Optimization II Homework 3 Solution

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Problem 1

```
load('/Users/nicklu/JHU/2022Spring/Intro to Opt2/hw03/hw1_1.mat');

thecolors = color(edgelist,2)

thecolors =

     2
     1
     2
     1
     1
     2
     1
     2


load('/Users/nicklu/JHU/2022Spring/Intro to Opt2/hw03/hw1_2.mat');


thecolors = color(edgelist,3)

thecolors =

     3
     1
     2
     3
     1
     2
     3
     3
     2


load('/Users/nicklu/JHU/2022Spring/Intro to Opt2/hw03/hw1_3.mat');


thecolors = color(edgelist,3)
```

```
thecolors =

     3
     1
     3
     2
     1
     2
     3
     1
     1
     2
```

For the last graph, I get a 6-coloring in 2.841599 seconds and 5-coloring in 27.283567 seconds using GUROBI. 4-coloring seems, and indeed infeasible. I tried to run it on my computer, having 100% CPU usage and it became super hot, but no result came out after 20 minutes.

Problem 2

I affirm that I read and understand the solution for 2021 Exam1, Problem 5.

Problem 3

Since paths and even cycles of half-present edges can be broken down, selecting alternating edges to be disjoint fully-present edges in the matching, with cardinality at least as great. The only way to make $\alpha'_f(G) > \alpha'(G)$ different is when there are odd cycles in a graph. In an odd cycle graph, $\alpha'_f(G) = \alpha'(G) + \frac{1}{2}$:

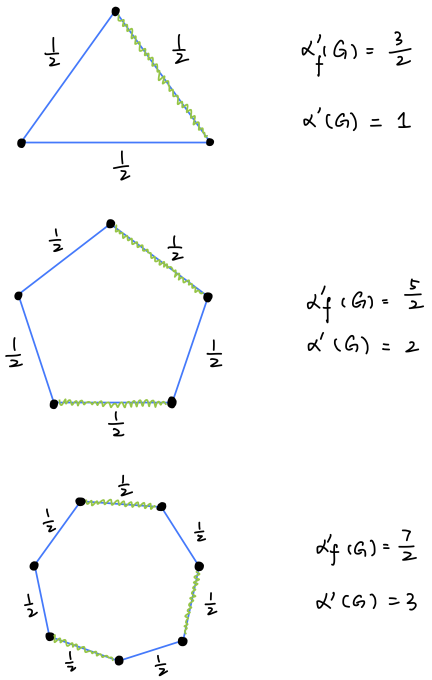

Figure 1: Odd cycle examples

Observe that as the number of edges of an odd cycle increases, the ratio $\dfrac{\alpha'_f(G)}{\alpha'(G)}$ decreases. The largest ratio $\dfrac{3}{2}$ occurs when number of edge is 3. Therefore $\alpha'_f(G) \leq \dfrac{3}{2}\alpha'(G)$.