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Imperfect information, algorithmic price discrimination, and collusion*

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Abstract

We analyze the ability of firms to sustain collusion in a setting in which horizontally differentiated firms can price discriminate based on private information. Firms receive private, noisy signals regarding customers' preferences. We find that there is a non-monotonic relationship between signal quality and the sustainability of collusion. Starting from a low level, an increase in signal precision first facilitates collusion. There is, however, a threshold beyond which any further increase renders collusion less sustainable. Our analysis provides important insights for competition policy, particularly in light of firms' growing reliance on increasingly sophisticated computer algorithms to analyze consumer data and to make pricing decisions. In contrast to previous findings, our results reveal that a ban on price discrimination can help to prevent collusive behavior as long as signals are sufficiently noisy.

Keywords: Algorithm; big data; collusion; private information; signal; third-degree price discrimination
JEL classification: *D*43; *L*13; *L*41

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1. Introduction

The digitalization of the economy has led to an ever-growing, massive accumulation of data – especially consumer data – and the development of tools to analyze big data. Furthermore, important decisions are increasingly supported by machine learning and artificial intelligence, or are autonomously made by algorithms. The growing use of technologically advanced tools poses a potential challenge to competition in a wide variety of sectors and industries, because it may give rise to anti-competitive conduct. With regard to price setting, the new opportunities resulting from the technological progress, such as pricing algorithms, have led to concerns that collusion could be facilitated, and customers could be hurt. Given the widespread collection of data as a result of the use of Internet-based goods and services (e.g., smartphones, social media) and the increasing importance for customers to purchase goods and services over the Internet, these concerns are relevant for most groups of society – see, for instance, the case studies on retail and electric utility in McKinsey Global Institute (2017) – and their relevance will become even greater in the future. Competition authorities and regulators around the world have started to look into this issue (see, e.g., OECD, 2017). Nevertheless, there has been only very little research into whether or how algorithms affect the ability of firms to sustain collusion. To guide the ongoing and intensifying discussion on this issue, we propose a theoretical model that captures one key aspect of the use of algorithms for pricing: improving data analysis for better price discrimination (see, e.g., Gupta and Pathak, 2014; Beneke and Mackenrodt, 2021).

We analyze the sustainability of tacit collusion in a setting in which horizontally differentiated firms can price discriminate based on private, imperfect information. More precisely, prices for different customer groups can be based on private, but imperfect signals on customers' preferences. These aspects are of high relevance, for example, in traditional brick-and-mortar and online retailing markets. To price discriminate between different customer segments, most firms in these industries collect data on their own customers through different channels (e.g., loyalty programs, cookies), or buy data from data-collection firms. In the US, for instance, the second-largest discount store retailer, Target, uses a data-mining program to assign many different predictors to customers.[1] The quality of the data and the precision of the predictions, however, can crucially affect firms' pricing

[1]See the article "How Companies Learn Your Secrets", by C. Duhigg in the *New York Times*, http://www.nytimes.com/2012/02/19/magazine/shopping-habits.html. See also Esteves (2014) for these and other examples. Further real-life examples are provided in Gautier et al. (2020).

decisions.[2] In particular, data quality and/or data analyses are rarely perfect, and algorithms are used to improve them. In the case of Target, for example, its so-called "pregnancy prediction" was flawed; pregnancy-related mailers were sent out to women for months after they had suffered a miscarriage.[3]

At the same time, antitrust authorities have already expressed growing concern over the use of such technology to aid collusive behavior in online retailing. The acuteness of this issue was underscored by the recent stern warning from the Competition and Market Authority (CMA) in the UK. The warning was issued after the CMA had found signs of price coordination among retailers in different markets on platforms such as Amazon.[4,5]

We shed light on this issue by examining the relationship between price discrimination and firm behavior, with an eye toward understanding the conditions under which collusion is most likely to occur. We find that the critical discount factor necessary to sustain tacit collusion is non-monotonic in signal precision. In particular, an increase in precision reduces the critical discount factor whenever the level of signal precision lies below a certain threshold. From levels above this threshold, an increase in precision leads to a higher critical discount factor. The intuition behind this finding is as follows. In our model, collusive profits are independent of signals, whereas deviation profits and competitive profits, which serve as punishment for deviations, can depend on how precise signals are. Deviation profits are weakly increasing in signal quality, because price discrimination allows firms to target customers more effectively. At the same time, competitive profits are falling in signal quality. Competition grows fiercer because both firms can price discriminate more effectively. Hence, improvements in signal precision have opposing effects on the critical discount factor because both the gains from deviation and the losses from punishment increase. We show that, below the threshold, the gain from defecting does not outweigh the loss from punishment. Intuitively, potential misrecognition of customers renders deviation from collusive prices relatively unprofitable. But above the threshold, the reverse is true. Because customers can be targeted

[2]For discussions of this issue in different contexts, see Liu and Serfes (2004), Esteves (2004, Chapter 2), and Colombo (2016), among others.
[3]As Charles Duhigg, a journalist with the *New York Times*, puts it: "I can't tell you what one shopper is going to do, but I can tell you with 90 percent accuracy what one shopper is going to do if he or she looks exactly like one million other shoppers. You expect that there is some spillage there, and as a result that you will give the wrong message to a certain number of people." See https://6thfloor.blogs.nytimes.com/2012/02/21/behind-the-cover-story-how-much-does-target-know/.
[4]For more details, see, for instance, https://www.theguardian.com/business/2016/nov/07/online-sellers-price-fixing-competition-and-markets-authority-amazon.
[5]In the first anticompetitive e-commerce case, *US v. Topkins*, a poster seller on Amazon was accused of writing computer code for an algorithm to coordinate price setting with his competitors. See Mehra (2016) for details.

effectively, deviation becomes relatively tempting. As a consequence, we find that, in contrast to previous findings, banning price discrimination can be an effective means to prevent collusion when signals are sufficiently uninformative. This holds true for most robustness checks.

We contribute to the literature as follows. This paper first adds to the combination of two strands of the theoretical industrial organization literature: third-degree price discrimination and collusion, both among horizontally differentiated firms. In the first strand, Bester and Petrakis (1996) show that third-degree price discrimination through the use of coupons intensifies competition in markets that are segmented exogenously by customer preferences. In a similar set-up, Shaffer and Zhang (1995) illustrate that the possibility of third-degree price discrimination leads to a prisoner's dilemma. Corts (1998) then generalizes these findings; under best-response asymmetries (firms find different groups of customers most valuable), third-degree price discrimination leads to profits lower than under uniform pricing. Fudenberg and Tirole (2000) analyze the impact of third-degree price discrimination in a dynamic context in which learning about customer preferences is endogenous from the purchasing history. After the first period, firms learn about the preferences of their own customers. In the second period, poaching can take place through price discrimination. The authors also find that third-degree price discrimination, which they refer to as behavior-based price discrimination, results in more intense competition and, hence, lower profits. Villas-Boas (1999) extends their set-up to long-lived firms and overlapping customer generations, and finds that competition is intensified if firms and customers are patient.

In these contributions, firms either have or obtain perfect information about customer preferences; by contrast, Esteves (2009, 2014) analyzes the impact of imperfect information. She shows that improving the quality of information also results in lower competitive profits under third-degree price discrimination. Colombo (2016) investigates the impact of imperfect information in the dynamic context of Fudenberg and Tirole (2000). First-period learning is noisy because firms cannot recognize every first-period customer, and, hence, only learn the preferences of a portion of their customers. He finds that there is an inverse U-shaped relationship between quality of information and competitive profits. Following Stole (2007), fiercer competition due to third-degree price discrimination creates incentives to commit to uniform pricing. That is, firms may seek to collude. In our paper, we focus on how potential misrecognition – as in Esteves (2009, 2014) – affects the scope for tacit collusion.

Combining the two strands, Liu and Serfes (2007) consider the impact of information on collusion. In their set-up, however, information is publicly available, and its quality is defined by the number of market segments. Then, an increase in information quality is equivalent to an increase of the number of perfectly distinguishable segments and, hence, the number of segment-specific prices. The authors analyze different collusive schemes. Their main finding is that collusion becomes harder to sustain as the number of market segments increases. Helfrich and Herweg (2016), whose work is closest to ours, consider two settings with perfect information and price discrimination under either best-response symmetries or best-response asymmetries. They show that, compared with the situation in which there is a ban on price discrimination, third-degree price discrimination helps to fight collusion under both best-response symmetries and best-response asymmetries.

The findings from the theoretical literature on the relationship between collusion and third-degree price discrimination can thus be summarized as follows. When price discrimination is based on perfect information, theory predicts that third-degree price discrimination renders collusion less likely.[6] Therefore, the implication for antitrust policy is that a legal ban on third-degree price discrimination facilitates tacit collusion. In most markets, however, information is not perfect. Thus, we contribute to this literature by relaxing the assumption of third-degree price discrimination under perfect information. By generalizing parts of the results in Helfrich and Herweg (2016), our analysis provides an important insight, namely that the outcomes can be fundamentally different when firms' information about customer preferences is private and imperfect.

Moreover, we contribute to the emerging literature that analyzes the relationship between collusion and the use of algorithms. This literature has mainly addressed the question of how prices develop in an experimental setting when simple learning algorithms are used (Calvano et al., 2020; Klein, 2021). It has been documented that supra-competitive prices can indeed be sustained, although it takes quite some time for algorithms to coordinate.[7] A less skeptical picture is drawn by Miklós-Thal and Tucker (2019), who investigate the impact of better demand predictions and algorithms on the sustainability of collusion.

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, we derive the relevant payoffs for the case in which firms can price discriminate and for the case of uniform pricing, and we determine the critical discount factors. Then, we compare the sustainability of collusion in the two pricing regimes. In Section 4, we discuss the robustness of our results. We conclude in Section 5.

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[6]Note that results are less clear-cut for the case of quantity discounts – that is, for second-degree price discrimination (see Gössl and Rasch, 2020).

[7]Technical challenges with regard to tacit collusion (and price discrimination) are presented in Gautier et al. (2020). Legal aspects and issues with regard to a change in competition policy are discussed in, for example, Harrington (2018), Schwalbe (2018), and Ezrachi and Stucke (2020).

2. Model

We consider the following stage game of incomplete information developed in Armstrong (2006), which is a variant of Bester and Petrakis (1996), Esteves (2004, see Chapter 2), and Esteves (2014). Consider a linear city, following Hotelling (1929), with two symmetric firms, A and B, that are located at each end of the unit interval $[0, 1]$ (i.e., $\ell_A = 0$ and $\ell_B = 1$, respectively). Firms' fixed and marginal costs are normalized to zero. Firms compete in prices p_A and p_B, respectively.

A customer wishes to buy a single unit from either firm A or firm B. Buying from firm i (with $i \in \{A, B\}$) gives a customer located at $x \in [0, 1]$ a net surplus of

$$U(x; p_i) = 1 - p_i - \tau|\ell_i - x|,$$

where we normalize the customer's valuation for the unit (which is independent of the firm) equal to one. $|\ell_i - x|$ measures the distance the customer travels to firm i, and τ is the transport cost per unit of distance incurred. Customers of mass one are uniformly distributed along the unit interval.

We assume that firms have private information about brand preferences. This might be because firms use different algorithms, or because they purchase customer data from different marketing companies. When a customer is located in the left half at $x \in [0, 1/2]$, and hence prefers firm A, we assume that firm i receives the signal $s_i = s_L$ with probability $\sigma \in [1/2, 1]$. That is, the signal is weakly informative. Firm i receives the signal $s_i = s_R$ with probability $1 - \sigma$. When a customer at $x \in (1/2, 1]$ prefers firm B instead, firm i receives the signal $s_i = s_R$ with probability σ and the signal $s_i = s_L$ with probability $1 - \sigma$. Conditional on a customer's location, the signals are symmetric in the sense that signal precision is not better for closer customers, and that signals are drawn independently for each firm.[8] In line with our motivation in the introduction, the parameter σ can be interpreted as the accuracy of the algorithm. Advances in technology result in a higher value for σ.

Note that our set-up nests the following two extreme cases: (i) the signal does not convey any information (i.e., $\sigma = 1/2$); and (ii) market segments are perfectly distinguishable (i.e., $\sigma = 1$).[9]

[8]We relax the assumption of independent signals in Section 4.3 and that of symmetric signals in Section 4.4.

[9]The first case represents the classic model in which price discrimination is not possible (or banned), whereas the second case corresponds to a segmented market with two distinguishable segments (see Liu and Serfes, 2007; Helfrich and Herweg, 2016).

The timing of the game is as follows: first, firms receive signals; second, firms simultaneously set prices; and finally, customers decide from which firm to buy, and payoffs are realized.

Because signals are private, firms do not know their competitor's payoff function. Hence, we consider a game of incomplete information. To solve the stage game, we use the notion of Bayesian Nash equilibrium. Throughout the analysis, we focus on equilibria in which the market is covered – that is, the situation in which all customers along the line buy from one of the two firms. For this purpose, we impose the following assumption on customers' transport costs.

Assumption 1. $\tau \in [0, 2/3]$.

Assumption 1, which is common in the related literature, guarantees that the market is fully served, absent price discrimination.[10, 11]

3. Analysis and results

In this section, we analyze collusion under two pricing scenarios: price discrimination and uniform pricing. From the comparison of these two cases, we provide policy implications for a ban on price discrimination.

3.1. Price discrimination

Firms can condition their prices on their signals as long as the signals are informative: after observing signal s_L, firm i charges $p_{i,L}$, and after observing s_R, it charges $p_{i,R}$.

3.1.1. Demand. To derive firms' expected demands conditional on their private signals, we need to distinguish between four possible outcomes, where an outcome is characterized by a tuple (s_j, s_k) with $j, k \in \{L, R\}$, and where the first (second) element is the signal of firm A (firm B). Because signals are independently drawn for a customer, firms can either receive

[10] Under price discrimination, the market is covered for larger values of the transport costs because prices tend to be lower. To ensure better comparability, we use the more restrictive upper bound on the transport-cost parameter.

[11] Instead, one could follow Bénabou and Tirole (2016) by assuming that the outside option is costly – and not equal to zero as we assume. That is, one could assume that the outside option is located at either end of the linear city. Then, Assumption 1 would not be needed. Though this would not change our results qualitatively, it would make the comparison with the above-mentioned literature less clean.

identical signals ($j = k$) or different signals ($j \neq k$). The possible signal realizations are summarized in set $S := \{(s_L, s_L), (s_L, s_R), (s_R, s_L), (s_R, s_R)\}$.

In the following, we determine those customers who are indifferent between buying from firm A and from firm B given one of the four signal realizations. Let $\tilde{x}_1$ denote the indifferent customer for the tuple (s_L, s_L):

$$1 - p_{A,L} - \tau\tilde{x}_1 = 1 - p_{B,L} - \tau(1 - \tilde{x}_1) \Leftrightarrow \tilde{x}_1 = \frac{1}{2} - \frac{p_{A,L} - p_{B,L}}{2\tau}.$$

Now let $\tilde{x}_2$ denote the indifferent customer for the tuple (s_L, s_R):

$$1 - p_{A,L} - \tau\tilde{x}_2 = 1 - p_{B,R} - \tau(1 - \tilde{x}_2) \Leftrightarrow \tilde{x}_2 = \frac{1}{2} - \frac{p_{A,L} - p_{B,R}}{2\tau}.$$

Similarly, let $\tilde{x}_3$ denote the indifferent customer for the tuple (s_R, s_L):

$$1 - p_{A,R} - \tau\tilde{x}_3 = 1 - p_{B,L} - \tau(1 - \tilde{x}_3) \Leftrightarrow \tilde{x}_3 = \frac{1}{2} - \frac{p_{A,R} - p_{B,L}}{2\tau}.$$

Finally, let $\tilde{x}_4$ denote the indifferent customer for the tuple (s_R, s_R):

$$1 - p_{A,R} - \tau\tilde{x}_4 = 1 - p_{B,R} - \tau(1 - \tilde{x}_4) \Leftrightarrow \tilde{x}_4 = \frac{1}{2} - \frac{p_{A,R} - p_{B,R}}{2\tau}.$$

We assume, for the moment, that prices are such that $\tilde{x}_1 \leq \tilde{x}_3 \in [0, 1/2]$, and $\tilde{x}_2 \leq \tilde{x}_4 \in [1/2, 1]$, which – as will become clear later – holds true in the symmetric equilibrium.

The notion of Bayesian Nash equilibrium requires that firm i – after receiving a signal – updates its beliefs with regard to the respective customer's actual preference and with regard to the signal of its competitor. Because signal realizations are independent across firms and periods, the updating process is independent in each stage game. Let L (R) denote the case in which the customer is located in the left (right) half. Then, a firm's posterior belief that a customer prefers firm A given signal s_L is

$$\Pr(L|s_L) = \frac{\Pr(s_L|L)\Pr(L)}{\Pr(s_L|L)\Pr(L) + \Pr(s_L|R)\Pr(R)} = \sigma,$$

where $P(s_L|L) = \sigma$, and $P(s_L|R) = 1 - \sigma$. Moreover, the probability of a customer being of type L or R is equal to $\Pr(L) = \Pr(R) = 1/2$.

Note that the posterior is equal to the precision of the signal due to the equal probability of L or R occurring. Conditional on this, firm i's posterior belief that its competitor has received signal s_L is equal to the conditional probability of the customer being of type L, namely σ. In the remaining cases, beliefs are updated analogously.

We can now determine firms' expected demands. Denote by $D_{i,j}$ firm i's demand when it receives the signal s_j. Suppose firm A receives the signal s_L, and suppose that, given prices charged by firm B, it sets a price $p_{A,L}$, such that $p_{B,L} \leq p_{A,L} \leq p_{B,R}$, which indeed holds in equilibrium. Then, firm A's demand[12] can be derived as

$$D_{A,L}\left(p_{A,L}; p_{B,L}, p_{B,R} | s_L\right) = \sigma\left(\sigma\tilde{x}_1 + (1-\sigma)\frac{1}{2}\right) + (1-\sigma)\left(\sigma\left(\tilde{x}_2 - \frac{1}{2}\right) + (1-\sigma)\cdot 0\right).$$

The first term represents the case in which firm A obtains the true signal (which happens with probability σ). In the associated bracket, the first expression gives the demand when firm B also gets the true signal s_L, and the second expression is the demand when the rival gets the false signal s_R. In the latter case, the rival is not attractive for the customers who would have to pay a higher price, and travel a longer distance. The second term represents the case in which firm A gets a false signal s_L. In the associated multiplicative bracket, the first expression denotes the demand when firm B obtains a true signal s_R, and the second expression is the demand when firm B obtains the false signal. When firm B receives a false signal, and firm A also receives a false signal, then firm A is more aggressive in its pricing strategy, but firm B is more aggressive as well. Because the preferred firm is firm B, the resulting demand for firm A is zero.

Substituting the expressions for $\tilde{x}_1$ and $\tilde{x}_2$, and simplifying, we arrive at the following demand:

$$D_{A,L}\left(p_{A,L}; p_{B,L}, p_{B,R} | s_L\right) = \frac{\sigma}{2}\left(1 - \frac{p_{A,L} - \sigma p_{B,L} - (1-\sigma)p_{B,R}}{\tau}\right).$$

Similarly, conditional on receiving signal signal s_R, and given $p_{B,L} \leq p_{A,R} \leq p_{B,R}$, which again holds in equilibrium, firm A's demand can be derived as

$$D_{A,R}\left(p_{A,R}; p_{B,L}, p_{B,R} | s_R\right) = \sigma\left(\sigma\left(\tilde{x}_4 - \frac{1}{2}\right) + (1-\sigma)\cdot 0\right) + (1-\sigma)\left(\sigma\tilde{x}_3 + (1-\sigma)\frac{1}{2}\right).$$

The first term is the case when firm A obtains the true signal (which happens with probability σ). In the associated bracket, the first term is the demand when firm B gets the true signal s_R, and the second term represents the demand when the rival obtains the false signal s_L. The latter demand is zero, because now the rival prices conservatively to attract customers

[12]Note that we derive the *expected* demand $\mathbb{E}[D_{i,j}]$ here. This is also the case for all other demands and profits that are derived in the following. For simplification, we speak of "demands" and "profits", and drop $\mathbb{E}[\cdot]$.

who it believes prefer firm A. This is also done by firm A. Because both firms price to attract customers who prefer the rival, and customers are truly located in the right half, firm A is unable to attract any demand. The second term represents the case in which firm A gets a false signal s_R. In the associated multiplicative bracket, the first term gives the demand when firm B obtains a true signal s_L, and the second term gives the demand at firm A when firm B also obtains the false signal. When firms A and B both receive a false signal s_R, firm A is more conservative in its pricing strategy to attract customers, and firm B is more aggressive. Because the preferred firm is firm A, firm A attracts all the demand.

Substituting the expressions $\tilde{x}_3$ and $\tilde{x}_4$, and simplifying, we find

$$D_{A,R}\left(p_{A,R}; p_{B,L}, p_{B,R} | s_R\right) = \frac{1-\sigma}{2} - \frac{\sigma\left(p_{A,R} - (1-\sigma)p_{B,L} - \sigma p_{B,R}\right)}{2\tau}.$$

The demand for firm B conditional on its signal realization can be derived analogously. In the following, we analyze the three cases of competition, collusion, and deviation.

3.1.2. Competition. We start by analyzing the competitive payoffs, that is, the static Bayesian Nash equilibrium payoff of the stage game as defined in Section 2. These are later used as punishment payoffs in the dynamic game.[13] The maximization problem of firm i is given as

$$\max_{p_{i,L}, p_{i,R}} \pi_i = p_{i,L} D_{i,L}\left(p_{A,L}; p_{B,L}, p_{B,R} | s_L\right) + p_{i,R} D_{i,R}\left(p_{A,R}; p_{B,L}, p_{B,R} | s_R\right).$$

Differentiating with respect to prices, and solving the system of first-order conditions gives symmetric equilibrium prices of

$$p^*_{A,L} = p^*_{B,R} = \frac{2\tau}{1+2\sigma} \quad \text{and} \quad p^*_{A,R} = p^*_{B,L} = \frac{\tau}{\sigma(1+2\sigma)},$$

where $p^*_{A,R} < p^*_{A,L}$ and $p^*_{B,L} < p^*_{B,R}$ hold as long as the signal is (slightly) informative. Hence, price discrimination allows firms to set higher prices for those customers who are signaled to be located more closely to their own location – in other words, those customers with a higher willingness to pay for their product. Thereby, the market is divided into four segments, as in Fudenberg and Tirole (2000), under informative signals. In each half, there are customers served by their preferred firm, and customers poached by the less preferred firm. That is, $0 < \tilde{x}^*_1 < \tilde{x}^*_2 = \tilde{x}^*_3 = 1/2 < \tilde{x}^*_4 < 1 \; \forall \sigma \in (1/2, 1]$. The equilibrium payoff for each firm amounts to

[13]The results from this part are equivalent to Armstrong (2006).

$$\pi^* = \frac{\tau(1 + 4\sigma^2)}{2\sigma(1 + 2\sigma)^2}.$$

We observe that firms' payoffs are decreasing in the signal precision. As Esteves (2014) points out, an increase in signal precision has two opposing effects. On the one hand, misrecognition of customers decreases, which means that a firm can charge closer customers more, while reducing the price for customers who prefer its rival. In other words, a firm can poach more effectively. On the other hand, because the rival behaves more aggressively as well when poaching loyal customers, a firm optimally responds by reducing its prices. In this set-up, it turns out that the competition effect outweighs the increase in prices due to reduction in misrecognition (information). Hence, competition is intensified with a rise in signal precision. As a result, for any $\sigma \in (1/2, 1]$, payoffs are strictly lower than static Bayesian Nash equilibrium payoffs under uniform pricing, because we have best-response asymmetries.

3.1.3. Collusion. Under full collusion, firms maximize industry profits by minimizing total transport costs. That is, firms divide the market in two, and each firm serves its own turf. In our game, this allocation can only be induced by charging symmetric prices. Because firms try to extract the maximal surplus from customers, net of transport costs, it is not optimal to attract customers in the competitor's half. Put differently, firms will not price discriminate based on private information about customers' preferences. Instead, they will set a single price for all customers, such that the customer located at the center is indifferent between buying and not buying – that is, when $1 - p^c - \tau|\ell_i - 1/2| = 0$. We summarize these considerations in the following lemma.

Lemma 1. *Collusive prices and payoffs are given by*

$$p^c = 1 - \frac{\tau}{2}$$

and

$$\pi^c = \frac{1}{2} - \frac{\tau}{4}.$$

We observe that price discrimination cannot lead to higher payoffs compared with uniform pricing, because firms can only distinguish two customer groups (left and right halves of the market).[14]

[14]This is no longer the case when we allow for partial collusion or more than two segments; see Sections 4.1 and 4.2.

3.1.4. Deviation. To characterize the optimal deviation strategy, we need to define the following thresholds for τ:[15]

$$\tau_1 := \frac{2(1-\sigma)}{5-3\sigma}, \quad \tau_2 := \frac{2\sigma}{2+3\sigma}, \quad \text{and} \quad \tau_3 := \frac{2(1-\sigma)}{1+\sigma}.$$

It is easily checked that $\tau_1, \tau_2, \tau_3 \in [0, 2/3]$ for any $\sigma \in [1/2, 1]$, and that $\tau_2 \lesseqqgtr \tau_3$ for $\sigma \lesseqqgtr 1/\sqrt{2} \approx 0.7071$. The following lemma characterizes the optimal deviation behavior.

Lemma 2. *The optimal deviation from collusive prices yields the following prices and payoffs, which are continuous and differentiable in both σ and τ:*

$$p^d_{A,L} = p^d_{B,R} = \begin{cases} 1 - \dfrac{3\tau}{2} & \text{if } \tau \in [0, \tau_1] \\ \dfrac{1}{2} - \dfrac{\tau(1-3\sigma)}{4(1-\sigma)} & \text{if } \tau \in (\tau_1, \tau_3], \\ 1 - \dfrac{\tau}{2} & \text{if } \tau \in \left(\tau_3, \dfrac{2}{3}\right], \end{cases}$$

$$p^d_{A,R} = p^d_{B,L} = \begin{cases} 1 - \dfrac{3\tau}{2} & \text{if } \tau \in [0, \tau_2], \\ \dfrac{1}{2} + \dfrac{\tau(2-3\sigma)}{4\sigma} & \text{if } \tau \in \left(\tau_2, \dfrac{2}{3}\right], \end{cases}$$

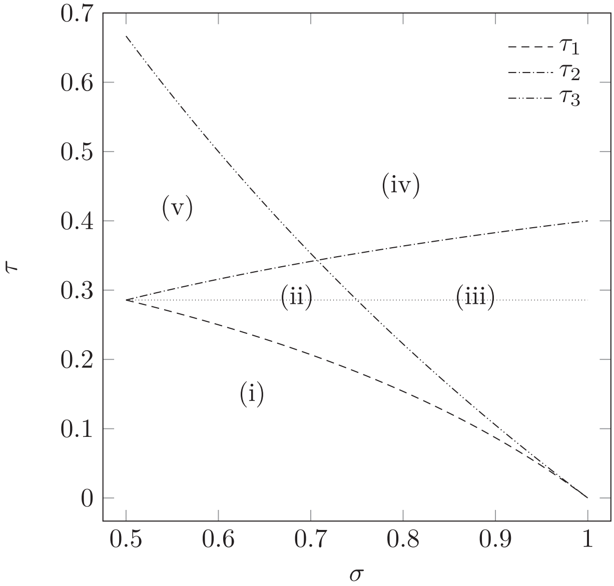
and

$$\pi^d = \begin{cases} 1 - \dfrac{3\tau}{2} & \text{if } \tau \in [0, \tau_1], \\ \dfrac{(2(1-\sigma) + 3\tau(1+\sigma))^2 - 32\tau^2}{32\tau(1-\sigma)} & \text{if } \tau \in (\tau_1, \min\{\tau_2, \tau_3\}], \\ \dfrac{\sigma(1-\sigma)(4(1+\tau) - 15\tau^2) + 4\tau^2}{32\sigma\tau(1-\sigma)} & \text{if } \sigma \in \left[\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}\right] \wedge \tau \in (\tau_2, \tau_3], \\ \dfrac{2 + \sigma(2-\tau) - 3\tau}{4} & \text{if } \sigma \in \left(\dfrac{1}{\sqrt{2}}, 1\right] \wedge \tau \in (\tau_3, \tau_2], \\ \dfrac{4\sigma^2(1+\tau) + 8\sigma\tau(1-\tau) + \tau^2(2-\sigma)^2}{32\sigma\tau} & \text{if } \tau \in \left(\max\{\tau_2, \tau_3\}, \dfrac{2}{3}\right]. \end{cases}$$

Corresponding to the cases in Lemma 2, Figure 1 divides all combinations of parameter values of σ and τ into regions (i)–(v). Intuitively, the regions are divided corresponding to the following two considerations:

[15]The derivation of these thresholds is part of the proof of Lemma 2, which is given in the Online Appendix.

Figure 1. Characterization of deviation strategies



Notes: The dotted horizontal line at $2/7 \approx 0.2857$ gives the threshold below which a deviating firm wants to serve the whole market in the case of uniform pricing (see Section 3.2).

a deviating firm wants to serve all customers in its competitor's turf (regions (i)–(iii)) or only some of them (regions (iv) and (v)); a deviator wants to charge uniform prices (region (i)), price discriminate cautiously (regions (ii) and (v)), or price discriminate aggressively (regions (iii) and (iv)). The first consideration is well known from the related literature on uniform pricing (see, e.g., Chang, 1991). The second consideration is unambiguous when information is perfect (see Helfrich and Herweg, 2016; Liu and Serfes, 2007). Then, a deviator price discriminates aggressively by charging the collusive price from close customers, while poaching more distant customers with low prices. When information is imperfect, however, the quality of information plays an additional role. When signals are relatively noisy, the firm might misrecognize customers' preferences. This can be costly. In this case, the firm prefers to charge rather similar prices conditional on the signal received. Thereby, it avoids losing infra-margins by offering a relatively low price to a close customer, and foregoing demand when offering a relatively high price to a distant customer. When the signal is sufficiently reliable, however, misrecognition becomes less likely, and hence the firm prefers to act more aggressively by charging the collusive price to the customers it expects to prefer its own product, and rather low poaching prices to the customers it expects to prefer the product of its competitor.

The behavior of a deviating firm in each region is explained in detail as follows. In region (i), transport costs are very low, and hence a deviating firm captures the entire market by setting a uniform price that is independent of the signal quality. In region (ii), transport costs are still sufficiently low, such that the deviator wants to capture all customers, whereas it prefers to price discriminate between the different groups depending on the signal it receives. To be precise, it still wants to charge a price that is independent of the signal quality to its competitor's customers, while the price it wants to charge to its own customers rises in the signal quality. This results in an increasing price difference between groups. In region (iii), the deviator still captures all customers, and wants to price its product for its competitor's customers as before. The relatively precise signal, however, makes it profitable for the deviator to charge the collusive price to customers it expects to prefer its own product. In region (iv), transport costs are high, such that the firm finds it too costly to capture all of its competitor's customers. Because it can price discriminate between groups, and because signals are relatively precise, it still wants to charge the collusive price to close customers. The price difference, however, increases in the signal quality, because the price it wants to charge to its competitor's customers decreases in the signal quality. In region (v), transport costs are again high, such that it is too costly for the deviating firm to serve all customers whose signal indicates a preference for its competitor. At the same time, it is too costly to charge the collusive price to customers it expects to prefer its own product, because the signal is relatively noisy. This price, however, increases in the signal quality, and hence the price difference between the groups also increases.

3.2. Uniform pricing

The above case nests the scenario in which firms are not allowed to price discriminate, because outcomes are the same as in the situation in which signals are uninformative (i.e., $\sigma = 1/2$). Hence, punishment profits reduce to

$$\pi_u^* = \frac{\tau}{2},$$

collusive profits to

$$\pi_u^c = \pi^c = \frac{1}{2} - \frac{\tau}{4},$$

and deviation profits to

$$\pi_u^d = \begin{cases} 1 - \dfrac{3\tau}{2} & \text{if } \tau \in \left[0, \dfrac{2}{7}\right], \\ \dfrac{(2+\tau)^2}{32\tau} & \text{if } \tau \in \left(\dfrac{2}{7}, \dfrac{2}{3}\right]. \end{cases}$$

We can now proceed with the dynamic game to compare firms' collusion incentives in the two pricing scenarios.

3.3. Dynamic game and critical discount factors

To study the scope for collusive behavior, we extend our set-up to a game of infinite horizon.

3.3.1. Dynamic game. In the infinitely repeated game, the stage game described in Section 2 is played in each period $t = 0, \ldots, \infty$. Firms are long-lived – that is, they play over the entire sequence of the infinitely repeated game. Expected payoff in period t is defined as the stage game payoff plus the discounted value of the stream of future payoffs determined by the continuation game strategy profile. Firms' common discount factor is $\delta \in (0, 1)$. Customers are short-lived – that is, they only play for a single period, and they are replaced by a new cohort of customers in the subsequent period.[16] As a consequence, intertemporal price discrimination is not possible, because firms cannot use information about customers' purchase histories.[17] Their payoff is given by their net utility in the respective period. All players are payoff maximizing. Customers are perfectly informed. Hence, their payoff is deterministic.

Because the stage game is Bayesian, we use the notion of perfect Bayesian equilibrium when analyzing the dynamic game. We refrain from explicitly stating the set of players' beliefs as part of the equilibrium description. In addition, because customers are short-lived, firms cannot learn their preferences over time. The same holds true for beliefs regarding the signals of the competitor, because these are independent across periods.

Further, we assume that firms use grim-trigger strategies, as defined in Friedman (1971) to support collusive outcomes. Thereby, we follow the related literature and can compare results. However, we want to focus on the impact of signal quality on the following trade-off for a firm: (i) long-term gains from collusive behavior compared with competitive outcomes against (ii) short-term gains by deviating unilaterally from collusive behavior.[18] The stationary strategy profile can be summarized as follows.

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[16]As argued in Section 4.3, asymmetric signal quality can be interpreted as relaxing the assumption of short-lived customers.

[17]Note that this assumption is also made by all the contributions mentioned above, and hence ensures comparability.

[18]It is important to note here that we assume that deviation from a collusive agreement is perfectly observed. This assumption is valid for the industries we consider, such as online retail marketplaces where prices are in the public domain, and where a deviation from a collusive strategy is observed at little or no cost.

- In the starting period $t = 0$, each firm charges the collusive price. In any subsequent period $t = 1, \ldots, \infty$, each firm:
  - charges the collusive price as long as it does not observe any other price in period $t - 1$;
  - or else plays Bayesian Nash equilibrium strategies.
- Customers buy from the firm providing the highest net utility when this utility weakly exceeds the value of the outside option (zero). If customers are indifferent between the two firms, they choose randomly.

To verify whether the suggested strategy profile constitutes a perfect Bayesian equilibrium, we need to verify that the one-shot-deviation principle is satisfied (for a formal argument, see Hendon et al., 1996). Given firms' strategies and beliefs over customers' preferences and the respective competitor's private information, this is true if and only if the following inequality is satisfied in any period t:

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta \pi^*}{1-\delta}. \tag{1}$$

From this, it follows that the critical discount factor is defined by

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^*} =: \bar{\delta}. \tag{2}$$

All things equal, a lower (higher) punishment or deviation profit facilitates collusion (makes collusion harder to sustain). That is, the set of discount factors that satisfy the one-shot-deviation principle becomes larger (smaller). The opposite is true for the respective change in the collusive profit. Put differently, lower (higher) gains from defecting (that is, $\pi^d - \pi^c$) and higher (lower) losses from punishment (that is, $\pi^c - \pi^*$) make collusion easier (harder) to sustain.

3.3.2. Critical discount factors. We start with the well-documented case of uniform pricing (Chang, 1991). Given the respective profits and Condition (2), the critical discount factor is given as

$$\bar{\delta}_u = \begin{cases} \dfrac{2-5\tau}{4(1-2\tau)} & \text{if } \tau \in \left[0, \dfrac{2}{7}\right], \\[2ex] \dfrac{2-3\tau}{2+5\tau} & \text{if } \tau \in \left(\dfrac{2}{7}, \dfrac{2}{3}\right]. \end{cases} \tag{3}$$

By construction, $\bar{\delta}_u$ is independent of σ. Note that it decreases in the transport-cost parameter (i.e., $\partial \bar{\delta}_u / \partial \tau < 0$).

Using the payoffs derived in the three scenarios under price discrimination, we can characterize the critical discount factor in the following proposition (with the proof given in the Online Appendix).

Proposition 1. *When firms can price discriminate, the critical discount factor $\bar{\delta}$ is a continuous and differentiable function of signal quality σ and transport cost τ with the following properties: (a) it is non-monotonic in the signal quality, such that $\partial\bar{\delta}/\partial\sigma < 0$ (> 0) holds for low (high) values of the signal quality; and (b) it is non-monotonic in the transport costs, such that $\partial\bar{\delta}/\partial\tau < 0$ (> 0) holds for low (high) values of the transport cost.*

Let us have a closer look at the intuition behind these findings. By construction, the collusive profit is independent of signal quality, whereas the deviation profit and the Bayesian Nash equilibrium profit depend on it, as we can see from the analysis. More precisely, for a given value of the transport-cost parameter, the deviation profit is weakly increasing in signal quality, because targeting customers becomes easier. At the same time, the Bayesian Nash equilibrium profit is falling in signal quality, because competition becomes fiercer. Hence, increasing signal quality has opposing effects on the critical discount factor, because both the gains from deviation and the losses from punishment increase. Helfrich and Herweg (2016) and Liu and Serfes (2007) find that, for perfect signal quality, the destabilizing effect dominates. As a consequence, collusion is harder to sustain under price discrimination than under uniform pricing, that is, $\bar{\delta}(1/2,\tau) < \bar{\delta}(1,\tau)$.

Now consider the case in which signal quality is imperfect. Starting from a low level of signal precision, as precision increases, the gain from defecting increases relatively slower than the loss from punishment. For the case with relatively low transport costs, this is intuitive. A deviating firm finds it profitable to capture the entire market irrespective of the signal precision (see region (i)). Meanwhile, competition intensifies as σ increases, and hence the loss from punishment increases.

The case in which transport costs are relatively high is more involved. On the one hand, punishment profits are decreasing as before. On the other hand, deviation profits increase in σ (see regions (ii), (iv), and (v)). Because signal quality is relatively low, a deviating firm expects to misrecognize customers often, and hence the firm price discriminates cautiously. That is, the difference in prices conditional on signals is rather low. In our model, this misrecognition effect slows down the increase in deviation profit relative to the increase in loss from punishment. As a result, collusion is facilitated.

Starting from a high level of signal precision, as precision increases, the misrecognition effect becomes less pronounced. The deviating firm price discriminates more aggressively. In other words, the difference in prices conditional on signals is rather high. Thereby, the increase in deviation profits outweighs the increase in loss from punishment impeding collusion. We thus shed light on the intermediate cases between uniform pricing ($\sigma = 1/2$) and price discrimination conditional on perfect information ($\sigma = 1$), and we show that sustainability of collusion is non-monotonic in signal quality. Moreover, it turns out that there is a non-monotonic relationship between sustainability of collusion and transportation cost. The logic behind this result can similarly be derived from Figure 1.

Proposition 1 provides new insights for competition policy. In our set-up, an increase in signal precision leads to lower customer prices under competition due to best-response asymmetries. If signals are perfect, both competitive prices and the likelihood of collusive behavior are lowest. Either effect benefits customers. We know from the analysis that an increase in signal quality from a relatively low level facilitates collusion. In this area, any policy that deregulates access to or usage of customer data, resulting in a gain in predictive power of firms' algorithms,[19] can also support collusive behavior. In particular, regulators should be alarmed if the industry in question lobbies for such deregulation. While a single firm always gains from an increase in its predictive power, an increase of all firms' predictive power drives down competitive profits. Deregulation, however, might enable firms to coordinate their prices. From a relatively high level, an increase in predictive power impedes collusive behavior. In this area, any policy concerned with customer privacy that restricts predictive power of firms can come at the cost of collusive behavior.

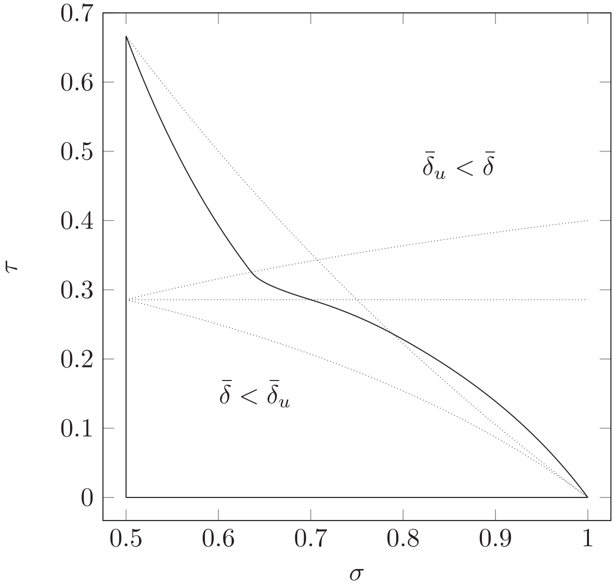
3.3.3. Comparison. Figure 2 provides a full comparison of the critical discount factors in the two pricing scenarios for all permissible parameter values of signal quality and transport costs.

When signal quality does not provide any information (i.e., $\sigma = 1/2$), price discrimination is not feasible. Hence, the critical discount factors are equal. As pointed out before, when signal quality is perfect (i.e., $\sigma = 1$), collusion is harder to sustain under price discrimination than with uniform prices.

For $\sigma \in (1/2, 1)$, there is a non-monotonic relationship of the critical discount factor in signal quality under price discrimination as stated in Proposition 1. In particular, starting from $\sigma = 1/2$, the critical discount factor first decreases, and then increases in signal quality after a cut-off. At

[19]To a certain extent, it seems natural to assume a positive relation between the amount and variety of available data and predictive power. Yoganarasimhan (2020) provides evidence for this relationship in the context of search queries. She finds that personalized search, especially long term and across sessions, helps to improve the accuracy of suggested results significantly.

Figure 2. Comparison of the critical discount factors with and without price discrimination for all permissible parameter values



Notes: For those parameter combinations represented by the solid lines, the two critical discount factors coincide (i.e., $\bar{\delta} = \bar{\delta}_u$). The dotted lines separate the different regions with respect to the deviation strategies for the cases with and without price discrimination.

the same time, the critical discount factor under uniform pricing remains unchanged. We can thus state the following corollary.

Corollary 1. *For any $\tau \in (0, 2/3)$, there exists a threshold $\tilde{\sigma}(\tau) \in (1/2, 1)$ such that for $\sigma = \tilde{\sigma}(\tau)$, we have $\bar{\delta} = \bar{\delta}_u$. Moreover, for any $\sigma \lesseqgtr \tilde{\sigma}(\tau)$, it holds true that $\bar{\delta} \lesseqgtr \bar{\delta}_u$.*

This finding has an important implication. We find that a ban on price discrimination facilitates collusion as documented in Liu and Serfes (2007) and Helfrich and Herweg (2016) as long as signal quality is relatively high; otherwise, a ban on price discrimination hinders collusion, which is in stark contrast to the previous literature.

4. Robustness

In this section, we test the robustness of our main results. To this end, we consider the case of partial collusion and a situation with more than just two segments. Furthermore, we consider the cases of asymmetric signal

Table 1. Comparison of collusive profits

Low signal quality/	$\delta = 0$	$\bar{\delta} \le \delta < \bar{\delta}_u$	$\bar{\delta}_u < \delta$
transport costs	$\pi^* < \pi_u^*$	$\tilde{\pi}_u^c < \pi^c$	$\pi^c = \pi_u^c$
High signal quality/	$\delta = 0$	$\bar{\delta}_u \le \delta < \bar{\delta}$	$\bar{\delta} < \delta$
transport costs	$\pi^* < \pi_u^*$	$\tilde{\pi}^c < \pi_u^c$	$\pi^c = \pi_u^c$

quality and correlated signals. Last, we comment on the use of optimal penal codes.

4.1. Less-than-maximum collusion

Up to this point, we have only analyzed a situation in which firms are able to sustain collusion at monopoly prices. These prices are the same in the two pricing scenarios that we consider. However, whenever such collusion is not feasible, because the actual discount factor is too low, firms have the possibility to set lower collusive prices to reduce deviating incentives. Note, first, that in the standard set-up, which corresponds to the case in which $\sigma = 1/2$, Chang (1991) shows that supra-competitive prices can always be sustained for a strictly positive discount factor as long as firms are (slightly) differentiated. Then, the comparison of the two regimes with and without price discrimination can be reduced to a comparison of the collusive profits. To this end, recall the results for two extreme cases. First, when collusion at maximum prices is sustainable in both scenarios (i.e., $\delta \ge \max\{\bar{\delta}, \bar{\delta}_u\}$), firms make the exact same profits, because they charge the same price. Second, when the industry discount factor goes to zero, "collusion" can only be sustained at competitive prices. In this case, we know that uniform pricing results in higher profits. Hence, we know that for low values of the discount factor, uniform pricing results in higher collusive profitability, whereas there is no difference for high values of the discount factor (see Table 1).

For intermediate values of the discount factor, things are less obvious. Note, first, that a decrease in the value of the discount factor, once it is below the critical discount factor, results in lower collusive prices, and hence lower profits, because deviation becomes relatively more attractive. Now consider the scenario in which transport costs are low, and/or signal quality is low. In this case, we know from our benchmark analysis of full collusion that the critical discount factor under price discrimination is lower than under uniform pricing. Hence, for any value of the discount factor between the two critical discount factors (i.e., $\delta \in [\bar{\delta}, \bar{\delta}_u)$), firms under uniform pricing are forced to adjust their collusive prices downward, such that collusive profits fall (see Table 1, which indicates less-than-maximum collusive profits with a tilde). At the same time, firms that price discriminate can still charge the monopoly prices. As a result, and in line with the previous findings, colluding firms benefit from price discrimination – at least so long as the discount factor is sufficiently high. Moreover, we can see that when the discount factor goes to zero, price discrimination will eventually yield lower profits. By continuity, we can conclude that profits will intersect at some point.

In contrast, when transport costs are high, and/or when signal precision is high, collusion at maximum prices can be sustained for a greater range of the discount factor under uniform pricing. Now, whenever the discount factor lies in between the two critical discount factors (i.e., $\delta \in [\bar{\delta}_u, \bar{\delta})$), the collusive profit under price discrimination must decrease, whereas the collusive profit under no discrimination stays at the maximum level. Hence, collusion is more profitable under uniform pricing (see Table 1). Because punishment profits are also higher in this case, collusion tends to always be more profitable under uniform pricing (i.e., for lower values of the discount factor).

We can thus state that our main result – that price discrimination can have a collusion-facilitating effect – is robust to introducing less-than-maximum collusion. An alternative interpretation of the above-mentioned mechanism is the following. Fix the collusive profit at a level that is sufficiently high but also lower than the profit under full collusion. Then, for very low signal precision and/or transport costs, this profit can be attained for a larger range of the discount factor when firms can price discriminate. The opposite holds for high signal quality and/or transport costs. This is exactly what we find when considering the sustainability of the highest possible collusive price. Clearly, for (very) low values of the discount factor, this result no longer holds. Take, for example, a collusive profit that is equal to the punishment profit without price discrimination. Of course, this profit is sustainable for every discount factor without price discrimination. Because the profit lies above the one-shot profit under price discrimination, a strictly positive discount factor is required to sustain it in this case. Hence, the range of the discount factor for which the profit can be attained reduces with price discrimination. Note that this observation is independent of the signal quality and transport costs.

Another aspect is that profits are identical, and firms do not serve customers at different prices in the main analysis. Let us stress that whenever collusion at maximum prices is not sustainable in at least one of the two pricing scenarios analyzed, collusive profits differ for all or almost all discount factors. Moreover, from the analysis in Liu and Serfes (2007) with perfect signals and two market segments, we know that when price discrimination is possible, firms might charge different less-than-maximum collusive prices to the two groups in a situation in which full collusion cannot be sustained, and in which less-than-maximum collusive profits are not too low.[20] Such differentiated pricing implies that firms serve market segments together. That is, a firm also serves customers who are located closer to the competitor. By doing so, less-than-maximum collusion is facilitated, because deviation becomes more difficult. At the same time, this means that the allocation of customers becomes inefficient due to the fact that when some customers buy from the firm that is farther away, transport costs go up. We are confident that this insight also holds at least for those signals that are very good (though not perfect). In these cases, we know that allowing price discrimination makes collusion more difficult to sustain, but it can have negative welfare effects.

4.2. More than two segments

We acknowledge that, given our approach to modeling information, even when signal quality is perfect (i.e., $\sigma = 1$), firms only have relatively crude information because there are just two customer groups. Clearly, this is fairly restrictive, and might fit certain markets or industries better than others. To address this shortcoming, and to show that our main insight from the above analysis continues to hold, consider this extension, which follows Liu and Serfes (2007). The number of segments equals any integer $n = \{1, 2, 4, 8, \ldots\}$, and segments are of equal length. Then, uniform pricing results for $n = 1$, and the case considered in the main analysis is represented by $n = 2$. Here, we again assume that firms receive a signal of quality σ that a customer is located in a specific segment; with probability $1 - \sigma$, the signal incorrectly indicates that the customer is located in one of the other three segments with equal probability.[21]

In what follows, we abstain from a fully fledged analysis due to tractability issues, and restrict our attention to a number of limit cases. Nevertheless, these cases illustrate that our most important finding from the main analysis – that negative, collusion-facilitating effects from price discrimination can arise – continues to hold. Note first that, of course, an increase in market segments does not have any effect on the outcomes for collusion, deviation, and competition under uniform pricing (see the results

[20] See their comment in their footnote 26 for the results that are discussed in this paragraph. Because of the complexity of this issue, they restrict their attention to a simpler – albeit not the optimal – approach where firms do not share demand in the different segments; that is, firms only serve their closer half of the market.

[21] Clearly, other assumptions can be made. For example, a wrong signal can mean that the customer is located in the one or in one of the two adjacent segment(s). In light of the mechanisms in the deviation case, we are confident that our argument is also valid for alternative assumptions.

in Section 3.2). As a consequence, Expression (3) for the critical discount factor under a ban of price discrimination does not change.

Next, consider the case with potential price discrimination and a worthless signal (i.e., $\sigma = 1/2$). In this situation, price discrimination is not feasible, which means that the profits and the critical discount factor are the same as under uniform pricing independent of an increase in the number of market segments. This is what we also have in the scenario with only two segments.

Turning to the other extreme situation in which the signal precision is perfect (i.e., $\sigma = 1$), we can borrow from Liu and Serfes (2007) with regard to the consequences of price discrimination and a change in the number of market segments. For tractability reasons, they only consider those cases in which a deviating firm serves the whole market. This is true for sufficiently low transport costs: $\tau \leq n/(3+n)$. Then, profits in the three different scenarios are derived as

$$\pi^* = \frac{\tau(9n^2 - 18n + 40)}{36n^2},$$
$$\pi^c = \frac{1}{2} - \frac{\tau(2+n)}{8n},$$
$$\text{and} \quad \pi^d = 1 - \frac{\tau(2+n)}{2n}. \tag{4}$$

Hence, the critical discount factor is given as

$$\bar{\delta} = \frac{9n[4n - 3\tau(2+n)]}{2[36n^2 - \tau(27n^2 + 18n - 40)]}.$$

Liu and Serfes (2007) point out that an increase in the number of market segments leads to a higher critical discount factor. In other words, this means that collusion becomes harder to sustain – whenever the market can be perfectly segmented in at least two parts (i.e., whenever price discrimination is possible). Hence, the qualitative result for perfect signal quality continues to hold: collusion is facilitated when price discrimination is banned.

So far, we know that for sufficiently low transport costs, results do not qualitatively change with a variation in the number of segments at the two extremes when signals are worthless, or when signals are perfect. We now turn to the situation in which signal quality lies between these two limit cases. As Liu and Serfes (2007) point out, competitive profits under price discrimination are always below those in a scenario without price discrimination. Now if we allow for imperfect signals, this effect continues to hold, but – as in the main analysis – it is less pronounced. Consider the scenario with four segments. In this case, each firm sets an individual price for each of the four market segments; the farther away the segments are, the lower prices are set. The analysis is a straightforward (but a lot more involved) extension of that in the main analysis.[22] The resulting competitive profit for signal quality $\sigma \in [1/4, 1]$ can be shown to amount to

$$\pi^* = \frac{1}{24(1856\sigma^7+7600\sigma^6+4092\sigma^5+4025\sigma^4+1420\sigma^3+525\sigma^2+140\sigma+25)^2} \times \left[\begin{array}{l} \tau(2228224\sigma^{16} + 22970368\sigma^{15} + 85819392\sigma^{14} + 147306496\sigma^{13} \\ + 188551168\sigma^{12} + 171301248\sigma^{11} + 421881856\sigma^{10} + 255587848\sigma^9 \\ + 261708678\sigma^8 + 135983804\sigma^7 + 72704735\sigma^6 + 27904419\sigma^5 \\ + 10593401\sigma^4 + 2600669\sigma^3 + 702981\sigma^2 + 103295\sigma + 13700) \end{array}\right].$$

With worthless signals, which imply that $\sigma = 1/4$ in this case, the profit equals the profit under uniform pricing of $\tau/2$. For a perfect signal, the competitive profit equals $7\tau/36$ (see also Expression (4) for $n = 4$). For any signal in between, the competitive profit monotonically decreases as the signal precision improves.

At the same time, any higher number of market segments greater than two results in higher collusive profits with perfect information. Note that this effect, in principle, is also present under imperfect signals, but it is smaller in size, and it strongly depends on signal precision. To see this, consider again the case with four segments. Colluding firms must weigh two options against each other when signals are neither worthless nor perfect.

First, they can set the uniform collusive price with one segment or two segments (i.e., $p^c = 1 - \tau/2$), and serve half of the market each. Hence, the resulting profit equals $1/2 - \tau/4$. Such price setting implies that firms forgo profits from setting a higher price for customers who are signaled to be located in the closer segment. From the following comparison with the profits in the next regime, we see that this price is indeed profitable when signal precision and transport costs are not too high.

Second, firms decide to price discriminate. In this case, however, they forgo demand from those customers who are mistakenly signaled to be located in the closer segment; hence, these customers are offered the higher price that they are not willing to pay. In this second scenario, the market is split in half; that is, each firm serves the two closest segments, and the price addressed at customers who are located in the segment farther away again equals p^c. The price for customers who are located in the closer segment depends on the signal quality and transport costs. For intermediate values of signal precision and transport costs, firms charge a lower price of

[22] We thank Jana Gieselmann for excellent research assistance.

$1/2 - \sigma(1-\tau)/2(1-\sigma)$ to customers signaled to be located in the closer segment. The profit in this situation equals[23]

$$\frac{16 + 16\sigma^2 - \sigma(32 + 72\tau - 48\tau^2) + 72\tau - 39\tau^2}{192\tau(1-\sigma)}.$$

Hence, we can conclude that when signal quality and transport costs are not too high, firms stay away from setting different prices for different segments. In other words, they set a single, signal-independent collusive price targeted at customers from both segments in the market half that is closer to their own location. Importantly, this means that, in this situation, collusive prices and profits are the same as those that occur under no discrimination.

For this situation, we analyze deviation incentives. The simplest option is to set a single deviation price, which implies that the deviating firm captures the whole market. As in the main analysis, the deviating price and profit are given as $p^d = \pi^d = 1 - 3\tau/2$. To check whether this can be an optimal deviating strategy, suppose that signal precision and/or transport costs gradually increase. At some point, it might become optimal to price discriminate. The first option in this direction is to charge a higher price only to those customers who are signaled to be from the closest segment. The price for the other segments does not change compared with the previous situation. When the price is increased only slightly above the previously mentioned deviating price, the deviating firm does not lose any demand in the three closest segments; the loss only surfaces in the segment that is farthest away, where signal precision affects demand erosion. Hence, the optimal price for the closest segment will depend on signal quality, and can be derived as $1/2 + 3\sigma\tau/4(1-\sigma)$. The signal-dependent profit amounts to

$$\frac{-4 - \sigma^2(2-3\tau)^2 + \sigma(8 + 60\tau - 108\tau^2) - 72\tau + 108\tau^2}{96\tau(1-\sigma)}.$$

Comparing the two deviation options reveals that a deviating firm sets a single, signal-independent deviation price when the signal quality is very poor and transport costs are very low.[24] In this situation, and in line with what we observed for the two-segment case, the deviation profit is the same as under no discrimination.

We can thus conclude that our main result still holds whenever transport costs are very low and signals are not too precise. In this case, as argued

[23]For sufficiently high signal precision and transport costs, firms set the highest possible price of $1 - \tau/4$ for the closer segment. The resulting profit then amounts to $5/12 + \sigma(4+\tau)/48 - 5\tau/24$.
[24]Note that the critical threshold for the signal quality and/or transport costs from this comparison is below that resulting from the comparison of collusive profits.

above, collusive and deviation profits are the same with and without price discrimination. However, as in the main analysis, price discrimination always strengthens the punishment; the effect increases with signal quality. As a consequence, banning price discrimination can make it more difficult for firms to sustain collusion also if more market segments are available. However, from the analysis in Liu and Serfes (2007) for the case with perfect information, we know that when opportunities to price discriminate improve (the number of segments grows), it becomes harder to sustain collusion. This means that there is a non-monotonic relationship between the critical discount factor and signal quality. As in the main analysis, we conjecture that this result also holds for the case with higher transport costs and lower signal precision.

4.3. Asymmetric signal quality

In this subsection, we relax the assumption of symmetric information accuracy. Similar to the approach used by Esteves (2014), we assume that the signal a firm receives is a function of the respective customer's preference. We consider the following case. The signal a firm receives when facing a customer in its own half is weakly more precise than the signal it receives when facing a distant customer. We denote the probability that the signal is correct when the customer is close (distant) by σ_1 (σ_2), and assume that $1/2 \leq \sigma_2 \leq \sigma_1 \leq 1$. Thereby, we address the concern that a firm might know most about the characteristics of those customers who prefer its product, and hence should be able to identify these with higher probability. This can also be interpreted as a short-cut approach to modeling customers who live for more than a single period, and firms that have access to an imperfect tracking technology similar to that defined in Colombo (2016).

Consider set S, which contains all possible signal tuples (s_j, s_k), and let $f(s_j, s_k|x \in l)$ denote the joint probability density function conditional on customer x's preference $l \in \{L, R\}$. We impose the following assumption on the functional form of $f(\cdot)$.

Assumption 2.

$$f(s_j, s_k|x \in L) = \begin{cases} \sigma_1\sigma_2 & \textit{for} \quad (s_L, s_L), \\ \sigma_1(1-\sigma_2) & \textit{for} \quad (s_L, s_R), \\ (1-\sigma_1)\sigma_2 & \textit{for} \quad (s_R, s_L), \\ (1-\sigma_1)(1-\sigma_2) & \textit{for} \quad (s_R, s_R), \end{cases}$$

and

$$f(s_j, s_k|x \in R) = \begin{cases} (1-\sigma_2)(1-\sigma_1) & \textit{for} \quad (s_L, s_L), \\ (1-\sigma_2)\sigma_1 & \textit{for} \quad (s_L, s_R), \\ \sigma_2(1-\sigma_1) & \textit{for} \quad (s_R, s_L), \\ \sigma_2\sigma_1 & \textit{for} \quad (s_R, s_R). \end{cases}$$

The density function under Assumption 2 is well defined, and nests the extreme case of symmetric signals (for $\sigma_1 = \sigma_2 = \sigma$). As before, after observing signal s_i, firm i has to infer on the customer's actual preference and on the signal s_j received by its competitor. Suppose firm i receives signal s_L. Applying Bayes' rule, its updated belief that a customer prefers firm A, and that its competitor has received the same signal is

$$\Pr(s_L, L|s_L) = \frac{f(s_L, s_L|L)\Pr(L)}{f_{s_i}(s_L|L)\Pr(L) + f_{s_i}(s_L|R)\Pr(R)} = \frac{\sigma_1\sigma_2}{1+\sigma_1-\sigma_2},$$

where f_{s_i} denotes the marginal distribution of s_i. In the remaining cases, beliefs are updated analogously. Given beliefs, we can specify each firm's maximization problem, and determine mutual best responses in a fashion similar to that used in the main analysis (see the Online Appendix). Firms optimally set prices equal to

$$p^*_{A,L} = p^*_{B,R} = \frac{2\tau}{1+\sigma_1+\sigma_2}$$

and

$$p^*_{A,R} = p^*_{B,L} = \frac{\tau(1-\sigma_1+\sigma_2)}{1+\sigma_1+\sigma_2},$$

where $p^*_{A,R} < p^*_{A,L}$ and $p^*_{B,L} < p^*_{B,R}$ as long as the signal is (slightly) informative. The resulting equilibrium profit for each firm amounts to

$$\pi^* = \frac{\tau(1-\sigma_1^2+6\sigma_1\sigma_2-\sigma_2^2)}{2\sigma_2(1+\sigma_1+\sigma_2)^2(1+\sigma_1-\sigma_2)}.$$

This profit serves as punishment profit in the dynamic game as defined in Section 2, and equals that derived in Section 3 for $\sigma_1 = \sigma_2 = \sigma$ by construction. The intuition from the symmetric case can initially be misleading here by suggesting a similar relation between punishment payoffs and average signal quality. In fact, we observe that the more asymmetric the signal quality is, the higher the punishment payoffs are – namely, the punishment payoffs rise in the signal quality for close customers, and fall in the signal quality for distant customers. When the signal quality for close customers increases, firms can better identify these customers, allowing for an increase of the price they set for them. However, when the signal quality for distant customers decreases, firms more often misrecognize these customers, leading to less aggressive poaching because costly mistakes become more likely. Overall, signal asymmetry softens competition. Because deviation payoffs are affected in the same way (see the proof of Proposition 2), it is not clear from an *ex ante* perspective how signal asymmetry translates into the critical discount factor $\bar{\delta}_{\text{asy}}$. The following proposition summarizes our result.[25]

Proposition 2. *For any $\sigma_2 < \sigma_1$, the critical discount factor $\bar{\delta}_{\text{asy}}$ is strictly larger compared with both cases of symmetric signal quality ($\sigma = \sigma_1$ and $\sigma = \sigma_2$). In addition, $\bar{\delta}_{\text{asy}}$ is non-monotonic in the signal quality for close customers σ_1 and in the signal quality for distant customers σ_2.*

Note that the proof will be omitted because of the analogy to the proof of Proposition 1.

By construction, the critical discount factors in the symmetric and asymmetric cases are equivalent for $\sigma_1 = \sigma_2 = \sigma \in [1/2, 1]$. Starting from $\sigma_1 = \sigma_2 = 1/2$, we can see from the proof of Proposition 2 that a marginal increase in both dimensions leads to a marginal reduction of $\bar{\delta}_{\text{asy}}$. By continuity of $\bar{\delta}_{\text{asy}}$, and from Proposition 1, it immediately follows that we can always find $1/2 \leq \sigma_2 < \sigma_1$, such that $\bar{\delta}_{\text{asy}} < \bar{\delta}_u$. Then, collusion is more likely in terms of set inclusion if price discrimination is permitted, compared with the case of no price discrimination. In particular, at low levels of precision, an increase in precision reduces the critical discount factor, because the misrecognition effect that facilitates collusion dominates the collusion-hindering effect of price discrimination. The following corollary summarizes this argument.

Corollary 2. *For $\sigma_2 < \sigma_1$, a ban on price discrimination helps fight collusion if signals are sufficiently noisy.*

4.4. Correlated signals

In this subsection, we relax the assumption of independent signal realizations by allowing for positive correlation of the private signals received by the firms. This is natural because firms might obtain customer data from similar sources, or use similar algorithms to infer on customer types from available data.

Consider the set of all signal tuples S, and let $g(s_j, s_k | x \in l)$ denote the joint probability density function conditional on customer x's preference $l \in \{L, R\}$. We assume the following functional form of $g(\cdot)$.

[25] It determines mutual best responses in a fashion similar to that used in the main analysis.

Assumption 3.

$$g(s_j, s_k|x \in L) = \begin{cases} \sigma^2 + \chi & \textit{for} \quad (s_L, s_L), \\ \sigma(1-\sigma) - \chi & \textit{for} \quad (s_L, s_R), (s_R, s_L), \\ (1-\sigma)^2 + \chi & \textit{for} \quad (s_R, s_R), \end{cases}$$

and

$$g(s_j, s_k|x \in R) = \begin{cases} (1-\sigma)^2 + \chi & \textit{for} \quad (s_L, s_L), \\ \sigma(1-\sigma) - \chi & \textit{for} \quad (s_L, s_R), (s_R, s_L), \\ \sigma^2 + \chi & \textit{for} \quad (s_R, s_R), \end{cases}$$

where $\chi \in [0, \sigma(1-\sigma)]$ *measures the degree of correlation.*

The density function under Assumption 3 is well defined, and nests the two extreme cases: (i) independent signals, for $\chi = 0$, and (ii) perfectly correlated signals, for $\chi = \sigma(1-\sigma)$. The second case is equivalent to a model with imperfect public information about customer preferences. It is easily checked that the interval $[0, \sigma(1-\sigma)]$ is non-empty for $\sigma \in [1/2, 1)$. In the following, we solve for the Bayesian Nash equilibrium of the stage game. As before, after observing signal s_i, firm i has to infer the customer's actual preference and the signal of its competitor. For illustration, suppose that firm i receives signal s_L. Applying Bayes' rule, its posterior belief that a customer prefers firm A, and firm j receives the same signal is

$$\Pr(s_L, L|s_L) = \frac{g(s_L, s_L|L)\Pr(L)}{g_{s_i}(s_L|L)\Pr(L) + g_{s_i}(s_L|R)\Pr(R)} = \sigma^2 + \chi,$$

where g_{s_i} denotes the marginal distribution of s_i. In the remaining cases, beliefs are updated similarly. Given beliefs, we can specify each firm's maximization problem and determine mutual best responses analogously to the main analysis (see the Online Appendix). Firms optimally set prices equal to

$$p^*_{A,L} = p^*_{B,R} = \frac{\tau(2\sigma^2 + \chi)}{\sigma(2\sigma^2 + \sigma + 2\chi)}$$

and

$$p^*_{A,R} = p^*_{B,L} = \frac{\tau(\sigma + \chi)}{\sigma(2\sigma^2 + \sigma + 2\chi)},$$

where $p^*_{A,R} < p^*_{A,L}$ and $p^*_{B,L} < p^*_{B,R}$ when the signal is informative. Resulting equilibrium payoffs for each firm are

$$\pi^* = \frac{\tau(4\sigma^4 + \sigma^2 + 2\sigma\chi(1 + 2\sigma) + 2\chi^2)}{2\sigma(2\sigma^2 + \sigma + 2\chi)^2}.$$

These payoffs are the punishment payoffs in the dynamic game as defined in Section 2. By construction, punishment payoffs are equal to those derived in Section 3 for $\chi = 0$. Furthermore, we observe that these payoffs fall as χ is rising; that is, gains from collusion are higher. Because collusive prices are set uniformly, and hence optimal deviation only depends on a firm's private signal, collusive and deviating payoffs remain unchanged compared with the case with symmetric signals. Therefore, we arrive at the following proposition.

Proposition 3. *For any $\chi > 0$, the critical discount factor $\bar{\delta}_{\text{cor}}$ is strictly lower compared with the case of independent signal quality σ. In addition, $\bar{\delta}_{\text{cor}}$ is non-monotonic in σ.*

Note that the proof will be omitted because of the analogy to the proof of Proposition 1.

At the lower and upper bounds of σ, the cases of correlated and independent signals are equivalent by construction, and hence the critical discount factors are equal. The following corollary directly results from Propositions 1 and 3:

Corollary 3. *For any $\chi > 0$, the probability that a ban on price discrimination facilitates collusion is strictly lower compared with the case of independent signal quality σ. Furthermore, the difference strictly increases in χ.*

4.5. Optimal punishments

So far, we have assumed that firms punish deviating behavior by reverting to competition forever. To us, this appears to be plausible – especially when thinking about tacit collusion without a certain punishment mechanism, a situation in which defection might lead to competition for an undetermined time horizon. Grim-trigger strategies generate exactly this trade-off because punishment coincides with competitive outcomes. If, instead, optimal penal codes as in Lambson (1987) and Abreu (1988) are employed in our set-up, punishment payoffs become deterministic and finite.

Under optimal punishments, firms still trade off gains from deviation against losses from punishment, but, by construction, they do not take competitive outcomes into account at all. The harshest punishment is independent of the signal precision, and a punishing firm sets a single price that is equal to the marginal cost – zero in the present case. Then, however, because of the transport costs, the competitor can attract only those customers whose locations are in the same half as its own location. Hence, the competitor also charges a single price that is independent of the signal precision. As a result, the punishment profit does not depend on the signal quality. With regard to the collusive price, we point out that there is no change. This means that neither the collusive price nor the collusive profit depend on the signal. As a consequence, the optimal deviating price and the resulting deviation profit remain unchanged. Recall that these can depend on signal quality, where – if transport costs are sufficiently high – deviation becomes more profitable as signal precision grows.

Taken together, we can thus state that when firms use optimal penal codes, there is an unambiguous relationship between the critical discount factor and the signal quality. Because signal precision only has a positive – if any – impact on the deviation profit, the critical discount factor can only increase in signal quality. As a direct consequence, and in contrast to our findings from the main analysis, banning price discrimination has no effect, or it facilitates collusion.

5. Conclusions

In this paper, we focus on the impact of data-driven price-discrimination strategies on the scope for tacit collusion among firms. We find that enhanced prediction of customer preferences results in a U-shaped effect on firms' ability to sustain collusion. We find that for low levels of predictive capabilities, collusion is easier to sustain under price discrimination than under uniform pricing; this result is robust to a number (though not all) of alternative modeling assumptions. For sufficiently high levels, collusion is harder to sustain under discriminatory pricing than under uniform pricing. Thereby, potential misrecognition of customers plays a crucial role.

We provide the following policy implications: data regulation should consider adverse effects on competition, even though this mission is not central to designing data policy. In particular, deregulation of access to or usage of customer data facilitates the coordinated behavior of firms as long as initial predictions of customer preferences are weak. By contrast, for relatively strong predictions, policies intending to restrict access to and usage of customer data facilitate the coordinated behavior among firms. Moreover, the effect of a legal ban on price discrimination on firms' ability to collude crucially depends on the quality of predictions.

The model we employ does not allow us to draw conclusions with regard to welfare because we do not take into account customer preferences for privacy, or other adverse effects due to discrimination of customers. However, one may argue that the findings reveal certain conditions that warrant lower and higher degrees of concern for regulators. For example, when the exchange of customer data leads to higher signal precision toward perfect information, competition authorities should be less concerned with regard to collusive activity than would be warranted for a case in which firms exchange data on prices and demands. Furthermore, given the positive, destabilizing effect of better technology in Miklós-Thal and Tucker (2019) and the (partly) facilitating effect that we find, we conclude that it appears to be very important for competition authorities to evaluate where and how better algorithms become effective.

Our results can also be useful in the context of merger policy. When antitrust agencies and competition authorities review a merger case, potential coordinated effects resulting from the merger play an important role. There is a checklist of typical market characteristics to watch out for because they facilitate coordination. On this list, for instance, are high concentration and product homogeneity (see, e.g., Sibley and Heyer, 2003). Our analysis also underscores that decision-makers should pay attention to data quality and make better opportunities to use algorithms in the context of merger review. Our set-up applies to markets in which firms collect and/or buy customer information, in which product differentiation occurs on a horizontal dimension, and in which firms use customer information to make targeted offers and to offer discounts. If a merger results in better targeting due to pooling or technological collaboration, our results indicate that, all else equal, collusion can be more likely depending on technological status.

Clearly, our approach is limited in several respects. We do not investigate the direct effect that the use of better algorithms can have for the sustainability of collusion, such as monitoring or parallel behavior. Furthermore, we do not address hub-and-spoke-style collusion with a common algorithm. Both aspects feature prominently in recent competition-policy debates (see, e.g., OECD, 2017). Lastly, we do not study the empirical side of the issue. Despite these limitations, we are confident that our focus on more powerful algorithms to improve price discrimination can contribute to a better understanding of how such technological advances may shape the incentives – and the ability – of firms to collude.

Supporting information

Additional supporting information may be found online in the supporting information section at the end of the article.

Online Appendix

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