Quiz Review

1) $y'' + 36y = 0.$

Solve the DE.

$r^2 + 36 = 0$

$r^2 = -36$

$r = \pm 6i$

$y = e^{0t}\left(C_1 \cos(6t) + C_2 \sin(6t)\right)$

$$y = C_1 \cos(6t) + C_2 \sin(6t)$$

2) Solve the IVP for Q1.

$$y(0) = 6$$

$$\rightarrow y'(0) = -6$$

$$C_1 = 6$$

$$C_2 = -1$$

$$6 = C_1 \cos(0) + C_2 \sin(0))$$

$$\boxed{6 = C_1} \quad + 0$$

$$C_1 \cos(6t) + C_2 \sin(6t)$$

$$y' = -6 C_1 \sin(6t) + 6 C_2 \cos(6t)$$

$$-6 = -6 C_1 \sin(0) + 6 C_2 \cos(0))$$

$$-6 = 6 C_2 \quad \boxed{C_2 = -1}$$

3) $y'' + y' - 6y = 0$

Solve the DE.

$$r^2 + r - 6 = 0$$

$$(r + 3)(r - 2)$$

$$r = -3, 2$$

$$y = C_1 e^{-3t} + C_2 e^{2t}$$

4) Cauchy-Euler Equations.

$$\rightarrow ax^2 y'' + bxy' + cy = 0$$

$y' = \dfrac{dy}{dx}$ $\qquad a \neq 0, \; b, c$

$$\downarrow \quad t = \ln(x) \; \leftarrow$$

$$\rightarrow aY'' + (b-a)Y' + cY = 0$$

$Y' = \dfrac{dY}{dt}$

$$x^2 y'' + xy' + 4y = 0$$

$$\downarrow \quad t = \ln(x)$$

$$a = 1 \qquad b = 1 \qquad c = 4$$

$$y'' + (0)y' + 4y = 0$$

$$y'' + 4y = 0$$

$$r^2 + 4 = 0$$

$$r^2 = -4$$

$$r = \pm 2i$$

$$y(t) = C_1 \cos(2t) + C_2 \sin(2t)$$

$$\downarrow \quad t = \ln(x)$$

$$y(x) = C_1 \cos\left(2\ln(x)\right) + C_2 \sin\left(2\ln(x)\right)$$