Análise Estrutural: Treliças

Exemplo:

nó B

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
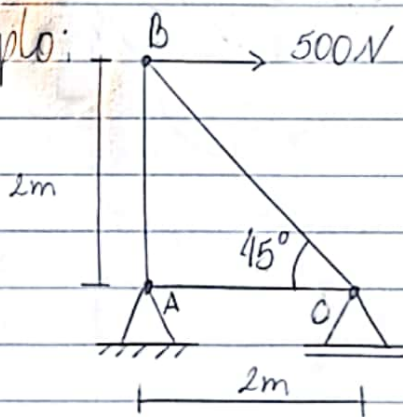
$$\xrightarrow{\oplus} \Sigma F_x = 0 \Rightarrow 500 + N_1 \operatorname{sen} 45 = 0$$
$$N_1 = 707,11 \, N \, (C)$$

$$\underset{\oplus}{\downarrow} \Sigma F_y = 0 \Rightarrow N_1 \cos 45 + N_2 = 0$$
$$N_2 = 500 \, N \, (T)$$

nó C

$$\overset{\oplus}{\leftarrow} \Sigma F_x = 0 \Rightarrow N_3 - 707,11 \cos 45 = 0$$
$$N_3 = 500 \, N \, (T)$$

$$\underset{\oplus}{\downarrow} \Sigma F_y = 0 \Rightarrow V_C + 707,11 \operatorname{sen} 45 = 0$$
$$V_C = -500$$
$$V_C = 500 \, N \, \uparrow$$

nó A

$$\sum F_x = 0 \Rightarrow H_a - 500 = 0$$
$$H_a = 500N \leftarrow$$
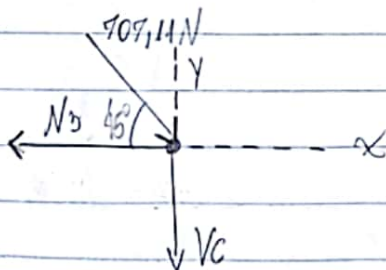
$$\sum F_y = 0 \Rightarrow V_a - 500 = 0$$
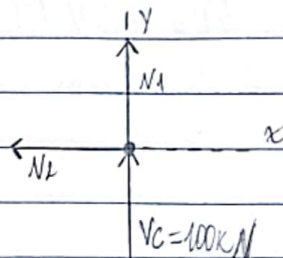$$V_a = 500N \downarrow$$
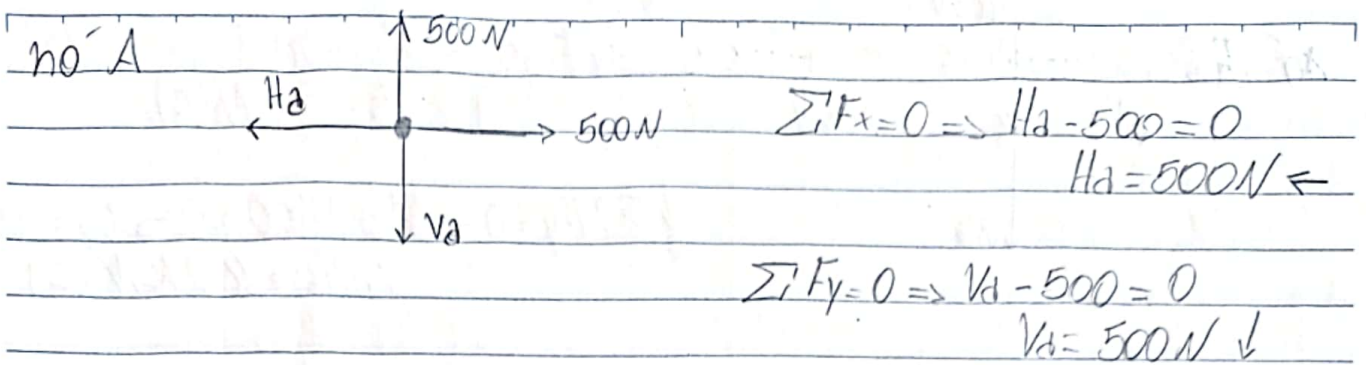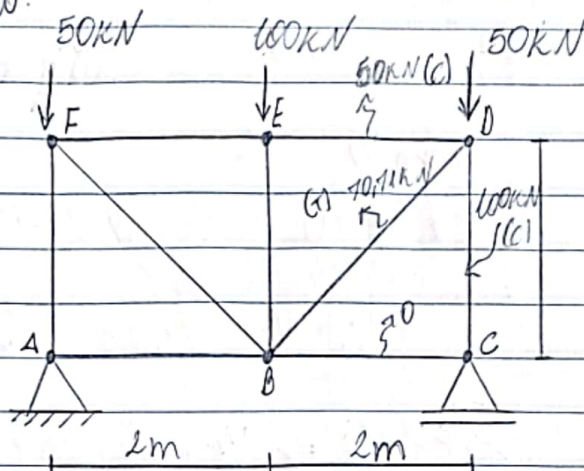
Exemplo:

$$\sum F_x = 0 \Rightarrow H_d = 0$$

$$\left(\sum M_d = 0 \Rightarrow -4V_c + 400 = 0\right.$$
$$V_c = 100kN = V_d$$

nó C:

$$\sum F_x = 0 \Rightarrow N_2 = 0$$

$$\uparrow \oplus \sum F_y = 0 \Rightarrow N_1 + 100 = 0$$
$$N_1 = -100 \Rightarrow N_1 = 100kN \, (c)$$

nó D:
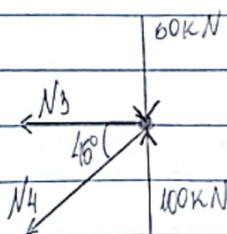
$$\uparrow \oplus \sum F_y = 0 \Rightarrow N_4 \, \text{sen} \, 45 - 100 + 50 = 0$$
$$N_4 = 70,71 \, kN \, (T)$$

$$\rightarrow \oplus \sum F_x = 0 \Rightarrow N_3 + N_4 \cos 45 = 0$$
$$N_3 = -50 \; \therefore \; N_3 = 50kN \, (c)$$
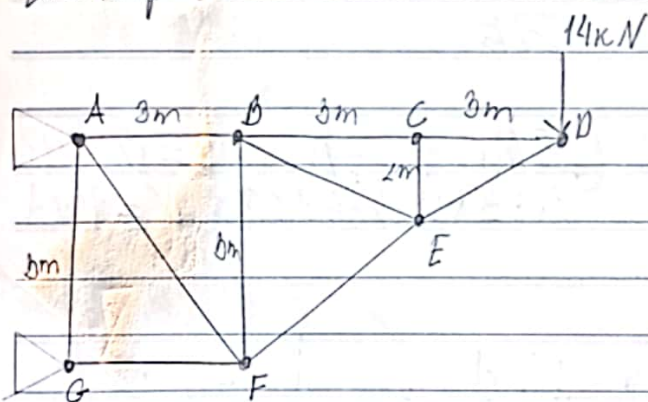
nó E

100kN

←N5   ×   60kN

↓N6

$\sum F_x = 0 \Rightarrow N_5 + 50 = 0$
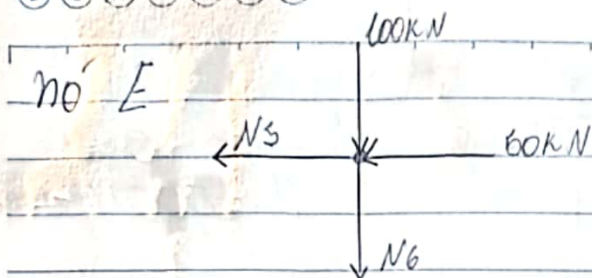
$N_5 = 50 kN (c)$

$\downarrow \oplus \sum Fy = 0 \Rightarrow N_6 + 100 = 0 \Rightarrow N_6 = -100$

$\therefore N_6 = 100 kN (c)$

Exemplo:

14kN

A 3m B 3m C 3m D

2m

3m 3m E

G F

nó D

$\downarrow \oplus \sum Fy = 0 \Rightarrow N_2 \, sen\theta + 14 = 0$

$N_2 = -25,23$

$\therefore N_2 = 25,23 kN (c)$

$\rightarrow \oplus \sum Fx = 0 \Rightarrow N_1 + N_2 \cos\theta = 0$

$N_1 = 21 kN (T)$

nó C
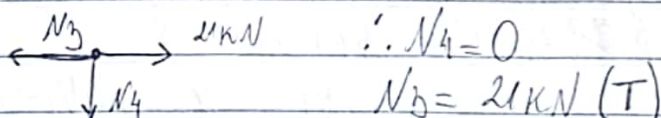
←N3→ 21kN
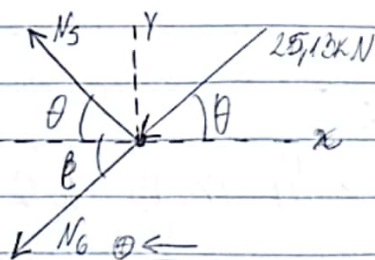
↓N4

$\therefore N_4 = 0$

$N_3 = 21 kN (T)$

nó E

↖N5 ↑Y ↗25,13kN

θ ( )θ

l →x

↙N6

$\sum Fx = 0 \Rightarrow \begin{cases} 0,832 N_5 + 0,949 N_6 + 25,13 \cdot 0,832 = 0 \\ \end{cases}$

$\uparrow \oplus \sum Fy = 0 \Rightarrow 0,555 N_5 + 0,316 N_6 - 25,13 \cdot 0,555 = 0$

$N_5 = 8,43 kN (T)$

$N_6 = 29,51 kN (c)$

Digitalizado com CamScanner

Exemplo:
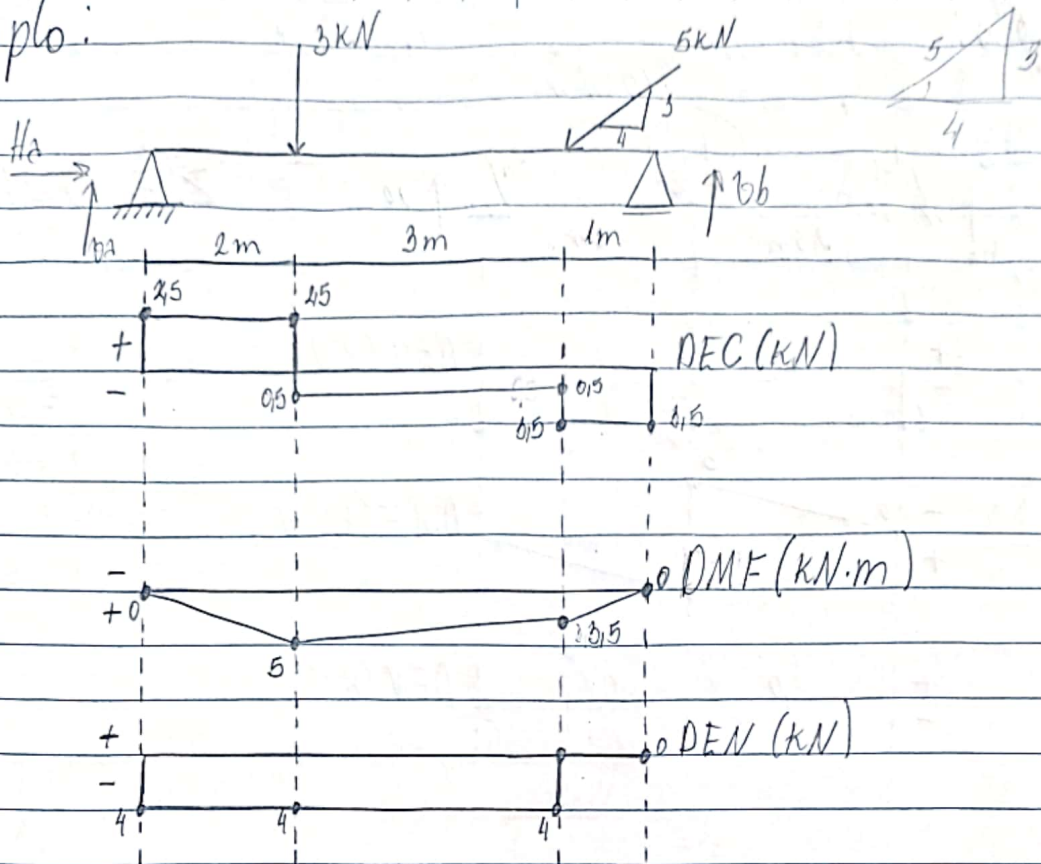
$$\sum F_x = 0 \Rightarrow H_a - 5\sin\theta = 0 \Rightarrow H_a = 4kN \rightarrow$$

$$\sum M_a = 0 \Rightarrow -6\,v_b + 3 \cdot 5 + 3 \cdot 2 = 0 \Rightarrow v_b = 3,5\,kN.m \uparrow$$

$$\sum F_y = 0 \Rightarrow v_a - 3 - 3 + 3,5 = 0 \Rightarrow v_a = 2,5\,kN \uparrow$$
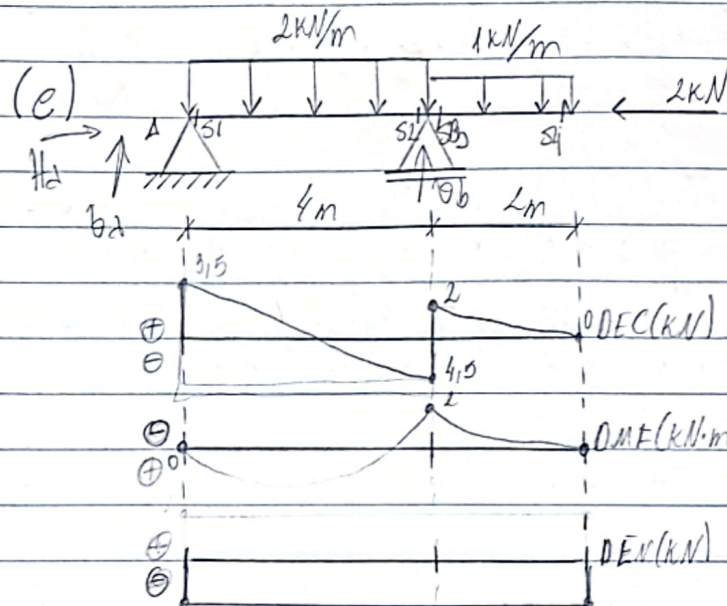
Exercícios:

(d)
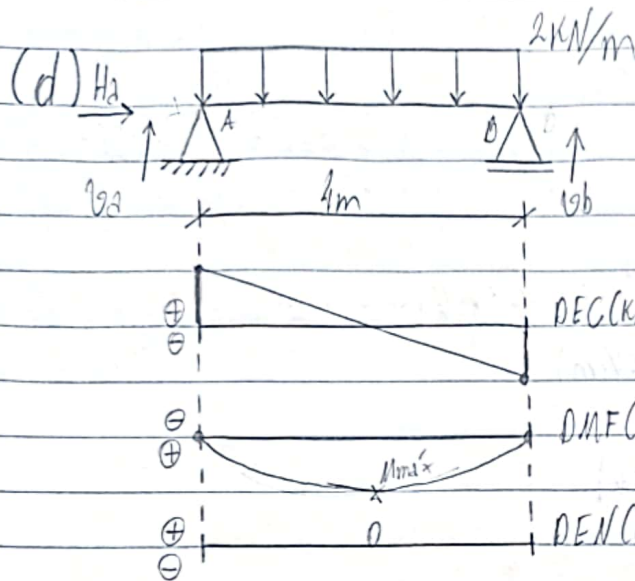
$$\sum F_x = 0 \Rightarrow H_a = 0 \qquad F_{\square} = 8 kN$$

$$\left(\sum M_a = 0 \Rightarrow -4 v_b + 8 \cdot 2 = 0 \Rightarrow v_b = 4 kN\right.$$

$$\uparrow \sum F_y = 0 \Rightarrow v_a - 8 + 4 \Rightarrow v_a = 4 kN$$

$$\frac{dM}{dx} = -V$$

$$M_{máx} = área \triangle = \frac{2 \cdot 4}{2} = 4 kN \cdot m$$

(e)

$$F_{\square 1} = 2 \cdot 4 = 8 kN$$
$$F_{\square 2} = 2 \cdot 1 = 2 kN$$

$$\sum F_x = 0 \Rightarrow H_a - 2 = 0 \Rightarrow H_a = 2 kN \rightarrow$$

$$\left(\sum M_a = 0 \Rightarrow v_b = 6,5 kN \uparrow\right.$$

$$\uparrow \sum F_y = 0 \Rightarrow v_a = 3,5 kN \uparrow$$

$$\frac{x}{4} = \frac{3,5}{8}$$

$$x = 1,75 m$$

$$M_{máx} = área \triangle = \frac{1,75 \cdot 3,5}{2} = 3,06 kN \cdot m$$

f)

2kN/m $F_R = 4kN$

S_R S_i

Lm

4   0   DEC (kN)

4   0   DMF (kN·m)

0   DEN (kN)

h) $H_d \rightarrow$

5kN   5kN   9kN

b_d 4m   4m   3m   2m $\uparrow b_b$

$H_d = 0$

$\left(\sum_i M_d = 0 \Rightarrow -13\, b_b + 9 \cdot 11 + 3 \cdot 8 + 5 \cdot 4 = 0 \right.$

$$b_b = 11\,kN \uparrow$$

$\uparrow \sum_i F_y = 0 \Rightarrow b_d - 5 - 3 - 9 + 11 = 0$

$$v_d = 6\,kN \uparrow$$

DEC
(kN)

6   6

1   1

-2   -2

-11

$\oplus$
$\ominus$

DMF
(kN·m)

24   22
23

$\ominus$
$\oplus$

DEN
(kN)

0   0
0

$\oplus$
$\ominus$

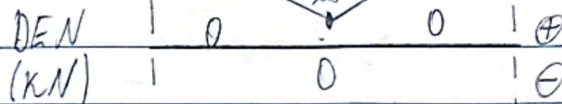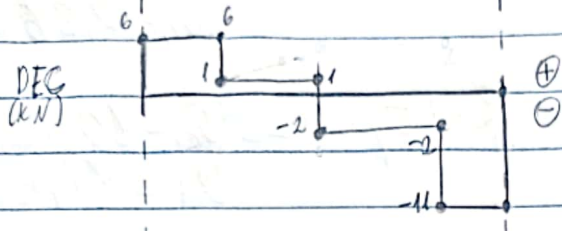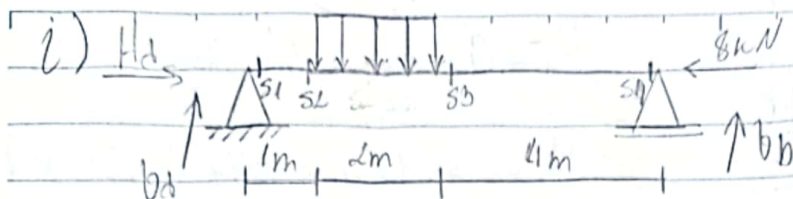$$F_\square = 2 \cdot 5 = 10 KN$$

i) H_d

5kN/m

8kN

$$\Sigma F_x = 0 \Rightarrow H_d - 8 = 0$$
$$H_d = 8 KN \rightarrow$$

S1 S2 S3   S4

v_d 1m   2m   4m v_b

$$\left(\Sigma M_d = 0 \Rightarrow -10b + 2 \cdot 10 = 0\right)$$
$$v_b = 0,86 KN \uparrow$$

DEC
(kN)

7,14

V=0

10m

2m

2,86

⊕
⊖

$$\Sigma F_y = 0 \Rightarrow v_d - 10 + 2,86 = 0$$
$$v_d = 7,14 KN \uparrow$$

$M_{máx}$ 10

1m

$$\frac{x}{2} = \frac{7,14}{10}$$
$$x = 1,43 m$$

DMF
(kN·m)

⊖
⊕

$M_{máx}$

DEN
(kN)

⊕
⊖

8

$$M_{máx} = (7,14 \cdot 1) + \left(\frac{1,43 \cdot 7,14}{2}\right)$$
$$M_{máx} = 12,25 kN \cdot m$$

⊖

$F_\square = 6 kN$

6 kN

2 kN/m

8 kN·m   2 kN

H_d

b_d

S9

b_b

S1 S2 S3 S4 S6 S6 S7 S8

1,5m | 1,5m | 1m | 1m | 1m

$\sum F_x = 0 \Rightarrow H_d = 0$

$\sum M_1 = 0 \Rightarrow -6b_b + 10 + 8 + 9 + 9_c$

$b_b = 6 kN \uparrow$

$\uparrow \sum F_y = 0 \Rightarrow b_d - 6 - 6 - 2 + 6 = 0$

$b_d = 8 kN \uparrow$

DEC
(kN)

8

5

1

4   4

6

$\oplus$
$\ominus$

DMF
(kN·m)

2

6

9;15

10

$\ominus$
$\oplus$

DEN
(kN)

0   0   0   0

$\oplus$
$\ominus$

Exemplo:

$$\sum F_x = 0 \Rightarrow H_a = 0$$

$10 kN \cdot m$

H_a

S_1 S_2 S_3 S_4

V_a $\uparrow V_b$

$2,5m$ $2,5m$

$+$
$-$ DEC (kN)
2 2 2

5

$-$ 0 DMF (kN·m)
$+$ 5

$+$ 0 0 DEN (kN)
$-$

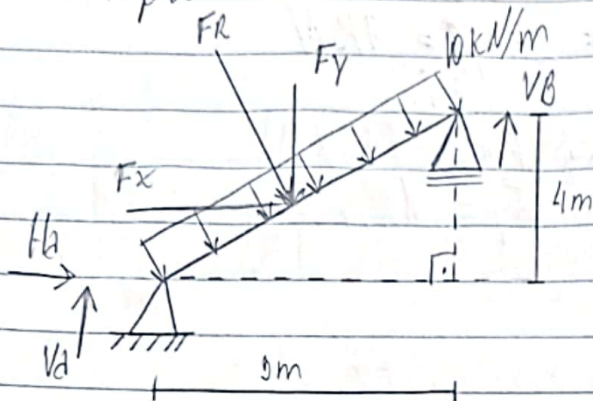$$\sum M_a = 0 \Rightarrow -5V_b + 10 = 0 \Rightarrow V_b = 2kN \uparrow$$

$$\sum F_y = 0 \Rightarrow V_a + 2 = 0 \Rightarrow V_a = -2 \Rightarrow V_a = 2kN \downarrow$$

Exemplo:

$$F_R = 10\frac{kN}{m} \cdot 5\,m = 50\,kN$$

$$\begin{cases} \Sigma\, F_x = 0 \\ \Sigma\, F_y = 0 \\ \Sigma\, M = 0 \end{cases}$$

usando:

$\operatorname{sen}\theta = \dfrac{4}{5} = 0{,}8$ $F_y = F_R \cos\theta = 50 \cdot 0{,}6 = 30\,kN$

$\cos\theta = \dfrac{3}{5} = 0{,}6$ $F_x = F_R \operatorname{sen}\theta = 50 \cdot 0{,}8 = 40\,kN$

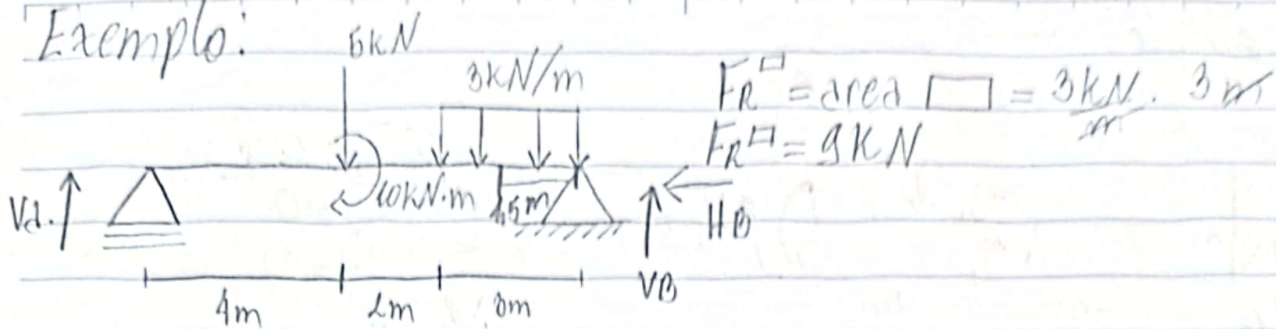$\Sigma\, F_x = 0 \Rightarrow H_a + F_x = 0 \Rightarrow H_a = -40\,kN \Rightarrow H_a = 40\,kN \leftarrow$

$\stackrel{\curvearrowright \oplus}{\left(\Sigma\, M_a = 0 \Rightarrow -V_B \cdot 3 + 50 \cdot 2{,}5 = 0 \Rightarrow V_B = 41{,}67\,kN \uparrow \right.}$

$\uparrow^{\oplus} \Sigma\, F_y = 0 \Rightarrow V_d - 30 + V_B = 0 \Rightarrow V_d = 30 - 41{,}67 \Rightarrow V_d = -11{,}67$

$$\boxed{V_d = 11{,}67\,kN \downarrow}$$

Exemplo:

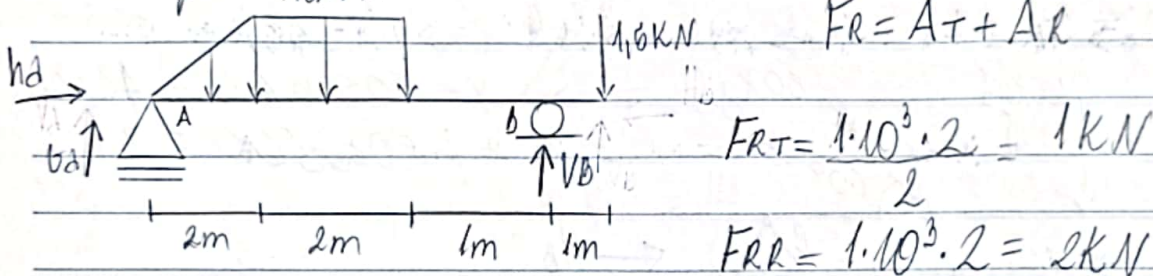$$F_R^{\square} = \text{area } \square = 3kN \cdot 3m$$
$$F_R^{\square} = 9 kN$$

$$\sum F_x = 0 \Rightarrow H_B = 0$$

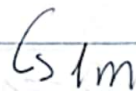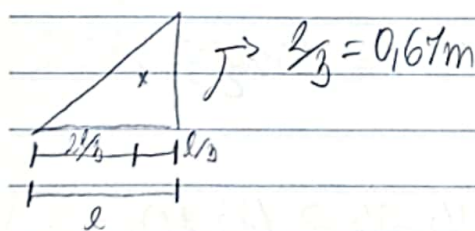$$\sum M_B = 0 \Rightarrow 9 V_A + 10 - 5 \cdot 5 - 9 \cdot 1{,}5 = 0 \Rightarrow V_A = 3{,}17 kN \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow 3{,}17 + V_B - 9 - 5 = 0 \Rightarrow V_B = 10{,}83 kN \uparrow$$

Exemplo: 1kN/m

$$F_R = A_T + A_R$$

$$F_{RT} = \frac{1 \cdot 10^3 \cdot 2}{2} = 1 kN$$

$$F_{RR} = 1 \cdot 10^3 \cdot 2 = 2 kN$$

Centroide Triângulo

$$\frac{2}{3} = 0{,}67 m$$
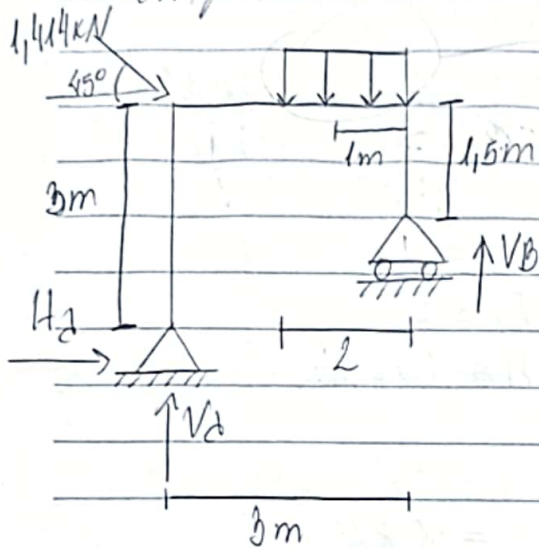
centroide Retângulo

$$\hookrightarrow 1m$$

$$\sum F_x = 0 \Rightarrow h_A = 0$$

$$\sum M_A = 0 \Rightarrow 5 V_B - 1{,}5 \cdot 6 - 2 \cdot 3 - 1 \cdot 1{,}33 = 0 \Rightarrow V_B = 3{,}27 kN \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow V_A + 3{,}27 - 1{,}5 - 2 - 1 = 0 \Rightarrow V_A = 1{,}23 kN \uparrow$$
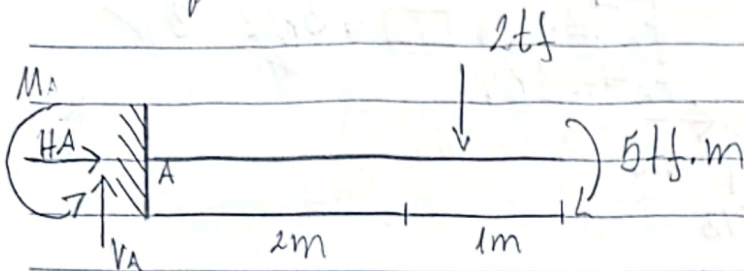
Exemplo:

$F_R = 2k \cdot 2 = 4KN$

$F_x = 1 KN$

$F_y = 1 KN$

$$\sum F_x = 0 \Rightarrow H_A + 1k = 0 \Rightarrow H_A = -1k \Rightarrow H_A = 1k \leftarrow$$

$$\sum M_a = -3 V_b + 4k \cdot 2 + 1 \cdot 3 = 0 \Rightarrow V_B = 3,67 KN \uparrow$$

$$\uparrow \sum F_y = 0 \Rightarrow V_a - 1k + V_b - 4k = 0 \Rightarrow V_a = 1,33 kN \uparrow$$

Exemplo.

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_z = 0$$

$$\sum F_x = 0 \Rightarrow H_A = 0$$

$$\overset{\oplus}{\uparrow} \sum F_y = 0 \Rightarrow V_a - 2 = 0 \Rightarrow V_a = 2tf \uparrow$$

$$\sum M_a = 0 \Rightarrow M_a - 5 - 2 \cdot 2 = 0 \Rightarrow \boxed{M_a = 9\,tf \cdot m} \,\,\circlearrowright$$

Exemplo:

$$y = 50\,sen\,60 = 43,30$$
$$x = 50\,cos\,60 = 25$$
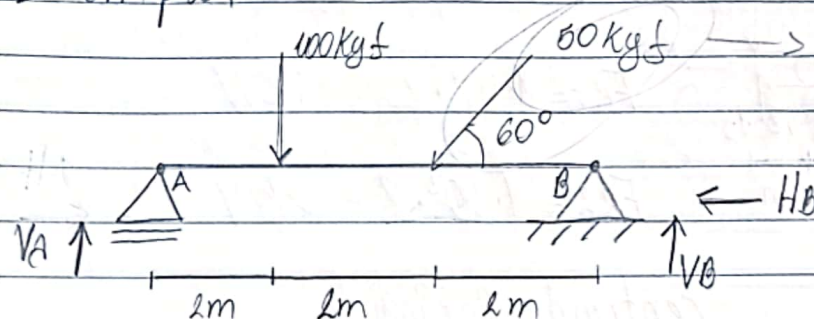
$$\overset{\oplus \leftarrow}{\sum F_x = 0} \Rightarrow H_B + 25 = 0 \Rightarrow H_B = -25 \quad \therefore H_B = 25\,kgf$$

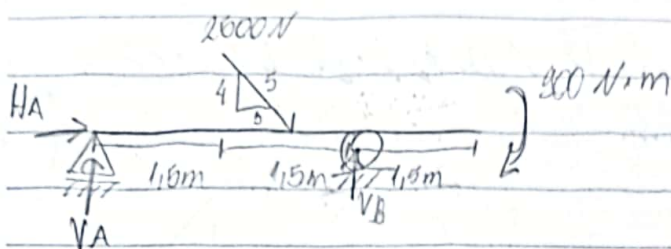$$\overset{\oplus}{\uparrow} \sum F_y = 0 \Rightarrow V_a - 100 + V_B - 43,30 = 0 \Rightarrow V_a + V_B = 143,30$$

$$\sum M_a = 0 \Rightarrow 6 \cdot V_B - 100 \cdot 2 - 43,30 \cdot 4 = 0 \Rightarrow V_B = 62,2\,kgf \uparrow$$

$$V_a = 143,30 - 62,2 = 81,1\,kgf$$

Exemplo:

$$\vec{\sum} \overset{\oplus}{F_x} = 0 \Rightarrow H_A + 2500\cos\theta = 0 \Rightarrow H_A = -1500 = 1500N \leftarrow$$

$$\overset{\oplus}{C} M_a = 0 \Rightarrow 900 - V_B \cdot 3 + 2000 \cdot 1,5 = 0$$
$$V_B = 1300 N \uparrow$$

$$\uparrow \sum \overset{\theta}{F_y} = 0 \Rightarrow V_a + 1300 - 2000 = 0$$
$$V_a = 700 N \uparrow$$

Exemplo:
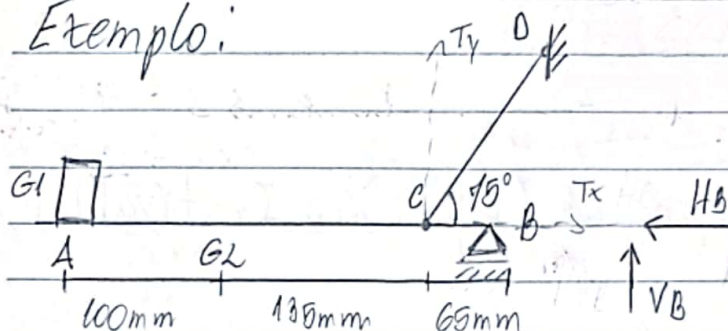
$$\vec{\sum} \overset{\oplus}{F_x} = 0 \Rightarrow -H_B + T_x = 0 \quad H_B = T_x \Rightarrow H_B = T\cos 75$$
$$H_B = 34,62$$

$$\uparrow \sum \overset{\theta}{F_y} = 0 \Rightarrow -G_1 - G_2 + T_y + V_B = 0 \Rightarrow T sen 75 + V_B = 32$$
$$V_B = -97,22 = 97,22N \downarrow$$

$$\overset{\oplus}{(\sum} M = 0 \Rightarrow (T sen 75)65 - 20 \cdot 300 - 12 \cdot 200 = 0$$
$$T = 133,78 N \quad \nearrow 75°$$