{P}_{\zeta,c}}\left\{\frac{1}{M_{\rm pl}^4}\frac{\rm d}{{\rm d}k}\left[\frac{V^2}{V_\phi^2}\left(\frac{H_*}{2\pi}\right)^2\right] + \frac{\rm d}{{\rm d}k}\left[Q_{\sigma_c}^2\left(\frac{H_*}{2\pi}\right)^2\right]\right\}. \tag{10}$$

This is further simplified using chain-rule relations such as $\frac{{\rm d}V}{{\rm d}k} = \frac{{\rm d}\phi}{{\rm d}k}V_\phi$, $\frac{{\rm d}V_\phi}{{\rm d}k} \equiv \frac{{\rm d}\phi}{{\rm d}k}V_{\phi\phi}$, and $\frac{{\rm d}H_*}{{\rm d}k} = \frac{1}{a}$. After doing the algebra and defining the first-order slow-roll parameters (SRP) as

$$\epsilon_V \equiv \frac{1}{2}M_{\rm pl}^2\left(\frac{V_\phi}{V}\right)^2 \qquad \eta_V \equiv M_{\rm pl}^2\frac{V_{\phi\phi}}{V}, \tag{11}$$

an expression for the spectral index can be obtained [2] (to first order in SRP) in terms of the SRP,

$$n_{s,c} - 1 = -2\epsilon_V - \frac{4\epsilon_V - 2\eta_V}{1 + 2\epsilon_V M_{\rm pl}^2 Q_{\sigma_c}^2}. \tag{12}$$

The running of the spectral index is defined as

$$n_{{\rm run},c} = \frac{{\rm d}n_{s,c}}{{\rm d}\ln k} = k\frac{{\rm d}n_{s,c}}{{\rm d}k}, \tag{13}$$

and can be obtained using a similar process, where the SRP parameters within $n_{s,c}$ can be differentiated with respect to k using their definitions and the chain-rule relations used in (10). This gives

$$n_{{\rm run},c} = -4\epsilon_V(2\epsilon_V - \eta_V) - \frac{2\left[8\epsilon_V^2 - 6\epsilon_V\eta_V + 2\epsilon_V(2\epsilon_V\eta_V - 2\eta_V^2)M_{\rm pl}^2 Q_{\sigma_c}^2\right]}{\left(1 + 2\epsilon_V M_{\rm pl}^2 Q_{\sigma_c}^2\right)^2} \quad [2]. \tag{14}$$

Now, to find r_c, note that during inflation, *tensor* perturbation modes are generated following a power spectrum of

$$\mathcal{P}_T \equiv \frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2 \quad [2]. \tag{15}$$

The tensor to scalar ratio is defined to be the ratio between power spectra of tensor and scalar perturbations, so I find that (using the definition for ϵ_V)

$$r_c \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\zeta,c}} = \frac{\frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2}{\left(\frac{1}{M_{\rm pl}^4}\frac{V^2}{V_\phi^2} + Q_{\sigma_c}^2\right)\left(\frac{H_*}{2\pi}\right)^2} = \frac{\frac{8}{M_{\rm pl}^2}}{\frac{1}{2M_{\rm pl}^2\epsilon_V} + Q_{\sigma_c}^2} = \frac{16\epsilon_V}{1 + 2\epsilon_V M_{\rm pl}^2 Q_{\sigma_c}^2}. \tag{16}$$

These results will be considered in section 4.

3 The Modulated Reheating Scenario

The second spectator field scenario that I will consider is known as the *modulated reheating scenario* [3]. In traditional inflation theory, the inflaton decays uniformly during reheating; the crux of this theory is that the inflaton can have a spatially modulating decay rate if an additional so-called *modulus* spectator field, denoted with σ_m, is present (with an energy density much lower than that of the inflaton).

To begin, I'll use the same δN formalism presented in equations (1) - (8) (which can still be applied because I did not specify the spectator field being the curvaton until section 2), and apply (8) to σ_m to obtain

$$\mathcal{P}_{\zeta,m} = \left(\frac{\partial N}{\partial \sigma_{m,*}}\frac{H_*}{2\pi}\right)^2, \tag{17}$$

where N denotes the number of e-folds, H_* is the value of H upon horizon entry, and I assume that the modulus field is light. To proceed, I will assume that the Universe transitions from inflaton oscillation to radiation dominated instantaneously, i.e. it enters reheating instantly, following [3]. Now, in order to obtain an expression for N, I will follow the definition of N below equation (1), and partition time with milestones denoted by $*$, denoting horizon entry, *end*, denoting the end of inflation, and *reh*, denoting the beginning of reheating. This gives

$$N(t_f, t_*, \vec{x}) = \int_{t_*}^{t_{\rm end}} H\,\mathrm{d}t + \int_{t_{\rm end}}^{t_f} H\,\mathrm{d}t = \int_{t_*}^{t_{\rm end}} H\,\mathrm{d}t + \int_{t_{\rm end}}^{t_f} H\frac{\mathrm{d}\rho}{\mathrm{d}\rho}\,\mathrm{d}t \equiv \int_{t_*}^{t_{\rm end}} H\,\mathrm{d}t + \int_{\rho_{\rm end}}^{\rho_f} \frac{H}{\dot{\rho}}\,\mathrm{d}\rho, \tag{18}$$

with ρ the energy density and f again denoting some final time (as in (1)). Defining $N_* = \int_{t_*}^{t_{\rm end}} H\,\mathrm{d}t$ (the number of e-folds between t_* and $t_{\rm end}$), the rightmost integral can be split up into the two eras of inflaton (denoted ϕ) domination and radiation (denoted γ) domination, such that

$$N(t_f, t_*, \vec{x}) = N_* + \int_{\rho_{\rm end}}^{\rho_{\rm reh}} \frac{H}{\dot{\rho}_\phi}\,\mathrm{d}\rho_\phi + \int_{\rho_{\rm reh}}^{\rho_f} \frac{H}{\dot{\rho}_\gamma}\,\mathrm{d}\rho_\gamma. \tag{19}$$

Next, given the energy conservation equation $\dot{\rho} + 3H(1+w)\rho = 0$ derived from Einstein's equations (where w is the equation of state parameter for whichever species dominates the energy density), I obtain $\dot{\rho}_\phi = -3\rho_\phi H$ and $\dot{\rho}_\gamma = -4\rho_\gamma H$ (given that the inflaton behaves as matter while oscillating [3]) during the respective eras. This result can be plugged into (19) to obtain

$$N(t_f, t_*, \vec{x}) = N_* - \int_{\rho_{\rm end}}^{\rho_{\rm reh}} \frac{\mathrm{d}\rho_\phi}{3\rho_\phi} - \int_{\rho_{\rm reh}}^{\rho_f} \frac{\mathrm{d}\rho_\gamma}{4\rho_\gamma} = N_* - \frac{1}{3}\ln\frac{\rho_{\rm reh}}{\rho_{\rm end}} - \frac{1}{4}\ln\frac{\rho_f}{\rho_{\rm reh}}, \tag{20}$$

which can then be simplified using Friedmann's first equation ($\rho \propto H^2$) to get

$$N(t_f, t_*, \vec{x}) = N_* - \frac{1}{6}\ln\frac{H(t_{\rm reh})}{H(t_{\rm end})} - \frac{1}{2}\ln\frac{H(t_f)}{H(t_{\rm reh})} \equiv N_* - \frac{1}{6}\ln\frac{\Gamma(\sigma_{m,{\rm reh}})}{\beta H(t_{\rm end})} - \frac{1}{2}\ln\frac{H(t_f)}{H(t_{\rm reh})}, \tag{21}$$

where I have adopted the notation of [3] to represent the decay rate of the *inflaton* with $\Gamma(\sigma_m)$, and used $\beta \equiv \Gamma(\sigma_{m,{\rm reh}})/H(t_{\rm reh})$ as a constant near unity. From here, it is easy to find that (using the chain rule and the fact that N only depends on the modulus field through $\Gamma(\sigma_{m,\mathrm{reh}})$),

$$\frac{\partial N}{\partial \sigma_{m,*}} = \frac{\partial \sigma_{m,\mathrm{reh}}}{\partial \sigma_{m,*}} \frac{\partial N}{\partial \sigma_{m,\mathrm{reh}}} = \frac{\partial \sigma_{m,\mathrm{reh}}}{\partial \sigma_{m,*}} \left(-\frac{1}{6}\right) \frac{\beta H(t_{\mathrm{end}})}{\Gamma(\sigma_{m,\mathrm{reh}})} \frac{\Gamma'(\sigma_{m,\mathrm{reh}})}{\beta H(t_{\mathrm{end}})} = -\frac{1}{6} \frac{\partial \sigma_{m,\mathrm{reh}}}{\partial \sigma_{m,*}} \frac{\Gamma'(\sigma_{m,\mathrm{reh}})}{\Gamma(\sigma_{m,\mathrm{reh}})}, \quad (22)$$

where $'$ denotes differentiation with respect to σ_m. Now, I can finally obtain the full formula for the primordial power spectrum of the pertubations due to the inclusion of the modulus field (using (17)),

$$\mathcal{P}_{\zeta,m} = \left(\frac{1}{6} \frac{\Gamma'(\sigma_{m,\mathrm{reh}})}{\Gamma(\sigma_{m,\mathrm{reh}})} \frac{\partial \sigma_{m,\mathrm{reh}}}{\partial \sigma_{m,*}} \frac{H_*}{2\pi} \right)^2 . \quad (23)$$

In order to obtain a more explicit expression for the general $\mathcal{P}_{\zeta,m}$ than (23), the derivative within (23) needs to be evaluated. Going through the technical specifics are beyond the scope of this work, so I will go over everything schematically, following [3]. In summary, the equation of motion governing σ_m is approximated based on the assumption that the modulus is light (compared to the inflaton), and is then integrated to obtain

$$\boxed{\mathcal{P}_{\zeta,m} = \left[\frac{1}{6} \frac{\Gamma'(\sigma_{m,\mathrm{reh}})}{\Gamma(\sigma_{m,\mathrm{reh}})} \frac{H_*}{2\pi} \right]^2 \left[\frac{1}{1 - X(\sigma_{m,\mathrm{reh}})} \frac{V'(\sigma_{m,\mathrm{reh}})}{V'(\sigma_{m,*})} \right]^2}, \quad (24)$$

where

$$X(\sigma_{m,\mathrm{reh}}) \equiv -\frac{2\beta}{3} \frac{\dot{\Gamma}(\sigma_{m,\mathrm{reh}})}{\Gamma(\sigma_{m,\mathrm{reh}})^2}, \quad (25)$$

$V(\sigma_m)$ is the modulus potential. Now, the spectral index $n_{s,m}$ and its running $n_{\mathrm{run},m}$ can be computed as in section 2, as

$$n_{s,m} - 1 \equiv \frac{\mathrm{d} \ln \mathcal{P}_{\zeta,m}}{\mathrm{d} \ln k} = \frac{k}{\mathcal{P}_{\zeta,m}} \frac{\mathrm{d}\mathcal{P}_{\zeta,m}}{\mathrm{d}k}, \quad (26)$$

$$n_{\mathrm{run},m} = \frac{\mathrm{d} n_{s,m}}{\mathrm{d} \ln k} = k \frac{\mathrm{d} n_{s,m}}{\mathrm{d}k}. \quad (27)$$

The expressions for $n_{s,m}$ and $n_{\mathrm{run},m}$ can be further simplified as in section 2 with chain-rule relations, but it turns out that these two parameters are not at all affected by the dynamics of the modulus *or* the inflaton (following horizon crossing), creating no change to these observables from the single inflaton scenario [3].

4 Observational Implications and Discussion

One of the most important results of the curvaton theory is that inflaton potentials that were previously all but ruled out by observations, such as chaotic quadratic, quartic and even sextic models, can produce perturbations that match observational data if alongside a curvaton during inflation [2]. As shown in (12) and (16), the presence of a curvaton (and therefore some Q_{σ_c}) modifies the parameters $n_{s,c}$ and r_c, which can be constrained with observations, and given certain values of ϵ_V and Q_{σ_c} determined by the mixed-field model, these previously ruled-out inflaton models can be relaxed to produce n_s and r that are within reasonable confidence for current data [2]. A strong example of this is given by [4] and shown in Fig. 1, left, which illustrates how for a mixed (quartic) inflaton and curvaton scenario, the observable parameters r and n_s can be noticeably affected by the introduction of the curvaton.

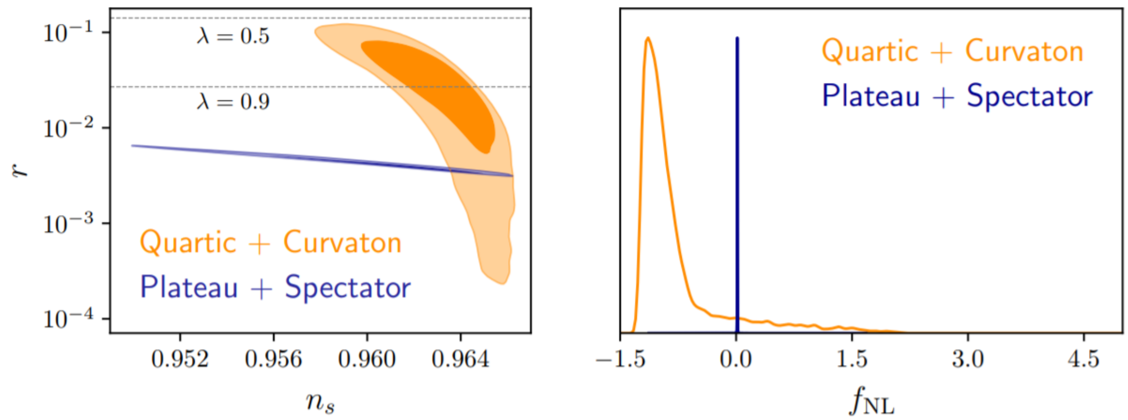

Figure 1: Left: Parameter space plot of the marginal posterior probability distributions of tensor-to-scalar ratio r vs. spectral index n_s for two models, the relevant one being in orange, which is for a quartic inflaton and curvaton model. Right: probability distribution of the bispectrum non-linearity parameter $f_{\rm NL}$ for the two models. Figure from [4].

Now, consider $\mathcal{P}_{\zeta,m}$ (24), and the term $X(\sigma_{m,\rm reh})$ (25) in the denominator. This $X(\sigma_{m,\rm reh})$ term is the only factor within $\mathcal{P}_{\zeta,m}$ that involves the time derivative of $\Gamma(\sigma_m)$, and as such, any dynamics in the modulus field will show up in this term, and modify $\mathcal{P}_{\zeta,m}$. In [3], it is shown that specifically, if $\dot{\Gamma}(\sigma_m)$ is high during reheating, i.e. that the decay rate of the inflaton is quickly changing, and/or the effect mass of the modulus $V''(\sigma_m)$ changes between inflation and reheating, the power spectrum can be noticeably suppressed, following (24) and (25), which could be easily seen in observations [3].

Another topic to consider, although not the primary focus of this paper, is the non-Gaussianity of observed perturbations that can be generated by the inclusion of such spectator fields. Such deviation from linearity is much higher than what could be attributed to a single-field inflaton theory, which can only produce little to no non-Gaussianity [2]. One of the primary quantifiers of non-Gaussianity is $f_{\rm NL}$, a term derived from the *bispectrum* of primordial perturbations, which is essentially the power spectrum to one higher order (i.e. where the power spectrum is equivalent to a two-point correlation function of angularly separated observations, the bispectrum is equivalent to a three-point correlation function). As such, a nonzero $f_{\rm NL}$ indicates non-Gaussianity in the perturbations, with a higher magnitude of $f_{\rm NL}$ corresponding to a stronger degree of non-Gaussianity. Both [2] and [3] consider the effect of their respective spectator fields on the bispectrum as well as the power spectrum, and derive the $f_{\rm NL}$ generated in the two scenarios. In the modulus field scenario, $f_{\rm NL}$ is found to be heavily dependent on the dynamics of the modulus, similar to the effect of the modulus on $\mathcal{P}_{\zeta,m}$, so much that some models can generate non-Gaussianity so high that it could be all but excluded by recent observations; specifically, $|f_{\rm NL}| \gtrsim 10$, while Planck has measured only $f_{\mathrm{NL}} = 2.7 \pm 5.8$ [1][3]. Similarly, in the mixed curvaton-inflaton scenario, [2] found that $f_{\mathrm{NL}} \gg 1$ can be generated, even if the contribution of the overall perturbations from the curvaton is on order of or smaller than the perturbations sourced by the inflaton.

Conclusion

In this work, I considered the effects of introducing an additional spectator scalar field alongside the inflaton during inflation and/or reheating. Such spectator fields can produce noticeable changes to observables such as power spectra, spectral indices, and tensor-to-scalar ratios, which in some scenarios, can bring certain models of the inflaton potential that were previously ruled out in single-field inflation theory by observations back into the realm of possibility. These spectator field scenarios can also create noticeable non-Gaussianity of observed perturbations, even when the contribution of the spectator field to the overall perturbations are small compared to that of the inflaton.

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