Instead of calling your  $\Delta(w,t)$  Dirac delta, it is better to call it Heaviside function  $H\left(x-\frac{1}{2}\right)=\frac{1}{2}(1+sgn(x-\frac{1}{2}))$  https://en.wikipedia.org/wiki/Heaviside\_step\_function

Good thing about Heaviside function is that is can be approximated by the smooth logistic function  $H(x) = \frac{1}{1+e^{-2kx}}$ , with k large enough (along with other smooth approximations). This may both be related to entropies and used as a differentiable loss function in optimizing ML models (not sure how to at the moment).

 $N_p(w,*) = \sum_t N^p(w,t)$  technically is not a norm, as it does not satisfy homogeneity property (except for p = 1). It is a pseudonorm though. Similarly to the above  $\Delta(w,*)$  is a pseudonorm and is Hamming distance of such vector from zero.

It may be useful to consider a vector space over vocabulary  $T=\{t\}$  of size |T| and a vector N in this space with |T| components N(w,t). It is interesting to note that an entropy-like expression can be obtained for actual  $l_1$ norm, because

$$\frac{d}{dp} \|N\|_{l_p} = \frac{d}{dp} \left( \sum_{t} N^p(w, t) \right)^{1/p} = \log \sum_{t} N^p(w, t) \left( \sum_{t} N^p(w, t) \right)^{1/p} \sum_{t} \log N(w, t) N^p(w, t) =$$

$$= H(w, *) \|N\|_{l_p} \log \sum_{t} N^p(w, t)$$

I don't have a viable physical interpretation for this, but it may be highly far-reaching.