Basics
$H_0 = 100h \quad z = \frac{\Delta\lambda}{\lambda} \simeq \frac{v}{c}$
$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}} = 1+z \quad \vec{v} \simeq H_0\vec{d}$
$\vec{d} = a(t)\vec{x} \quad H = \frac{\dot a}{a} \quad \Omega_i = \frac{\rho_{i,0}}{\rho_c}$

Metrics and Tensors
$d\ell^2 \equiv d_{phys} = a^2(t)[d^3\vec{x}^2]$ (flat)
closed: sphere, $k=1$
curved: $d\ell^2 = \frac{a^2}{a_0^2}[d^3\vec x^2 + k\,du^2]$
$k(x^2+y^2+z^2)+u^2 = R_c^2$
$d\ell^2 = \frac{a^2}{a_0^2}[\frac{dr^2}{1-k(r/R_c)^2} + r^2 d\Omega^2]$
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2 \quad \tilde r \equiv r/R_c$
$d\chi \equiv \frac{d\tilde r}{\sqrt{1-k\tilde r^2}} \quad ds^2 = -dt^2 + d\ell^2$
$\chi \equiv \arcsin(h)\tilde r, k=(-)1 \quad \nabla_\mu g_{\alpha\beta}=0$
$ds^2 = -dt^2 + \frac{a^2}{a_0^2}[\frac{dr^2}{1-k(r/R_c)^2}+r^2 d\Omega^2]$
$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} \equiv 1+z \quad x^\mu = (t,r,\theta,\phi)$
$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ diag: $g^{\mu\nu} = 1/g_{\mu\nu}$
$x^\alpha \to \hat x^\mu : \hat T^\mu_\nu = \frac{\partial \hat x^\mu}{\partial x^\alpha}\frac{\partial x^\beta}{\partial \hat x^\nu}T^\alpha_\beta \quad V_\alpha = \frac{\partial S}{\partial x^\alpha}$
$\nabla_\mu T^\alpha_\beta = \partial_\mu T^\alpha_\beta + \Gamma^\alpha_{\mu\lambda}T^\lambda_\beta - \Gamma^\lambda_{\mu\beta}T^\alpha_\lambda$
$\Gamma^\sigma_{\alpha\beta} = \frac12 g^{\sigma\rho}(\partial_\alpha g_{\beta\rho} + \partial_\beta g_{\alpha\rho} - \partial_\rho g_{\alpha\beta})$

Geodesics and Curvature
$P^\mu \equiv \frac{\partial x^\mu}{\partial\lambda} \quad \frac{\partial^2 x^\alpha}{\partial\lambda^2} + \Gamma^\alpha_{\mu\beta}P^\beta P^\mu = 0$
$ds \equiv \sqrt{|g_{\mu\nu}dx^\mu dx^\nu|} \quad U^\alpha \equiv \frac{dx^\alpha}{ds}$
$=(1,0,0,0)$ for local flat frame.
$U^\alpha U_\alpha = -1 \quad P^\alpha = mU^\alpha$ (massive)
$P^\mu P_\mu = -m^2$ (ms) $P^\mu P_\mu = 0$ (msls)
$E \equiv -P_\alpha U^\alpha_{obs}$ usually $U^\alpha_{obs} = (1,0,0,0)$
$\Rightarrow E = P^0 \quad E^2 - m^2 = g_{ij}P^iP^j \equiv p^2$
$v^i_{phys} = a\frac{dx^i}{dt} \quad R_{\mu\nu} \equiv R^\alpha_{\mu\alpha\nu} \quad R^\alpha_{\mu\beta\nu} \equiv$
$\partial_\beta\Gamma^\alpha_{\mu\nu} - \partial_\nu\Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\beta\lambda}\Gamma^\lambda_{\mu\nu} - \Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\beta}$
$R = g^{\mu\nu}R_{\mu\nu} \quad G_{\mu\nu} = R_{\mu\nu} - \frac12 g_{\mu\nu}R$
$\nabla_\mu G^{\mu\nu}=0 \quad T^{\mu\nu} = \frac{\Delta P^\mu}{(\Delta x)^3} \quad \nabla_\mu T^{\mu\nu}=0$
$\equiv$flux of P^μ across a surface of const x^ν
$T^{\mu\nu} = \mathrm{diag}(\rho,P,P,P)$ (perfect, flat FRW)
$T^{\mu\nu} = Pg^{\mu\nu} + (\rho+P)U^\mu U^\nu$
$T^\mu_\nu = \mathrm{diag}(-\rho,P,P,P) \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$

Content and Distances
$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{R_c^2 a^2} \quad \frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho+3P)$
$\rho_c = 3H_0^2/8\pi G \quad \dot\rho + 3H(1+\omega)\rho = 0$
$a = 1/(1+z) \quad P = \omega\rho \quad H(a) =$
$H_0\sqrt{\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2}}$
or DE term is $\Omega_\Lambda a^{-3(1+w)}$
$t = \int_{a_{em}}^{a_{obs}} \frac{da'}{a'H(a')} \quad \chi = \int_{a_{em}}^{a_{obs}}\frac{da'}{a'^2H(a')}$
$\chi = r\ (k=0)$
$\chi = R_c \arcsin(h)[r/R_c]\ (k=-1)$
$F_{obs} = \frac{L_{em}}{4\pi d_L^2} \quad \theta_{obs} = \frac{r_{ruler}}{r_{dist}} = \frac{R_{phys}}{d_A}$
$d_L = (1+z)r \quad d_A = (1+z)^{-1}r$

Particle Statistics & Early Universe
$N = \int d^3x\, n(\vec x) = \int d^3x \frac{d^3p}{(2\pi)^3} f(\vec x,\vec p,t)$
$n(\vec x) = \int\frac{d^3p}{(2\pi)^3}f \quad \rho(\vec x) = \int\frac{d^3p}{(2\pi)^3}Ef$
$E = \sqrt{p^2+m^2} \quad P(\vec x) = \int \frac{d^3p}{(2\pi)^3}\frac{p^2 f}{3E}$
$f = \frac{g}{e^{(E-\mu)/T}\pm1}$, + F, - B
equil: $\mu=0$ rel, eq: $\rho = A\frac{\pi^2}{30}gT^4$,
$A_{f/b} = \frac78, 1 \quad g_\gamma = 2, g_e = 4, g_\nu = 6$
$g_* \equiv \sum_{b,rel} g_i\left(\frac{T_i}{T_\gamma}\right)^4 + \frac78\sum_{f,rel}g_i\left(\frac{T_i}{T_\gamma}\right)^4$
$g_* = 106.75, 10.75, 3.36 \quad \rho = mn$
nonrel: $n = g\left(\frac{mT}{2\pi}\right)^{3/2}e^{-(m-\mu)/T}$
$dE = T\,dS - P\,dV + \mu\,dN \quad s = \frac{\rho+P-\mu n}{T}$
$s \propto a^{-3} \quad g_{*s}\equiv g_*$ but pwr $4\to3$
$g_{*s}a^3T_\gamma^3 = const$ KE: $T_i = T_\gamma$
$P_{rel} = \frac{\rho_{rel}}{3} \quad T < 0.51\mathrm{Mv}: \frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$
$H = 1.66\sqrt{g_*}\frac{T^2}{m_{pl}} \quad a = \frac{T_0}{T}\left(\frac{3.91}{g_{*s}}\right)^{1/3}$

$w_m = 0 \quad w_r = 1/3$

(Unperturbed) Boltzmann Equations
$\frac{Df}{dt} = C[f] \quad \frac{Df}{dt} = \frac{\partial f}{\partial t} + \frac{dp}{dt}\frac{\partial f}{\partial p} \quad p\propto 1/a:$
$\to \dot p = -Hp:$
$\int\frac{d^3p}{(2\pi)^3}\frac{Df}{dt} = \frac{dn}{dt} + 3Hn = \int\frac{d^3p}{(2\pi)^3}C[f]$
Collisionless: $\frac{dn}{dt}+3Hn = 0 = \frac{1}{a^3}\frac{d(na^3)}{dt}$
$1+2\Leftrightarrow 3+4: \quad \frac{1}{a^3}\frac{d(na^3)}{dt} =$
$= \int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^32E_2}\int\frac{d^3p_3}{(2\pi)^32E_3}\int\frac{d^3p_4}{(2\pi)^32E_4}$
$\times(2\pi)^4\delta^3(\vec p_1+\vec p_2-\vec p_3-\vec p_4)\delta(E_1+E_2-E_3-E_4)$
$\times|\mathcal M|^2[f_3f_4 - f_1f_2]$
$\int\frac{d^4p}{(2\pi)^4} \equiv \int\frac{d^3p\,dE}{(2\pi)^4}\delta(E^2-(p^2+m^2))\frac{dE}{dE^2}\frac{1}{2E}$
KE, $e^{(E-\mu)/T}\gg\pm1: f\to e^{\mu/T}gf_{eq}:$
$f_3f_4 - f_1f_2 = e^{-E_{tot}/T}\left(\frac{n_3n_4}{n_3^{eq}n_4^{eq}} - \frac{n_1n_2}{n_1^{eq}n_2^{eq}}\right)$
$\frac{1}{a^3}\frac{d(na^3)}{dt} = n_1^{eq}n_2^{eq}\left(\frac{n_3n_4}{n_3^{eq}n_4^{eq}} - \frac{n_1n_2}{n_1^{eq}n_2^{eq}}\right)\langle\sigma v\rangle:$
$\sim Hn_1 = n_1n_2\langle\sigma v\rangle \sim \Gamma n_1 \quad H\ll\Gamma: (\cdots)=0$
$H\gg\Gamma: n\propto a^{-3}$
$\Gamma_1 = n_{\text{target (2)}}\langle\sigma v\rangle = H$

Dark Matter Freezeout (FO)
$\chi+\chi\Leftrightarrow\gamma\gamma$ (TE≡KE). assume $1\equiv2\to$
$n_1n_2 = n_\chi^2$ FO: $H(T_f) = \Gamma = n_\chi^{eq}\langle\sigma v\rangle \equiv n_\chi\langle\sigma v\rangle$
$\rho_{\chi,0} = \rho_{\chi,f}a_f^3$ FO: $T = m_\chi \quad T\simeq\frac{m_\chi}{20}$
$\Omega_\chi = \frac{0.3}{h^2}\left(\frac{m_\chi/T_f}{10}\right)\frac{10^{-37}\mathrm{cm}^2}{\langle\sigma v\rangle}$
$\langle\sigma v\rangle = G_F^2(4\text{ GeV})^2 \simeq 3\times10^{-26}\frac{\mathrm{cm}^3}{\mathrm s}$
nonrel: $n_{eq} = g\left(\frac{mT_f}{2\pi}\right)^{3/2}e^{-m/T_f}$

BBN Stage 1: n-p Freezeout
($t\sim1$ s),
still radiation dominated $e^\pm,\nu$ in eq: $g_* = 10.75$
$pn,\bar p\bar n$ annihilated, leftover p,n:
$\nu+n\Leftrightarrow p+e^-$ & $e^++n\Leftrightarrow p+\bar\nu$
$e^\pm,\nu$ in eq: $\frac{n_n}{n_p} = \frac{n_n^{eq}}{n_p^{eq}} = \left(\frac{m_n}{m_p}\right)^{3/2}e^{-Q/T}$
$Q = m_n - m_p = 1.293$ MeV
p-n FO: $\Gamma_{p\leftrightarrow n} = \frac{14}{\tau_n}\left(\frac{T}{m_e}\right)^5 = H$
$T_f = 0.8\text{ MeV}\left(\frac{g_*}{10.75}\right)^{1/6}\left(\frac{\tau_n}{886\text{ s}}\right)^{1/3}$
$X_n \equiv \frac{n_n}{n_n+n_p} = 0.166$ normally

BBN Stage 2: Nucleosynthesis
Next ($t\sim270$ s), (He forms)
$n+p\Rightarrow D$ & $D+D\Rightarrow{}^3\text{He}+n$ &
& ${}^3\text{He}+D\Rightarrow{}^4\text{He}+p \quad g_D = 3$
$Y_{\text{He}}\equiv\frac{4n_{\text{He}}}{n_n+n_p}\simeq\frac{4(\frac12 n_n)}{n_n+n_p} = 2X_n$**?** (all $n\to$He)
WRONG; from 1 to 270 s, n decayed:
$Y_{\text{He}} = 2X_n(t_D) = 2X_n(T_f)e^{-(t-t_f)/\tau_n} = 0.24$
$n+p\leftrightarrow D+\gamma$:
$n_D = n_pn_n\frac{g_D\left(\frac{m_DT}{2\pi}\right)^{3/2}e^{-m_D/T}}{g_ng_p\left(\frac{m_NT}{2\pi}\right)^3e^{-(m_n+m_p)/T}}$
$n_D = X_nX_pn_b^2\frac34\left(\frac{2\pi(2m_N)}{m_N^2T}\right)^{3/2}e^{B/T}$
$N\equiv$nucleon, $m_D = 2m_N$
$B=$binding E$\equiv m_p+m_n-m_d\simeq2.22$ MeV
$X_d\equiv\frac{2n_D}{n_b} = X_nX_p\eta n_\gamma\frac32\left(\frac{4\pi}{m_NT}\right)^{3/2}e^{B/T}$
$n_\gamma = \frac{\zeta(3)}{\pi^2}2T^3 \quad \eta = n_b/n_\gamma\simeq5.5\times10^{-10}\frac{\Omega_bh^2}{0.020}$
$X_D = 16.3X_nX_p\eta\left(\frac{T}{m_N}\right)^{3/2}e^{B/T}$
eventually these FO: $D+D\Leftrightarrow{}^3\text{He}+\eta+\gamma$ &
&${}^3\text{He}+D\Leftrightarrow{}^4\text{He}+p$

Messing with BBN
BBN depends on:
1) $\Omega_bh^2\uparrow\Rightarrow D$ forms earlier $\Rightarrow n$: less t to dcy$\Rightarrow$
$\Rightarrow$ more He. Also $\Rightarrow$ later $D+D\to$ He FO $\Rightarrow$
less D
2) (nuclear physics/weak interactions):
$\Gamma_{pe\leftrightarrow n\nu}\propto G_F^2, \tau_n\propto G_F^{-2}$
if $G_F\downarrow\Rightarrow\Gamma_{pe\leftrightarrow n\nu}\downarrow\Rightarrow$n FO earlier$\Rightarrow$ more n
also $\Gamma_{pe\leftrightarrow n\nu}\downarrow\Rightarrow$ fewer n decays$\Rightarrow$ more n
both more $n\Rightarrow$ more He
3) (other rel. content of univ):
for $T>m_e\Rightarrow g_* = 10.75+2\frac78(N_\nu-3)$
$N_\nu\uparrow\Rightarrow g_*\uparrow\Rightarrow H\uparrow\Rightarrow$ earlier FO $\Rightarrow$ more n
also $g_*\uparrow\Rightarrow\rho\uparrow\Rightarrow H\uparrow$ and $\rho\uparrow\Rightarrow$ earlier FO
also $H\uparrow\Rightarrow$ decreasing t of $T=0.07$ MeV$\Rightarrow$
$\Rightarrow$ more $n\Rightarrow$ more He

$\eta_{FO}\neq\eta_{today}$

Recombination
$T\gtrsim$eV: $e^-+p\Leftrightarrow H+\gamma \quad n_H\ll n_e\sim n_p$
need $\langle\sigma v\rangle$ for $n=2$ since $n=1$ effectless
$H_{n=2}\Rightarrow H_{n=1}+\gamma \quad X_e\equiv\frac{n_e}{n_e+n_H} = \frac{n_e^{free}}{n_e^{tot}}$
$\frac{X_e^2}{1-X_e} = \frac{n_en_p}{n_Hn_\gamma\eta} \quad n_b = n_p+n_H \quad g_H = 4$
$\frac{n_en_p}{n_\gamma n_H}$ in eq: $\frac{n_H}{n_H^{eq}} = \frac{n_en_p}{n_e^{eq}n_p^{eq}}\Rightarrow$
$\frac{n_en_p}{n_H} = \left(\frac{m_em_p}{m_H}\frac{T}{2\pi}\right)^{3/2}\times e^{(m_H-m_p-m_e)/T}$
$\frac{X_e^2}{1-X_e} = \frac1\eta\frac{\pi^2}{2\zeta(3)T^3}\left(\frac{m_eT}{2\pi}\right)^{3/2}e^{-B_H/T}$
$B_H = m_p+m_e-m_H = 13.6$ eV $\quad X_e\in[0,1]$
Saha: $\frac{X_e^2}{1-X_e} = \frac{0.26}{\eta}\left(\frac{m_e}{T}\right)^{3/2}e^{-B_H/T}$

CMB
Photon scattering: $\gamma+e^-\Rightarrow\gamma+e^-$
(free n all decayed)
$\Gamma = n_e\sigma_Tc = \frac{n_e}{n+p+n_H}n_b\sigma_T = X_en_b\sigma_T$
$\Gamma<H$: scatter stop:
$\frac{\Gamma}{H} = \frac{n_b\sigma_T}{H}X_e = 113X_e\left(\frac{1+z}{1000}\right)^{3/2}$
$\Rightarrow\Gamma>H$ if $X_e\simeq1$ at $z\gtrsim1000$
$\Gamma<H$ if $X_e\ll1$ at $z\lesssim1000$

CMB Mysteries (Lead to Inflation)
1) Horizon Problem: homogeneous universe
but should be causally disconnected
2) CMB: very nearly flat univ
$\Omega_k = 1-\Omega_{tot} \quad 1 = \Omega_{tot}(a) - \frac{k}{a^2R_c^2H^2}$
$\frac{d}{dt}|\Omega_k| = -2\frac{\ddot a}{\dot a^3}\frac{1}{R_c^2}>0$ without inflation
3) Initial Fluctuations: CMB flucs w/
angular size $\gg$ horizon size

Inflation Motivations
soln to **1)**: $\ddot a>0\Rightarrow\frac{d}{dt}|\Omega_k|<0$
so can make univ arbitrarily flat initially
comoving Hubble radius $\equiv1/(aH)$
largest scale at which physics can happen at some time
$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{\ddot a}{\dot a^2}$
H const$\to\chi_{hor} = \int_0^a\frac{d(\ln a')}{a'h(a')}$ diverges
longer D prod. equil: less D left

"Derivation" of the Inflaton
Need $P<-\rho/3$ for $\ddot a<0$ from Friedmann 2
$S_{EH} = \frac{1}{16\pi G}\int\sqrt{-g}R\,d^4x$
$\delta S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R_{\mu\nu}-\frac12Rg_{\mu\nu}\right]\delta g_{\mu\nu}$
add S_{matter} to S_{EH} gives S_{tot}
$\delta S_{tot} = \int d^4x\sqrt{-g}\left[\frac{G_{\mu\nu}}{16\pi G}+\frac{\delta\mathcal L_m}{\sqrt{-g}\delta g_{\mu\nu}}\right]\delta g_{\mu\nu} = 0$
EOM: $G_{\mu\nu} = -16\pi G\frac{\delta\mathcal L_m}{\sqrt{-g}\delta g_{\mu\nu}}\equiv8\pi GT_{\mu\nu}$
$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta\mathcal L_m}{\delta g^{\mu\nu}}$
$S_\phi = \int d^4x\left[\sqrt{-g}\left(-\frac12g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right)\right]$
$T^\mu_\nu = g^{\alpha\mu}\nabla_\alpha\phi\nabla_\nu\phi - \delta^\mu_\nu\left(\frac12\nabla^\alpha\phi\nabla_\alpha\phi+V\right) \quad g^\mu_\nu = \delta^\mu_\nu$
$\rho = \frac12\dot\phi^2+V(\phi) \quad P = \frac12\dot\phi^2-V(\phi) \quad \ddot\phi+3H\dot\phi+V'(\phi)=0$

The Evolution of Inflation
$\frac{\ddot a}{a} = H^2+\dot H \quad \varepsilon_H\equiv-\frac{\dot H}{H^2} \quad \dot H = -4\pi G\dot\phi^2$ SR: $\varepsilon_H,\eta_H\ll1$
$V\gg\dot\phi^2: w\simeq-1$ SR: $\ddot\phi\ll3H\dot\phi: \quad 3H\dot\phi\simeq-V'(\phi)$
$\eta_H\equiv-3\frac{\ddot\phi}{3H\dot\phi}$ "eta problem": tough to make infl model
with SR conds. SR: $H^2\simeq\frac{8\pi G}{3}V \quad \varepsilon_V\equiv\frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2\simeq\varepsilon_H$
$\eta_V\equiv\frac{1}{8\pi G}\frac{V''}{V} \quad \eta_H\simeq\eta_V-\varepsilon_H$
SR stops @ $1=\varepsilon_V$
1) Chaotic infl: inflaton could get up initially from qntm fluc,
but lack of prim. grav. waves ruled this out
2) New infl: high ρ gives quadratic V
3) Eternal inf: tunnel into inflation, multiverse
$\frac{1}{aH}\propto\sqrt a$ in MD, $\propto a$ in RD, $\propto a^{-1}$ in ID. i is begin of infl
Solve Horizon Prblm: $\frac{1}{a_iH_i}\geq\frac{1}{a_0H_0}\Rightarrow\frac{a_eH_e}{a_iH_i}\geq\frac{a_eH_e}{a_0H_0}$
Quadratic Inflaton: $\ln(a_e/a_i)\gtrsim60\Rightarrow$
$\ln(a_e/a_i)\equiv\int_{a_i}^{a_e}\frac{da}{a}\equiv\int_{t_i}^{t_e}\frac{\dot a}{a}dt\equiv\int_{\phi_i}^{\phi_e}H\frac{d\phi}{\dot\phi}$

After Inflation
Out of SR: ϕ oscillates, $V\simeq\frac12m^2\phi^2$
$\ddot\phi+3H\dot\phi+m^2\phi=0 \quad H\downarrow\Rightarrow\ddot\phi\to-m^2\phi\Rightarrow\phi=\phi_0\sin(mt)$
$\langle V\rangle = \frac12m^2\langle\phi^2\rangle = \frac14m^2\phi_0^2 \quad \langle T\rangle\sim\langle\frac12\dot\phi^2\rangle = \frac14m^2\phi_0^2$
$P = T-V = 0$ so it's matterlike: $\rho_\phi\simeq a^{-3}$
Energy trans to reheating: $\dot\rho_\phi+3H\rho_\phi = -\Gamma_\phi\rho_\phi$
$\dot\rho_r+4H\rho_r = \Gamma_\phi\rho_\phi$

Perturbation Theory
$ds^2 = -(1+2A)\,dt^2 - 2aB_i\,dx^i\,dt$
$+a^2(t)\left[\delta_{ij}(1+2\phi)+2E_{ij}\right]dx^i\,dx^j$
$\vec B = \vec\nabla B + \vec B_\perp \quad E_{ij} \equiv E_{ij}^{\|} + E_{ij}^{\perp} + E_{ij}^{\mathrm{TT}}$
grav wave along z: $E^{\mathrm{TT}}_{ij} = \begin{pmatrix} h_+ & h_\times & 0\\ h_\times & -h_+ & 0 \\ 0&0&0\end{pmatrix}$
Gauge Freedom:
$t\to \hat t + \xi^0(t,\vec x) \quad \vec x + \hat{\vec x} = \vec x + \vec\xi(t,\vec x)$
$\vec\xi = \vec\nabla\xi + \vec\xi_\perp$
$\hat g_{\mu\nu} = \left[\delta^\alpha_\mu - \partial_\mu\xi^\alpha\right]\left[\delta^\beta_\nu - \hat\partial_\nu\xi^\beta\right] g_{\alpha\beta}$
in general, can set 2 scalar perts to 0
and 2 DOF of metric perts to 0
1) Synchronous Gauge: $A=B=\vec B_\perp = 0$
2) Poisson Gauge: $B=E=\vec E^\perp=0$
$\rho = \bar\rho(t)+\delta\rho(t,\vec x) \quad P = \bar P(t) + \delta P(t,\vec x)$
$ds^2 = -(1+2A)\,dt^2 - 2aa_iB\,dt\,dx^i$
$+a^2\left[\delta_{ij}(1+2\phi) - 2\left(\nabla_i\nabla_j - \frac13\nabla^2\right)E\right]$
Bardeem 1: $\Phi_A = A + \frac1a\frac{\partial}{\partial\tau}[aE'-B]$
Bardeem 2: $\Phi_H = -\phi + aH[B-E']$
Dodelson Gauge
$A=\Psi \quad \phi=\Phi \quad E=B=0$
DIAGONAL!
$ds^2 = -(1+2\Psi)\,dt^2 + a^2\delta_{ij}(1+2\Phi)\,dx^i\,dx^j$
$G^{\mu\nu} = 8\pi G T^{\mu\nu} \quad \delta G^{\mu\nu} = 8\pi G\delta T^{\mu\nu}$
$\bar G^{\mu\nu} + \delta G^{\mu\nu} = 8\pi G\left(\bar T^{\mu\nu} + \delta T^{\mu\nu}\right)$
$f(\vec x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec k\cdot\vec x}\tilde f(\vec k) \quad \delta G^0_0 = 8\pi G\delta T^0_0$:
$-\nabla^2\Phi + 3aH(\Phi' - aH\Phi) = 4\pi G a^2\delta\rho$
FT: $k^2\tilde\Phi + 3aH(\tilde\Phi' - aH\tilde\Phi) = 4\pi Ga^2\delta\tilde\rho$
subhorizon: $\lambda \ll \frac{1}{aH}$: newtonian gravity
superhorizon: $\lambda\gg\frac{1}{aH}$: non-Newt grav
take $\nabla_i\nabla_j - \frac13\delta_{ij}\nabla^2$ both sides of ii eqn:
get $\tilde\Phi = -\tilde\Psi$. $\tilde v_B \equiv ik\tilde B + \frac{k^i\tilde T^0_i}{ka(\bar\rho+\bar P)}$
$\zeta \equiv -\Phi_H - \frac{iaH}{k}v_B$ (Dropping ~)
curvature of uniform density slices
superhrzn: $k\ll aH$: $\zeta = \phi - H\frac{\delta\rho}{\dot{\bar\rho}}$
Adiabatic: $\zeta_{rad} = \zeta_{mat} = \zeta_i$
Isocurvature (opp): $\bar\rho$ =const
adiabatic⇒ ζ const on superhrzn
Perturbations from Inflation
our gauge: $\delta\phi = E = 0 \quad T^0_i = 0 \quad \zeta = \Psi$
$ds^2 = -(1+2A)\,dt^2 - 2aa_iB\,dt\,dx^i$
$+a^2\left[\delta_{ij}(1+2\zeta)\,dx^i\,dx^j\right]$
vary S: gives A, B, can plug back into S
$S^{(2)} = \frac12\int d^4x\frac{\dot\phi^2}{H^2}\left[a^3\dot\zeta^2 - a(\partial_i\zeta\partial_i\zeta)\right]$
Step 1: EOM for ζ
$S^{(2)} = \frac12\int dt\,d^3x\,a^3 \left(\frac{\dot\phi}{H}\dot\zeta\right)^2 - a\left[\vec\nabla\left(\frac{\dot\phi}{H}\zeta\right)\right]^2$
$f \equiv \frac{\dot\phi}{H}\zeta$: $S\simeq \frac12\int dt\,d^3x\,a^3\dot f^2 - a(\vec\nabla f)^2$
$d\tau\equiv dt/a \quad f' = a\dot f \quad v\equiv af$
$S = \int d\tau\,d^3x\left[\frac12(v')^2 - \frac12(\vec\nabla v)^2 + \frac{a''}{a}v^2\right]$
vary wrt v: $v'' - \nabla^2 v - \frac{a''}{a}v = 0$
Step 2: Use sbhrzn QFT to get ζ flucs
$\Pi = \frac{\partial\mathcal L}{\partial v'} = v' \quad [v(\vec x), v'(\vec y)] = i\delta^3(\vec x - \vec y)$
$[\hat a_{\vec k}, \hat a^\dagger_{\vec k'}] = (2\pi)^3\delta^3(\vec k - \vec k') \quad \langle 0|\,\hat a^\dagger_{\vec k} = 0 \quad \hat a_{\vec k}\,|0\rangle = 0$
$\hat v(\tau,\vec x) \equiv \int\frac{d^3k}{(2\pi)^3}\left[v_k(\tau)e^{-i\vec k\cdot\vec x}\hat a_{\vec k} + v_k^*(\tau)e^{-i\vec k\cdot\vec x}\hat a^\dagger_{\vec k}\right]$
plug into EOM, go to Fourier space
$v_k''(\tau) + \left(k^2 - \frac{a''}{a}\right)v_k(\tau) = 0 \quad \omega^2\equiv k^2 - \frac{a''}{a}$
during infl: $\frac{a''}{a} = 2a^2H^2 \quad k\gg aH$:
$\omega = kv_k'' + \omega^2 v_k = 0$
from commutators: $v_kv_k^{*'} - v_k^*v_k' = i$
$v_k = Ae^{-i\omega\tau} + Be^{i\omega\tau} \quad A^2 - B^2 = \frac{1}{2\omega}$
$v_k'' + V'(v_k) = 0 \Rightarrow V(v_k) = \frac12\omega^2v_k^2 \Rightarrow$
$E_k = \frac12\left|v_k'\right|^2 + \frac12\omega^2|v_k|^2 = \omega^2\left[\frac{1}{2\omega} + 2B^2\right]$
$B = 0: E_{min}\Rightarrow A = \frac{1}{\sqrt{2\omega}} \Rightarrow$
$v_k = \frac{1}{\sqrt{2\omega}}e^{-i\omega\tau} \simeq \frac{1}{\sqrt{2k}}e^{-i\omega\tau}$
Step 3: Use EOM to get ζ at hrzn exit
superhrzn: $k\ll aH$. Inflation: $\tau = -\frac{1}{aH}$
$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right)v_k = 0 \Rightarrow v_k = \frac{e^{-k\tau}}{\sqrt{2k}}\left[1 - \frac{i}{k\tau}\right]$
sprhrzn: $k\tau\ll 1$: $\left|v_k^{spr}\right|^2 = \frac{1}{2k^3\tau^2} = \frac{a^2H^2}{2k^3} \Rightarrow$
$\left|f_k^{spr}\right|^2 = \frac{H^2}{2k^3} \equiv \left|\delta\phi_k^{spr}\right|^2 \quad \hat v_k \equiv v_k\hat a_{\vec k} + v_{-k}\hat a^\dagger_{-\vec k}$:
$\sigma^2_{v_k} = \langle 0|\,v_k^\dagger v_k\,|0\rangle = (2\pi)^3\left|v_{-k}\right|^2$
$\left\langle\hat v_k^\dagger\hat v_{k'}\right\rangle \equiv (2\pi)^3\delta^3(\vec k - \vec k')P(k) \quad P(k) = |v_k|^2$
$(2\pi)^3P_v(k)\sim\overline{(\delta v_k)^2}$
$\langle\hat v(\vec x)\hat v(\vec y)\rangle \equiv \int\frac{dk}{k}\left(\frac{k^3P_v}{2\pi^2}\right)\frac{\sin kr}{kr}$

Power Spectra for Inflation
$f = \frac{\dot\phi}{H}\zeta \quad |f_k|^2 = \frac{H^2}{2k^3} = P_f(k)$
$\mathcal P_f(k) = \frac{k^3}{2\pi^2}P_f(k) = \left(\frac{H}{2\pi}\right)^2$
$\mathcal P_\zeta = \frac{G}{\pi\varepsilon_H}H^2 = \frac{1}{\pi\epsilon_H}\left(\frac{H^2}{m^2_{\mathrm{pl}}}\right)\Big|_{k=aH}$
ζ const wrt time
$n_s - 1 \equiv \frac{d\ln\mathcal P_\zeta}{d\ln k} = -4\varepsilon_H + 2\eta_H$
$n_s = 1$: scale-inv perts
$\mathcal P_\zeta = A_s\left(\frac{k}{k_0}\right)^{n_s-1} \quad \alpha_s = \frac{dn_s}{d\ln k}$
Tensors from Inflation
flat, vacuum metric: $-\partial_t^2h_{ij} + \nabla^2h_{ij}$
$h''_{+,\times} + 2\frac{a'}{a}h'_{+,\times} + k^2h_{+,\times} = 0$
do same process with S, let $f\equiv\frac{h_{+,\times}}{\sqrt{16\pi G}}$
gives $P_{h_+} = P_{h_\times} = 16\pi G\left(\frac{H^2}{2k^3}\right)$
$P_h \equiv 4P_{h_+} = 16\varepsilon_H P_\zeta \Rightarrow r\equiv\frac{P_h}{P_\zeta} = 16\varepsilon_H$
The Case for Inflation
$\zeta = n_s - 1 + \frac12\alpha_s\ln\frac{k}{k_0} \quad H^2 = \frac{8\pi V}{3m^2_{\mathrm{pl}}}$
ζ re-enter horizon:
$3\Psi H^2 - \frac{k^2}{a^2}\Phi - 3\dot\Phi = -4\pi G\delta\rho$
subhrzn: $-\frac{k^2}{a^2}\Phi = -4\pi G\delta\rho$
sprhrzn: $\dot\Phi = 0 \quad \delta\rho = \Psi\left(-\frac{3H^2}{4\pi G}\right) = \Psi(-2\rho)$
$\delta\equiv\frac{\delta\rho}{\rho} = -2\Psi \quad \zeta = \Phi - H\frac{\delta\rho}{\dot\rho} = \Phi + \frac{\delta}{3(1+w)}$
$\zeta = \Phi + \frac{2\Phi}{3(1+w)}$ consider e baryons
Perturbed Boltzmann Equations
$f(\vec x,\vec p, t)\equiv f(\vec x, p, \hat p^i, t)$
$\frac{Df}{dt} = 0 = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial p}\frac{dp}{dt} + \mathcal O(2)$
$\frac{dx^i}{dt} = P^i/P^0 \quad p^2 = P^iP^jg_{ij} - P^iP^ja^2(1+2\Phi)\delta_{ij}$
$P^\mu P^\nu g_{\mu\nu} = -m^2 = (P^0)^2g_{00} + p^2$
Photons: $(P^0)^2 = -\frac{p^2}{g_{00}}{}^2 \simeq p^2(1-2\Psi)$
$\frac{dx^i}{dt} = \frac{\hat p^i}{a}(1-\Phi+\Psi)$
Temp Perts (Photons)
photons: $f(\vec x,p,\hat p,t)\equiv\frac{1}{e^{p/T(1+\Theta)}-1}$
$\Theta = \frac{\Delta T}{T}(\vec x,p,\hat p,t)$ Thompson sct: $\Theta\simeq\frac{\Delta T}{T}(\vec x,\hat p,t)$
$f(\Theta) = f^{(0)} + \Theta\frac{\partial f}{\partial\Theta}\Big|_{\Theta=0} = f^{(0)} - p\frac{\partial f^{(0)}}{\partial p}\Theta$
Z.O: $\frac{Df}{dt} = \frac{\partial f^{(0)}}{\partial t} - Hp\frac{\partial f^{(0)}}{\partial p}$
F.O: $\frac{Df}{dt} = -p\frac{\partial f^{(0)}}{\partial p}\left[\frac{\partial\Theta}{\partial t} + \frac{\hat p^i}{a}\frac{\partial\Theta}{\partial x^i} + \frac{\partial\Phi}{\partial t} + \frac{\hat p^i}{a}\frac{\partial\Psi}{\partial x^i}\right]$
=0 for neutrinos; don't collide
Massive Particles
$f(\vec x, E,\hat p, t) \quad E^2 = p^2 + m^2$
$\frac{Df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat p^i}{a}\frac{p}{E}\frac{\partial f}{\partial x^i} - \frac{\partial f}{\partial E}\left[H\frac{p^2}{E} + \frac{p^2}{E}\frac{\partial\Phi}{\partial t} + \frac{\hat p^i}{a}p\frac{\partial\Psi}{\partial x^i}\right]$
int. over $\int\frac{d^3p}{(2\pi)^3}$: $\dot n + \frac1a\frac{\partial(nv^i)}{\partial x^i} + \left[H + \frac{\partial\Phi}{\partial t}\right]3n = 0$
(=0 for collisionless) $\quad \delta_m = \frac34\delta_r$
$\delta = \frac{\delta n}{\bar n} \quad n = \bar n + \delta n = \bar n[1+\delta]$ Z.O: $\dot{\bar n} + 3H\bar n = 0$
Electron-Photon Interactions
$e^-(\vec q) + \gamma(\vec p) \Leftrightarrow e^-(\vec q') + \gamma(\vec p') \quad E_e\sim m_e$
$\frac{|\mathcal M|^2}{m_e^2} = 8\pi\sigma_T$
$\Theta_0(\vec x,t) = \frac{1}{4\pi}\int\sin\Theta_p\,d\Theta_p\,d\phi_p\,\Theta(\vec x,\hat p,t)$
$\sim$ avg of $\frac{\Delta T}{T}$ over all directions
$\dot\Theta + \frac{\hat p^i}{a}\frac{\partial\Theta}{\partial x^i} + \dot\Phi + \frac{\hat p^i}{a}\frac{\partial\Psi}{\partial x^i} = n_e\sigma_T\left[\Theta_0 - \Theta + \hat p\cdot\vec v_b\right]$
Go to Fourier: $\mu\equiv\hat p\cdot\hat k \quad \Theta\propto e^{i\vec k\cdot\vec x}\Theta(\vec k)$
$\Theta' + ik\mu\Theta + \phi' + ik\mu\Psi = n_e\sigma_T a[\Theta_0 - \Theta + v_b\mu]$
$\Theta_\ell\equiv\frac{1}{(-i)^\ell}\int_{-1}^1\frac{d\mu}{2}\mathcal P_\ell(\mu)\Theta(\mu)$
int PDE over $\mu/\,d\Omega$:
$\Theta_0' + k\Theta_1 + \Phi' = n_e\sigma_Ta[\Theta_0 - \Theta_0 + 0]$
int again:
$\Theta_1' - \frac k3\Theta_0 - \frac k3\Psi + \frac23k\Theta_2 = n_e\sigma_Ta\left[\frac i3v_b - \Theta_1\right]$
only care about γ: mfp short: ignore Θ_{2+}.
Just before horizon re-entry (superhorizon)
$k\ll aH\Rightarrow k\to0 \quad n_e\sigma_Tc\gg H: \Theta_1 = \frac i3v_b$
$\delta'_{dm} + 3\Phi' = 0 \quad \delta_b' + 3\Phi' = 0\Rightarrow\delta_{dm} + \delta_b + C_1$
$\Theta_0' + \Phi' = 0 \quad \mathcal N_0' + \Phi' = 0 \Rightarrow \Theta_0 = \frac13\delta_{dm} + C_2,$
$\mathcal N_0 = \Theta_0 + C_3 \quad \Theta_0 = \frac{\Delta T}{T} = \frac14\delta_\gamma$
adiabatic: $C_i = 0\Rightarrow\delta_{dm} = \frac34\delta_\gamma$
EFE: $3\Phi H^2 + \frac{k^2}{a^2}\Phi + 3H\dot\Phi = 4\pi G\delta\rho$
early times: $\delta\rho\equiv\delta\rho_r = 4(\delta_\nu\mathcal N_0 + \rho_\gamma\Theta_0)$
RD: $a\propto\tau\Rightarrow 4\Phi' + \Phi''\tau = 0\Rightarrow\Phi$ =const & Φ decay
EFE eqn: $\Phi = 2\left[f_\nu\mathcal N_0 + (1-f_\nu)\Theta_0\right]$
adiabatic: $\mathcal N_0 = \Theta_0 : \Phi = 2\Theta_0 \quad \Phi = \frac23\delta_{dm} = \frac23\delta_b$
$\Phi\to\frac{9}{10}\Phi$ for MD→RD $\quad f_\nu = \rho_\nu/(\rho_\nu+\rho_\gamma)$

Horizon Entry
RD: $\ddot\Theta_0 - \frac{k^2}{3}\Theta_0 = 0$ (SHO)
$\delta_{dm}\propto\log(a)$ from DE $x_{dm} = \int v\frac{dt}{a}$
after entry, $\delta_{dm}\propto a \quad \Phi_p = \frac23\zeta$
(prim: RD superhorizon)
Matter Power Spectrum
$\Phi(k,\tau_0) = \Phi_p(k)\frac{\Phi_{spr}(k,\tau_{md})}{\Phi_p(k)}\frac{\Phi(k,\tau_{md})}{\Phi_{spr}(k,\tau_{md})}\frac{\Phi(k,\tau_0)}{\Phi(k,\tau_{md})}$
subhorizon: $\frac{k^2}{a^2}\Phi = 4\pi G\delta\rho$
MD: gravity wins: growth factor D
$\Phi(k,\tau_0) = \frac{9}{10}\left(\frac23\zeta\right)T(k)\frac{D(a)}{a}$
$\delta\rho\simeq\rho_m\delta_m \quad \delta_m = \frac35\frac{k^2}{\omega_{m,0}H_0^2}\Phi_pT(k)D(a)$
MD: $\delta_m\propto D(a) = a \quad P = \left|\tilde\delta^2\right| \Rightarrow$
$P_{m,0}(k) = \frac{k^4}{H_0^4}\left(\frac{3}{5\Omega_{m,0}}\right)^2\left[\frac49\left(\frac{2\pi^3}{k^3}\right)A_s\left(\frac{k}{k_0}\right)^{n_s-1}\right]$
$\times T^2(k)D^2(a) \quad \propto k^{n_s}T^2(k)D^2(a)$
$T(k) = 1$ for $k<k_{eq}\Rightarrow P_m(k)\propto k^{n_s}$ (large scales)
small scales (enter during RD): coupled b,γ:
$\Theta_1 = \frac i3v_b \quad \Theta_0' + k\Theta_1 + -\Phi' \quad \Theta_1' - \frac k3\Theta_0 = -\frac k3\Phi$
$v_m' + aHv_ik\Phi \quad \delta_m' + ikv_m = -3\Phi'$
$k^2\Phi = 4\pi Ga\left[\rho_m\delta_m + 4\rho_r\Theta_0 + \frac{3aH}{k^2}(i\rho_mv + 4\rho_r\Theta_1)\right]$
Early Times: $(\rho_r\gg\rho_m, \rho_r\Theta_0\gg\rho_m\delta_m)$
Φ dcy on entry, Θ oscil, $\delta_m(k,\tau) = A\Phi_p(\ln Bk\tau)$
Stage 2: $\rho_m\delta>\rho_r\Theta$, etc: ignore rad perts, use Fried:
$\frac{d^2\delta}{dy^2} + \frac{2+3y}{2y(y+1)}\frac{d\delta}{dy} - \frac{3}{2y(y+1)}\delta = 0 \quad y = \frac{a}{a_{eq}}$
(Meszaros; pert evol. inside and outside horizon)
Solns: RD($y\ll1$): D_1 =const, $D_2 = \frac23\ln\frac4y$ −const
MD ($y\gg1$): $D_1 = y, D_2$ =decaying mode
$\delta = C_1D_1(a) + C_2D_2(a)\Rightarrow$
$\delta_{(dm)}(k,a) = \frac32A\Phi_p\ln\left[\frac{4B}{e^3}\frac{a_{eq}}{a_{hor}}\right]\frac{a}{a_{eq}} \quad \begin{pmatrix}A = 9.11\\B = 0.54\end{pmatrix}$
MD: $k\gg k_{eq}$: $\frac{a_{eq}}{a_{hor}} = \frac{\sqrt2k}{k_{eq}}\Rightarrow$
$T(k\gg k_{eq}) = \frac{12k^2_{eq}}{k^2}\ln\left(\frac{k}{8k_{eq}}\right)\propto\frac{\ln k}{k^2}$
$P_\delta\propto k^{n_s}\Rightarrow P_\delta\propto\begin{pmatrix}k^{n_s} & k\ll k_{eq}\\ \ln^2k/k^{4-n_s} & k\gg k_{eq}\end{pmatrix}$
$\sigma^2(R)\equiv\left\langle\delta^2_R\right\rangle = \int\frac{dk}{k}\mathcal P_\delta(k)F^2(kR)$
$F\sim\begin{pmatrix}1 & kR\ll1\\0 & kR\gg1\end{pmatrix} \quad \delta\propto a\Phi$
DE era: Φ decays, Meszaros Eqn solvable:
$D_1 = \frac{5\Omega_{m,0}}{2}\frac{H(a)}{H_0}\int_0^a da'\left(\frac{a'H(a')}{H_0}\right)^{-3}$
The CMB and Perturbations
we see $\Theta(\vec x_{here},\tau_0,\hat p)\equiv\frac{\Delta T}{T}(\theta,\phi) \quad \theta,\phi$ viewing angls
$\frac{\Delta T}{T}(\theta,\phi) = \sum_{l=0}^\infty\sum_{m=-l}^l a_{lm}Y_{lm}(\theta,\phi)$
$a_{lm} = \int\sin\theta\,d\theta\,d\phi\,Y^*_{lm}(\theta,\phi)\frac{\Delta T}{T}(\theta,\phi)$
$= \int_{-1}^1 d\mu\int_0^{2\pi}d\phi\,Y^*_{lm}(\theta,\phi)\int\frac{d^3k}{(2\pi)^3}e^{i\vec k\cdot\vec x_0}\Theta(\vec k,\tau_0,\mu)$
$\langle a_{lm}\rangle = 0 \quad \left\langle a_{lm}a^*_{l'm'}\right\rangle\equiv\delta_{ll'}\delta_{mm'}C_l$
$\left\langle\Theta(\vec k,\hat p)\Theta^*(\vec q,\hat n)\right\rangle\equiv(2\pi)^3P_{\delta,p}(k)\delta^3(\vec k - \vec q)$
$C_l = \frac2\pi\int_0^\infty dk\,k^2P_{m,p}(k)\left|\frac{\Theta_l(k,\tau)_0}{\delta_p(k)}\right|^2$
cosmic var: only $2l+1$ samples per C_l
$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}} \quad C_l$ domn by k with $l = kx_{cmb}$
but all $k > x_{cmb}$ contribute. sprhrzn at CMB : $l\lesssim 80$
$E_{obs} = E_{lss} + \Psi\Rightarrow\frac{\Delta T}{T} = \Theta_0(\vec x_{lss}) + \Psi(\vec x_{lss})$
sprhrzn: $\Theta_0' = -\Phi'$ and RD: $\Phi = 2\Theta_0$
$\Theta_0(k,\tau_{rd}) = \frac12\Phi_p(k) \quad \Theta_0(\tau) = -\Phi(\tau) + \frac32\Phi_p(k)$
$\Phi_p = \frac{10}{9}\Phi(\tau_*)\ \Theta_0(\tau_*) = -\frac23\Psi(\tau_*)\ \left(\frac{\Delta T}{T}\right)_{obs} = \frac13\Psi(\tau_*$
cold spots we see were hotspots.
Adiabatic: $\delta_{dm} = \frac34\delta_\gamma = 3\Theta_0\Rightarrow\left(\frac{\Delta T}{T}\right)_{obs} = -\frac16\delta(\tau_*)$
(SW): $l+(l+1)\frac\pi2\left(\frac{\Omega_m}{D_1(a_0)}\right)\delta_H^2$
(subhrzn): TS Boltzmann. $R\equiv\frac{3\rho_b}{4\rho_\gamma}$
$\ddot\Theta_0 + aH\left(\frac{R}{1+R}\right)\dot\Theta_0 + \frac{k^2\Theta_0}{3(1+R)} = F(DM)$
(slight γ) forced oscil: $\Theta_0(\tau_*) = A\cos(kc_s\tau) + \frac{F_0}{kc_s}$
$c_s = \sqrt{\frac{1}{3(1+R)}} = \sqrt{\frac P\rho}$ also doppler effect seen
$\lambda^{com}_{mfp} = \frac{1}{n_e\sigma_Ta} \quad \lambda_D\sim\frac{1}{\sqrt{n_e\sigma_Ta^2H}} \quad k\gg1/\lambda_D$ smoot
$n_e\uparrow\Rightarrow\lambda_D\downarrow\Rightarrow k_D\uparrow \quad \Delta n_b\Rightarrow c_s$,rebounds,
spacing, amplitude, diff tail.
Photons after CMB to us
can scatter: op depth
ISW can come from changing Φ (dom trans)
CMB params: $A_s, n_s, \Omega_{cdm}h^2, \Omega_bh^2, \theta_s, \tau$
$(\theta_s\sim d_A\Rightarrow H_0,\Omega_k,\Omega_\Lambda)$