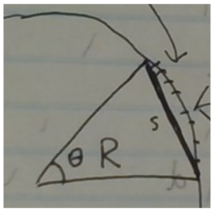
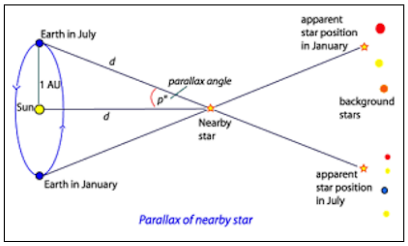
0.0.1 Geometry and Basics



arc length $l = \theta R$ **IN RADIANS**
$v_t = \omega r$, $\omega = 2\pi f$, $F_c = m\omega^2 r$
1 as $= 4.85 \times 10^{-6}$ rad
$s = 2R\sin\frac{\theta}{2} \simeq R\theta$ at $R \gg l$
parallax angle α, $\sin\alpha = R/d$
R = 1 AU = 1.5×10^8 km
$\alpha \simeq R/d$ as $d \ll R$ **IN RADIANS**
1 pc = d such that α= 1 as
$pc \propto \frac{1}{as}$

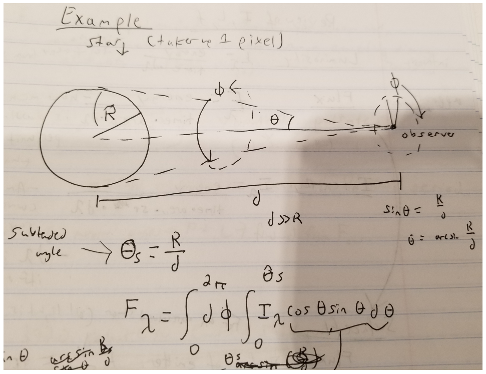


Center of Mass: $x_{COM} = (\sum_i M_i x_i)/M_{total}$
Where x=0 can be defined however.
Solid Angle: $\Omega \equiv A/R^2$ where A, R some surface area outlined by Ω, R is r of sphere.
$SA_{sphere} = 4\pi R^2$ $\quad d\Omega = \sin\theta \, d\theta \, d\phi$
$V_{sphere} = \frac{4}{3}\pi r^3$
OBAFGKM in dec. order of T_{surf}.
1 dyne = 10^{-5} N
1 erg = $10^{-7} J$
Luminosity classes: I through V, on HR diagram, lines from top downwards, sorted by width of spectral lines.

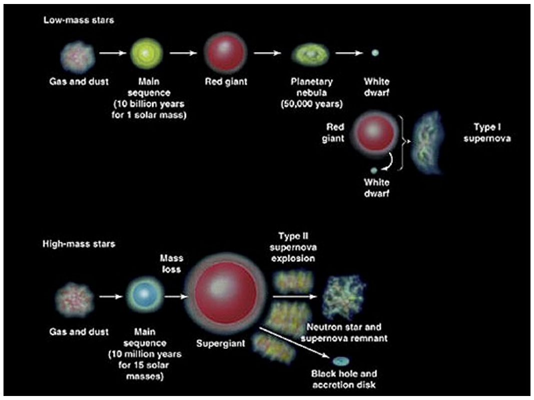
0.0.2 Gravitation and Kepler's Laws

Shell Theorem:
1. No grav force on objects inside **spherical** shell
2. Spherically symmetric objects can be treated as point masses

Kepler's Laws
1st The C.O.M. of a system does not accel.; can choose a ref frame where it's stationary.
3rd: For m orbiting M, with orbital radius R:
If $M \gg m$: $P^2 = \frac{4\pi^2}{GM}R^3$
If M is **not** $\gg m$: $P^2 = \frac{4\pi^2}{G(M+m)}a^3$; $MR = mr$; $a = r + R$
$R_M = ma/(m+M)$ $\quad r_m = Ma/(M+m)$
where a is the *distance between the two masses* and P is the orbital period of both.
$F = \frac{GMm}{r^2}$
Motions Find Mass: $v_t^2 = \frac{GM(<r)}{r}$
Sun BB curve lower on side of lower WL cause these get absorbed by n=2 H gas in sun atmos.

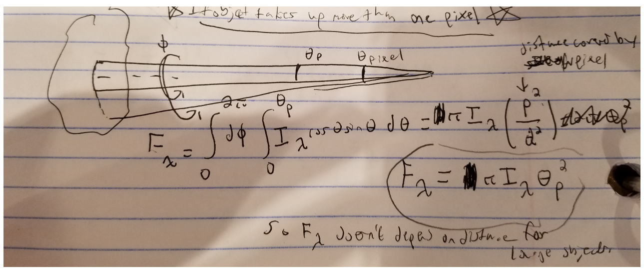


Eventually derives: $F_\lambda = \pi I_\lambda \frac{R^2}{d^2}$ (unresolved)



0.0.3 Intensity, Flux, Luminosity, Etc (Light Stuff)

$I_\lambda = energy/(dt \, dA_\perp \, d\Omega \, d\lambda)$
$dA_\perp$ is the area vector for a chosen ray.
$F = energy/(dt \, dA \, d\lambda)$
is the integral of I over all solid angles.
$L = energy/dt$
I and F don't have to be per λ.
$F_{obs\,(unresolved)} = L/4\pi d^2 = (R^2 F_{surf}/d^2)$
$F_{\lambda(resolved)} = \pi I_\lambda \theta^2_{pixel}$
$F_{surf} = \pi I$
$L = 4\pi R^2_{object} F_{surf}$
SB Law: $F_{surface} = \sigma T^4$
$Period_{variable\,star} \propto L$
Blackbody: $B_\lambda = (2hc^2/\lambda^5)/(e^{hc/\lambda kT} - 1)$
$B_\nu = (2h\nu^3/c^2)/(e^{h\nu/kT} - 1)$
$\therefore E$ **is linear wrt to** ν, **but not** λ.
Wein's Law: $\lambda_{max} = (2.9\,mm)/T$ (max of BB curve)
Doppler (Angular other side): $\lambda_{obs} = \lambda_{em}\sqrt{\frac{1-v/c}{1+v/c}}$
$\Delta\lambda/\lambda = \gamma - 1$
If $v \ll$ c: $\lambda_{obs} = \lambda_{em}(1 - v/c)$
$\frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{-v}{c} = z$, sign of v is neg if moving away
Rot Broadening: $\Delta\lambda/\lambda = 2v_{rot}/c$
Thermal Broadening: $\frac{\Delta\lambda}{\lambda} \approx \frac{2}{c}\sqrt{\frac{2kT}{m_{particle}}}$
Pressure Broadening: $\frac{\Delta\lambda}{\lambda} \propto P$
local thermodynamic equil (local blackbody:) $l_{mfp} \ll h_T$
h_T is the temperature scale height



So for resolved (> 1 pixel): $F_\lambda = \pi I_\lambda \theta^2_{pixel}$

Eclipsing Binaries
1. Went through files to determine highest stdev in Mag (time originally in MJD)
2. Supersmoother metric was made to measure the deviation of adjacent phased-up points:
3. Looped through possible periods, phased up epochs for each, plotted ssm data (average of abs. dif b/w adjacent, "zeroed" mags (mags-median[mags]) vs possible periods to determine period
4. Period used to phase epochs correctly and plot light curve (zeroed mags vs. phased epochs)

Binary systems: redshift reveals P and v, that leads to masses.

0.0.4 Microscopic Physics

$E_n = \frac{-13.6\,eV}{n^2}$ $\quad E_\gamma = hf$ $\quad f = c/\lambda$ $\quad \Delta p_x \Delta x > h$
Δn requires exact amount of E (like ladder) besides ionization.
$P_{degen} = \kappa\rho^{m+1}$, m =2/3 for non-rel, m=1/3 for rel.

Hydrogen Emission

n_f	Name, Type
1	UV, Lyman
2	Visible, Balmer
3	IR, Paschen

MFP stuff
cross section $\sigma = \frac{No.\,of\,events}{(No.\,density\,of\,targets)*(l\,traveled)}$
Prob of event $= \sigma l_{path} n$
$l_{mfp} = 1/(n\sigma)$
$\tau = \frac{l_{medium}}{l_{mfp}}$, $\tau \gg 1$: optically thick

$\int \frac{dI_\lambda(l)}{I_\lambda} = -\int n\sigma_\lambda \, dl$ $\quad \tau_\lambda = nl\sigma_\lambda$
$\downarrow$
$I_\lambda = I_{(\lambda,0)}e^{-\tau_\lambda}$

$\Delta M = M_f - M_i = -2.5\log_{10}(F_1/F_2)$ $\quad \gamma = 1/\sqrt{1 - v^2/c^2}$ $\quad t' = t\gamma$ $\quad X_{element} = \rho_{element}/\rho_{total}$ (Mass fraction)
hyperfine hydrogen n=1: $f_{up\to down} = 1420$ MHz, $\lambda = 21$ cm

0.0.4 Stellar Physics

Virial Thm Stuff and More
$U_{grav} + 2K = 0$
$K_{system} = \frac{3}{2}NkT = \frac{3}{2}\frac{M_{system}}{m_{particle}}kT$
$U_{grav} = -\int_0^R \frac{GM(<r)\rho(r)4\pi r^2 dr}{r}$
U is negative!, increases (less negative) as dist. increases
If ρ is constant, $U_{grav} = -\frac{3GM^2_{star}}{5R_{star}}$
$E_{total} = U_{grav}/2 = \frac{-GM^2}{2R}$ (also neg)
$E_{thermal} = -E_{total}$
$PV = NkT$ $\quad P = nkT$ $\quad PV = \frac{2}{3}K$ Relativ: $PV = \frac{1}{3}K$
$P = -\frac{1}{3}\frac{U_{grav}}{V}$ $\quad |U_{grav}| \approx \frac{GM^2}{R}$ $\quad P \approx \left(\frac{4\pi}{3^4}\right)^{1/3} GM^{2/3}\rho^{4/3}$
$P_{radiational} = \frac{1}{3}aT^4$ $\quad N = M_{total}/\overline{m}$
Isotropic Pressure: both radiational and kinetic
$L \propto T^4 R^2$
$L \propto T^8_{surf}$
$\uparrow T \;\rightarrow \uparrow R$ due to increased P and V

Useful TPS

Stellar Interior ODEs
$\frac{dM(<r)}{dr} = 4\pi r^2 \rho$
$\boxed{\frac{dP}{dr} = -\frac{GM(<r)}{r^2}\rho}$
$P = n_{(no.density)}kT = \frac{\rho}{m_{particle}}kT$
$\frac{dL}{dr} = 4\pi\rho r^2\epsilon$, $\quad \epsilon \equiv \frac{E\,produced}{mass}$
$\frac{dT}{dr} = -\frac{1}{4\sigma_{S-B}T^3} \cdot \frac{1}{l_{mfp-\gamma}} \cdot \frac{L}{4\pi r^2}$
Approximating these as power laws gives:
$P \approx \frac{M\rho}{r}$ $\quad M \approx r^3\rho$ $\quad L \approx \frac{T^4 r}{\kappa\rho}$

Thermostat Cycle in core:
fusion rate $\propto T \rightarrow T\uparrow \rightarrow P\uparrow \rightarrow$ core expands $\rightarrow$ core cools $\rightarrow$ T/fus rate $\downarrow \rightarrow P\downarrow \rightarrow$ core contracts $\rightarrow T\uparrow$: REPEAT.

Stellar Reactions
p-p chain: $6p^+ 2e^- \rightarrow 2p^+ +^4 He + 2\nu_e$
$\Delta m = 0.7\%\,of\,4m_p$, $\quad E = 25.71\,MeV$
For $M > 1.2M_\odot$: (also a bit for lesser masses)
CNO Cycle: same net cycle as p-p, just diff. steps

0.0.5 Stellar Evolution A: Main Sequence, White Dwarfs, Supermassive Stars

Main Sequence
$L \approx M^{3.5}$
MS lifetime $\tau \propto 1/M^{2.5}$
$L \propto T^8_{surf}$

Massive Stars In chronological order of fusion reactant shells in deep core (if hot/massive enough):
H, He, C, O, Ne, Si, Fe
end in Core Collapse Supernovae, (type II, or type Ib/Ic if no H left)

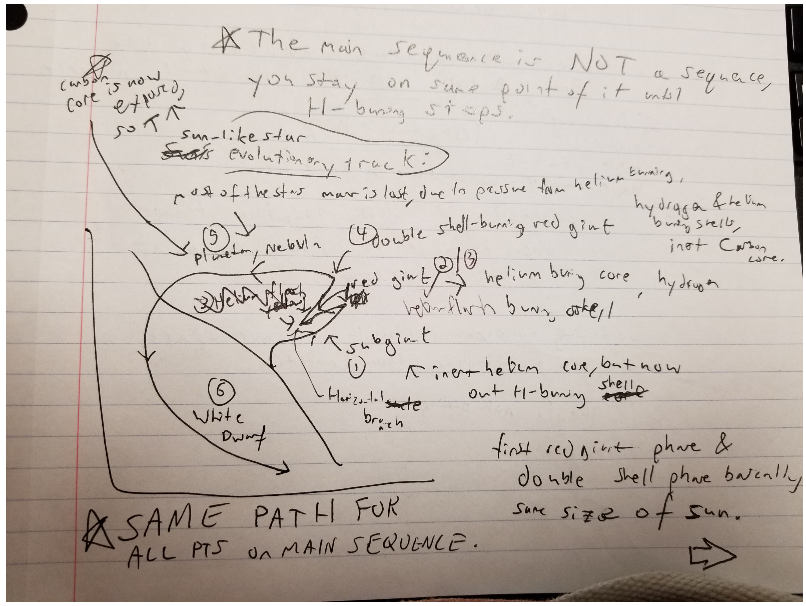
White Dwarfs
$R \propto R_c, \quad \overline{\rho} \propto \rho_c, \quad L \propto R^2 T^4, \quad R_{WD} \approx 7000\,km \left(\frac{M}{M_\odot}\right)^{-1/3} \propto \frac{1}{m_e}$
Non-rel: $M \propto \rho_c^{1/2} \rightarrow R_{WD} \propto M^{-1/3}$
$\downarrow$ as M added
Rel: $M \propto \rho_c^0 \rightarrow$trying to increase ρ_c will not increase M
Max $M_{WD} = M_{ch} = 1.4 M_\odot$
Using Virial Thm, and formula for P_{degen} & $\rho^{4/3} = \frac{M^{4/3}}{V^{4/3}}$:

$$\overline{P}V = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{Z}{A}\right)^{4/3} \frac{M^{4/3}}{V^{1/3}} = -\frac{1}{3}U_{grav} = -\frac{1}{3}\frac{GM^2}{R}$$

Where Z is proton count, A is nucleon count

Solve for M, gives: $M = \left[\frac{3}{10}\left(\frac{9}{32\pi^2}\right)^{1/3} \frac{hc}{Gm_p^{4/3}}\right]^{3/2} = 0.8 M_\odot$
This is off, actually $M_{ch} = 1.4 M_\odot$, minimum mass for Ia novae

Once WD gets enough mass via accretion, carbon in core begins fusing, except not, thermostat is broken, as unlike normal stars, T does not affect P.
Surface grav: $g = \frac{GM}{R^2}$



Stars peel of main sequence like a banana, starting from highest L

0.0.6 Stellar Evolution B: Core Collapse Supernovae, Neutron Stars

Core Collapse Timeline
FOR STARS M$> 8M_\odot$
1. iron core grows until reaching M_{ch},
after which e^- degen pressure can't support it anymore.
2. Core collapses, hot γ's break up iron nuclei into He, then p and n, cooling the core
(these rxns use E, at the end basically only p, n left):
$\gamma +^{56} Fe \rightarrow 13^4 He + 4n$
then $\gamma +^4 He \rightarrow 2p + 2n$
3. e^- absorbed into p (and other nuclei) making ν_e & n, further cooling core:
$e^- + p \rightarrow n + \nu_e$
4 Now the core is mostly neutrons.

Energy released in core collapse
Total change in $U_{grav} = 5 \times 10^{53}\,ergs$
Energy used in:
10% for nuclei breakup
1% for debris KE
0.01% for luminosity
89% in released ν's

Other Stellar Interior Equations
Where L is luminosity and dL is luminosity of shell, $\epsilon(r)$ is energy produced per unit mass:
$L + \mathrm{d}L = L + M_{shell} \times \epsilon(r)$
$\mathrm{d}L = 4\pi\rho r^2\,\mathrm{d}r \times \epsilon(r)$

$$\boxed{\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi\rho r^2 \epsilon(r)}$$

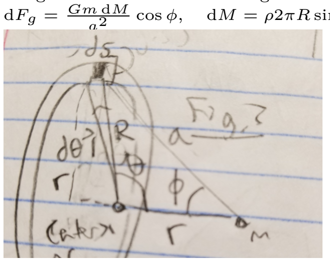
Neutron Stars
(the collapsed iron cores of supergiants)
$R_{NS} \simeq \frac{m_e}{m_n} \simeq 11\,km \left(\frac{M}{1.4M_\odot}\right)^{-1/3}$
$M_{max} \approx 0.2 \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} \times m_p$
Where $\frac{Z}{A} = 1$ (no protons)
So $M_{max} \approx 5.6 M_\odot$ **WRONG!**
Accounting for strong force b/w neutrons, and degen pressure contributing to gravitation (due to GR):
$\boxed{M_{NS,max} \simeq 2 - 3M_\odot}$
Above this, a black hole will form, as degen press. can't beat gravity
Pulsars
Jets created by strong magnetic field

Degeneracy Pressure
Using the stellar interior equations, and $P_{degen} = \kappa\rho^{m+1}$, you arrive at:
$\frac{\mathrm{d}P}{\mathrm{d}r} = \kappa(m+1)\rho^m \frac{\mathrm{d}\rho}{\mathrm{d}r}$
To solve this, switch to dimensionless variables, $x = \rho/\rho_{center}$
$R \propto R_{core}; \overline{\rho} \propto \rho_{core}$

Wien's Law Derivation
set $\frac{\mathrm{d}B_\lambda}{\mathrm{d}\lambda} = 0 = \frac{xe^x}{e^x - 1} - 5$
Where $x = hc/\lambda kT$
Solved numerically to find $x \simeq 4.965$

0.0.7 Derivations

Shell Thm
1. Divide spherical shell of mass M, radius R into individual rings with masses dM. a is dist from test mass to given ring.
Use F_g formula to determinedF_g b/w test mass m and dM.
2. dM can be writtin in terms of sphere surface density ρ. Use this to write dF_g in terms of integratable differentials. Integrate over all rings (θ) to find F_g.
$\mathrm{d}F_g = \frac{Gm\,\mathrm{d}M}{a^2}\cos\phi, \quad \mathrm{d}M = \rho 2\pi R\sin\theta R\,\mathrm{d}\theta$



3. Law of cosines used to simplify integral. F_g is found to be 0.

B_λ from B_ν
Using chain rule, $B_\lambda = B_\nu \cdot \frac{\partial\nu}{\partial\lambda}$

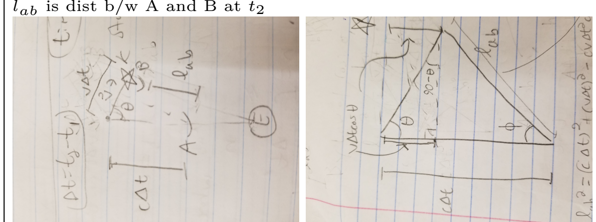
Stefan Boltzmann
$F_{surf} = \pi \int_0^\infty B_\nu\,\mathrm{d}\nu$, use $u(\nu) = \frac{h\nu}{kT}$

Sun core temp
Find P(0) from Hydro equil eqn
Use $PV = NkT = \frac{M_{core}}{m}$
$T \approx \frac{P_{center}\overline{m}}{\rho_{sun}k}$

Radiative Transport:
Consider a shell with thickness dr.
$f_{shell_{net}} = f_{up} - f_{down}$
$f_{shell_{net}} = \sigma_{SB}[T^4(r) - T^4(r + \mathrm{d}r)]$
Simplified with Taylor expansion:
$\frac{L(r)}{4\pi r^2} = \sigma_{SB}[T^4(r) - [T^4 + \frac{\mathrm{d}T^4}{\mathrm{d}r}\,\mathrm{d}r]]$
We are talking about how much energy a photon can carry (radiative transport) therefore $\mathrm{d}r \equiv l_{mfp-\gamma}$

$$\boxed{\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{1}{4\sigma_{S-B}T^3} \cdot \frac{1}{l_{mfp-\gamma}} \cdot \frac{L}{4\pi r^2}}$$

Angular (and regular) Doppler
A star with constant $\vec{v}$ releases wave A at t_1, then B at t_2
$c_\Delta t$ is, at t_2, dist b/w A and where star was at t_1.
l_{ab} is dist b/w A and B at t_2



At small $\Delta t, l_{ab} \approx c\Delta t - v\Delta t\cos\theta$
$\lambda_{obs} = \frac{l_{ab}}{n} \quad n$ (no. of peaks and troughs b/w waves)$= \frac{c\Delta t}{\lambda_{em}\gamma}$
$\rightarrow \boxed{\lambda_{obs} = \lambda_{em}\gamma(1 - \frac{v}{c}\cos\theta)}$

Virial Thm
$\mathrm{S} = \sum_\alpha \vec{p}_\alpha \cdot \vec{r}_\alpha \quad \left\langle\frac{dS}{dt}\right\rangle \equiv \frac{1}{N\,measurements}\sum_{n=0}^{N-1}\frac{dS}{dt} =$

$\boxed{\frac{1}{T}\int_0^T \frac{dS}{dt}dt = 0\,as\,T \rightarrow \infty}$ T: total time of all measurements.

$\left\langle\frac{dS}{dt}\right\rangle = 0 = \left\langle\sum_\alpha \frac{d\vec{p}_\alpha}{dt} \cdot \vec{r}_\alpha + \vec{p}_\alpha \cdot \frac{d\vec{r}_\alpha}{dt}\right\rangle = \left\langle\sum_\alpha \vec{F}_\alpha \cdot \vec{r}_\alpha\right\rangle + 2\left\langle K_{total}\right\rangle$
$\left\langle\frac{dS}{dt}\right\rangle = 0 = \left\langle\sum_\alpha \left(\sum_{i\neq\alpha} \vec{F}_{i\rightarrow\alpha} \cdot \vec{r}_\alpha\right)\right\rangle + 2\left\langle K_{total}\right\rangle$
The inner summation term can be divided up into $\alpha > i$ and $\alpha < i$ sums. These are just mirrored, $\therefore$
$\left\langle\frac{dS}{dt}\right\rangle = 0 = \left\langle\sum_\alpha \left(\sum_{i<\alpha} \vec{F}_{\alpha,i} \cdot (\vec{r}_\alpha - \vec{r}_i)\right)\right\rangle + 2\left\langle K_{total}\right\rangle$
$\left\langle\frac{dS}{dt}\right\rangle = 0 = \left\langle\sum_\alpha U_{i\alpha}\right\rangle + 2\left\langle K_{total}\right\rangle \rightarrow \boxed{\langle U\rangle + 2\left\langle K_{total}\right\rangle = 0}$

Hydrostatic Equilibrium
For cylinder slice of star w/ thickness dr, mass dm,
$F_g = \frac{G\,\mathrm{d}m M(<r)}{r^2}$
$F_{up} = AP(r) \quad F_{down} = AP(r - dr)$
$F_{down} + F_g = F_{up} \rightarrow A(P(r) - P(r + \mathrm{d}r)) = \frac{G(\rho(r)A\,\mathrm{d}r)M(<r)}{r^2}$
Cancel out A's, simplify left side to become H.E. eqn.

Hydrostatic Eq. to Virial Thm
Multiply both sides of HS eqn by r, then integrate from 0 to R wrt r, right side is then U_{grav}.
Left side becomes $-3\overline{P}V$ (integrating P(r) over star V), so $U_{grav} = -3\overline{P}V$.
Using $PV = \frac{2}{3}K$, $U_{grav} + 2K = 0$: Virial Thm