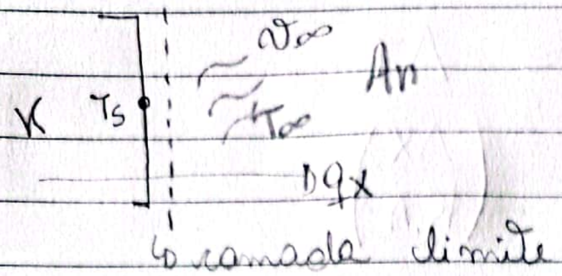
Condução Transiente (cap 5)



→ Bal. de energia

- $E_E - E_S + \cancel{E_{gerado}}^{\,0} = E_{acumulada}$

- $$\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)}^{\,0} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}^{\,0}\right]$$

$$= \rho V_c \frac{\partial T}{\partial t} \quad (1)$$

- $$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho V_c \frac{\partial T}{\partial t} \quad (2)}$$

A taxa de calor é:

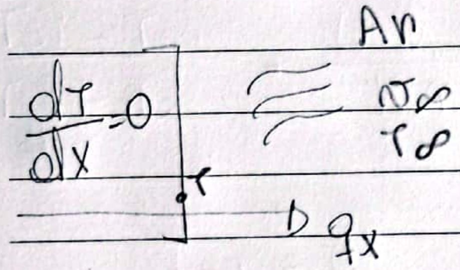
$$q_x = -KA\frac{dT}{dx} \approx \frac{\Delta T}{\left(\frac{\Delta x}{K.A}\right)}$$

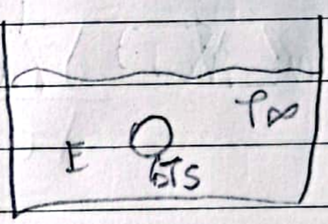
• Onde:

- Isolante: $K \approx 10^{-2}$ kcal/m·K
- metais: $K \approx 10^{2}$ kcal/m·K

Método da capacitância Global:

• T_∞ = const.





$T_s > T_\infty$

→ $-E_s$ = E acumulada

→ $-hA\underbrace{(T - T_\infty)}_{\theta} = \rho V_c \, dT/dt$

$$\left.\begin{aligned} T - T_\infty &= \theta \\ d\theta/dt &= dT/dt \end{aligned}\right\} -h.A.\theta = \rho V_c \, d\theta/dt$$

$$\boxed{-h \cdot A \cdot \theta = \rho V c \, d\theta/dt} \quad (3)$$

$$\theta = -\frac{\rho V c}{h A} \, d\theta/dt$$

$$\int_0^t dt = -\frac{\rho V c}{h A} \int_{\theta_i}^{\theta} \frac{d\theta}{\theta}$$

$$t = -\frac{\rho V c}{h \cdot A} \ln\left(\frac{\theta}{\theta_i}\right)$$

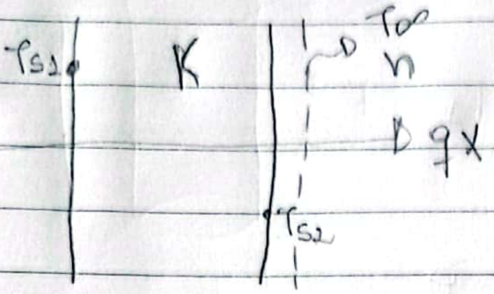
$$t = \frac{\rho V c}{h \cdot A} \ln\left(\frac{\theta_i}{\theta}\right)$$

$$t = -\frac{\rho V c}{h \cdot A} \ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right)$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{h \cdot A}{\rho V c} \cdot t\right)$$

$$\boxed{\bullet \; T = T_\infty + \exp\left(-\frac{h \cdot A \cdot t}{\rho V c}\right)(T_i - T_\infty)} \quad (4)$$

Validação do método da capacitância global.



$T_{s1} > T_{s2} > T_\infty$

→ Dedução:

$$\frac{K \cdot A\,(T_{s1}-T_{s2})}{L} = hA\,(T_{s2}-T_\infty)$$

$$\frac{T_{s1}-T_{s2}}{T_{s2}-T_\infty} = \frac{L\,h\,A}{K\,A} = \frac{(hA)}{(K/L)} = \frac{hA}{(1/R_{cond})} = \frac{(1/R_{conv})}{(1/R_{cond})}$$

$$\therefore \frac{T_{s1}-T_{s2}}{T_{s2}-T_\infty} = \frac{R_{cond}}{R_{conv}}$$

Se R_{cond} é baixo e R_{conv} é alta.

$$Bi = \frac{R_{cond}}{R_{conv}} \approx 0 \rightarrow T_{s1}-T_{s2} \approx 0$$

$$= \frac{L/KA}{1/hA} <<< 1 \quad \therefore \quad \frac{h \cdot L}{K} <<< 1$$

Assim, obtemos uma adimensionalização

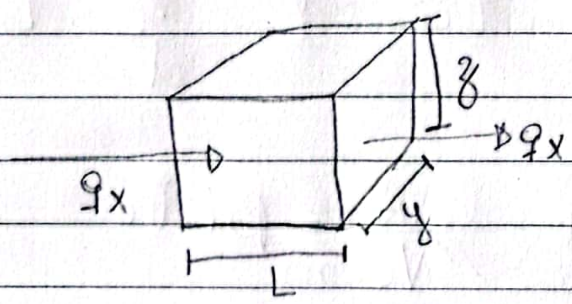
$$\boxed{Bi = \frac{h \cdot L}{K} <<< 1}$$

• OBS: Condição que deve ser satisfeita p/ validar a eq. 4.

• Biot:

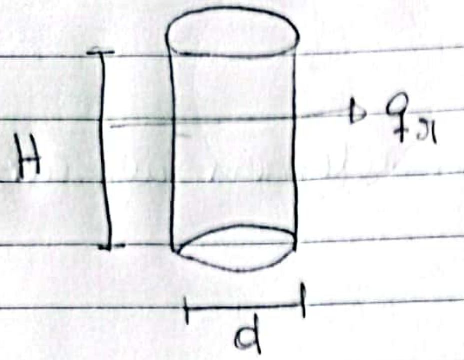
$$Bi = \frac{h\,L_c}{K} < 0,1 \qquad \boxed{\bullet\, L_c = \frac{V}{A_s}}$$

a) Placa



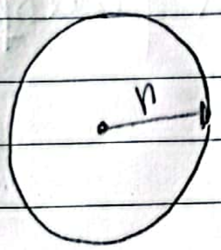
$$L_c = L\frac{\cancel{y}\cancel{z}}{\cancel{y}\cancel{z}} = L$$

b) Cilindro



$$\boxed{L_c = \frac{\cancel{\pi}(d^{\cancel{2}}/4)\cancel{H}}{\cancel{\pi}\cancel{d}\cancel{H}} = \frac{d}{4} = \frac{r}{2}}$$

c) Esfera



$$L_c = \frac{\cancel{4}/3\,\cancel{\pi}\,r^3}{\cancel{4}\,\cancel{\pi}\,\cancel{r^2}} = \frac{r}{3} = \frac{d}{6}$$

* Determinando Bi na eq. 4:

$$\frac{h.A.t}{\rho V_c} \cdot \frac{K}{K} \cdot \frac{L_c}{L_c^2} = Bi \cdot \underbrace{\frac{K}{\rho c}}_{\alpha} \cdot \frac{t}{L_c^2}$$

$$Bi \underbrace{\frac{\alpha\, t}{L_c^2}}_{Fo:\ Fourier} \rightarrow \frac{h.A}{\rho V_c} = Bi . Fo$$

se $Bi < 0,1$:

$$T = T_\infty + (T_i - T_\infty) \cdot \exp[-Bi \cdot Fo]$$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp[-Bi \cdot Fo]} \quad (5)$$

Tal que: $\cdot Bi = \frac{h \cdot Lc}{K} < 0,1$

$\cdot Fo = \frac{K}{\rho c} \frac{t}{Lc^2} = \frac{\alpha \cdot t}{Lc^2}$

$$\boxed{Lc = \frac{V}{A_{sup}} = \frac{4/3 \pi r^3}{4 \pi r^2} = \frac{r}{3}}$$

$\cdot Lc = d/6$ ou $Lc = r/3$ $\quad \cdot \alpha = K/\rho \cdot c$

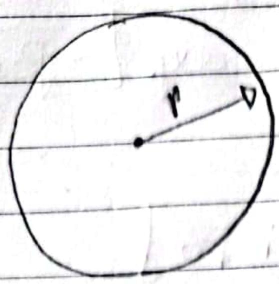
Eq. para sólidos:

$T \neq f(x)$: $\frac{dT}{dx} = 0$ $\quad \cdot \rho = cte$

Exemplo: para cobre:

T (K)	K [W/m·°C]	c [J/kg·°C]
300	401	385
339	397,96	389,56
400	393	397
500	379	417

Esfera de cobre



$T_\infty = 298\ K$

Ar, h

$$T_i = 500\ K \xrightarrow{\tau = 500\ s} T = ?$$

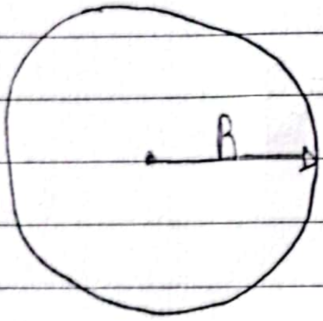
Eq. 05: mét. da capacitância global

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[\frac{-h \cdot \tau}{L_c\, \rho\, c}\right]$$

Passos:

1. Chutar: T
2. $\bar{T} = \frac{T + T_i}{2}$
3. $c = f(\bar{T})$
4. $k = f(\bar{T})$
5. $Bi = \frac{h \cdot L_c}{k} < 0,1$
6. $\frac{|A - B|}{A} \cdot 100\% < 10\%$

Exemplo:



Aço

Dados:

$T_i = 1150 K$

$T = 450 K$

$T_\infty = 325 K$

$R = 0{,}005 m$

$h = 25 W/m^2 K$

$t = ?$

$K = 40 W/m°C$

$\rho = 7800 Kg/m^3$

$C = 600 J/Kg°C$

Solução:

$$\alpha = \frac{K}{\rho C} = 8{,}35 \cdot 10^{-6} \, m^2/s$$

$$L_c = \frac{r}{3} = 0{,}001667 \, m$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{h L_c}{K} \cdot \frac{K}{\rho C} \cdot \frac{t}{L_c^2} \right)$$

$t = 588{,}77 s$

simplificando eq(5):

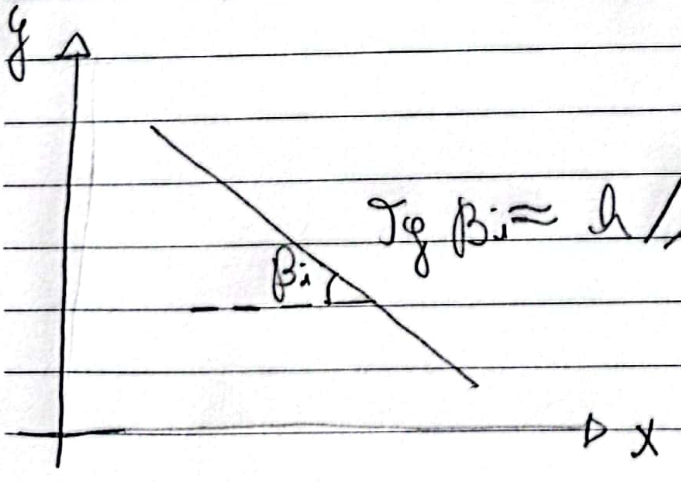
$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{ -Bi \, Fo \right\}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{ -\frac{h \cancel{L_c}}{\cancel{K}} \frac{\cancel{K}}{\rho C} \frac{t}{L_c^{\cancel{2}}} \right\}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{ \frac{-h}{\rho C L_c} \cdot t \right\}$$

→ linearizando:

$$\underbrace{\ln\left[\frac{T - T_\infty}{T_i - T_\infty}\right]}_{y} = \underbrace{\frac{-h}{\rho C L_c}}_{a} \cdot \underbrace{t}_{x}$$



SULTS