0.0.1 Star Formation, Black Hole Types, Hawking Radiation

Jeans Mass
The first criterion for gas cloud collapse:
$M > \frac{5R}{G}\frac{k_B T}{\overline{m}} \equiv M_{\text{jeans}}$
($\overline{m}$ is avg. gas particle mass, R is cloud radius)
$R_{\text{jeans}} = \left(\frac{15k_bT}{4\pi G\overline{m}\rho}\right)^{1/2} = \left(\frac{3M_{\text{jeans}}}{4\pi\rho}\right)^{1/3}$
Fragmentation
For some gas cloud above M_{j}, as cloud collapses,
$\rho\uparrow$, so $M_{\text{j}}\downarrow$, so now regions of the cloud have $M > M_{\text{j}}$
This cycle repeats for smaller and smaller subregions,
Only if T is constant
Once ρ increases enough, $l_{\text{mfp}}\downarrow$: cloud can't cool, then
$T\uparrow$ means $M_j\uparrow$; fragmentation stops
this is one reason why stars form in groups

Star Birth and Getting on the Main Sequence
1) Dark cloud cores, then gravitational collapse
2) Clump of gas becomes protostar, then T-Tauri
3a) Pre MS Star: If massive enough, collapses to begin fusion
3b) If not massive enough, stays as a brown dwarf
the more massive the star, the faster it forms
stars can take 10s of thous to 10s of mil yrs to land on MS

Star Clusters
Open Clusters: (stellar nurseries): mostly MS stars,
drift apart as supernovae start popping off
Globular Clusters: Dense, bound for life
rare star-star collisions thought to form blue stragglers
Metallicity: What the star formed from

Gas Cooling Mechanisms:
- Dissociation of molecular H, - Ionization of Hydrogen
- Hydrogen recombination (Balmer and lower E γ
can escape (not too high E that would just
be absorbed by surrounding H)
- Metals: molecular lines, free emission
The more metallic, the better at cooling, the more
fragmentation, smaller stars form
Diff cooling mechanisms happen at diff temps:
From low T to high, in order:
molecular line emission, low ionization metals,
H & He recomb., highly ionized metals,
free-free (deceleration of a charged particle deflected by
other charged particle; loses KE as photon)

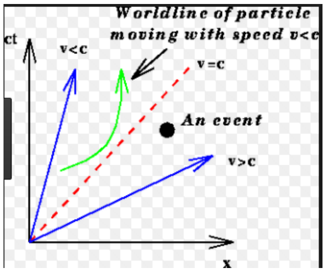
Types of Black Holes
Stellar Mass: form by core collapse SN, NS mass transfers
and mergers ($\sim 10M_\odot$)
Supermassive: formed by merging, and acreetion, found in
centers of galalxies and grow over cosmic time (mils of $M_\odot$)
Intermediate: 1000's of $M_\odot$, tentative evidence
Primordial: no clear evidence

Hawking Radiation
in empty space, particle-antiparticle pairs
exist for instant before annihilating into energy
near event horizon, one can fall in, as the other escapes,
but energy debt must be paid, so BH loses mass.
Therefore, BHs have temps and BB spectrum:
$T = 6.2\times10^{-8}\ \text{K}\times\frac{M_\odot}{M}$
Black holes evaporate if hotter than background CMB T:
($M < 0.008M_{\text{earth}}$)

IMPORTANT: $M(<r) = \int_0^r \rho(r')\,\text{d}V$, $\text{d}V = 4\pi r'^2\,\text{d}r'$
see interstellar gas visibly with:
diffuse cool gas: reflection/scattering
dense cool gas: extinction/reddening (objects appear redder)
hot gas: emits lines (red: Hα)

0.0.2 Relativity and Accretion Disks

Einstein's Postulates (1915 (GR))
1) Physics is the same in all ref frames that move at a const vel
1b) The speed of light in vacuum is same in all ref frames
2) **A freefall frame in a grav field is indist. from an iner. frame in no grav. field**
2b) An accelerating frame is indist. from a frame at rest in a grav field with g=a
Spacetime Diagrams



units: s and ls

Spacetime Interval
All observers agree on Δs
$(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x^2 + \Delta y^2 + \Delta z^2)$
Proper Time: τ: time between 2 events
as seen by obs who sees them happen
in same place
$(\Delta s)^2 < 0$: timelike
$(\Delta s)^2 = 0$: null, or lightlike
$(\Delta s)^2 > 0$: spacelike

Gravitational Redshift (less rigorous)
$z = \frac{\Delta\lambda}{\lambda_{\text{em}}} = \frac{-v}{c} = \frac{-g\Delta t}{c} = \frac{\Delta\lambda}{\lambda_{\text{em}}} = \frac{gh}{c^2}$
h is d traveled by γ
according to outside observer
redshifted as it exits grav well

SC metric: $\text{d}s^2 = -c^2\,\text{d}t^2\left(1-\frac{2GM}{rc^2}\right) + \text{d}r^2\left(1-\frac{2GM}{rc^2}\right)^{-1} + r^2\,\text{d}\theta^2 + r^2\sin^2\theta\,\text{d}\phi^2$
In flat spacetime: $\text{d}s^2 = -c^2\,\text{d}t^2 + \text{d}r^2 + r^2\,\text{d}\theta^2 + r^2\sin^2\theta\,\text{d}\phi^2$
Gravitational Time Dilation: $\text{d}\tau = \text{d}t\sqrt{1-\frac{2GM}{rc^2}}$
Where $\text{d}t$ is time seen by somebody infinitely far away.

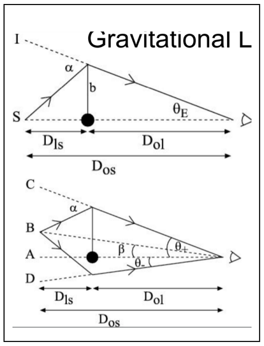
Grav Redshift: $\lambda_{\text{obs}} = \lambda_{\text{em}}\left(1-\frac{2GM}{rc^2}\right)^{-1/2}$ $\quad \frac{\Delta\lambda}{\lambda_{\text{em}}} = \frac{\sqrt{1-\frac{2GM}{r_2c^2}}}{\sqrt{1-\frac{2GM}{r_1c^2}}} - 1$

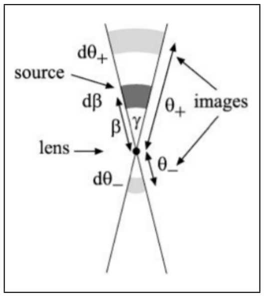
SC Radius (event hori): $R_s = \frac{2GM}{c^2}$, lowest possible STABLE orbit: $r > \frac{3GM}{c^2}$

Accretion Disks
From MFM, as $M(<r) \simeq M_{BH}$: $v_{\text{rot}} \simeq \sqrt{GM/r}$
For ring of disk: $T = r^{-3/4}\times\left(\frac{GM\frac{\text{d}M}{\text{d}t}}{8\pi\sigma_{SB}}\right)^{1/4}$ Where M is BH Mass
Lotal disk luminosity: $L_{\text{tot}} \simeq \frac{1}{2}\frac{GM\frac{\text{d}M}{\text{d}t}}{r_{\text{inner}}}$
Where r_{inner} is the innermost stable orbit ($3R_S$ for BH)
efficiency: how much of M going in is being radiated away: $\eta = \frac{L_{\text{tot}}}{\frac{\text{d}M}{\text{d}t}c^2} = \frac{1}{2}\frac{GM}{r_{\text{inner}}c^2}$
Eddington Luminosity Limit: $L_{\text{tot, max}} = \frac{4\pi cGMm_{\text{proton}}}{\sigma_T}$
$\sigma_T = 6.65\times10^{-29}\ \text{m}^2$, $m_p = 1.673\times10^{-27}$ kg

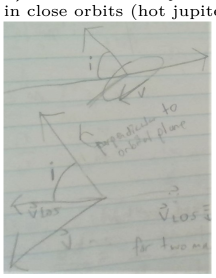
0.0.3 Gravitational Lensing and Finding Planets

For top right situation (aligned):
$\alpha = \frac{4GM}{c^2b}$ (Where b is dist of closest approach)
$\alpha D_{ls} = \theta D_{os}$
angular disk radius: $\theta_E = \left(\frac{4GM}{c^2}\frac{D_{\text{ls}}}{D_{ol}D_{os}}\right)^{1/2}$
Maybe: $\theta_E = 10^{-4}$ as $\left(\frac{M}{M_\odot}\right)^{1/2}\times\left(\frac{D_{ol}}{10\ \text{kpc}}\right)^{1/2}$
For bottom right situation (nonaligned):
$D_{os}\beta + D_{is}\alpha = D_{os}\theta \rightarrow \theta^2 - \beta\theta - \theta_E^2 = 0$
$\theta_\pm = \frac{1}{2}\left[\beta \pm (\beta^2 + 4\theta_E^2)^{1/2}\right]$
Lensing: brightness$\uparrow$; more light focused on us
Magnification (rightmost pic):
Relative increase in angular size of each image (+ and -):
$a_\pm = \frac{\text{img angular size}}{\text{source angular size}} = \frac{\theta_\pm\,\text{d}\theta_\pm}{\beta\,\text{d}\beta} = \frac{\theta_\pm}{2\beta}\left[1 \pm \frac{\beta}{(\beta^2+4\theta_E^2)^{1/2}}\right]$
$a_{\text{tot}} = a_+ + a_- = \frac{u^2+2}{u(u^2+4)^{1/2}}$, where $u = \beta/\theta_E$
For small u, $a_{\text{tot}} \simeq 1/u = \theta_E/\beta$ $\qquad \frac{\tau_{\text{star}}}{\tau_{\text{planet}}} \simeq \sqrt{\frac{M_{\text{star}}}{M_{\text{planet}}}}$
Duration of lensing event: $\tau = \frac{\theta_E D_{ol}}{v} \propto M_{\text{planet}}$ where v is
v b/w source and lens (planet) in the plane of the lens





1) Microlensing: find rockey planets
w/ long period: one time only
2) Radial Velocity of star: best finding massive plans
in close orbits (hot jupiters) (didn't form that close)



$M_{\text{planet}}\sin i \approx \left(\frac{\tau}{2\pi G}\right)^{1/3}\times|v_{1,\text{ obs}}|M_{\text{star}}$
Where τ is orbital period, i is shown in pic
3) Transit Detection: $\frac{\Delta f}{f} = \left(\frac{r_{\text{planet}}}{r_{\text{star}}}\right)^2$, $P = \frac{r_* + r_p}{d_{(p,*)}}$
4) Direct Imaging:
Only possible for big, young planets, far away from star

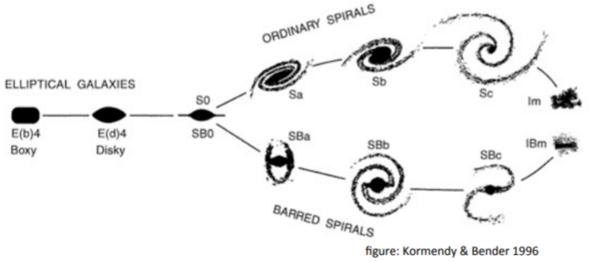
0.0.4 Galaxies

General Galaxy Stuff $\quad \rho \propto 1/r^2$
From Virial Thm, for ell. gals and gal clusters:
$M = \frac{5\sigma_{\text{LOS}}^2R}{G} \qquad \sigma_{\text{LOS}}^2 = \sum_N \frac{1}{N}(v_i - \overline{v})^2$

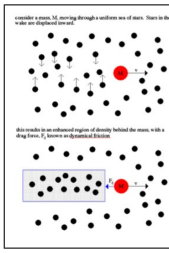
Active Galactic Nuclei
Size of the solar system
gas moving relativistically
center has Supermassive Black Hole
Emit mostly x-ray and radio (unlike stars)
Eddington Luminosity applies
AGN Variability: $R < c\Delta t \simeq 37.5$ AU
AGN Jets: extend up to one Mpc
emits at all wavelengths, peaks in IR, UV
Types:
Seyfert Galaxies: $10^8 - 10^{11}\ L_\odot$
Quasars: $3\times10^{10} - 4.3\times10^{14}\ L_\odot$
Some Non-AGN galaxies have SMBHs
(non-accreting) But not all gals has BHs
SMBH mass correlates to central bulge mass
(but SMBH does not effect gal structure)
$\uparrow$ due to merging galaxies merging SMBHs too

Isolated Galaxy Formation
(Galaxies don't really form in isolation)
1) 1st stars form spheroid: Es, S0s
tend to have old stellar populations
2) Gas comes in, forms disk as it is
the config of lowest energy that maintains
$\vec{L}$. New stars form from disk gas
Galactic Mergers
Steps: 1) random dispersion of stars
sphereoids develop (violent relaxation)
Dark Matter increases "target size"
Dynamical Friction helps galaxies merge.
More dyn fric if: slower relative
motion, greater satellite mass, larger
DM halos (mergers start when halos touch)
Gas flows inward due to tidal forces, new
stars form in huge burst due to replenished
gas, now a lot of stars in center
Most star form. was 10-11 Gyr ago
Galaxy Clusters: filled w hot gas,
gravitational lensing implies mass, all
indicates lots of DM. mostly elliptical gal
because of many mergers

Classifying Galaxies: Hubble "Sequence"



Left to Right: Gas infall & Outer disk growth, less spheroid,
more thin disk, more rotation, more cold gas & young stars, $\therefore$ bluer
Right to Left: Galaxy interactions (bulge/inner disk growth)
Ellipticals: Red and Dead; Irr: Most common, most gas vs. stars
Spiral: Most star formation (blue light means young stars)
Ellipticals: "early type" (misnomer)



0.0.5 Cosmology, the Big Bang, Dark Matter, and more Quasars

More on Quasars
1) Quasars are SMBH perpetually at Edd. Lum. 2) Quasars are not near MW b/c they were only common during first few Gyr after the BB because they require a lot of gas & merger activity.
3) Now, gas is mostly used up and mergers are less common
4) Not all are strong radio sources

Star Cosmology Players:
Hubble (extra-gal distances)
Leavitt (Cephied Period-L relation)
Slipher (Redshifts of "spiral nebulae")

The Evolving Universe
Hubble's Law: $v \simeq H_0 d$ $\quad H_0 \simeq 70$ km/(s Mpc)
$z = \frac{\text{dist today}}{\text{dist at emission}} - 1 = \frac{\Delta\lambda}{\lambda} \simeq \frac{v}{c}$
ABOVE ARE ONLY VIABLE AT SMALL z
$d = a(t)x$ Where d is actual distance b/w two things, x is their constant comoving distance
$H = \frac{\dot{a}(t)}{a(t)}$ $\quad H_0$ is evaluated at t_0, today; $a(t_0) = 1$
So, as H_0 is always increasing, so is $\dot{a}(t_0)$. (a-DOT)
Cosmol. Redshift:
$\frac{c\delta t_{obs}}{c\delta t_{em}} = \frac{a(t_{\text{obs}})}{a(t_{\text{em}})} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = 1 + z > 1$
a(t) is the const. of proportionality b/w comoving dists & actual proper dists.
If some a is 5 at some time, that means the object was 1/5 the distance it is today back then
most star formation was 10-11 Gyr ago

The Geometry of the Universe
Observations & Cosmo Principle imply the universe is homogenous and isotropic (no special positions are directions)
FRW Metric: $ds^2 = -c^2\,dt^2 + a^2(t)[\text{geometry of space}]$
$ds^2 = -c^2\,dt^2 + a^2(t)[dx^2 + dy^2 + dz^2]$ **Where dx, dy, dz are all comoving dist.**
Spherical Coords: $ds^2 = -c^2\,dt^2 + a^2(t)[dr^2 + r^2\sin^2\theta\,d\phi^2 + r^2\,d\theta^2]$

Possible Universe Geometries
ρ_{crit} is the total energy density needed for the universe to be flat
Closed (Spherical): $\rho_{\text{tot}} > \rho_{\text{crit}}$, parallel lines cross, triangle angles $> 180°$, will collapse: big crunch (this could be altered by DE though)
Open (Hyperbolic): $\rho_{\text{tot}} < \rho_{\text{crit}}$, parallel lines diverge, triangle angle $< 180°$, will expand forever (not altered by DE)
Flat: $\rho_{\text{tot}} = \rho_{\text{crit}}$, parallel lines stay parallel, triangle angles $= 180°$, will expand forever, but at slower and slower rate (altered by DE)
Evidence points to this geometry
Today: $\rho_{\text{matter}} = 0.3\rho_{\text{crit}}$

The Friedmann Equation (In a flat universe)
$\boxed{\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho_{\text{tot}}}$, $\rho_{\text{tot}} \equiv \rho_{\text{crit}}$ (either/both are not fixed with time, evaluated at a chosen H)
$\rho_{\text{crit},0}$ is simply evaluated at today (t_0,) H_0.
$\rho_{\text{crit},0} = 9 \times 10^{-27} kg/m^3$ $\Omega_i \equiv \frac{\rho_{\text{i},0}}{\rho_{\text{crit}}}$
for $i \in$ {matter, radiation, dark energy(Λ)}
So Ω_i is ALWAYS evaluated at today
today, $\Omega_{\text{m}} \simeq 0.3, \Omega_{\text{de}} \simeq 0.7, \Omega_{\text{r}} \simeq 9 \times 10^{-5}$
Second Friedmann Equation: (double dot a in numer.)
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\text{tot}} + \frac{3P}{c^2}\right)$ Where P is pressure
$\rho_{\text{m}} \propto \frac{1}{a^3}$, $\rho_{\text{r}} \propto T^4$, $T \propto \frac{1}{a} \propto (1+z)$
Density and More
General Case: $\rho \propto a^{-3(1+\omega)}$, $\omega_m = 0, \omega_r = \frac{1}{3}, \omega_{de} = -1$
ω is the eqn. of state parameter: $\omega = \frac{P}{c^2\rho}$
From this, we find: $\rho_{\text{de}} \simeq$ constant,
$\rho_m = \rho_{(m,0)}a^{-3} \to \rho_m \propto a^{-3}$
$\rho_r = \rho_{(r,0)}a^{-4} \to \rho_r \propto a^{-4}$
From homework, we found:
$H(t) = H_0\sqrt{\Omega_m a(t)^{-3} + \Omega_r a(t)^{-4} + \Omega_{\text{de}}}$
Calculating Age of Universe:
$t_0 = \int_0^{a(t_0)} \frac{da'}{\dot{a}'} = \int_0^1 \frac{da'}{a'H(a')}$
For radiation only universe: $H = H_0 a^{-2}$
Calculating Size of (Observable) Universe:
(derived from FRW metric for some γ traveling to now)
radius$= c\int_{t_{em}}^{t_0} \frac{dt'}{a} = c\int_{a_{em}}^{1} \frac{da'}{a'^2 H(a')}$
Where ' just indicates dummy variables

The Origins of Matter:
γ's $\leftrightarrow e^+, e^-$ if $E_\gamma = h\nu > E_{e^+/e^-} = mc^2$
$e^+ ande^-$ continuously annihilate
Equilibrium stops when radiation too cold but annihilation contiues, and fast
Freeze-out: annihilations stop b/c the partics. are dilute enough; b/c so few partics. left, and b/c the expans. of the universe
Matter-Antimatter Asymmetry
Most likely because of some unexplained matter-antimatter asymmetry (like 1,000,000 vs 1,000,001)
Dark Matter Origins:
The freeze-out process predict the observed DM density if DM particles interact via Weak F.
IF DM was produced by freeze-out, (only possible if it has antiparticles,) then there should be equal amts of DM and DantiM If both DM and M freeze out, they won't necessarily have same density today, as freezeout is dependent on MFP of particles annihilating

Big Bang Nucleosynthesis
NOT full atoms, just nuclei
$T = 10^9$ K $\quad 2p^+ + 2n^0 \to 2D^2 \to \text{He}^4$
$t_0 = 10$ s, lasted about 200 s
Universe was hot and dense enough to trigger this fusion, but cool enough:
$E_\gamma = k_b T = 0.08$ MeV, $E_{\text{deuterium}} = -2.22eV$
Universe had to be cool enough for so not even one in a billion photons (due to freeze-out) would have enough energy to destroy deuterium D
That fusion only possible with free neutrons, which decay quickly, so doesn't last long; this is why no elements heavier than Be formed; window was too short before too cool
observations of primordial He and D show that universe is only 4% atoms

The Cosmic Microwave Background
He, p^+, e^- became atoms
Surface of Last Scatter: 380,000 yrs after BB, when T = 3000 K, hydrogen atoms formed, after which photons could travel freely:
$l_{mfp} = \left(\frac{n_e}{n_{e\text{ free}}}\right) \times \frac{7\times10^{28}\text{ m}}{(1+z)^3}$
Because more of the free electrons joined nuclei (low enough energy free γ to not ionize atoms) so now universe was transparent from then on
$l_{universe} \approx \frac{c}{H}$
Our motion actually redshifts & blueshifts the CMB
CMB is the perfect BB radiator, T=2.725 confirms hot big bang. there are very tine mK flucs. for in this

Dark Matter Candidates
WIMPS are leading candidate; leading candidate for WIMP: SUSY fermion partners of Weak force carriers (partners of W, Z, Higgs bosons). Other proposals struggle to explain DM's current density
MACHOS: They cant be DM (at least more than 10% of it). Looking to the DM halo, MACHOS would produce much more gravitational microlensing than is actually seen, if it was all MACHOS.
Neutrinos: ruled out because they're too hot: the large scale structure that neutrionis would make is not what is observed.
Ultra-cold H2 globules: DM can't be made out of regular matter
MOND: Can explain some things, but not everything.
Still continues to not die.
1° disks in CMB, casually separated by photon path since BB; yet, still all have almost exactly same temp

0.0.6 Misc Info, Derivations

Misc Info
HII Regions: when O, B stars form, they emit radiation that can ionize surrounding gas, making HII region
$\rho/n = \overline{m}$ $\quad g_{surf} \simeq \frac{GM}{R^2}$
Press. broad comparing 2 stars: $\frac{\Delta\lambda_1}{\Delta\lambda_2} = \frac{n_1}{n_2} = \frac{P_1/kT_1}{P_2/kT_2}$
$N = M/\overline{m}$, Metallicity is frac. that's not H or He

Roche Lobe, Tidal:
$\frac{M_1}{(a-R_L)^2} - \frac{M_2}{R_L^2} = \frac{M_1}{a^2} - \frac{R_L(M_1+M_2)}{a^3}$
$R_L \simeq a\left(\frac{M_2}{3M_1}\right)^{1/3}$ $\quad F_{\text{tide}} \simeq \frac{2GMmh}{r^3}$
Jean's Mass Deriv:
From V thm: collapse if $U > 2K$: $\frac{3GM^2}{5R} > 2 \times \frac{3Mk_bT}{2m}$
Accretion Disk Temp:
For falling dm: $\Delta U_{\text{grav}} = \frac{-GM\,dm}{r^2}\,dr$
From MFM: $\Delta KE = \Delta E_{thermal}$
$\frac{1}{2}\,dm\left(\sqrt{\frac{GM}{r}}\right)^2 - \frac{1}{2}\,dm\left(\sqrt{\frac{GM}{r+dr}}\right)^2 = \frac{1}{2}\frac{GM\,dm}{r^2}\,dr$
$\frac{\Delta E_{\text{thermal}}}{dt} = \frac{1}{2}\frac{GM\,dm\,dr}{r^2\,dt}$
$\frac{\Delta E_{\text{thermal}}}{dt} = L_{\text{ring}} = \sigma_{SB}T^4(2\pi r\,dr) \times 2$
Solve these for each other to get answer.
Total Disk Luminosity:
$L_{\text{tot}} = \int_{r_{\text{inner}}}^{r_{\text{outer}}} 2(2\pi r\,dr)\sigma_{SB}T^4$
Eddington Luminosity:
Consider some e^- trying to fall into object.
There's pt where rad. press overcomes grav. attrac on e^-
$n_\gamma = \frac{L_\nu}{4\pi r^2 ch\nu}$
Scattering rate of $e^- = n_\gamma\sigma_T c$ Each scatter: $\Delta p_e = \Delta p_\gamma = \frac{h\nu}{c}$
$F_\gamma = \frac{dp}{dt}$, $F_{\gamma tot} = \frac{L_{tot}\sigma_T}{2\pi r^2 c}$; $F_\gamma > \frac{GMm_p}{r^2}$
(m_p because electrons bound to protons)

Cosmo Redshift Deriv:
For photon going towards Earth (FRW Metric):
$c^2\,dt^2 = a^2\,dr^2 \to dr = \pm c\frac{dt}{a}$
$\int_{r_{gal}}^{r=0} dr = \pm c\int_{t_{em}}^{t_{obs}} \frac{dt}{a} = c\int_{t_{em}+\delta t_{em}}^{t_{obs}+\delta t_{obs}}$
Double taylor expand on t_{em} and t_{obs}
$r_{gal} = c\int_{t_{em}}^{t_{obs}} \frac{dt}{a} + \left(\frac{-c}{a}\right)_{t_{em}}\delta t_{em} + \left(\frac{c}{a}\right)_{t_{obs}}\delta t_{obs}$
$0 = \frac{-c}{a_{em}}\delta t_{em} + \frac{c}{a_{obs}}\delta t_{obs}$
densities:
$\frac{d\rho}{dt} = \frac{3}{8\pi G}\left(2\frac{\ddot{a}}{a^2} - 2\frac{\dot{a}^2}{a^3}\right) = -3H(\rho + \frac{P}{c^2}) = -3H(1+\omega)\rho$