GeoChronR – an R framework to model and analyze age-uncertain paleogeoscientific timeseries.

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Abstract. Chronological uncertainty is a hallmark of the paleosciences. While many tools have been made available to researchers to produce age models suitable for various settings and assumptions, disparate tools and output formats often discourage integrative approaches. In addition, associated tasks like propagating age model uncertainties to subsequent analyses, and visualizing the results, have received comparatively little attention in the literature and available software. Here we describe GeoChronR, an open-source R package to facilitate these tasks. GeoChronR is built around emerging data standards for the paleosciences (Linked Paleo Data, or LiPD), and offers access to four popular age modeling techniques (Bacon, BChron, Oxcal, BAM). The output of these models is easily stored in LiPD, enabling paleo-aware synchronization, regression, correlation, principal component, and spectral analyses. Five real-world use cases illustrate how to use GeoChronR to facilitate these tasks, and to visualize the results in intuitive ways.

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1 Introduction

1.1 Background

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Quantifying chronological uncertainties, and how they influence the understanding of past changes, is a unique and fundamental challenge of the paleogeosciences. Without robust error determination, it is impossible to properly assess the extent to which past changes occurred simultaneously across regions or the duration of abrupt events, both of which limit our capacity to apply paleoscientific understanding to modern and future processes. The need for better infrastructure to both characterize uncertainty, and to explicitly evaluate how age uncertainty impacts the interpretation of records of past climate, ecology or landscapes, has been long recognized (Noren et al., 2013, (more)). In response to this need, the paleogeoscience community has made substantial advances toward improving geochronological accuracy by:

1. Improving analytical techniques that allow for more precise age determination on smaller and context-specific samples (?, ?, ?, ?, ?)

- 2. Refining our understanding of how past changes in the Earth system impact the age accuracy, for example: improvements to the radiocarbon calibration curve (?, ?,intcal references) and advances in our understanding of spatial variability in cosmogenic production rates used in exposure dating (?, ?)
- 3. Dramatic improvement in the level of sophistication and realism in age-depth models used to estimate the ages of sequences between dated samples (e.g. ?, Parnell et al. (2008), ?, ?).

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Over the past 20 years, these advances have been widely adopted in the paleogeosciences. However, despite the progress made in quantifying uncertainty in ages and in age models, few studies have formally evaluated how chronological uncertainty may have affected their results. For instance, whereas the algorithms presented by (?, Parnell et al. (2008), ?, ?) have been broadly used, the overwhelming majority of these studies calculate the single best-estimate model (often a median or mean), use this model to put measured paleoclimatic or paleoenvironmental data on a timescale, and then proceed to analyze the record with little to no reference to the age modeling exercise (?, ?). Typically, any discussions of chronological uncertainties remain qualitative.

This paradigm is beginning to change. In recent years a handful of studies have taken advantage of approaches that generate ensembles of age models to evaluate how the results of their analyses and conclusions vary given differences between ensemble members (?, ?,?,?,Marcott et al. (2013), ?). By using each ensemble age model to create a time-uncertain ensemble records, and then carrying that ensemble through the analysis, the precise impact of age uncertainty can be formally evaluated. This approach, of course, does not address all aspects of uncertainty, but it does offer the broad potential to ascertain which results are robust to chronological uncertainty, and which are not.

Despite its potential to substantially improve uncertainty quantification for the paleogeosciences, this framework is not widely utilized. The majority of studies utilizing this approach have been regional [e.g., ?,?,?, ?} or global-scale (e.g., ?, Marcott et al. (2013),?) syntheses. Occasionally, primary publications of new records incorporate time-uncertain analysis into their studies (?, more), but this remains rare. We suggest that there are several reasons for the lack of adoption of these techniques:

- 1. For synthesis studies, the necessary geochronological data are not publicly available for the vast majority of records. Even when they are available, the data are archived in diverse and unstructured data formats. Together, this makes what should be a simple process of aggregating and preparing data for analysis prohibitively time-consuming;
- 2. For studies of new and individual records, few tools for ensemble analysis are available, and those that are require a degree of comfort with coding languages and scientific programming that is rare among paleogeoscientists;
- 3. There is a disconnect between age-model development and time-uncertain analysis. Published approaches have utilized either simplified age-modeling approaches (?), or specialized approaches not used elsewhere in the community (?, ?,Marcott et al. (2013),?,?).

Extracting the relevant data from commonly-used age-modelling algorithms, creating time-uncertain ensembles, then reformatting those data for analysis in available tools typically requires the development of extensive custom codes. An integrative approach is needed to facilitate this work.

1.2 Motivation

GeoChronR was built to lower the barriers to broader adoption of these emerging methods. It provides an easily-accessibly, open-source and extensible software package of industry-standard and cutting-edge tools that provides users with a single environment to create, analyse, and visualize time-uncertain data. GeoChronR is designed around emerging standards in the paleogeosciences that connects users to growing libraries of standardized datasets formatted in the Linked PaleoData format (?) and (?).

10 1.3 Outline of manuscript

This manuscript describes the design, analytical underpinnings and most common use cases of GeoChronR. Section 2 describes the integration of age modelling algorithms with GeoChronR. Section 3 details the methods implemented for age uncertain analysis. Section 4 goes through the principles and implementation of age-uncertain data visualization in GeoChronR, and section 5 provides five real-world examples of how GeoChronR can be used for paleoscientific workflows.

15 2 Age Uncertainty Quantification in GeoChronR

GeoChronR does not introduce any new approaches to age uncertainty quantification; rather, it integrates existing, widely-used packages while streamlining the acquisition of age ensemble members. Fundamentally, there are two types of age models used in the paleogeosciences: tie-point and layer-counted. Most of the effort in age uncertainty quantification in the community has been focused on tie-point modelling, where the goal is to estimate ages (and their uncertainties) along a depth profile given chronological estimates (and their uncertainties) at multiple depths downcore. Over the past 20 years, these algorithms have progressed from linear or polynomial regressions with simple characterizations of uncertainty (?, ?) to more rigorous techniques, particularly Bayesian approaches: as of writing, the three most widely used algorithms are Bacon (Blaauw and Christen, 2011), BChron (Parnell et al., 2008), and OxCal (Ramsey, 2008), which are all Bayesian age-deposition models that estimate posterior distributions on age-depth relationships using different assumptions and methodologies. Trachsel and Telford (2017) reviewed the performance of these three algorithms, as well as a non-Bayesian approach (?), and found that the three Bayesian approaches generally outperform previous algorithms, especially when appropriate parameters are chosen (although choosing appropriate parameters can be challenging). Bacon, BChron and Oxcal all leverage Monte Carlo Markov Chain (MCMC) techniques to sample the posterior distributions, thereby quantifying age uncertainties as a function of depth in the section. GeoChronR interfaces with each of these algorithms through their R packages (?, Martin et al. (2018)) (cite other R Packages), streamlining input and output. %data input and the extraction of the age ensembles from the MCMC results for further analysis.

In addition to working with ensembles from tie-point age models, GeoChronR connects users to probabilistic models of layer-counted chronologies. BAM (Comboul et al., 2014) was designed to probabilistically simulate counting uncertainty in banded archives, such as corals, ice cores, or varved sediments, but can be used to crudely simulate age uncertainty for any record, and is useful when the data or metadata required to calculate an age-depth model are unavailable. Here we briefly describe the theoretical basis and applications of each of the four approaches integrated in GeoChronR.

2.1 Bacon

The Bayesian ACcumulatiON (Bacon) algorithm (Blaauw and Christen, 2011) is one of the most broadly used age-modelling techniques, and was designed to take advantage of prior knowledge about the distribution and autocorrelation structure of sedimentation rates in a sequence to better quantify uncertainty between dated levels. The algorithm employs an adaptive Markov Chain Monte Carlo algorithm that allows for Bayesian learning to update the sedimentation rate distribution. Bacon has two key parameters: the shape of the accumulation prior, and the segment length, which can interact in complicated ways (Trachsel and Telford, 2017). In our experience, the segment length parameter has the greatest impact on the ultimate shape and amount of uncertainty simulated by Bacon, as larger segments result in increased flexibility of the age-depth curve, and increased uncertainty between dated levels. Bacon is written in C++ and R, with an R interface. More recently, the authors released an R package rbacon (?), which GeoChronR leverages to provide access to the algorithm. Bacon will optionally return a subset of the MCMC accumulation rate ensemble members with high *a posteriori* probabilities, which GeoChronR uses to form age ensemble members for subsequent analysis.

maybe comment on the depositional model?

2.2 BChron

BChron (Haslett and Parnell, 2008, Parnell et al. (2008)) uses a similar approach, using a continuous Markov monotone stochastic process coupled to a piecewise linear deposition model. This simplicity allows semi-analytical solutions that make BChron computationally efficient. BChron was originally intended to model radiocarbon-based age-depth models in lake sedimentary cores of primarily Holocene age, but its design allows broader applications. In particular, modeling accumulation as additive independent gamma increments is appealing for the representation of hiatuses, particulary for speleothem records, where accumulation rate can vary quite abruptly between quiescient intervals of near-constant accumulation (Parnell et al., 2011, Dee et al. (2015),Hu et al. (2017)). The downside of this assumption is that BChron is known to exaggerate age uncertainties in cases where sedimentation varies smoothly (Trachsel and Telford, 2017).

Adjustable parameters? {Deborah}

2.3 Oxcal

The OxCal software package has a long history and extensive tools for the statistical treatment of radiocarbon and other geochronological data (Bronk Ramsey, 1995). In Ramsey (2008), age-depth modelling was introduced with three options for

modelling depositional processes that are typically useful for sedimentary sequences: uniform, varve, and Poisson deposition models, labeled U-sequence, V-sequence and P-sequence, respectively. The Poisson-based model is the most broadly applicable for sedimentary, or other accumulation-based archives (e.g. speleothems), and although any sequence type can be used in GeoChronR, most users will use a P-sequence, which is the default. Analagously to segment length parameter in Bacon, the k parameter (called "eventsPerUnitLength" in GeoChronR), controls how many events are simulated per unit of depth, and has a strong impact on the flexibility of the model, as well as the amplitude of the resulting uncertainty. As the number of events increases, the flexibility of the model, and the uncertainties decrease. Trachsel and Telford (2017) found that this parameter has large impact on the accuracy of the model, and moreso than the choices made in Bacon or Behron. Fortunately, ? made it possible for k to be treated as a variable, and the model will estimate the most likely values of k given a prior estimate and the data, however this calculation can greatly increase the convergence time of the model. Oxcal is written in C++, with an interface in R (Martin et al., 2018). Oxcal doesn't calculate posterior ensembles for a depth sequence, but can optionally output MCMC posteriors at specified levels in the sequence. GeoChronR uses this feature to extract ensemble members for subsequent analysis.

2.4 Banded Age Model (BAM)

Comboul et al. (2014) is a probabilistic model of age errors in layer-counted chronologies. The model allows a flexible parametric representation of such errors (either as Poisson or Bernoulli processes), and separately considers the possibility of double-counting or missing a band. The model is parameterized in terms of the error rates associated with each event, which are intuitive parameters to paleogeoscientists, and may be estimated via replication (DeLong et al., 2013). In cases where such rates can be estimated from the data alone, an optimization principle may be used to identify a more likely age model when a high-frequency common signal can be used as a clock (Comboul et al., 2014). As of now, BAM does not consider uncertainties about such parameters, representing a weakness of the method. Bayesian generalizations have been proposed (Boers et al., 2017), which could one day be incorporated into GeoChronR if the code is made public. BAM was coded in MATLAB and R, and it is this latter version that GeoChronR uses.

3 Age-uncertain data analysis in GeoChronR

Some theoretical work has attempted to quantify how chronological uncertainty may affect various paleoscientific inferences (e.g. Huybers and Wunsch, 2004); however, such an approach is necessarily piecemeal, and therefore impractical given the variety of age-uncertainty structures in real-world paleogeoscientific data. Consequently, GeoChronR follows a pragmatic and broadly-used approach that leverages age ensembles, and optionally ensembles in climate proxy or paleoenvironmental data, to propagate uncertainties through all steps of an analysis. Effectively, this is done by randomly sampling from the ensemble(s) and then repeating the analysis on each sample (typically hundreds to thousands) to build an output ensemble that quantifies the impact of those uncertainties on a particular inference. These output ensembles often do not lend themselves to binary significance statistics (e.g., a p-value below), but are readily used to provide quantiles that estimate probability density ranges,

and can provide strong evidence for which results are robust to age and proxy uncertainty (and which are not). Version 1.0.0 of GeoChronR has implemented ensemble analytical techniques for four of the most common analyses in the paleogeosciences: correlation, regression, spectral and principal component analyses.

3.1 Correlation

5 Pearson's product-moment correlation is the most common measure of association between variables.

Its computation is fast, lending itself to ensemble analysis, with a handful of pretreatment and significance considerations that are relevant for ensembles of paleogeoscientific data.

First, correlation analysis for timeseries is built on the assumption the datasets can be aligned on a common timeline. Age-uncertain data violate this assumption. We overcome this by treating each ensemble member from one or more age uncertain timeseries as valid for that iteration, then "bin" each of the timeseries into coeval intervals. The "binning" procedure in GeoChronR sets up an interval, which is typical evenly spaced, over which the data are averaged. Generally, this intentionally degrades the median resolution of the timeseries, for example, a timeseries with 37-year median spacing could be reasonably "binned" into 100- or 200-year bins. The binning procedure is repeated for each ensemble member, meaning that between different ensembles, different observations will be placed in different bins.

Following binning, Pearson correlation is calculated and recorded for each ensemble member. In addition to the correlation coefficients, GeoChronR calculates their significance for each correlation. Paleogeoscientific timeseries are often highly autocorrelated, which can lead to spurious assessments of significance (Hu et al., 2017). Therefore, in addition to the standard Student's T-test p-value calculation, we also calculate a p-value that is adjusted for autocorrelation to reflect the reduction in degrees of freedom due to autocorrelation following? However, repeating this test over multiple ensembles raises the issue of test multiplicity (Ventura et al., 2004), or "look elsewhere effect". To overcome this problem, we control for this false discovery rate using the simple approach of Benjamini and Hochberg (1995), coded in R by Ventura et al. (2004). NM: I think here we need to discuss when FDR is needed, and when it's not. In my view, if you're surveying the full breadth on an ensemble, you're not searching for p-value, just trying to quantify the range. However, if the users want to subset some best-case or worst-case ensembles, and then proceed with analysis, FDR seems appropriate.

25 3.2 Regression

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Linear regression is a commonly used tool to model the relationships between paleogeoscientific data and instrumental or other datasets. One application is calibration-in-time [references], whereby a proxy timeseries is calibrated to an instrumental series with a linear regression model over their period of overlap. This approach is particularly vulnerable to age uncertainties, as both the development of the relationship, and the reconstruction, are affected. GeoChronR propagates age (and optionally proxy) uncertainties through both the fitting of the ordinary least squares regression model, and the reconstruction "forecast" using the ensemble model results and age uncertainty. Like the correlation algorithm, ensemble regression uses an ensemble binning procedure that's analgous to correlation. GeoChronR then exports uncertainty structure of the modeled parameters (e.g. slope and intercept), as well as the ensemble of reconstructed calibrated data through time.

3.3 Principal Component Analysis

GeoChronR implements the age-uncertain principal component analysis (PCA) procedure introduced by Anchukaitis and Tierney (2013), with some minor modifications and additions, Like correlation and regression, PCA (or empirical orthogonal function {EOF} analysis) requires temporally aligned observations, and GeoChronR uses a binning procedure to achieve this across multiple ensembles. This differs from the implementation of Anchukaitis and Tierney (2013), who interpolated the data to a common timestep. In addition, traditional singular value decomposition approaches to PCA require a complete set of observations without any missing values. For paleoclimate data, especially when considering age uncertainty, this requirement is often prohibitive. To overcome this, GeoChronR implements multiple options for PCA analysis using the pcaMethods package. The default and most rigorously tested option is a probabilistic PCA (PPCA) approach that uses expectation maximization algorithms to infill missing values (Roweis, 1998). This algorithm assumes that the data and their uncertainties are normally distributed, which is often (but not always) a reasonable assumption for paleogeoscientific data. As in correlation and regression, GeoChronR propagates uncertainties through the analysis by repeating the analysis across randomly sampled age and/or proxy ensemble members to build output ensembles of the loadings (eigenvectors), variance explained (eigenvalues) and principal component timeseries. Because the sign of the loadings in PCA analyses is arbitrary and vulnerable to small changes in the input data, GeoChronR reorients the sign of the loadings for all PCs so that the mean of the loadings is positive. For well defined modes this effectively orients ensemble PCs, but loading orientation may be uncertain for lower order, or more uncertain, modes.

3.4 Spectral Analysis

Many research questions in the paleosciences revolve around spectral analysis: describing phase leads and lags among different climate system components over the Pleistocene (SPECMAP, IMBRIE et al., 1984), the hunt for astronomical cycles over the Holocene (?, ?) or in deep time (Meyers and Sageman, 2007, Meyers (2012), Meyers (2015)), or characterizing the continuum of climate variability (Huybers and Curry, 2006, Zhu et al. (in revision)). Yet, spectral analysis in the paleosciences faces unique challenges: chronological uncertainties, of course, as well as uneven sampling, which violates the assumptions of classical spectral methods (Ghil et al., 2002).

To facilitate the quantification of chronological uncertainties in such assessments, GeoChronR implements four spectral approaches: 1. the Lomb-Scargle periodogram (VanderPlas, 2018), which uses an inverse approach to harmonic analysis in unevenly-spaced timeseries.

2. REDFIT, a version of the Lomb-Scargle periodogram tailored to paleoclimatic data (Schulz and Mudelsee, 2002, Mudelsee (2002), Mudelsee et al. (2009)). The GeoChronR uses the implementation of REDFIT in the dplR package (Bunn, 2008). 3. the wavelet-based method of Mathias et al. (2004), called nuspectral. This method is quite similar to the Weighted Wavelet Z-transform algorithm of Foster (1996), though it is prohibitively slow in this implementation, and the fast version using a compact-support approximations of the mother wavelet did not perform well in our tests. 4. The multi-taper method (MTM) of Thomson (1982), a mainstay of spectral analysis (Ghil et al., 2002) designed for evenly spaced timseries. GeoChronR uses the

implementation of MTM in Meyers (2014), which couples MTM to efficient linear interpolation, together with various utilities to define autoregressive and power-law benchmarks for spectral peaks.

4 Visualization with GeoChronR

One of the challenges with age-uncertain analysis is that it adds at least one additional dimension to the results, which can be challenging to visualize. GeoChronR aims to facilitate simple creation of intuitive, publication-quality figures that provide multiple options for visualizing the impacts of age-uncertainty, while maintaining flexibility for users to customize their results as needed. To meet the multiple constraints of simplicity, quality and customization, GeoChronR relies heavily on the ggplot2 package (Wickham, 2016). High-level plotting functions in GeoChronR (e.g., plotTimeseriesEnsRibbons and plotPca) produce complete figures as ggplot2 objects, that can be readily customized by adding or changing ggplot2 layers. The figures in the Use Cases section (Section 5) are all produced directly from GeoChronR without modification and generally fall into a few categories.

{Light Mode vs Dark Mode}

4.1 Timeseries

The most common figure that users produce with GeoChronR are ensemble timeseries. GeoChronR uses two complementary approaches to visualize these ensembles. The first is the simplest, where a large subset of the ensemble members are plotted as semi-transparent lines. This approach, implemented in plotTimeseriesEnsLines, provides a faithful representation of the data, while the overlapping semi transparency provides a qualitative sense of the ensemble uncertainty structure. The second approach uses contours to more rigorlously visualize the structure of the time-value uncertainty space represented by the ensembles. plotTimeseriesEnsRibbons shows the quantiles of the ensembles at specified levels as shaded bands.

This approach provides the quantitative uncertainty structure, but tends to smooth out the apparent temporal evolution of the data. Fortunately, the two approaches are complementary, and often the best approach is to quantify the ensemble distribution with ribbons in the background, and then overlap them with a handful of ensemble lines to illustrate the structure in representive ensemble members.

4.2 Maps

GeoChronR has simple mapping capabilities built in that rely on the maps (code by Richard A. Becker et al., 2018) and ggmap (Kahle and Wickham, 2013) packages. The mapLipd and mapTs functions provide quick geospatial visualization of one or more datasets, but also serve as the basis for the visualization of ensemble spatial data produced by ensemble PCA analyses. In paleogeoscientific studies, the loadings (eigenvectors) of a PCA analysis are often portrayed as dots on a map, with a colorscale that highlights the sign and amplitude of the loadings. In ensemble PCA, the additional dimension of uncertainty in the loadings needs to be visualized as well. In GeoChronR, the median of the loadings is shown as a color, and the size of the symbol is inversely proportional to the spread of uncertainty across the ensemble. Consequently, large symbols depict

loadings that are robust to the uncertainties, whereas small symbols show datasets whose loadings change substantially across the analysis. An example is shown in Section ??

4.3 Spectra

It is customary to plot spectra on a log-log scale, which helps separate close-by frequencies. This choice also naturally highlights scaling laws (Lovejoy and Schertzer, 2013, Zhu et al. (in revision)) as linear structures in this reference frame. GeoChronR thus implements this convention. In addition, the abscissa $(\log_{10} f)$ is labeled according to the corresponding period, which are more intuitive than frequency to scientists reading the plot. To help identify significant periodicities, confidence limits can be superimposed, based on user-specified benchmarks (see 5). The plotSpectrum() function visualizes single ensemble members (e.g. a median age model), while plotSpectraEns() visualizes the quantiles of a distribution of ageuncertain spectra as ribbons, using the eponymous ggplot function. periodAnnotate() allows to manually highlight periods of interest, layered an existing plot.

5 Use cases

We now illustrate the use of these tools on five use cases. The first example shows how a user might create age ensembles on different archives, and how to visualize the timing of abrupt events with appropriate uncertainty quantification. The second example walks through ensemble correlation of age-uncertain records. The third introduces the topic of age-uncertain calibration-in-time. The fourth provides an example of regional age-uncertain principal components analyses, and the fifth deals with spectral analysis.

5.1 Creating an age ensemble

A common first task when using geoChronR is to create an age ensemble, either because the user is developing a new record, or because the age ensemble data for the record they are interested is unavailable. As described in section 2, workflows for four published age quantification programs are integrated into geoChronR. All four methods can be used simply in geoChronR with a LiPD file loaded into R that contains the chronological measurements, and the high-level functions runBacon(), runBchron(), runOxcal() and runBam(). These functions take LiPD objects as inputs, and return updated LiPD objects that include age-ensemble data generated by the respective software packages, with these data stored in appropriate tables. Typically, additional parameters are needed for to optimally run the algorithms. When these parameters are not specified, geoChronR will run in interactive mode, asking the user which variables and parameters they would like to model. These parameter choices are printed to the screen during while the program runs, or are available later with the function getLastVarString(). By specifying these parameters, age model creation can be scripted and run in non-interactive mode. In this use case, we'll use geoChronR and BChron (Parnell et al., 2008) to calculate an age ensemble for the Hulu Cave δ^{18} O speleothem record (?), and BAM (Comboul et al., 2014) to simulate age uncertainties for the GISP2 ice core δ^{18} O dataset

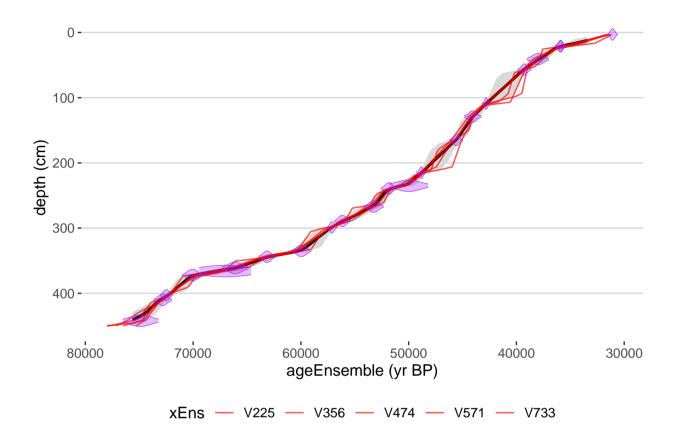


Figure 1. Need to write a caption

(?). The plotChronEns () function will plot an age-depth model and uncertainties derived from the age ensemble (figure 1).

After an age ensemble has been added to a LiPD object, the user can visualize the ensemble timeseries using plotTimeseriesEnsRi and plotTimeseriesEnsLines (). GISP2 δ^{18} O is plotted with age uncertainty, using both functions, in figure 2.

5 5.2 Correlation

Now that the user has generated age ensembles for the two datasets, they're interested to see if a correlation between the two datasets is robust to the age uncertainty modeled here. On multi-millennial timescales, the two datasets dislay similar features, and previous work has suggested that abrupt events of the Last Glacial period, which are observed in the GISP2 record, can impact the Asian Monsoon and be observed is speleothem records such as the Hulu Cave dataset. (NM: add references) The corens() function in geoChronR will calculate ensemble correlations across age-uncertain datasets, such as these. corens() will also sample across ensembles in the proxy data as well, if present. Here we calculate correlations

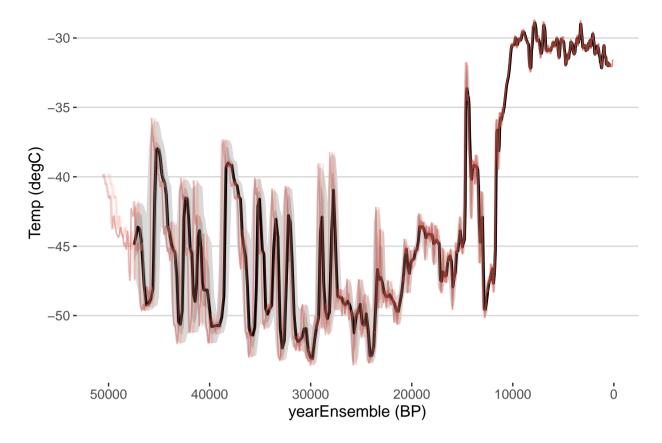
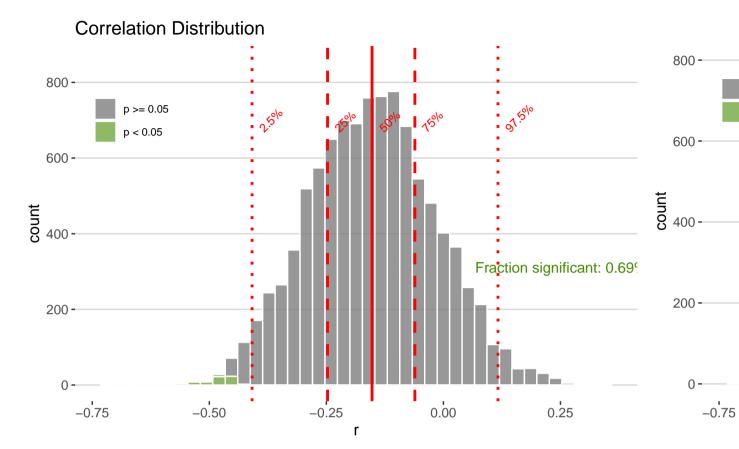


Figure 2. Need to write a caption

during the period of overlap in 500 yr steps, determining significance for each pair of ensemble members while accounting for autocorrelation.



The results are visualized as a histogram of the ensemble correlation results, with color shading to highlight significance (figure ??). The two timeseries are consistently negative correlations, although 13.7% of the ensemble members are positive. However, only 0.7% are significant after accounting for serial autocorrelation. Given the age models and correlation parameters used here, it seems pretty unlikely that these two datasets are significantly correlated. However, it's worth noting that evaluating these results remains subjective, as there is no theoretical expectation for what fraction of ensemble correlation results should be expected to pass a significance test in age-uncertain correlation. Two timeseries, which are actually correlated, each with it's own age-uncertainty, will commonly return insignificant results when random ensemble members are correlated against each other. Furthermore, the frequency of these "false negatives" depend on the structure of the age-uncertainties and the timeseries, and will vary to some extent by random chance. Therefore, it's appropriate to think of the ensemble correlation results produced by GeoChronR as a first-order estimate of the age-uncertain correlation characteristics between timeseries, rather than a binary answer to the question "Are these two datasets correlated?". For research applications that require rigourous determination of more nuanced uncertainties we recommend a surrogate timeseries null hypothesis approach (e.g. ?, others?).

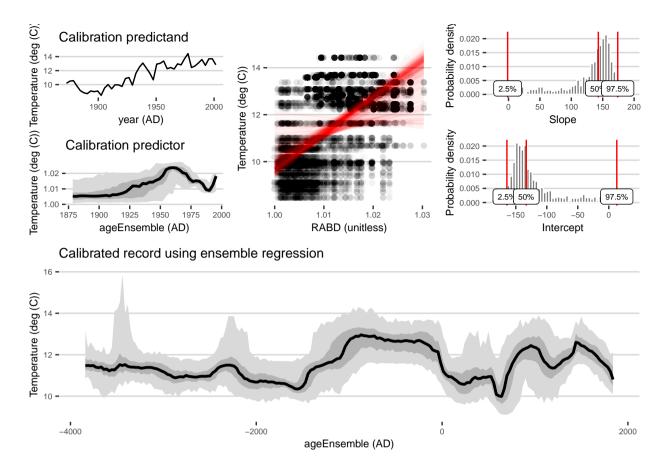


Figure 3. Need to write a caption

5.3 Age-uncertain Calibration

A natural extension of ensemble correlation is ensemble regression. Although there are use cases where regressing one age-uncertain variable onto another is called for, here we regress an age-uncertain paleoclimate proxy onto time-certain instrumental to develop a calibration-in-time. For this use case, we reproduce the results of Boldt, Brandon R. et al. (2015), where the authors calibrated a spectral reflectance measure of chlorophyll abundance, relative absorption band depth (RABD), to instrumental temperature in Northern Alaska. For each iteration in the analysis, a random age ensemble member is chosen and used to bin the RABD data onto a 3-year timescale. The instrumental temperature data, here taken from the nearest gridcell of the GISTEMP reanalysis product (?), are also binned onto the same timescale, insuring temporal alignment between the two timeseries. GeoChronR then derives an ordinary linear regression model, and then uses that model to "predict" temperature values from 3-year-binned RABD data back in time. This approach propagates the age uncertainties both through the regression and prediction process. plotRegressEns produces a number of individual plots that show the results of an age-uncertain regression, and additionally creates an overview "dashboard" that showcases the key results (figure 3).

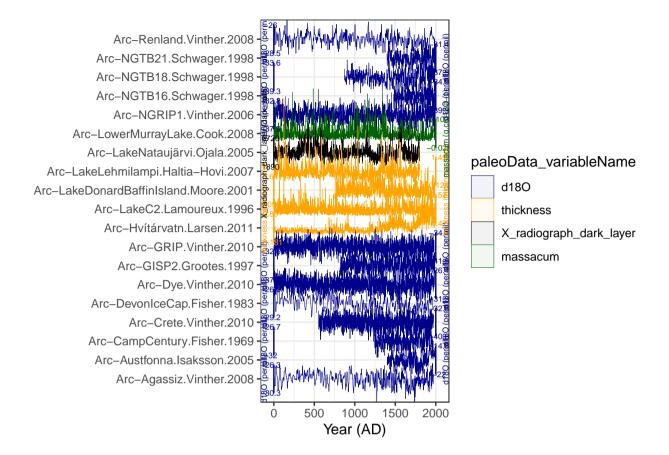


Figure 4. Need to write a caption

5.4 Principle Component Analysis (sec:pca)

Thus far, the use cases have highlighted age-uncertain analyses of one or two sites, however quantifying the effects of age uncertainty can be even more impactful over large spatial datasets. Here we showcase how to use geoChronR to perform age-uncertain principle components analysis (PCA), also known as Monte Carlo Empirical Orthogonal Function (MCEOF) analysis, pioneered by Anchukaitis and Tierney (2013). In this example, we examine the Arctic 2k database (McKay and Kaufman, 2014).

5.5 Spectral Analysis

To illustrate the use of spectral analysis functionalities, consider the following use case: a paleoceanographer wants to identify the relative energy of oscillations at orbital (Milankovitch) periodicities in a deep-sea sediment core, and quantify the impact of age uncertainties on this assessment. To do so, they select the benthic δ^{18} O record of the International Ocean Drilling Project core 846 (Mix et al., 1995, Shackleton (1995)), covering the past 4.7 million years. For this assessment, we use a new age

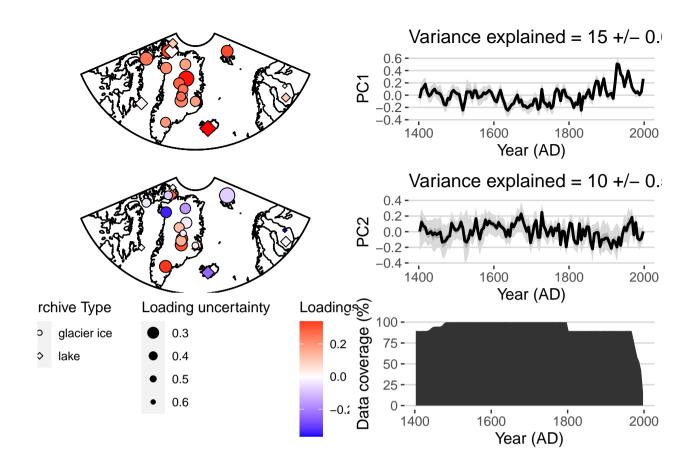
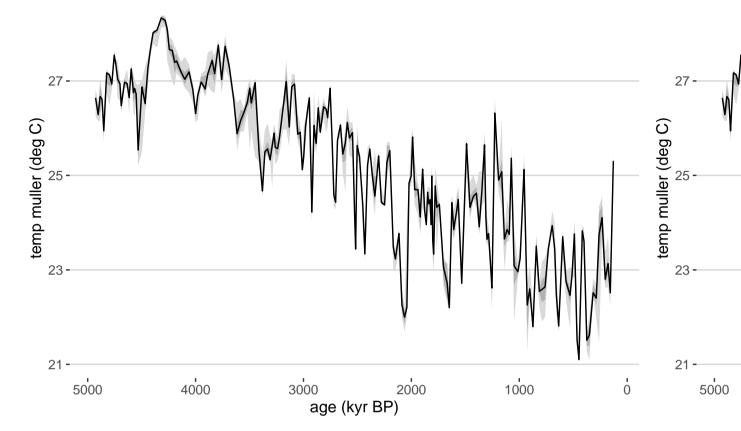


Figure 5. Need to write a caption

model obtained by alignment to the benthic $\delta^{18}O$ stack of Lisiecki and Raymo (2005) using the HMM-Match algorithm (Lin et al., 2014, Khider et al. (2017)). HMM-Match is a probabilistic method that generates an ensemble of 1000 possible age models compatible with the chronostratigraphic constraints; this ensemble was archived as a table in the associated LiPD file.

Now let us take a first look at the median age model:



This record displays three salient features:

- a long-term cooling trend characteristic of the late Neogene and Quaternary climate.
- quasi-periodic oscillations (the legendary Pleistocene Ice Ages)
- nonstationary behavior, related to the well-known mid-Pleistocene transition from a "41k world" to a "100k world" somewhere around 0.8 Ma (Paillard, 2001).

For tractability, let us focus on the last million years which cover the Quaternary Era. As mentioned in @ref(sec:spec_theory) a standard assumption of spectral analysis is that data are evenly spaced in time. In real-world paleo timeseries this is seldom the case. Indeed, over the past 1 Ma, the time increments (Δt) are sharply peaked around 2.5 ka, but they range from 0 to about 7.5 ka. From this point there are two ways to proceed: 1) use methods that explicitly deal with unevenly-spaced data, or 2) interpolate to a regular grid and apply standard methods (see @ref(sec:spec_theory)). Though GeoChronR supports four different methods, we will highlight only two here: REDFIT and MTM.

It is clear that the data contain significant energy (peaks) near, but not exactly at, the Milankovitch periodicities (100, 41, 23, and 19 kyr).

IODP 846 age ensemble, REDFIT

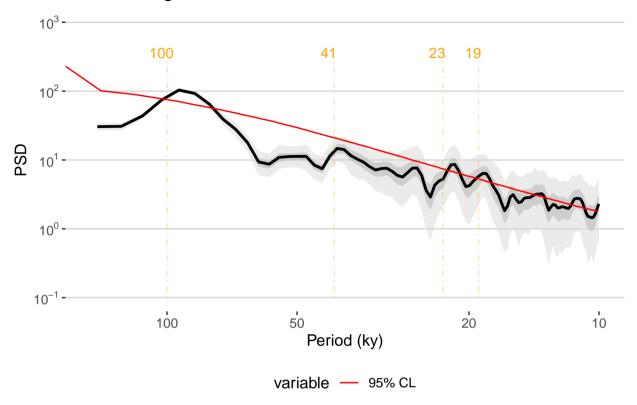


Figure 6. (#fig:REDFIT ensemble export)REDFIT spectrum for IODP 846, 1000-member age ensemble

These periodicities (particularly the eccentricity (100kyr) and precessional (23 and 19 kyr)) rise above the null (the 95% quantile from an autoregressive process of order one, see Mudelsee et al. (2009)). The obliquity periodicity is relatively weak, reaching just below the AR(1) benchmark.

The Lomb-Scargle periodogram used by REDFIT is a decent way to deal with unevenly-spaced timeseries, but it is still a periodogram, which is known to be inconsistent: the variance of each estimate goes to infinity as the number of observations increases. This is mitigated somewhat with the application of Welch's Overlapping Segment Averaging, but their parameter choice is fairly arbitrary. On the other hand, MTM (Thomson, 1982) is an optimal estimator, which is consistent (the more data, the better constrained the amplitude of the peaks). Formally, MTM optimizes the classic bias-variance tradeoff inherent to all statistical inference. It does so by minimizing speactral leakage outside of a frequency band with half-bandwidth equal to pf_R , where $f_R = 1/(N\Delta t)$ is the Rayleigh frequency, Δt is the sampling interval, N the number of measurements, and p is the so-called time-bandwidth product (Ghil et al., 2002). p can only take a finite number of values, all multiples of 1/2 between 2 and 4. A larger p means lower variance (i.e. less uncertainty about the power), but broader peaks (i.e. a lower spectral resolution), synonymous with more uncerainty about the exact location of the harmonic. So while MTM might not distinguish between

IODP 846 age ensemble, MTM

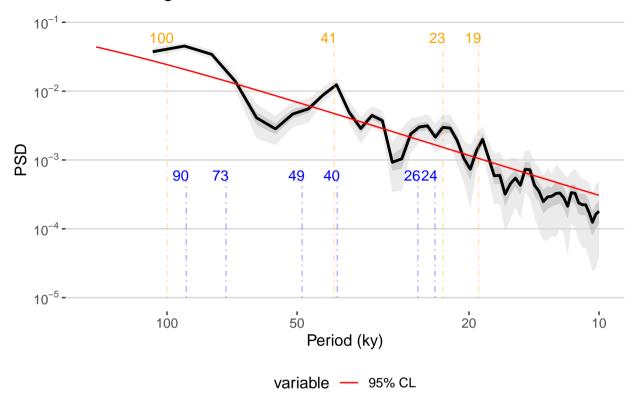


Figure 7. (#fig:MTM ensemble export)MTM spectrum for IODP 846, 1000-member age ensemble

closely-spaced harmonics, it is much less likely to identify spurious peaks, especially at high frequencies. In addition, a battery of formal tests have been devised with MTM, allowing under reasonably broad assumptions to ascertain the significance of spectral peaks. We show how to use this "harmonic F-test" below.

The classic version of MTM can only handle evenly-spaced data. As we saw above, the data are nearly evenly-spaced, so is is reasonable to interpolate them using standard methods. Conveniently, both the interpolation routine and MTM are available within the (astrochron Meyers, 2014) package, which is a dependency of GeoChronR.

To this we can add the periods identified as significant by MTM's F ratio test. GeoChronR estimates this by computing the fraction of ensemble members that exhbit a significant peak at each frequency. One simple criterion for gauging the level of support for such peaks given age uncertainties is to pick out those periodicities that are identified as significant more than 50% of the time. For consistency with REDFIT, we define the null as an AR(1) process fit to the data, but GeoChronR suports two other nulls: a power-law null and one derived with the method of Mann and Lees (1996). Both follow the astrochron implementation.

In orange are the Milankovitch frequencies and we label in blue the periodicities identified as significant by the test. You may notice a few differences to the REDFIT estimate. First, this ensemble of spectra exhibits a clear power law behavior from periods of 5 to 100 ky, which in this log-log plotting convention manifests as a linear decrease. This is part of the well-documented continuum of climate variability (Huybers and Curry, 2006, Zhu et al. (in revision)), which is conspicuously absent from the Lomb-Scargle (REDFIT) estimate, known to be extremely biased in its estimate of the spectral background.

Secondly, the MTM version with this time-bandwidth product is sharper than REDFIT, with more well-defined peaks, particularly for the obliquity period (41 ky), which pokes well above the 95% confidence limit. Here it is helpful to take a step back and contemplate our null hypothesis of AR(1) background, and the real possibility that without adjustments, we might be underestimating the lag-1 autocorrelation, hence making the test too lenient. There are alternatives to that in *Astrochron*, using either the robust method of Mann and Lees (1996) or a less error-prone version by the author (Meyers, 2012), called LOWSPEC. You might want to experiment with that.

Most important of all is the choice of null (Vaughan et al., 2011). The presence of scaling behavior (power law decrease) suggests that this should be a more appropriate null against which to test the emergence of spectral peaks. In GeoChronR, this can be done by specifying mtm null = "power law in the function call.

This may get rid of a few cycles, but many do remain (not shown). However, one should not be so foolish as to identify every single periodicity as independent, which affects assessments of significance, via the multiple comparisons problem (Vaughan et al., 2011). However, as pointed out by Meyers (2012), even that misses the point: sedimentary processes (and many processes in other proxy archives) tend to smooth out the signal over the depth axis, making comparisons at neighboring frequencies highly dependent. One solution is to use predictions made by a physical model about the frequency and relative amplitude of astronomical cycles (Meyers and Sageman, 2007). However this may not be applicable to all spectral detection problems. We thus refrain from implementing "cookie-cutter" solutions in GeoChronR, to force the user to think deeply about the null hypothesis and the most sensible way to test it.

There are four main takeways from this example:

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- 1. GeoChronR implements 4 different spectral methods, with different features and assumptions. This allows users to explore whether results obtained with one method are robust to these choices. More details of the implementation are provided in the vignette "spectral_analysis" that accompanies the package.
- 2. The methods are enabled for single age models, as well as age ensembles.
- 3. The choice of null hypothesis is always highly consequential, and should be made with care.
- 4. Hunting for astronomical (or other) frequencies in paleoclimate timeseries takes extreme care, and cannot be answered in one function call. In this particular example where the data are nearly evenly spaced, and interpolation has a negligible impact, the best choice is MTM for two reasons: (1) because it is optimal, in the sense that it minimizes spectral leakage, and (2) because the implementation used here (derived from *astrochron*) allows to flexibly apply different tests of significance. No such luxury is afforded by the other methods at present.

5. There is no one-size-fits-all method that will solve all spectral problems, so GeoChronR is primarily intended as an explorartory tool, designed to begin asking relevant questions. Readers are invited to consider the following references for a deeper exploration of these questions: Vaughan et al. (2011), Meyers (2012), Meyers (2015), Meyers and Malinverno (2018).

5 6 Conclusions

Version 1.0.0 of GeoChronR provides user-friendly access to common age-uncertain analysis tools in the paleogeosciences, along with intuitive visualization of the results. Although the focus has been on simplicity and ease of use, GeoChronR also has the underlying infrastructure to support customized analyses for users seeking to address a more nuanced or complex question. Nevertheless, in many ways the paleogeoscience community is only scratching the surface with age-uncertain analysis, and we look forward to working with the community to extend and expand this open-source package. Looking forward, we suggest that the next major direction for age-uncertain analysis is philosophical, not technical. Thus far, the community (and GeoChronR) has focused on quantifying the range of possibilities presented by age-uncertainty, not on developing approaches to constrain which ensemble members are most likely. Theoretically, additional information from nearby records, forcings or covariance structures could do so, but little has been done to date (?).

GeoChronR is open-source community-sofware, and has benefitted substantially from multiple contributors and input from early adopters and workshop participants. should we list them here? - In acknowledgements I think We welcome feedback and strongly encourage contributions and enhancements, via the GitHub issue tracker.

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6.0.1 Subsubsection Heading Here

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7 Content section with citations

See the R Markdown docs for bibliographies and citations.

Copernicus supports biblatex. I put the .bib entries from the Paleocube proposal into geochronr.bib. Citations work like this:

Read (Evans et al., 2013), and (see Dee et al., 2015).

8 Content section with list

If you want to insert a list, you must

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9 Examples from the official template

10 **9.1 FIGURES**

When figures and tables are placed at the end of the MS (article in one-column style), please add

between bibliography and first table and/or figure as well as between each table and/or figure.

9.2 TABLES

You can ad LATEX table in an R Markdown document to meet the template requirements.

9.3 PHYSICAL UNITS

Please use \unit{} (allows to save the math/\$ environment) and apply the exponential notation, for example 3.14km h^{-1} (using LaTeX mode: \((3.14\,\unit{\ldots}\)) or 0.872m s^{-1} (using only \unit{0.872\,\mu\,\s^{\ldots}-1}}).

Code and data availability. use this to add a statement when having data sets and software code available

Appendix A: Figures and tables in appendices

A1 Option 1

10 If you sorted all figures and tables into the sections of the text, please also sort the appendix figures and appendix tables into the respective appendix sections. They will be correctly named automatically.

A2 Option 2

If you put all figures after the reference list, please insert appendix tables and figures after the normal tables and figures. Regarding figures and tables in appendices, the following two options are possible depending on your general handling of figures and tables in the manuscript environment: \appendixfigures needs to be added in front of appendix figures \appendixtables needs to be added in front of appendix tables To rename them correctly to A1, A2, etc., please add the following commands in front of them: Please add \clearpage between each table and/or figure. Further guidelines on figures and tables can be found below.

Competing interests. The authors declare no competing interests.

20 Disclaimer, disc

Acknowledgements. ack

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