Optimization II Homework 4 Solution

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Problem 1

```
T =

  6×6 table

    \ n       2          3          4          5          6          7
    m \    _______    ________   ________   ________   _________  _________

    2       0.66695    0.49979    0.40014    0.33348    0.28561    0.24988
    3       0.50006    0.29984    0.19976    0.14281    0.10732    0.083268
    4       0.40005    0.19972    0.11433    0.071479   0.047723   0.033261
    5       0.3336     0.14288    0.071552   0.039695   0.023851   0.015121
    6       0.28546    0.10737    0.047567   0.023827   0.013006   0.0075634
    7       0.25017    0.083258   0.033272   0.015156   0.0075613  0.0041151
```

To compute the probability of a matrix A such that A has saddle point(s). We first assume A_{i^*,j^*} is a saddle point.

Then according to the definition of saddle point, $A_{i,j^*} < A_{i^*,j^*} < A_{i^*,j}$. This is true iff A_{i^*,j^*} is the mth element in a ordered set containing all elements from row i and column j . There are $(m+n-1)$ elements in the set, therefore, The probability to select A_{i^*,j^*} is $\frac{1}{m+n-1}$.

Next, we need to determine the rest elements to fill in row i and column j. Since choosing the elements for row or column automatically determines the other one, the probability is $\binom{m+n-2}{m-1}^{-1}$.

Lastly, since every element in the matrix have equal probablity to be a saddle point, we need to multiply the probability of one element be a saddle point by mn.

Therefore, the probability for matrix A to have saddle point(s) is:

$$\begin{aligned} P &= mn\frac{1}{m+n-1}\binom{m+n-2}{m-1}^{-1} \\ &= \frac{m!n!}{(m+n-1)!} \end{aligned}$$

When $m = 2$, the formula becomes:

$$P = \frac{2n!}{(n+1)!}$$

When $m = 3$, the formula becomes:

$$P = \frac{6n!}{(n+2)!}$$

Problem 2

```
M = [1 -3 6 -9 10;
     -8 6 4 2 -12;
     6 -2 8 -2 -1];

[pMefirst,qMefirst,pYoufirst,qYoufirst] = matrixgame(M)

v1 = expectedValue(pMefirst,qMefirst,M)
v2 = expectedValue(pYoufirst,qYoufirst,M)
v3 = expectedValue(pMefirst,qYoufirst,M)
v4 = expectedValue(pYoufirst,qMefirst,M)
```

```
pMefirst =            qMefirst =            pYoufirst =           qYoufirst =

         0                     0                     0                     0
    0.0667                     0                     0                     0
    0.9333                     0                     1                     0
                               0                                    0.7333
                               1                                    0.2667


v1 =                  v2 =                  v3 =                  v4 =

   -1.7333               -1.7333               -1.7333               -1
```

```
M = [1 -3 6 -9 10;
     -8 6 4 2 -12;
     10 12 8 11 20];

[pMefirst,qMefirst,pYoufirst,qYoufirst] = matrixgame(M)

v1 = expectedValue(pMefirst,qMefirst,M)
v2 = expectedValue(pYoufirst,qYoufirst,M)
v3 = expectedValue(pMefirst,qYoufirst,M)
v4 = expectedValue(pYoufirst,qMefirst,M)
```

```
pMefirst =            qMefirst =            pYoufirst =           qYoufirst =

         0                     0                     0                     0
         0                     0                     0                     0
         1                     1                     1                     1
                               0                                           0
                               0                                           0


v1 =                  v2 =                  v3 =                  v4 =

   8                     8                     8                     8
```