EMP Dome Theoretical Basis

1 Marx Generator Construction

This section discusses the theoretical performance of a hyper-thin Marx generator. The motivation for shrinking the axial dimension is to maximize energy density and minimize timing error for phase synchronization. Additionally, resistance is minimized, current is maximized, and the discharge and charge time constant is vastly reduced.

1.1 Geometry of Generator

Model a repeatable slice of the Marx generator as an air gap of height h_a, atop a copper plate of height h_c, atop a dielectric plate of height h_d, atop another copper plate of height h_c. All sections are of radius r, which is much greater than any particular height.

In this construction, the parallel plate capacitor expression for capacitance is sufficiently accurate. Therefore, we obtain the following expression for total generator capacitance, C.

$$C = \frac{\epsilon A}{h_d n} \tag{1}$$

where ϵ is the permittivity of the dielectric and n is the number of identical slices which make up the Marx generator. The total resistance of the slices, R, is given by

$$R = \frac{2n\rho h_c}{A} \tag{2}$$

where ρ is the resistivity of copper, which is approximately 10^{-8} Ωm. The RC time constant for such an array is then

$$\tau = CR = \frac{2\epsilon\rho h_c}{h_d} \tag{3}$$

For a device which has roughly uniform stack heights, one obtains an expression for the time constant

$$h_c \approx h_d \rightarrow \tau = 2 * (10^{-8}\ \Omega\ \mathrm{m})(8.85 * 10^{-12}\ \mathrm{F/m}) \tag{4}$$

$$\tau \approx 8.85 * 10^{-20} s \tag{5}$$

Ordinarily, skin effect in such a large radius conductor would attenuate the current transfer and elongate the time constant considerably. However, if the height h_c is comparable to or less than the skin depth, the induced eddy currents become restricted and the bulk conductor can participate in current transfer. For frequencies with associated time constants much larger than the time constant above, the skin depth is

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} \tag{6}$$

$$\begin{aligned} \delta &- \text{skin depth} \\ \rho &- \text{conductor resistivity} \\ \omega &- \text{signal frequency} \\ \mu &- \text{permeability of conductor} \end{aligned} \tag{7}$$

For copper, $\mu = 4\pi * 10^{-7}$ H/m and for a $\tau \approx 5$ ns $\rightarrow \omega = 1/(2\pi 10^{-9}) s^{-1}$, one obtains

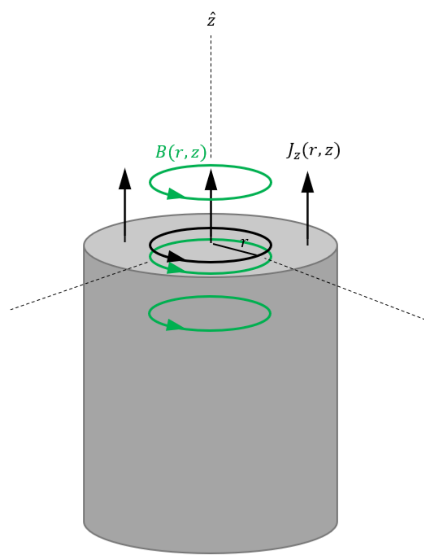
$$\delta = \sqrt{\frac{2 * 10^{-8}\Omega \text{ m} * 2 * \pi * 10^{-9}}{4\pi 10^{-7}\text{H/m}}} = 10\mu\text{m} \tag{8}$$

Therefore, for $h_c < 10\mu$m, the skin effect is reduced and the copper plates and the effective cross sectional area of the copper plates is not restricted to the skin depth (allowing the device's resistance to approach the expression for R above). Additionally, such a small conductor length approaches a few times the mean free path (for copper, 0.38 nm) which gives rise to velocity overshoot which can increase the conductivity of the conductor (lower ρ).

1.2 Derivation of Skin Effect Extinction

Skin effect extinction is critical to this proposal, so a thorough derivation from first principles is presented here. Skin effect is often derived by analyzing two Amperian loops inside of a cylindrical conductor.

To begin, we derive the magnetic field inside the conductor as a function of position.



We can derive the magnetic field from Faraday's Law.

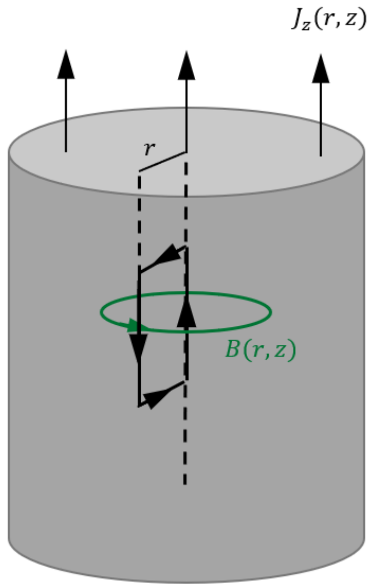
$$\nabla \times B = \mu J \tag{9}$$

$$\int_{\text{loop}} B \cdot dl = \mu \int_{\text{area}} J \cdot dA \tag{10}$$

$$2\pi r B = \mu \int_0^{2\pi} \int_0^r J_z(r', z) r' dr' dz \tag{11}$$

$$B(r, z) = \frac{\mu}{r} \int_0^r J_z(r', z) r' dr' dz \tag{12}$$

With this result we can begin the analysis of the first Amperian loop.



From the first loop we obtain an expression for the magnetic field inside the conductor as a function of distance from the central axis, r.

$$\nabla \times E = \frac{-\partial B}{\partial t} \tag{13}$$

$$\int_{\text{loop}} E \cdot dl = -\frac{\partial}{\partial t} \int_{\text{area}} B \cdot dA \tag{14}$$

$$-J_z(r)h + \int_r^0 J_r(\rho)d\rho + J_z(0)h + \int_0^r J_r(\rho) = -\frac{1}{\sigma}\frac{\partial}{\partial t}\int_0^r hB(\rho)d\rho \tag{15}$$

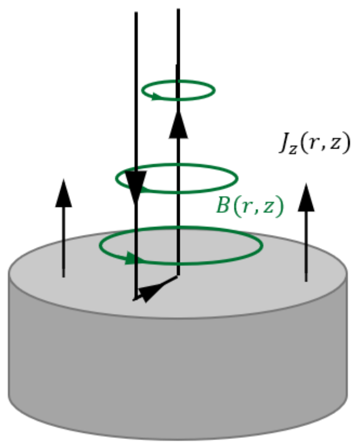
For wires, typically $h \gg r$, so $\int J_r(\rho)$ is presumed only a minor contribution. Additionally, for long wires, the assumption of isotropy with respect to translation along the axis of the wire suggests that $J_r = 0$ in the bulk wire. Therefore, the dropoff of J_z towards the center of the wire is is presumed

$$J_z(0,z) - J_z(r,z) = -\frac{1}{\sigma}\frac{\partial}{\partial t}\int_0^r B(r', z)dr' \tag{16}$$

$B(r', z)$ was determined earlier and can be expressed in terms of $J_z(r,z)$, which can be substituted above and a solution for $J_z(r,z)$ is found.

However, in the case that h is comparable in size or smaller than r, the influence of the ends can be considerable throughout the bulk. Examine an

Amperian loop which grazes the top surface of the conductor and extends out to infinity.



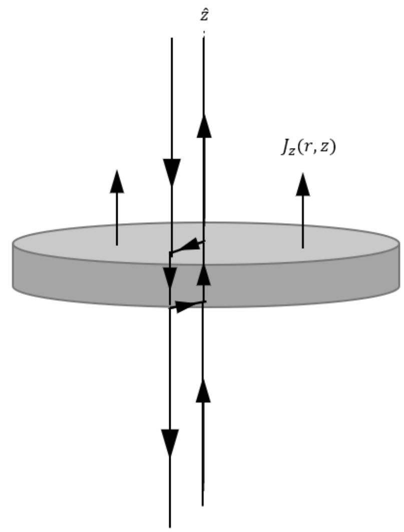
Because the current induces a non-zero field far from the wire, this loop must have a non-zero E field line integral. However, assuming a uniform external E is driving the current, the contribution from the portions of the loop parallel to the conductor axis cancel. The portion at infinity contributes nothing to the line integral since $B = 0$ at infinity. Therefore, the only contribution comes from the portion of the loop grazing the conductor surface. This permits the important result

$$\int_r^0 J_r(r', 0)dr' = -\frac{1}{\sigma}\frac{\partial}{\partial t}\int_0^\infty \int_0^r B(r', z)dr'dz \tag{17}$$

Therefore, we can express the radial current on the axial surfaces of the conductor in terms of the exterior field of the conductor. The exterior field can be found from Biot Savart Law. However, it is convenient to use an approximation. We will assume the B field is continuous at $z = 0$ and $z = -h_c$. We will also assume the field falls with the inverse square of z, the axial distance from the conductor. Thus we can find

$$B(r, z) \approx \frac{h_c^2}{4z^2}B(r, 0) \tag{18}$$

In the case of a thin conductor, we will take an Amperian loop that grazes the top and bottom of the conductor.



We can write the following expression

$$\int_{\text{loop}} E \cdot dl = -\frac{\partial}{\partial t}\int_{\text{area}} B \cdot dA \tag{19}$$

$$\begin{aligned}
&\int_{-h_c/2}^{h_c/2} J_z(r,z)dz - \frac{1}{\sigma}\frac{\partial}{\partial t}\int_{h_c/2}^{\infty}\int_0^r \frac{h_c^2}{4z^2} J_z(r',z)r'dr'dz \\
&+\int_{-h_c/2}^{h_c/2} J_z(0,z)dz - \frac{1}{\sigma}\frac{\partial}{\partial t}\int_{h_c/2}^{\infty}\int_0^r \frac{h_c^2}{4z^2} J_z(r',z)r'dr'dz \\
&\qquad = -\frac{1}{\sigma}\frac{\partial}{\partial t}\int_{-\frac{h_c}{2}}^{\frac{h_c}{2}}\int_0^r J_z(r',z)r'dr'dz
\end{aligned} \tag{20}$$

$$\begin{aligned}
&-\frac{1}{\sigma}\frac{\partial}{\partial t}\int_{-\frac{h_c}{2}}^{\frac{h_c}{2}}\int_0^r J_z(r',z)r'dr'dz + {\color{red}-\frac{1}{\sigma}\frac{\partial}{\partial t}\int_{\frac{h_c}{2}}^{\infty}\int_0^r B(r',z)r'dr'dz} \\
&\qquad = \int_{\frac{h_c}{2}}^{\frac{h_c}{2}} (J_z(r,z) - J_z(0,z))dz
\end{aligned} \tag{21}$$

The portion in red is the surface current normally ignored in skin effect derivations (since the derivation is taken far from the edges of the conductor where, from symmetry, one can infer the absence of radial current). More precisely, this term drops off in the bulk of long wires since the previous term (the magnetic field enclosed by the loop) is much larger than the field beyond the edges of the conductor. To demonstrate the extinction of skin effect for a thin conductor, we will apply the B field approximation described above.

$$-\frac{1}{\sigma}\frac{\partial}{\partial t}\int_{-\frac{h_c}{2}}^{\frac{h_c}{2}}\int_0^r J_z(r',z)r'dr'dz+\frac{h_c}{\sigma}\frac{\partial}{\partial t}\int_0^r J_z(r',z)r'dr'dz = \int_{-h_c/2}^{h_c/2}(J_z(r,z)-J_z(0,z))dz \tag{22}$$

For a relatively constant J_z along $\hat{z}$, we can find in the thin conductor case

$$-\frac{h_c}{\sigma}\frac{\partial}{\partial t}\int_0^r J_z(r',z)dr'dz+\frac{h_c}{\sigma}\frac{\partial}{\partial t}\int_0^r J_z(r')dr'dz = h_c(J_z(r)-J_z(0)) \tag{23}$$

$$0 = J_z(r)-J_z(0) \tag{24}$$

Therefore, in the case where the three Amperian loops enclose roughly equivalent magnetic flux (as in the thin conductor case), the skin effect vanishes. For long conductors, the central loop encloses vastly more magnetic flux and the skin effect holds.

1.3 Sufficient h_c for full dielectric charge

h_c should be large enough to ensure that there are enough free electrons in each electrode to fully charge the dielectric.

$$Q = \frac{C}{V} \tag{25}$$

$$\frac{\epsilon \pi r^2}{h_d}V \leq Nq = h_d \pi r^2 qn \tag{26}$$

where n is the volumetric concentration of free electrons in copper ($n = 8.3 * 10^{28}\text{m}^{-3}$).

$$h_d^2 \geq \frac{V\epsilon}{qn} \tag{27}$$

$$h_d \geq \sqrt{\frac{V\epsilon}{qn}} \tag{28}$$

For copper, we can determine the maximum voltage for which the height $h_d = 1\ \mu\text{m}$ will permit a full charge.

$$V \leq \frac{(10^{-6}\ \text{m})^2(1.6*10^{-19}\ \text{C})(8.3*10^{28}\ \text{m}^{-3})}{(8.85*10^{-12}\ \text{F/m})} \tag{29}$$

$$V \leq 1.5*10^9\ \text{V} \tag{30}$$

Therefore, free electron availability will not restrict the charging capacity of the slices. This limit is never reached since the dielectric breakdown of the capacitor dielectric will prevent the generator from reaching voltages of this magnitude.

1.4 Energy storage of generator

The generator can store a total energy, U_g, which is given by

$$U_g = \frac{1}{2} n C V^2 = \frac{\epsilon \pi r^2 n}{2 h_d} V^2 \tag{31}$$

where n is the number of slices that comprise the generator. We can determine the total stored energy of the array as well as energy density. First, identify the length, L_s, of a capacitor slice with air gap, h_a.

$$L_s = h_d + 2h_c + h_a \tag{32}$$

Under the current construction, $h_d = h_c$, so we can obtain an expression for n fitting into a total array length L.

$$n = \frac{L}{L_s} = \frac{L}{(3h_d + h_a)} \tag{33}$$

We would like to make h_a very small to keep conductivity of the device high. However, the length h_a has a lower bound dictated by the need to prevent dielectric breakdown of the air gap without external triggering. For the Marx generator to independently cascade after the first trigger, the constrain on h_a is

$$\frac{V}{E_{DB}} \geq h_a \geq \frac{V}{2E_{DB}} \tag{34}$$

where E_{DB} is the minimum electric field triggering a dielectric breakdown of air. Assuming we are somewhat constrained in the value of L, we can determine a maximum charged energy density (and, since L is fixed and known, a maximum total energy) from the following.

$$\frac{U_g}{L} = \frac{\epsilon \pi r^2 L V^2}{2(3h_d + h_a) h_d L} = \frac{\epsilon \pi r^2 V^2}{2(3h_d + \frac{V}{E_{DB}}) h_d} \sim V \tag{35}$$

We can obtain from this the total array peak voltage, V_T.

$$V_T = nV = \frac{LV}{(3h_d + \frac{V}{E_{DB}})} \tag{36}$$

From these equations there is motivation to maximize energy density and peak voltage by making h_a as large as possible to accommodate the highest possible slice voltage. However, doing so vastly increases the resistance of the current path through the generator, which makes a larger portion of the stored energy inaccessible. Ultimately, we must seek to maximize radiated energy, U_r. Radiated energy is maximized when power is maximized. To maximize the radiative power, we must impedance match the impedance of the generator with the impedance of the antenna. The total system impedance is

$$Z_T = Z_g + Z_l + Zr \tag{37}$$

where Z_g is the impedance of the generator, Z_l is the impedance of losses in the antenna and added losses for matching, and Z_r is the radiative impedance. Since we are tuning Z_l for impedance matching purposes, we will find

$$Z_T = Z_g + (Z_g - Z_r) + Z_r = 2Z_g \tag{38}$$

for the impedance matched case. We can obtain an expression for the total radiated power.

$$P_r = Z_r I^2 = Z_r \left(\frac{V_T}{Z_T}\right)^2 \tag{39}$$

$$P_r = \frac{Z_r V_T^2}{2Z_g^2} \tag{40}$$

The resistance, R_g of the generator can be obtained from the previous resistance expression for the slice plus the resistance from the plasma ($\rho_a \approx 10^{-3}\ \Omega$ m).

$$R_g = n\left(\frac{2\rho_c h_c + \rho_a h_a}{\pi r^2}\right) \tag{41}$$

$$R_g = \left(\frac{L}{(3h_d + \frac{V}{E_{DB}})}\right)\left(\frac{2\rho_c h_c + \rho_a h_a}{\pi r^2}\right) \tag{42}$$

We can expand this using a previous expression for V_T.

$$P_r = \frac{Z_r\left(\frac{LV}{(3h_d + \frac{V}{E_{DB}})}\right)^2}{2\left(\left(\frac{L}{(3h_d + \frac{V}{E_{DB}})}\right)\left(\frac{2\rho_c h_c + \rho_a h_a}{\pi r^2}\right)\right)^2} \tag{43}$$

$$P_r = Z_r\left(\frac{\pi r^2}{2(\rho_c h_c + \rho_a h_a)}\right)V^2 \tag{44}$$

Since power and energy are proportional by a time period, the expression above can be modified to obtain the fraction of stored energy which will be radiated away.

$$U_r = U_g \frac{Z_r \pi r^2}{2(\rho_c h_c + \rho_a h_a)} \tag{45}$$

$$U_r = \frac{\epsilon \pi r^2 V^2 L}{2(3h_d + \frac{V}{E_{DB}})h_d} \frac{Z_r \pi r^2}{2(\rho_c h_c + \rho_a \frac{V}{E_{DB}})} \tag{46}$$

We can obtain an expression for the high voltage limit

$$\boxed{\lim_{V\to\infty} U_r = \frac{\epsilon\pi^2 r^4 L Z_r E_{DB}^2}{4h_d\rho_a}} \tag{47}$$

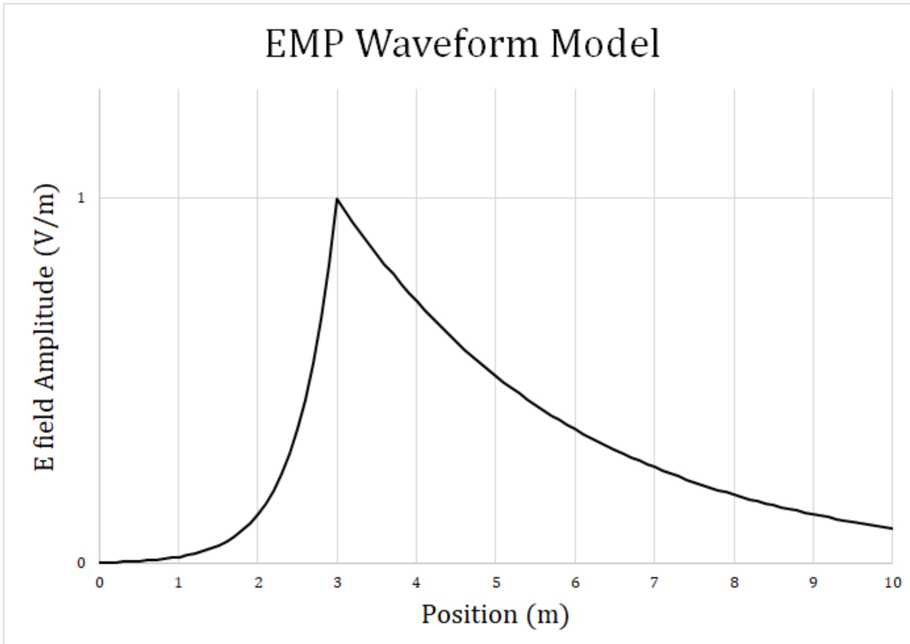
This is a very important result which we discuss here. The geometry of a generator array dictates the maximum energy it can radiate. Specifically, the radiated energy is proportional to the square of the array's conductive area (which ordinarily suffers from skin effect) and directly proportional to the capacitance (which ordinarily is kept low for a short discharge time constant). Herein lie the two major advantages which enable this generator design to deliver considerably more energy. First, this generator uses its scale to overcome the skin effect and have millions of times more conductive cross sectional area in the generator. Second, this generator design has a time constant which is invariant with capacitance (since capacitance and resistance are inversely proportional in this design). Therefore, the discharging time constant may be kept low while allowing the capacitance to rise, packing more energy in the generator and radiating more energy during a pulse. Theoretically, this design could easily impart 10^{15} times the energy of existing, compact Marx generators.

Under the assumption that $n = 300$, $Z_g = 0.0008\ \Omega$, $r = 0.5$ m, $h_d = 10^{-6}$ m, $L = 2$ m, and $Z_r = 0.01 * Z_g = 8\ \mu\Omega$, we find that $U_r \approx 1$ MJ for sufficiently high charge voltage ($V \geq 1000V$). This is the justification for analyzing megajoule range energies in subsequent sections.

2 Timing Constraints from Exponential Wave Model

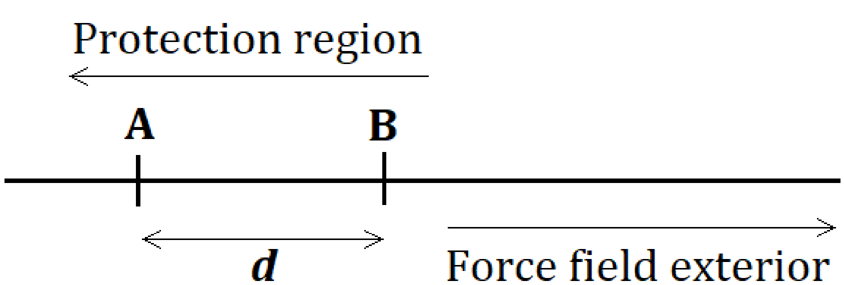
It is more accurate to model the EMP pulse as two joined exponentials

$$E(x,t) = E_0\left(u\left(x - \frac{t}{c}\right)\exp\left(\frac{x - t/c}{\tau_1}\right) + u\left(\frac{t}{c} - x\right)\exp\left(\frac{x - t/c}{\tau_2}\right)\right) \tag{48}$$



2.1 Constraint on Size of Array

Begin by focusing on the centerline that connects the protection region center, A, and B.



The time delay for the pulse at A to reach B $\frac{2d}{c}$ must be greater than the time constant of the rising portion of the pulse at B, τ_1. Thus we obtain

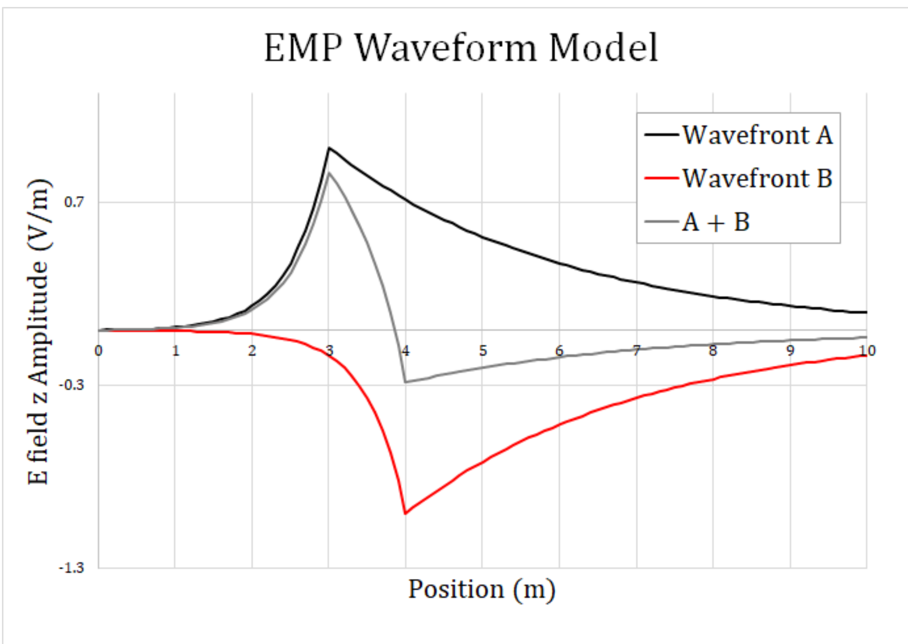
$$\frac{2d}{c} \geq \tau_1 \tag{49}$$

The concession on the exterior pulse amplitude for $\frac{2d}{c} \leq \tau_1$ is

$$\boxed{E = E_0 \text{exp}\left(\frac{-2d}{c\tau_1}\right)} \tag{50}$$

2.2 Constraint from Timing Error

If there is some non-zero timing error in when generator A fires relative to when it should fire, δt, one obtains an imperfect cancellation within the protection region



For an timing error, δt, the leaked amplitude is

$$\boxed{E_p = E_0 \left(1 - \text{exp}\left(\frac{-|\delta t|}{\tau_1}\right)\right)} \tag{51}$$

If we define a tolerability fraction $f_t = \frac{E_c}{E_0}$ we can obtain the tolerance condition

$$1 - f_t \leq \exp\left(\frac{-|\delta t|}{\tau_1}\right) \tag{52}$$

$$\ln(1 - f_t) \leq \frac{-|\delta t|}{\tau_1} \tag{53}$$

$$\tau_1 \leq \frac{-|\delta t|}{\ln(1 - f_t)} \tag{54}$$

$$\tau_1 \geq \frac{|\delta t|}{\ln\left(\frac{1}{1-f_t}\right)} \tag{55}$$

If we use the approximation $\ln(1 + x) \approx x$ and assume that f_t is small, we can use the approximate constraint

$$\tau_1 \geq \frac{|\delta t|}{f_t} \tag{56}$$

This is a useful relation, showing that the required time constant must be at least as large as to make the timing uncertainty the same fraction of the time constant as the tolerability level for the protection radius.

We can state the complete constraint equation as

$$\boxed{\frac{|\delta t|}{f_t} \leq \tau \leq \frac{2d}{c}} \tag{57}$$

Where we have taken $\tau_1 = \tau$ for simplicity since we only need to characterize the rising part of the EMP wave.

$$\begin{aligned} c &- \text{speed of light} \\ f_t &- \text{asset EMP tolerance} \\ \delta t &- \text{timing error} \\ \tau &- \text{pulse duration} \\ d &- \text{generator pair separation} \end{aligned} \tag{58}$$

For an example calculation, suppose

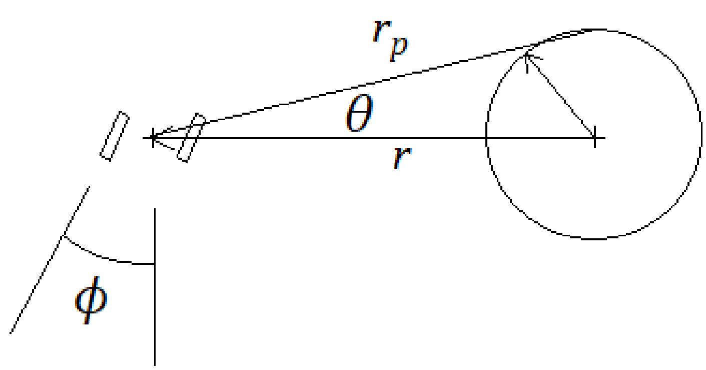
$$\tau = 10\text{ns},\ f_t = 5\% \implies \delta t \leq 500\text{ps},\ d \geq 30\text{m} \tag{59}$$

2.3 Incorporating Dipole Radiation Pattern

Thus far, the analysis on the propagating waveform has assumed an isotropic radiation pattern. Now we examine the effect of a dipole radiation pattern on constraints.

Most notably, pulse synchronization is not necessary for centerline protection if the dipoles are pointed towards the center of the protection radius (due to the zero radiation strength along the dipole axis). However, this permits significant holes in the outward wavefront which enemies could exploit. However, by aligning all dipole antennae with the center of the protection region and then tilting the dipole by an angle ϕ, one can tune the array's dependence on phase synchronization from none to complete.

Let r be the distance from the protection radius to the center of a pair of generator dipoles, let r_p be the desired protection radius, and let *theta* represent the angle subtended by the ray from the center of the generator pair to the center of the protection radius and the ray from the center of the generator pair to the end of the protection radius (such that *theta* is at maximum).



$$E = E_0 \sin^2(\phi - \theta) \left(1 - \exp\left(\frac{-|\delta t|}{\tau}\right)\right) \tag{60}$$

$$E = E_0 \left(\sin(\phi)\cos(\theta) - \sin(\theta)\cos(\phi)\right)^2 \left(1 - \exp\left(\frac{-|\delta t|}{\tau}\right)\right) \tag{61}$$

$$E = E_0 \left(\sin(\phi) \left(\frac{r}{\sqrt{r^2 + r_p^2}} \right) - \left(\frac{r_p}{\sqrt{r^2 + r_p^2}} \right) \cos(\phi) \right)^2 \left(1 - \exp\left(\frac{-|\delta t|}{\tau} \right) \right) \tag{62}$$

$$E = \left(\frac{E_0}{r^2 + r_p^2} \right) (r \sin(\phi) - r_p \cos(\phi))^2 \left(1 - \exp\left(\frac{-|\delta t|}{\tau} \right) \right) \tag{63}$$

For a fractional tolerance, $f_t = \frac{E_{\text{tolerable}}}{E_0}$, we can derive a modified constraint

$$f_t \geq \left(\frac{(r \sin(\phi) - r_p \cos(\phi))^2}{r^2 + r_p^2} \right) \left(1 - \exp\left(\frac{-|\delta t|}{\tau} \right) \right) \tag{64}$$

$$\frac{-|\delta t|}{\tau} \geq \ln \left(1 - f_t \left(\frac{r^2 + r_p^2}{(r \sin(\phi) - r_p \cos(\phi))^2} \right) \right) \tag{65}$$

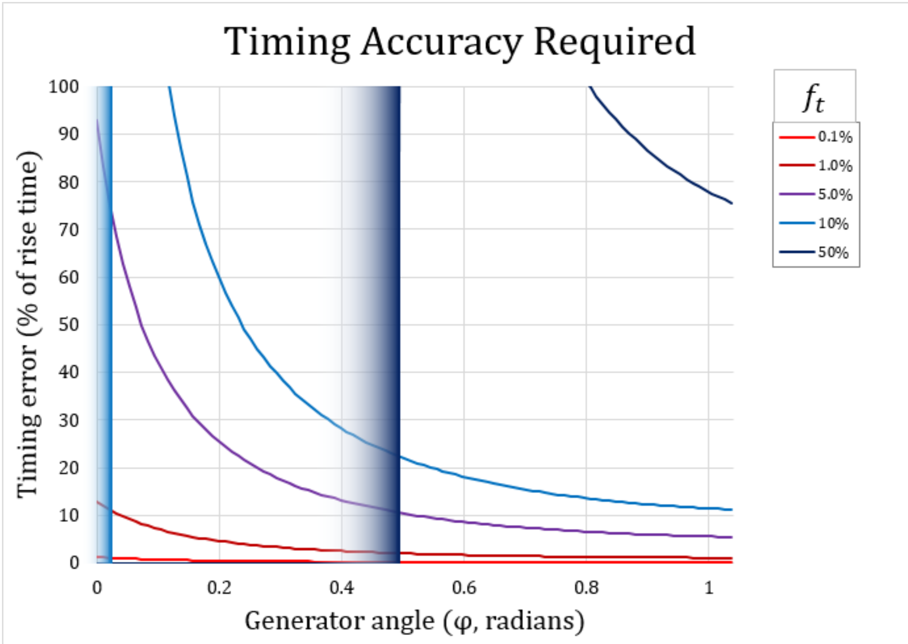
If we introduce a fractional utilization of the array, $m = \frac{r_p}{r}$, we can express the constraint

$$|\delta t| \leq -\tau \ln \left(1 - f_t \left(\frac{1 + m^2}{(\sin(\phi) - m \cos(\phi))^2} \right) \right) \tag{66}$$

This equation is only defined for situations in which sufficient cancellation of field inside the protection radius is not guaranteed by the geometry of the array. The geometry of the array guarantees cancellation in the protection radius for

$$f_t \leq \left(\frac{(\sin(\phi) - m \cos(\phi))^2}{1 + m^2} \right) \tag{67}$$

It is useful to examine required timing accuracy for a specified f_t. Note that on the plot below each line associated with a f_t reaches a threshold ϕ for which phase synchronization is no longer required (the attenuation due to the radiation pattern alone is sufficient). To the right of the faded area, no timing accuracy is required. The calculations for the plot below assumed $m = 0.5$.



3 Energy, Power, and Performance

3.1 Pulse intensity far from generator

The radiated energy from the EMP is roughly equally distributed in a radiation pattern shell of a thickness equal to the pulse width. For a spherical, isotropic radiation pattern (useful for initial approximation), the energy is given by

$$\int_{\text{shell}} \frac{\epsilon}{2}|E(R)|^2 dV = U_r \tag{68}$$

$$\frac{\epsilon}{2}|E(R)|^2 4\pi R^2 c\tau = U_r \tag{69}$$

$$|E(R)| = \sqrt{\frac{U_r}{2\epsilon\pi c\tau}}\frac{1}{R} \tag{70}$$

where τ is the pulse rising time, $|E(R)|$ is the magnitude of the electric field at peak, and R is the distance from the generator. For a quick calculation, taking $U_r = 1$ MJ, $\tau = 10$ ns, and $R = 1000$m, we find $|E(R)| = 1.37 * 10^5$ V/m. This is within the realm of electric field intensities necessary to interfere with small scale electronics.

3.2 Managing leakage into the protection region

Since there is a non-zero leakage into the protection region due to a non-zero timing error, $|\delta t|$, the intensity of the pulse will need to be restricted so that the assets can tolerate the leakage. This gives rise to the most restrictive constraint on range. Assume that a field intensity which is tolerable to the assets is also non-damaging to attackers. We obtain the following relation

$$E(R_{max}) = E_p \tag{71}$$

where E_p is the field leaked into the protection region. For N generator pairs aiming for constructive interference in a favored direction, we obtain

$$\sqrt{\frac{NU_r}{2\pi\epsilon c\tau}}\frac{1}{R_{max}}\exp\left(\frac{-|\delta t|}{\tau}\right) = \sqrt{\frac{NU_r}{2\pi\epsilon c\tau}}\frac{1}{r}\left(1-\exp\left(\frac{-|\delta t|}{\tau}\right)\right) \tag{72}$$

$$\frac{R_{max}}{r} = \frac{1}{\exp\left(\frac{-|\delta t|}{\tau}\right)-1} \tag{73}$$

For $|\delta t| = 0.5$ ns, we find

$$R_{max} = 20.5r \tag{74}$$

For $r = 50$m, $R_{max} = 1025$ m. Therefore, we have shown that, with modest phase synchronization, a sufficient effective range is possible without jeopardizing the assets in the protection radius.

3.3 Multiple pulse power

For a Marx generator that begins charging from zero, the stored energy is

$$U(p) = \frac{1}{2}CV\left(1-\exp\left(\frac{-p}{RC}\right)\right) \tag{75}$$

Where p is the period of time over which the generator has been charging. If the generator fires periodically with a period p, the output power is

$$P(p) = \frac{E(p)}{p} \tag{76}$$

This is a simple optimization problem which yields the maximum power output for

$$p = RC \tag{77}$$

$$\boxed{P_{\max} = \frac{CV}{2p}\left(1-e^{-1}\right)} \tag{78}$$