EE232E Homework #2 Report

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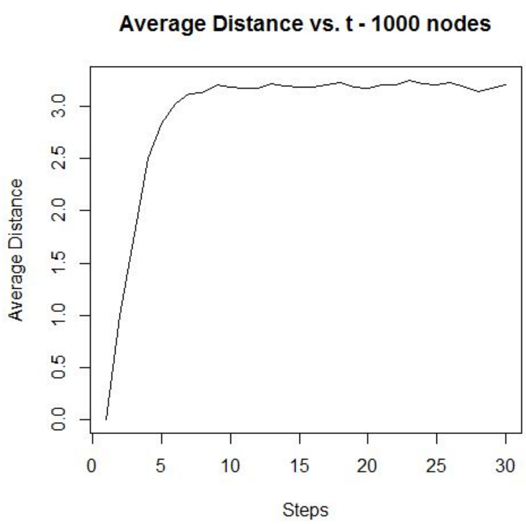
Problem 1.

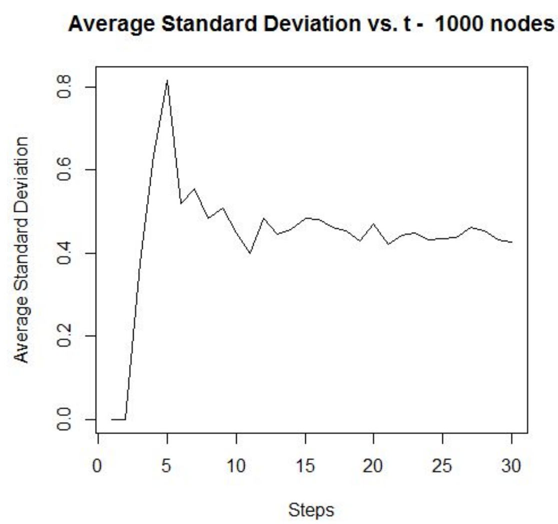
(a)

We created undirected random networks with 1000 nodes, with a probability 0.01 of drawing an edge between two arbitrary nodes, using the **`erdos.renyi.game`** function from **`igraph`** package.

(b)

Then we use **`netrw`** package to simulate random walk with 1000 walkers that starts with random locations. We use a variable `t` to represent the number of steps that the walkers have taken. We then measure and plot the average distance (using the **`shortest.paths`** function) of the walkers from the starting point with respect to `t`. The average standard deviation is is also measured as the following graphs:



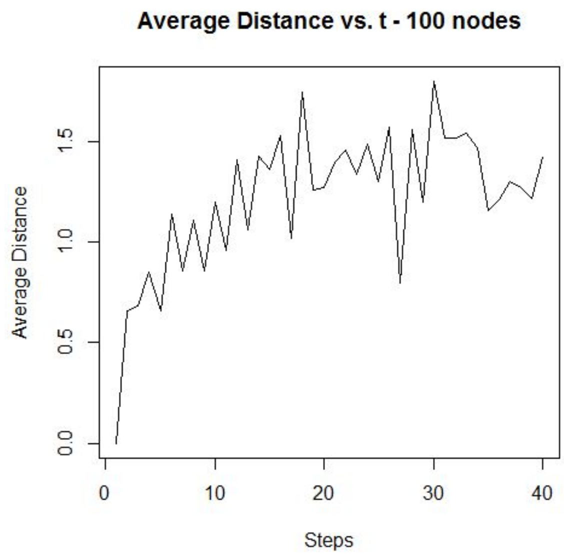


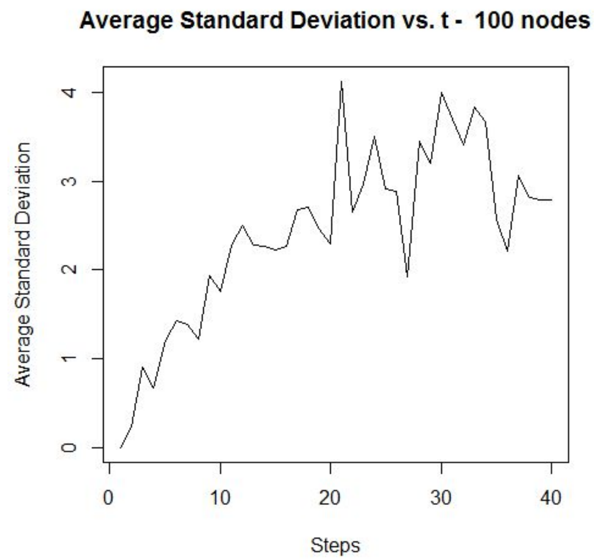
(c)

They do not have similar relations. In d-dimensional average distance can be negative, whereas in random networks all distances must be positive.

(d)

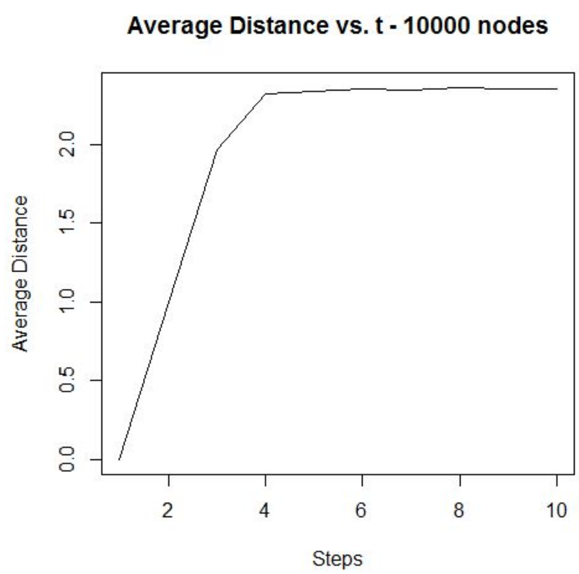
We repeated the first two steps with 100 nodes and 10000 nodes random networks. For 100 nodes network, we have the following results:

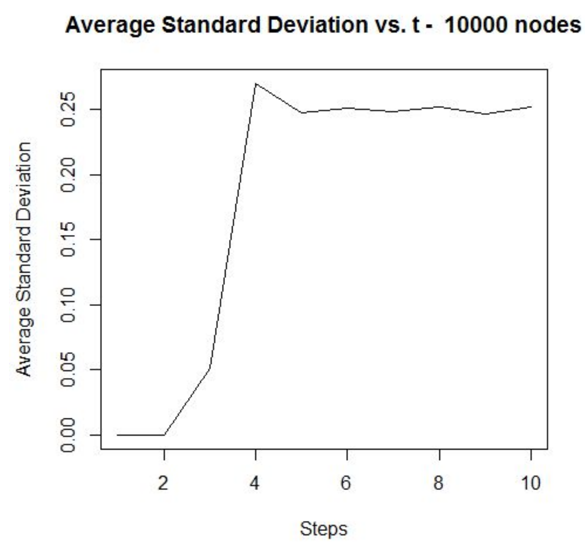




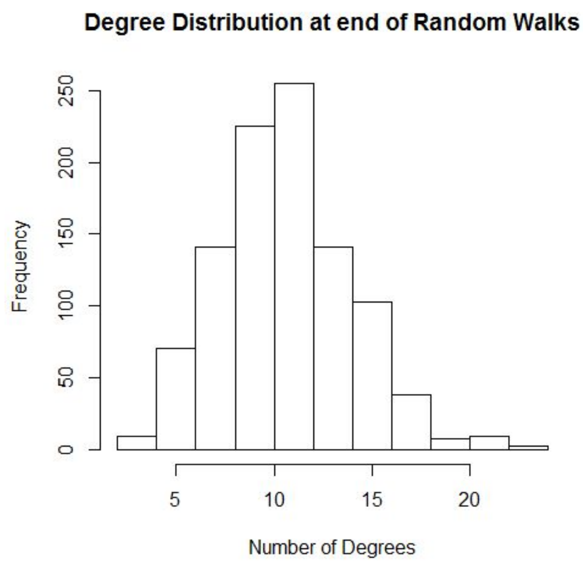
A random network with 100 nodes and p = 0.01 is a very disconnected network. As expected, most walkers are stuck around their starting point. The diameter of the network is 8.

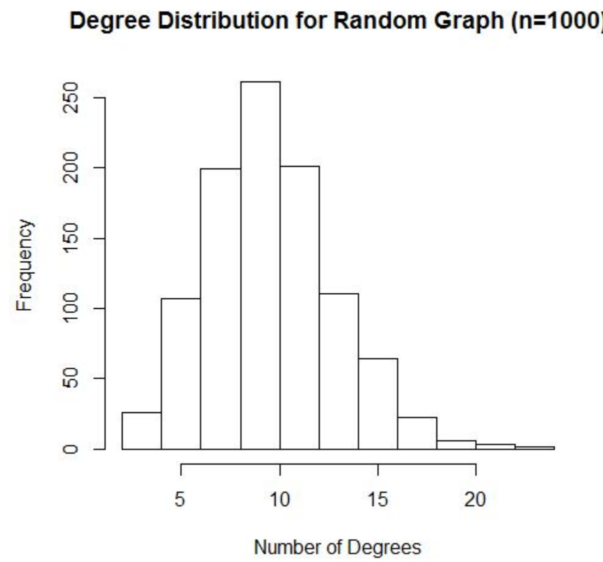
For the 10000 nodes network, we have the following results:





A random network with 10000 nodes and p=0.01 is a very dense network. The diameter is 3.
The distance is upper bounded by the diameter of the network.
(e)
The degree distribution at the end of random walk is measured below:





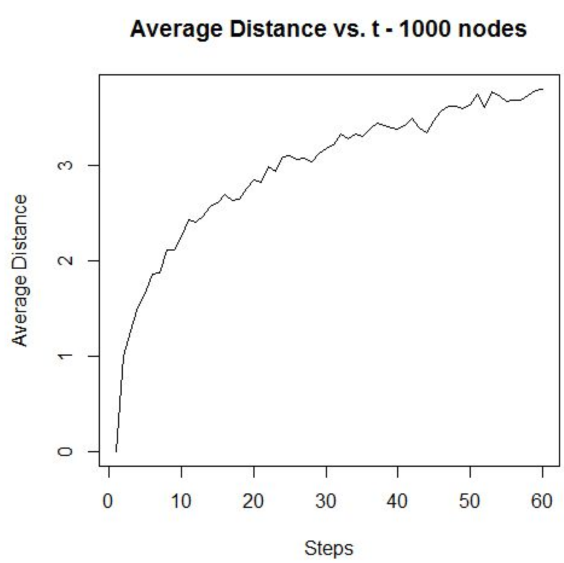
Problem 2.

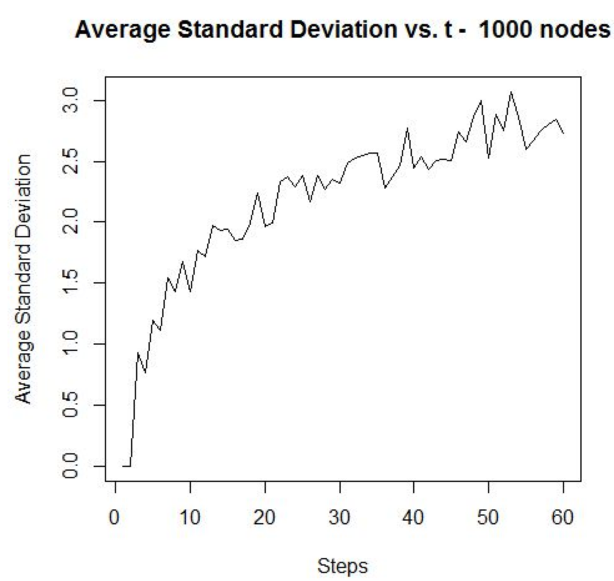
(a)

We use **`barabasi.game`** function to create undirected random graph with 1000 nodes. The degree distribution from this random network is proportional to `x^-3`

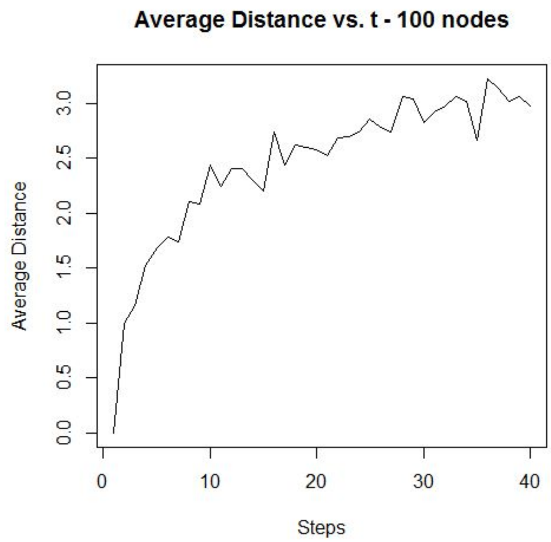
(b)

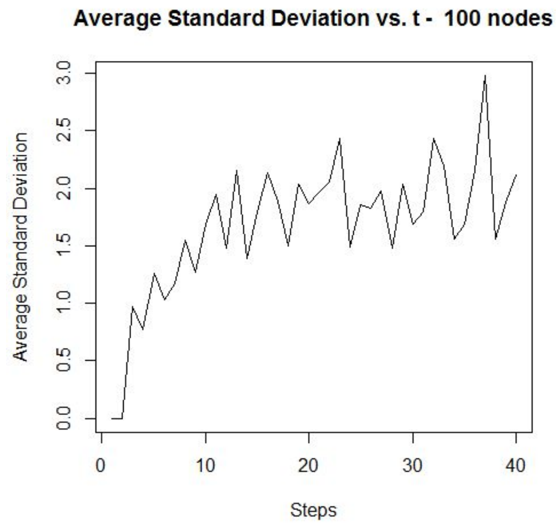
The average distance and average standard deviation over t is measured and shown in the following plots:



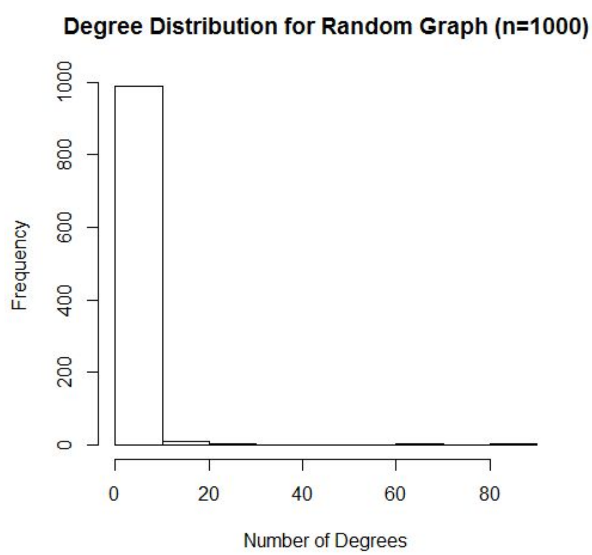


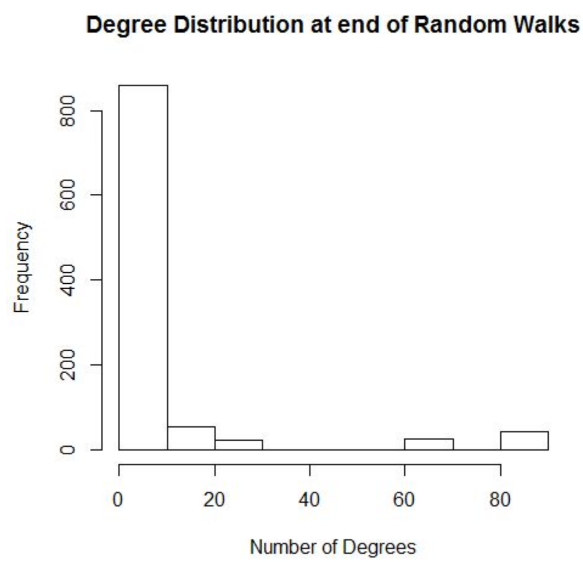
(d)





(e)



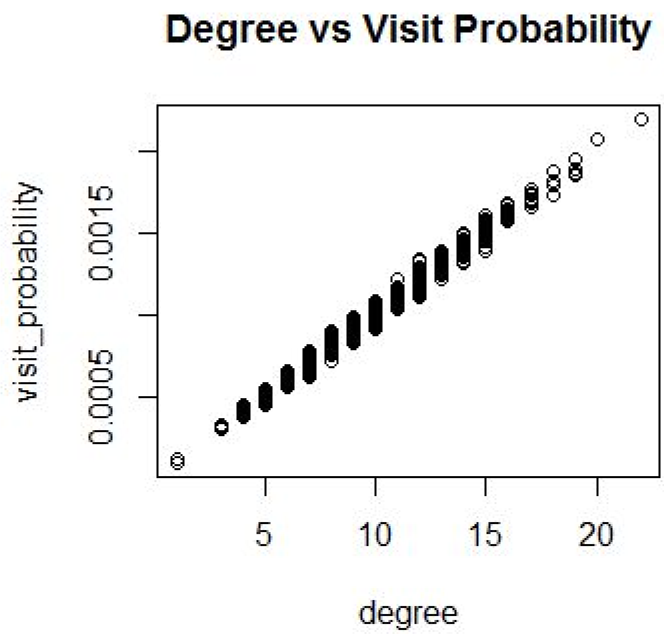


As we can see, these two results are very close.

Problem 3.

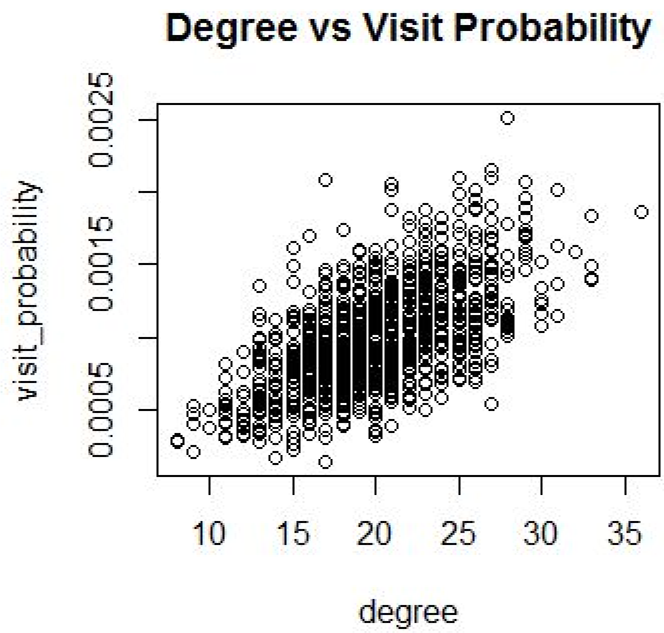
(a)

We created an undirected random network with 1000 nodes and 0.01 probability of drawing an edge between any pair of nodes using **`erdos.renyi.game`** function. Then, we simulated a random walk using **`netrw`** algorithm with a damping factor of 1. The following graph shows the relationship between degree and visit probability of an undirected random network with 1000 nodes and 0.01 probability of drawing an edge between any pair of nodes.



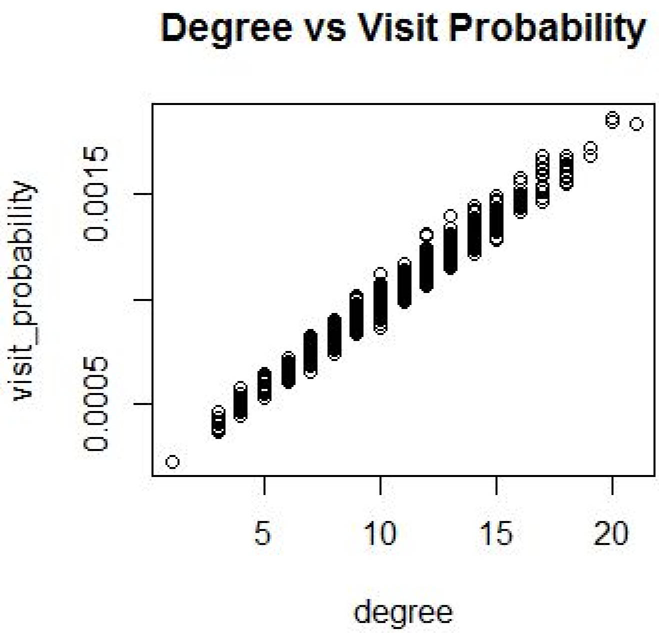
According to our graph, the visit probability is linearly related to the degree of the nodes. The correlation between degree and visit probability was 0.993701328327981.

(b)
We created a directed random network with 1000 nodes and 0.01 probability of drawing an edge between any pair of nodes using **`erdos.renyi.game`** function. Then, we simulated a random walk using **`netrw`** algorithm with a damping factor of 1. The following graph shows the relationship between degree and visit probability of an undirected random network with 1000 nodes and 0.01 probability of drawing an edge between any pair of nodes.



According to our graph, we could not observe any relationship between the visit probability and the degree of the nodes. The correlation between degree and visit probability was 0.596687312961693.

(c)
We created an undirected random network with 1000 nodes and 0.01 probability of drawing an edge between any pair of nodes using **`erdos.renyi.game`** function. Then, we simulated a random walk using **`netrw`** algorithm with a damping factor of 0.85. The following graph shows the relationship between degree and visit probability of an undirected random network with 1000 nodes and 0.01 probability of drawing an edge between any pair of nodes.

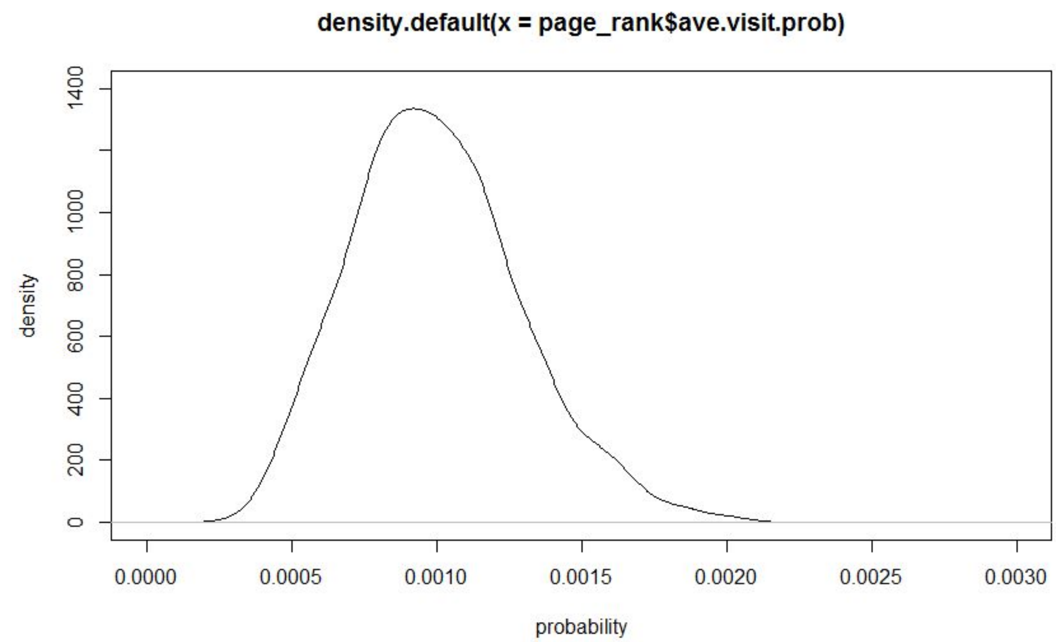


According to our graph, the visit probability is linearly related to the degree of the nodes. The correlation between degree and visit probability was 0.987800795685146, which means the relationship is not as strong as shown in part (a), but we still observed linear relationship. This concludes that if we have an undirected graph, the relationship between degree and visit probability is linear.

Problem 4.

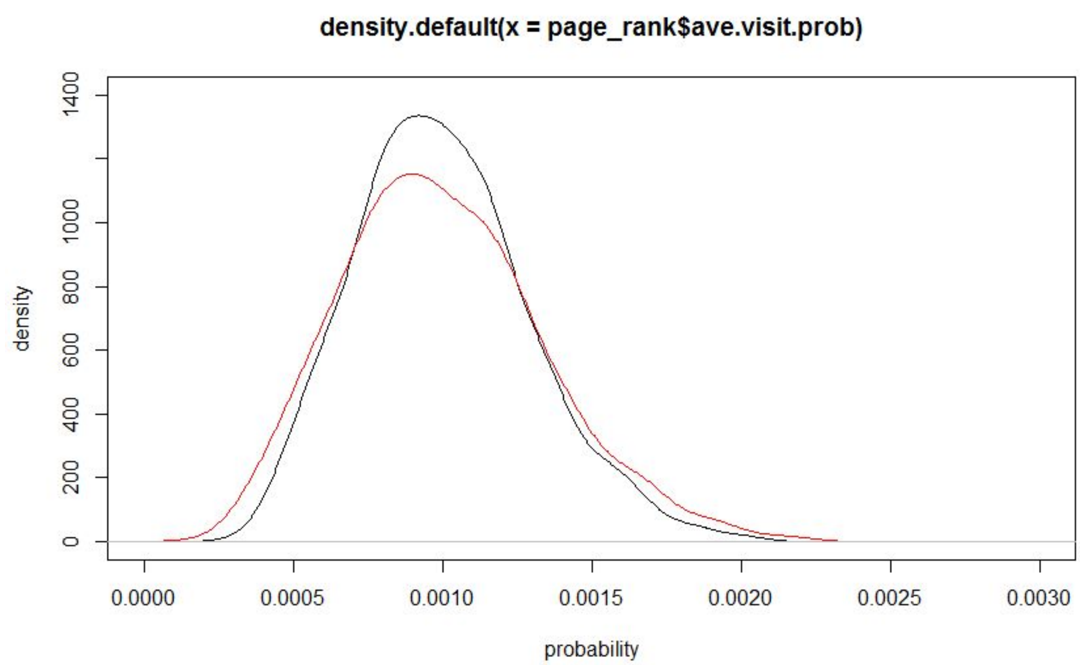
(a)

We used **`erdos.renyi.game`** function to create a directed random network with 1000 nodes, where the probability p for drawing an edge between any pair of nodes is 0.01. Then we used random walk netrw function with damping parameter 0.85 to simulate the PageRank of the nodes. The following plot is the teleportation probability density of the nodes in the random network:



(b)

In order to make teleportation probability to each node proportional to its PageRank, we reused the node visit probability from the previous pagerank to assign to the teleportation probability (instead of 1/N as normal pagerank). We compare the results by drawing the visit probability density of both pagerank on the same plot:



We can see that the second pagerank tends to have more large and small values than the normal pagerank, the density is stretching out more towards two ends than the first one does.

(c)
As we know, the normal pagerank equation is this:
$PR(A) = (1-d) / N + d (PR(T1)/C(T1) + ... + PR(Tn)/C(Tn))$, where the teleportation probability is $1/N$ supposedly equal across the nodes. To take into account the effect of the self-enforcement that the teleportation probability to each node is proportional to its PageRank, we need to recursively feeding the pagerank back to the equation as the teleportation probability.