Mon Feb 26

Quiz Wednesday

- Review Today
- LRC-Series Circuit

LRC-Series Circuit

Find the charge on Capacitor on LRC-series circuit

$L = 0.05$ Henry

$R = 1$ ohm

$C = 0.04$ Farads

$E(t) = 0$ Volts

$q(0) = 7$ Coulombs

$I(0) = 0$ amperes

$\downarrow$

$q'(0) = 0$

$$Lq'' + Rq' + \frac{q}{C} = E(t)$$

$$.05q'' + q' + \frac{q}{0.04} = 0$$

$$\frac{1}{20}q'' + q' + \frac{q}{0.04} = 0$$

$$\boxed{q'' + 20q' + 500q = 0}$$

Q: How to solve?

$$r^2 + 20r + 500 = 0$$

$$r = \frac{-20 \pm \sqrt{400 - 4(1)(500)}}{2}$$

$$r = \frac{-20 \pm \sqrt{400 - 4(500)}}{2}$$

$$r = \frac{-20 \pm \sqrt{400 - 2000}}{2}$$

$$r = \frac{-20 \pm \sqrt{-1600}}{2}$$

$$r = \frac{-20 \pm 40i}{2}$$

$$r = -10 \pm 20i$$

Solution to LRC-series

$$q(t) = e^{-10t}\left(C_1 \cos(20t) + C_2 \sin(20t)\right)$$

a) Find the charge at time $t = 0.02$.

$$q(0.02) = e^{-10(0.02)}\left(C_1\right.$$

Can't solve yet

$t = 0 \rightarrow q = 7$

$$7 = e^{0}\left(C_1 \cos(20(0)) + C_2 \sin(20(0))\right.$$

$$7 = e^{0}\left(C_1 + 0\right)$$

$$\boxed{7 = C_1}$$

$$q(t) = e^{-10t}\left(C_1\cos(20t) + C_2\sin(20t)\right)$$

$$q'(t) = (-10)e^{-10t}\left(C_1\cos(20t) + C_2\sin(20t)\right) + e^{-10t}\left(C_1\sin(20t) - C_2\cos(20t)\right)$$

$$0 = -10C_1 + 20C_2$$

$$0 = -70 + 20C_2$$

$$\boxed{C_2 = \frac{7}{2}}$$

$$\boxed{q(t) = e^{-10t}\left(7\cos(20t) + \frac{7}{2}\sin(20t)\right)}$$

$q(0.02)$

$t = 0.02$ s

$$e^{-.2}\left(7\cos(0.4) + \frac{7}{2}\sin(0.4)\right)$$

B) Determine the first time the charge = 0.

$q(t) = 0$

$$0 = e^{-10t}\left(7\cos(20t) + \frac{7}{2}\sin(20t)\right)$$

$$0 = \left(7\cos(2t) + \frac{7}{2}\sin(2t)\right)$$

$$0 = \frac{\left(7\cos(2t) + \frac{7}{2}\sin(2t)\right)}{\cos(2t)}$$

$$0 = 7 + \frac{7}{2}\tan(2t)$$

$$\frac{7}{2}\tan(2t) = -7$$

$$t = .1017$$