

# Estimating the Welfare Cost of Labor Supply Frictions\*

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## Abstract

This paper quantifies how much people would be willing to pay to remove frictions that impede them from working their ideal number of hours using two sufficient statistics: (1) the percentage difference between ideal (i.e., frictionless) and actual hours, and (2) the Hicksian elasticity of ideal hours with respect to the after-tax wage rate. We implement this method to construct estimates of the willingness-to-pay to remove frictions in the United States and Germany. There are three core findings: (1) the cost of adjustment frictions (an omnibus measure encompassing, for example, fixed costs of adjustment, discrete choice sets, and search costs) is large for any reasonable value of the Hicksian ideal hours elasticity, even when accounting for endogenous wages, multiple labor supply decisions, and dynamic decisions; (2) the cumulative cost of adjustment frictions and tax misperceptions is even larger: individuals would be willing to pay at least 10% of their income on average to remove these two frictions in hours worked; and (3) adjustment frictions appear to be much more costly than tax misperceptions.

*Keywords:* adjustment frictions, tax misperceptions, ideal hours worked, sufficient statistics

*JEL:* J22, J23, H24, H31

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# 1 Introduction

Individuals are often unable to work as much as they would like to due to various factors such as (1) high costs of changing jobs (Varejão and Portugal, 2007), (2) employer mandates on how many hours per week they must work (Dickens and Lundberg, 1993), (3) a lack of desirable jobs, and/or (4) misperceptions about the value of their time (DellaVigna, 2009). This paper seeks to explore the magnitudes and quantify the welfare cost of these sorts of labor market frictions.

More specifically, this paper seeks to quantify how much people would be willing to pay to remove frictions that impede them from supplying their optimal amount of labor. For instance, how costly is it (from the individual’s perspective) for a person who would like to work 35 hours/week but can only find a job that requires 40 hours/week? We begin by analyzing a standard, static labor supply model in which individuals have utility over consumption and hours worked. Individuals’ maximize utility with some choice of ideal (i.e., frictionless) hours  $h^*$ , but frictions lead individuals to end up working a different amount of actual hours with frictions,  $h^F$ . To begin with, we assume that agents correctly perceive wages, taxes, and their utility function, but otherwise do not take a stance on the type of frictions that generate this deviation: some examples which are consistent with our framework are fixed adjustment costs, search costs, or the presence of limited choice sets. We refer to these sorts of frictions collectively as “adjustment frictions”. First, we show how to recover an estimate of the willingness-to-pay (WTP) to remove adjustment frictions using two empirical objects (i.e., sufficient statistics): (1) the percentage difference between ideal and actual hours, and (2) the Hicksian elasticity of ideal hours with respect to the after-tax wage rate. This result, which builds on insights in Akerlof and Yellen (1985), Mankiw (1985), and Chetty (2012), exploits the fact that the cost of deviating from one’s optimal hours worked is determined by the curvature of utility and that the curvature of utility is inversely proportional to the Hicksian elasticity. Intuitively, utility functions with higher curvature generate smaller elasticities because people have a stronger preference to choose a given number of hours worked and are therefore less willing to change their hours substantially with the after-tax wage rate. Importantly, our framework is quite general insofar as we make no structural assumptions on the utility function or the type of adjustment frictions agents face.

Next, we take our theoretical framework to the data to recover the cost of adjustment frictions. Towards this end, we use data from the National Survey of the Changing Workforce (NSCW) in the U.S. and the German Socio-Economic Panel (GSOEP) which contain data on how many hours individuals work as well as how many hours they would ideally like to work. We document a considerable discrepancy between these two variables: across the employed pop-

ulation, the average absolute difference between ideal hours and actual hours is 9 (7) hours per week in the NSCW (GSOEP). Next, we leverage the panel nature of the GSOEP to estimate the Hicksian elasticity of *ideal hours* with respect to the after-tax wage rate using tax variation in the 1990’s and an empirical strategy similar to [Gruber and Saez \(2002\)](#) and [Kopczuk \(2005\)](#). While our estimates of this elasticity are noisy, the point estimates are consistent across specifications ( $\approx 0.16$ ) and, more importantly, all specifications can reject the hypothesis that the ideal hours elasticity is larger than one.<sup>1</sup> We then put these pieces together to compute the cost of adjustment frictions. We find that average WTP to remove hours frictions is around 20% (12%) of income in the NSCW (GSOEP) if the elasticity of ideal hours is 0.16; if this elasticity is 1, then the average WTP to remove hours frictions is 8% (3%) in the NSCW (GSOEP). Hence, the first key empirical takeaway is that the cost of adjustment frictions is relatively large for any reasonable value of the ideal hours elasticity.

The next section of the paper extends our theoretical framework to recover the cost of adjustment frictions in more general settings. In particular, we show how our framework can be extended to allow for an additional unobservable labor supply decision (effort-per-hour), many hours worked decisions (e.g., multiple earners per household), and dynamic decision making environments with savings as well as endogenous and stochastic wage growth. There are two findings: (1) we can still identify the cost of adjustment frictions using ideal hours, actual hours, and elasticities (although some of these more complex models require additional elasticities in addition to the Hicksian elasticity of ideal hours), and (2) empirically, the cost of adjustment frictions is still relatively large even accounting for any of these additional complexities.

Next, we relax the assumption that households correctly perceive the value of their time; more concretely, we allow agents to misperceive the tax schedule and investigate the cost of these “misperception frictions” which lead households to supply a sub-optimal level of labor. We show how to recover the cumulative cost of adjustment frictions and misperception frictions using data on ideal hours conditional on misperceptions, actual hours, the size of tax misperceptions, and the Hicksian elasticity of ideal hours. We then use NSCW data on ideal hours and actual hours (which, presumably are both chosen conditional on any underlying misperceptions of the tax schedule) along with data on the size of tax misperceptions from [Rees-Jones and Taubinsky \(2020\)](#) to estimate the cumulative cost of adjustment frictions and tax misperceptions. There are two key empirical takeaways from this exercise. First, adjustment frictions contribute far more to the cumulative cost of frictions than do misperceptions of the tax schedule for any reasonable value of the Hicksian ideal hours elasticity. Second, while the cost of adjustment

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<sup>1</sup>To the best of our knowledge, no paper has attempted to estimate the ideal hours elasticity (i.e., the hours elasticity in absence of frictions). However, several papers (e.g., [Kleven and Waseem \(2013\)](#), [Gelber et al. \(2020\)](#), [Tazhitdinova \(2020\)](#)) have estimated the frictionless earnings elasticity, which we typically expect to be larger than the frictionless hours elasticity. These papers also find that the frictionless earnings elasticity is less than 1.

frictions decreases with the Hicksian ideal hours elasticity (because higher elasticities imply less curvature in utility), the cost of tax misperceptions actually increases with the Hicksian ideal hours elasticity (because people change their behavior more in response to misperceptions). As a result, we find that the cumulative cost of adjustment frictions and tax misperceptions is at least 10% of income *regardless of the value of the Hicksian ideal hours elasticity*.

We are certainly not the first paper to explore the impacts of labor market frictions. A number of papers have proposed that frictions have important impacts on labor market outcomes (e.g. [Cogan \(1981\)](#), [Ham \(1982\)](#), [Altonji and Paxson \(1988\)](#), [Dickens and Lundberg \(1993\)](#), [Chetty et al. \(2011\)](#), [Chetty et al. \(2013a\)](#), [Chetty et al. \(2013b\)](#), and [Kleven and Waseem \(2013\)](#)). Fewer papers have attempted to explore the welfare costs of these frictions, although there are a few excellent recent papers which measure the welfare costs of specific frictions for specific subsets of the population. For example, [Søgaard \(2019\)](#) uses bunching evidence to estimate the cost of adjustment frictions for Danish students, finding a cost of optimization frictions around 2-3% of income. [Gelber et al. \(2020\)](#) estimate the size of frictions which generate a fixed cost of adjustment using changes in bunching in the context of the Social Security Earnings Test, finding that adjustment costs are around 0.5-2% of mean earnings for households around the Social Security Earnings Test threshold. [Gudgeon and Trenkle \(2024\)](#) apply this same method to a tax notch in Germany, finding an average adjustment cost of €400-500 (around 1% of household income) for households near a notch at the bottom of the income distribution. [Rees-Jones and Taubinsky \(2020\)](#) measure the cost of tax misperceptions, finding that the (household) welfare cost is around 0.5% of income.

Relative to previous work, our first contribution is methodological: we develop a framework to estimate the cost of frictions that can accommodate many different sorts of frictions (discrete choice sets, fixed costs of adjustment, demand side constraints, search costs, and misperceptions) and places few assumptions on the functional form of utility. Moreover, we show that our approach can be extended to incorporate many realisms (heterogeneous effort-per-hour, joint labor supply decisions, and dynamic decision making environments). Previous work has typically focused on estimating the cost of a specific type of friction (e.g., a fixed cost of adjusting hours or tax misperceptions) for a specific subset of the population and typically makes stronger functional form assumptions (e.g., quasi-linear iso-elastic utility); the primary contribution of this paper is developing and implementing a framework to quantify the costs of multiple different sources of frictions simultaneously for the entire employed population while making few structural assumptions.

Our theoretical results illustrate the importance of accounting for many sources of frictions simultaneously: the cost of frictions grows *quadratically* with the deviation from optimal hours

induced by frictions. For example, if three different frictions (e.g., fixed costs of adjustment, discrete choice sets, and misperceptions) each lead to successive 1% deviations from optimal hours, this will generate a cumulative cost of frictions that is 9 times as large as each of these frictions individually. While previous papers (e.g., [Gelber et al. \(2020\)](#), [Gudgeon and Trenkle \(2024\)](#), [Chetty \(2012\)](#), [Rees-Jones and Taubinsky \(2020\)](#)) all illustrate particular types of frictions that generate a cost valued at  $\approx 1\%$  of income, we find that the cumulative cost of adjustment frictions and tax misperceptions is at least 10% of income. Thus, the second contribution of this paper is to highlight that the *cumulative* cost of adjustment frictions and tax misperceptions is perhaps larger than previously believed; moreover, because our methodology can disentangle the costs of adjustment frictions versus misperceptions, we are able to illustrate that adjustment frictions are more costly to individuals than tax misperceptions.<sup>2</sup>

In the interest of transparency, our methodology has three key downsides relative to prior work: (1) we estimate a second order approximation for the cost of frictions (while we provide evidence that this is not driving our findings by showing that our baseline results are similar to what we would infer about the cost of frictions using our actual and ideal hours data and specifying two common functional forms for utility, this is nonetheless a limitation), (2) our analysis only estimates frictions for employed individuals (we would need counterfactual wages to apply our results for unemployed individuals),<sup>3</sup> and (3) our approach uses (unincentivized) survey data on both actual hours and ideal hours worked; while there are alternative ways to measure actual hours of work (e.g., time-use diary entries, cell phone tracking data), measuring ideal hours of work for a representative sample of the population is much more difficult.<sup>4</sup> While there are potentially tools to overcome some limitations of survey data in the future (e.g., using vignette methods as in [Eriksson and Kristensen \(2014\)](#)), the reliance on survey data is an important limitation compared to other approaches (e.g., [Gelber et al. \(2020\)](#)) which can be applied with administrative data.

Finally, from a policy perspective, there are many policy reforms that impact that size of labor market frictions, such as (1) laws and regulations that increase the availability of gig work, (2) regulations that impose costs on firms which do not scale with the number of hours worked (e.g., full benefit mandates for individuals who work a certain amount of hours or compliance costs that scale with the number of workers but not hours worked by each worker),

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<sup>2</sup>This paper also contributes tertiarily to the literature on estimating structural parameters in the presence of frictions (e.g., [Chetty \(2012\)](#), [Kleven and Waseem \(2013\)](#), [Gelber et al. \(2020\)](#), [Gudgeon and Trenkle \(2024\)](#), [Tazhitdinova \(2020\)](#)) by estimating a frictionless hours elasticity in the context of German tax reforms in the 1990's, finding that we can reject that this elasticity is larger than 1.

<sup>3</sup>[Mas and Pallais \(2019\)](#) implement a field experiment to elicit the value of employment for unemployed individuals, which can be used to recover a cost of frictions for the unemployed.

<sup>4</sup>For instance, [Mas and Pallais \(2017\)](#) run a field experiment to elicit preferences over hours worked but their results apply only to the population of job seekers in a particular industry.

(3) proposals for government employment guarantees, (4) laws regarding paid-time-off, or (5) initiatives that improve tax salience. While the present paper does not evaluate any specific policy, the framework developed herein can be used to calculate the WTP to reduce frictions from particular policy reforms. Moreover, given that our empirical findings suggest that the WTP to remove labor market frictions is substantial, this suggests that it is likely important to take account of the WTP for changes in labor market frictions when conducting welfare analysis.

The rest of the paper is structured as follows. Section 2 illustrates how to estimate the cost of frictions in a baseline labor supply model. Section 3 discusses our data on actual and ideal hours as well as presents a baseline estimate of the cost of frictions. Section 4 illustrates how our framework can be extended to account for endogenous wages, many hours decisions, and dynamic decision making environments. Section 5 extends our framework to allow for tax misperceptions. Section 6 concludes.

## 2 Adjustment Frictions Theory

Our goal is to understand how much worse off individuals are as a result of their actual hours of work deviating from their ideal hours of work due to frictions. We will separately analyze the impacts of two different kinds of frictions: adjustment frictions and misperceptions of the tax schedule. Adjustment frictions should be thought of as constraints which prohibit individuals from supplying their ideal amount of labor but are *not* related to misperceptions. For instance, limited choice sets arising from demand side constraints, search costs, and fixed costs of changing jobs would all fall under our broad definition of adjustment frictions.

### 2.1 Baseline Labor Supply Model

We begin with a standard labor supply model in which individuals have preferences over consumption,  $c$ , and hours worked,  $h$ . Individuals have a smooth utility function:

$$u(c, h)$$

Agents face a (potentially) non-linear tax schedule  $T(wh)$  where  $w$  is their hourly wage:

$$c = wh - T(wh) \tag{1}$$

To begin with, we assume that agents correctly perceive their environment; we discuss how the results change with misperceptions of the tax schedule in Section 5. Let  $h^*$  denote the solution which maximizes  $u(c, h)$  subject to the budget constraint, Equation (1); we will refer to  $h^*$  as “ideal hours”. However, suppose that agents face adjustment frictions and therefore the agent ends up supplying some “actual hours”,  $h^F$ , and thus consumes  $c(h^F) = wh^F - T(wh^F)$ . For instance, perhaps agents maximize  $u(c, h)$  s.t. Equation (1) and the additional constraint

that  $h \in H^F$  where  $H^F$  denotes some limited choice set. Or agents maximize  $u_F(c, h) = u(c, h) - \kappa \mathbb{1}[h \neq \tilde{h}]$  s.t. Equation (1) where  $\kappa$  denotes a fixed cost that agents must pay to deviate from a status quo hours  $\tilde{h}$ . Remaining agnostic as to the specific form, we are interested in understanding the cost of these adjustment frictions which lead individuals to work  $h^F$  hours rather than their frictionless optimum  $h^*$ . Precisely, we will be interested in estimating the parameter  $\delta(h^F)$  which represents the willingness-to-pay (WTP) to remove frictions defined implicitly by the following expression:

$$u(c(h^F) + \delta(h^F), h^F) = u(c(h^*), h^*) \quad (2)$$

$\delta(h^F)$  is the amount of money required to compensate an individual for frictions which prevent them from choosing their optimal hours,  $h^*$ . Henceforth, we will refer to  $\delta(h^F)$  as the “cost of frictions” and/or the “WTP to remove frictions”, recognizing that  $\delta(h^F)$  is a money-metric. First, we will show how to (approximately) measure  $\delta(h^F)$  in terms of two objects: the percentage difference between  $h^F$  and  $h^*$  as well as the compensated elasticity of ideal (i.e., frictionless) hours. Towards this end, let us explore how  $\delta(h)$  changes with  $h$  by applying the implicit function theorem to Equation (2) to calculate:

$$\left. \frac{d\delta}{dh} \right|_{h^*} = - \frac{u_c(c(h^*), h^*) w (1 - T'(wh^*)) + u_h(c(h^*), h^*)}{u_c(c(h^*), h^*)} = 0 \quad (3)$$

where the second equality follows from the FOC which is satisfied at  $h^*$  assuming  $T(wh)$  is differentiable at  $h^*$ . Equation (3) is a consequence of the envelope theorem and tells us that it is not costly for agents to experience frictions that move them infinitesimally far away from their optimal choice of hours. As a result of Equation (3), we can use the fundamental theorem of calculus twice to deduce:

$$\delta(h) = \int_{h^*}^h \frac{d\delta}{dh}(s) ds = \int_{h^*}^h \int_{h^*}^s \frac{d^2\delta}{dh^2}(t) dt ds \quad (4)$$

Hence, in order to understand the cost of frictions, we require an estimate of the second derivative of frictions as we move away from  $h^*$ . Applying the implicit function theorem to Equation (3), we can get an expression for  $\left. \frac{d^2\delta}{dh^2} \right|_{h^*}$  (see Appendix A.1 for the proof):

$$\left. \frac{d^2\delta}{dh^2} \right|_{h^*} = - \frac{u_{cc}w^2(1 - T')^2 - u_cw^2T'' + 2u_{ch}w(1 - T') + u_{hh}}{u_c} \quad (5)$$

Note that the (strict) second order condition for optimality of  $h^*$  requires that  $u_{cc}w^2(1 - T')^2 - u_cw^2T'' + 2u_{ch}w(1 - T') + u_{hh} < 0$  so that  $\left. \frac{d^2\delta}{dh^2} \right|_{h^*} > 0$ . Equation (5) tells us that  $\left. \frac{d^2\delta}{dh^2} \right|_{h^*}$  is equal to the curvature in  $u(wh - T(wh), h)$  w.r.t.  $h$  divided by the marginal utility of consumption. Thus, Equation (4) combined with Equation (5) highlight that costs of deviation from an optimum are a function of the second derivatives of utility as discussed in Akerlof and Yellen (1985) or Mankiw (1985). We will now show that  $\left. \frac{d^2\delta}{dh^2} \right|_{h^*}$  can be expressed in terms of the Hicksian elasticity of ideal hours of work w.r.t. the tax rate, denoted  $\xi_{h^*}$ . When faced with

a linear tax schedule,  $\xi_{h^*}$  is the elasticity of  $h^*$  w.r.t. the keep rate holding utility constant:  $\xi_{h^*} \equiv \frac{\partial h^*}{\partial(1-T')} \frac{1-T'}{h^*} |_u$  (we define this elasticity more generally when agents face a non-linear tax schedule in the proof of Lemma 1 in Appendix A.2). We have:

**Lemma 1.**  $\frac{d^2\delta}{dh^2}|_{h^*}$  is related to the Hicksian ideal hours elasticity  $\xi_{h^*}$  via:

$$\frac{d^2\delta}{dh^2}\bigg|_{h^*} = \frac{w(1-T')}{\xi_{h^*}h^*} \quad (6)$$

Lemma 1 tells us that  $\frac{d^2\delta}{dh^2}|_{h^*}$ , which equals the curvature in  $u(wh - T(wh), h)$  w.r.t.  $h$  divided by marginal utility of consumption, is inversely proportional to the Hicksian labor supply elasticity. The intuition is that utility functions with higher curvature must generate smaller elasticities because people have a stronger preference to choose a given hours worked and are therefore less willing to change their hours substantially with the tax rate. This brings us to our first result:

**Proposition 1.** Suppose that  $\frac{d^3\delta}{dh^3} \approx 0$ . The WTP to remove frictions expressed as a fraction of optimal income  $wh^*$  can be approximated as:

$$\frac{\delta(h^F)}{wh^*} \approx (1 - T'(wh^*)) \frac{\left(\frac{h^F - h^*}{h^*}\right)^2}{2\xi_{h^*}} \quad (7)$$

*Proof.* Starting from Equation (4), using the assumption that  $\frac{d^3\delta}{dh^3} \approx 0$  so that  $\frac{d^2\delta}{dh^2}$  is roughly constant at  $\frac{w(1-T')}{\xi_{h^*}h^*}$  by Lemma 1 between  $h^*$  and  $h$  yields:

$$\begin{aligned} \delta(h^F) &= \int_{h^*}^{h^F} \int_{h^*}^s \frac{d^2\delta}{dh^2}(t) dt ds \approx \int_{h^*}^{h^F} \int_{h^*}^s \frac{w(1-T')}{\xi_{h^*}h^*} dt ds \\ &= \int_{h^*}^{h^F} \frac{w(1-T')}{\xi_{h^*}h^*} (s - h^*) ds = \frac{w(1-T')}{2\xi_{h^*}h^*} (h^F - h^*)^2 \end{aligned}$$

□

Hence, one can recover an estimate of the cost of frictions with an estimate of the Hicksian elasticity of ideal hours with respect to the tax rate,  $\xi_{h^*}$ , along with the extent to which actual (observed) hours deviate from ideal hours in percentage terms:  $\frac{h^F - h^*}{h^*}$ . The key takeaway from Proposition 1, which is conveyed in Figure 1, is that the WTP to remove frictions increases quadratically with the distance between  $h^*$  and  $h^F$  (see Figure 1a) and increases with the inverse of the elasticity  $\xi_{h^*}$  (see Figure 1b). Proposition 1 is conceptually the same as Lemma 1 from Chetty (2012) with the additions that Proposition 1 allows for non-linear tax schedules and non-quasilinear utility. Proposition 1 should thus be viewed as a baseline for recovering the cost of frictions; the primary theoretical contribution of the present paper is to illustrate how the logic of Proposition 1 can be extended to much more general settings (Sections 4 and 5) and the primary empirical contribution is applying Proposition 1 and associated generalizations to data.



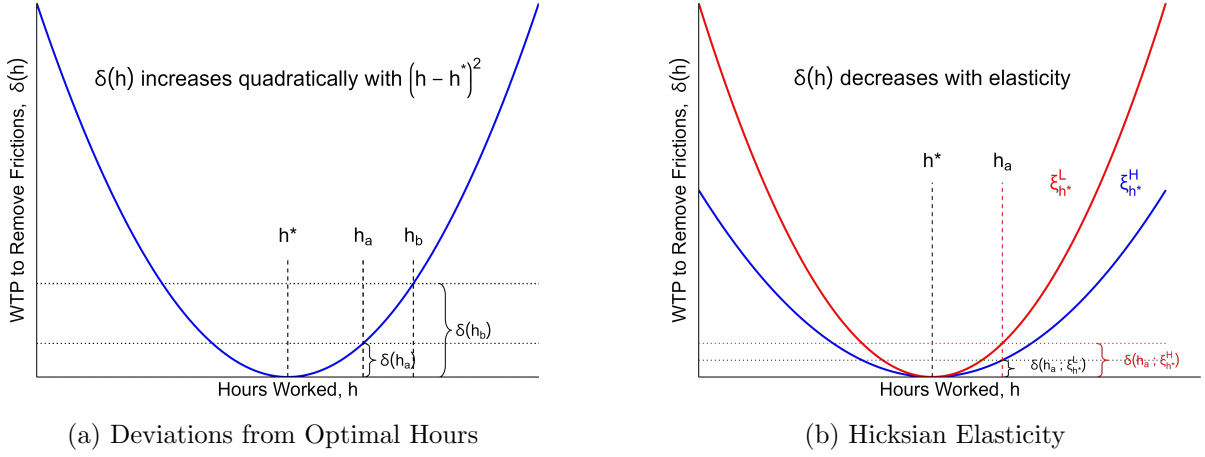


Figure 1: Intuition for Proposition 1

**Notes:** These graphs plot the WTP to remove frictions,  $\delta(h)$ , vs hours worked,  $h$ . Panel **a** shows  $\delta(h)$  for two values of hours worked,  $h = \{h_a, h_b\}$ , illustrating that  $\delta(h)$  increases quadratically with  $h - h^*$ . Panel **b** shows  $\delta(h_a)$  for two values of  $\xi_{h^*}$ :  $\xi_{h^*}^H$  (associated with the blue curve) and  $\xi_{h^*}^L$  (red curve) with  $\xi_{h^*}^H > \xi_{h^*}^L$ . Panel **b** illustrates that  $\delta(h)$  decreases with  $\xi_{h^*}$  for fixed  $h$ .

## 2.2 Revealed Preference Bounds on Cost of Frictions

Our empirical application of Proposition 1 will involve gathering data on  $h^F$  and  $h^*$  and exploring how the cost of adjustment frictions varies with  $\xi_{h^*}$ . However, we first illustrate that not all values of  $\xi_{h^*}$  are consistent with given choices of  $h^F$  and  $h^*$ : under mild assumptions we can bound the range of elasticities  $\xi_{h^*}$  that are consistent with given choices of  $h^F$  and  $h^*$  using revealed preference arguments (see Appendix A.3 for proof):

**Proposition 2.** Suppose  $\frac{\partial u(c, h)}{\partial h} \leq 0 \forall c, h$  and  $\frac{d\delta}{dh}(h) \leq 0 \forall h < h^*$ .<sup>5</sup> Then:

$$\frac{\delta(h^F)}{wh^*} \leq \frac{wh^* - T(wh^*) - [wh^F - T(wh^F)]}{wh^*} \quad \text{for } h^* > h^F \quad (8)$$

$$\frac{\delta(h^F)}{wh^*} \leq \frac{wh^* - T(wh^*) + T(0)}{wh^*} \quad \text{for } h^* < h^F \quad (9)$$

Furthermore, if  $\frac{d^3\delta}{dh^3} \approx 0$  then:

$$\xi_{h^*} \geq \frac{1}{2} \frac{(1 - T'(wh^*)) \left( \frac{h^F - h^*}{h^*} \right)^2}{\frac{wh^* - T(wh^*) - [wh^F - T(wh^F)]}{wh^*}} \quad \text{for } h^* > h^F \quad (10)$$

$$\xi_{h^*} \geq \frac{1}{2} \frac{(1 - T'(wh^*)) \left( \frac{h^F - h^*}{h^*} \right)^2}{\frac{wh^* - T(wh^*) + T(0)}{wh^*}} \quad \text{for } h^* < h^F \quad (11)$$

Intuitively, for individuals who would ideally like to work more than they currently do

<sup>5</sup>The condition that  $\frac{d\delta}{dh}(h) < 0 \forall h < h^*$  just means that the WTP to remove frictions is increasing as  $h$  gets farther away from  $h^*$ . This is loosely equivalent to utility  $u(c(h), h)$  being single-peaked in  $h$ . This condition always holds, for example, if  $u(c, h)$  is quasi-linear in  $c$  so that  $u(c, h) = c - v(h)$  with convex  $v(h)$  and linear taxes as in this case:

$$c(h) + \delta(h) - v(h) = c(h^*) - v(h^*)$$

$$\frac{d\delta}{dh}(h) = v'(h) - c'(h) = v'(h) - (1 - T')w$$

$v'(h^*) - (1 - T')w = 0$  by the FOC so  $v'(h) - (1 - T')w < 0 \forall h < h^*$  by convexity of  $v(\cdot)$ .

( $h^* > h^F$ ), the cost of frictions is bounded above by the gain in income generated from moving from  $h^F$  to  $h^*$  because labor supply is costly. Similarly, for individuals who would ideally like to work less than they currently do ( $h^* < h^F$ ), the cost of frictions is bounded above by the cost of moving to unemployment from  $h^*$ , which is in turn bounded above by the consumption loss of moving to unemployment from  $h^*$ . Hence, if  $\frac{d^3\delta}{dh^3} \approx 0$ , then the cost of frictions from Proposition 1,  $(1 - T'(wh^*))\frac{\left(\frac{h^F - h^*}{h^*}\right)^2}{2\xi_{h^*}}$ , must be less than the upper bounds in Equation (8) and (9); this creates a lower bound for the value of  $\xi_{h^*}$  consistent with (1) costly labor supply  $\frac{\partial u(c, h)}{\partial h} \leq 0$  and (2) the decision to work rather than be unemployed. To see the intuition for why this implies bounds on  $\xi_{h^*}$  for a given individual, note that for any values of  $h^*$  and  $h^F$  (given  $w$  and  $T(\cdot)$ ), there is a maximum value of  $\delta$  that can be reconciled with Equations (8) and (9). Returning to Figure 1b, we see that smaller  $\xi_{h^*}$  imply larger values of  $\delta$  for a given  $h^*$  and  $h^F$ ; hence, in order for  $\delta(h^F)$  to satisfy Equations (8) and (9),  $\xi_{h^*}$  cannot be too small.

### 3 Empirical Size of Adjustment Frictions

Next, we will now apply Proposition 1 (along with Proposition 2) to empirically assess the size of adjustment frictions. There are three elements that we will need to assess the magnitude of adjustment frictions: (1) how many hours individuals actually work in the presence of adjustment frictions,  $h^F$ , (2) how many hours individuals would ideally like to work in absence of adjustment frictions  $h^*$ , and (3) the Hicksian elasticity of ideal hours  $h^*$  w.r.t. the tax rate. We will begin by discussing two datasets which have information on  $h^F$  and  $h^*$ : the U.S. National Study of the Changing Workforce and the German Socio-Economic Panel.

#### 3.1 Data on $h^F$ and $h^*$

**National Study of Changing Workforce (NSCW):** The NSCW is a repeated cross-sectional survey conducted by The Families and Work Institute in 1992, 1997, 2002, and 2008 in the U.S. The survey contains a wide number of questions related to employment and demographics. For our purposes, we will use information on wages  $w$  and actual hours worked per week in the (potential) presence of frictions,  $h^F$ .<sup>6</sup> Most importantly, the survey also asks the question “If you could do what you wanted to do, ideally how many hours, in total, would you like to work each week?” To use this question to measure  $h^*$ , we need individuals to answer this question by considering a world in which their hours choice is fully flexible, their income scales linearly with their hours worked, and there are no other material changes to their circumstances. There are several concerns: individuals may respond to this question by entirely ignoring the fact that their income would change with hours, or individuals may respond by stating how many hours they think they could reasonably work while getting paid the same

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<sup>6</sup>We get  $h^F$  from responses to the question: “How many hours do you usually work each week at your job?”

amount, or individuals may interpret an “ideal” world as one in which their life situation was different above and beyond removing adjustment frictions (for instance, one might interpret an “ideal” world as one in which childcare was cheaper). To deal with these concerns, we first note that only 4.5% of all respondents answered that they’d ideally like to work 0 hours; we drop these individuals as this response suggests they certainly ignored the financial costs of working less when responding given that not working is always in one’s choice set. Moreover, in 2008, the NSCW contains a follow-up question that asks, “Why don’t you work fewer (more) hours per week?”. We use responses to this question to determine whether individuals interpreted the original question about ideal hours as “hours in absence of adjustment frictions”. For individuals who say that they would like to work less, we assume those who responded that they do not work less because they would lose their job or because they face too much pressure from their boss are interpreting the initial question as being about adjustment frictions. In contrast, we assume individuals who would like to work less but do not because they “need the money” are not interpreting the question about ideal hours as being about adjustment frictions (as monetary ramifications of working less should be internalized when determining ideal hours). For individuals who would like to work more, we assume those who responded that they do not work more because they cannot find a job with more hours or because they cannot find the needed flexibility in another job are interpreting the initial question as being about adjustment frictions; in contrast, we assume individuals who do not work more due to childcare or family responsibilities are not interpreting the question about ideal hours as being about adjustment frictions. Table 2 in Appendix C provides the full distribution of responses to the above questions and shows which responses we assume are related to adjustment frictions versus not related to adjustment frictions. Our main estimates of adjustment frictions will use only the 2008 survey and include only individuals who we believe interpreted the initial question about ideal hours as being about hours in absence of adjustment frictions.<sup>7</sup> <sup>8</sup> Figure 2a presents a binscatter of how ideal hours varies with actual hours in the NSCW and Figure 14a in Appendix C shows the distributions of  $h^F$  and  $h^*$  in the NSCW. Note that there is a strong relationship between actual and ideal hours; moreover workers who work fewer hours would like to work

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<sup>7</sup>The remaining sample is therefore representative if we assume that the excluded 37.4% of individuals would face the same distribution of frictions as the rest of the employed sample if they interpreted the question as being about adjustment frictions. Figure 13 in Appendix C provides estimates of adjustment frictions under two alternative assumptions: (1) all individuals interpreted the question on ideal hours as being solely about removing adjustment frictions (hence, no individuals are dropped and we can use the entire employed sample across all years), and (2) all individuals who we deem to have not interpreted the question as being about adjustment frictions have ideal hours equal to actual hours (i.e., instead of dropping these individuals, we set  $h^* = h^F$ ).

<sup>8</sup>One may be concerned that our estimates only use data from 2008, which was a recession year in the U.S. We believe this is unlikely to be problematic because interviews were conducted in early 2008 before unemployment ticked up substantially and because the mean and median absolute difference between ideal and actual hours worked (ignoring potential misinterpretations of the question about ideal hours) were virtually constant in 1997, 2002, and 2008 - see Figure 15 in Appendix C.

more (on average) and workers who work many hours would like to work less (on average).

**German Socio-Economic Panel (GSOEP):** The GSOEP is a longitudinal survey of approximately 15,000 households, representative of the German population, conducted by the *Deutsches Institut für Wirtschaftsforschung* since 1984. In addition to collecting information on hourly wages  $w$  and hours worked (possibly in the presence of frictions)  $h^F$ , we use the answer to the following question to infer  $h^*$ : “If you could choose your ideal working hours, *taking into account your income would change according to the number of hours*: How many hours would you work?” We believe that this question is phrased in a manner which enables easier interpretation of the answer as being related to adjustment frictions given that it explicitly states that income would go down with hours worked. We use the answer to this question as our measure of ideal hours worked in the absence of adjustment frictions,  $h^*$ .<sup>9</sup> Figure 2b presents a binscatter of how ideal hours varies with actual hours in the GSOEP and Figure 14b in Appendix C shows the distributions of  $h^F$  and  $h^*$  in the GSOEP. There is again a strong relationship between actual and ideal hours; moreover workers who work few hours would like to work more (on average) and workers who work many hours would like to work less (on average).

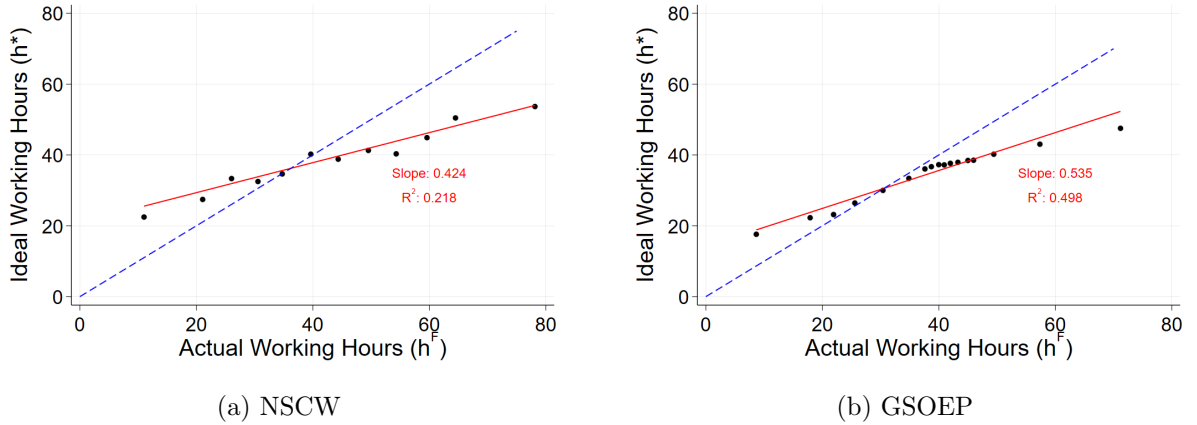


Figure 2: Ideal vs Actual Hours

**Notes:** This graph shows a binscatter of ideal hours  $h^*$  vs. actual observed hours  $h^F$  for the 2008 NSCW included sample and the 1990-2019 GSOEP. The red line represents the best linear predictor of ideal hours based on observed working hours and the dashed blue line shows the 45° line.

### 3.2 Estimating the Ideal Hours Elasticity

The final component we need to assess the size of frictions is the elasticity of ideal hours w.r.t. the tax rate. While many papers have estimated elasticities of actual hours (and income) w.r.t. tax changes, to the best of our knowledge, no paper has estimated the impact of tax changes on *ideal* hours worked, which is the relevant elasticity for understanding the cost of adjustment

<sup>9</sup>While the GSOEP does not ask a question about why people do not work their ideal hours, we believe this is not overly problematic because the question about ideal hours in the GSOEP explicitly mentions that income is reduced by working less and, in the NSCW, the primary reason we exclude individuals is because they do not appear to account for the financial ramifications of working less when giving their ideal hours of work: see Table 2 in Appendix C.

frictions. We now attempt to estimate this parameter using the GSOEP and variation from German tax reforms in the 1990s.<sup>10</sup> The variation in tax rates is discussed and depicted in Appendix B.1.

Our empirical strategy will explore how changes in ideal hours worked between pairs of years relates to tax changes between the same pairs of years for given individuals. We label the two years 1 and 2; following Feldstein (1999) and Gruber and Saez (2002), the length of time between “year 1” and “year 2” will be three years in our baseline specifications although we will show robustness to this time length. We use data on ideal hours worked from the GSOEP for years between 1991-2001, which comprises a 5 year window on either side of a large tax reform in 1996 (there were other smaller tax reforms between 1996 and 2001, see Appendix B.1). Hence, our dataset will consist of stacked three year differences between years 1991 and 1994, 1992 and 1995, etc.<sup>11</sup>

The vast majority of previous papers that have analyzed behavioral responses to tax reforms have explored changes to piecewise linear tax schedules. In contrast, the German tax schedule is a non-linear (quadratic) function. Hence, we need to derive a slightly different estimating equation (relative to prior empirical work) that accounts for the non-linearities in the tax schedule. We show in Appendix B.2 how to derive the following discretized Slutsky-esque equation to recover the Hicksian ideal hours elasticity,  $\xi_{h^*}$ , for non-linear reforms:

$$\Delta \log(h^*)_i = \xi_{h^*} \Delta \log(1 - T'(wh^*))_i + \eta_{h^*} \Delta \log(wh^* - T(wh^*))_i + \nu_i \quad (12)$$

where  $i$  denotes an individual,  $\Delta \log(1 - T'(wh^*))_i = [\log(1 - T'_2(w_1 h_1^*)) - \log(1 - T'_1(w_1 h_1^*))]_i$  represents the mechanical change in marginal tax rates between years 1 and 2 for individual  $i$ ,  $\Delta \log(wh^* - T(wh^*))_i = [\log(w_1 h_1^* - T_2(w_1 h_1^*)) - \log(w_1 h_1^* - T_1(w_1 h_1^*))]_i$  represents the mechanical change in after tax income between years 1 and 2 for individual  $i$ , and  $\nu$  represents an error term. The subscripts 1 and 2 represent values in year 1 and year 2, respectively.

There are two points to discuss. First, the terms  $\Delta \log(1 - T'(wh^*))_i$  and  $\Delta \log(wh^* - T(wh^*))_i$  in Equation (12) represent *mechanical* changes (often referred to as simulated changes) in the keep-rate and after-tax-income holding ideal hours (and therefore ideal income) constant at year 1 level. Previous papers (e.g., Gruber and Saez (2002) or Kopczuk (2005)) that have estimated similar specifications to Equation (12) have typically included the actual changes in the keep-rate and after-tax-income and instrumented for these changes with the mechanical changes that would have ensued if there were no behavioral effects. Using these simulated instruments is of course not a problem from an econometric standpoint, but for the purposes of

<sup>10</sup>We cannot estimate this elasticity in the U.S. using the NSCW because the NSCW is not a panel dataset.

<sup>11</sup>Unfortunately, the GSOEP did not collect data on ideal hours worked in 1996 so we can not include the 1993 to 1996 and 1996 to 1999 differences in our baseline regressions.

recovering the parameter  $\xi_{h^*}$ , theory tells us that we should *not* use the mechanical changes in taxes as simulated instruments and instead use them directly as regressors when  $T'' \neq 0$ .<sup>12</sup>

Second, estimating Equation (12) will yield an unbiased estimate of  $\xi_{h^*}$  assuming that changes in marginal tax rates and tax liabilities are assigned (as good as) randomly so  $\nu_i \perp \Delta \log(1 - T'(wh^*))_i$  and  $\nu_i \perp \Delta \log(wh^* - T(wh^*))_i$ . In practice, tax changes are *not* randomly assigned: the (mechanical) change experienced in both the marginal tax rate and tax liability from a reform is determined by one's income level in year 1. Hence, the key threat to identification is that  $\Delta \log(h^*)_i$  is correlated with  $(w_1 h_1^*)_i$ : this may occur, for instance, due to differential trends in  $h^*$  across the income distribution. We employ the solution commonly adopted in previous papers (e.g., Gruber and Saez (2002)), which is to control flexibly for income in year 1. Our estimating equation is as follows where  $f(w_1 h_1^*)_i$  is a flexible function of year 1 income,  $Year_j$  denote base year dummies, and incomes are adjusted for inflation:

$$\Delta \log(h^*)_i = \xi_{h^*} \Delta \log(1 - T'(wh^*))_i + \eta_{h^*} \Delta \log(wh^* - T(wh^*))_i + f(w_1 h_1^*)_i + Year_j + \nu_i \quad (13)$$

Table 1 shows results from estimating Equation (13).<sup>13</sup> All specifications (other than Column 7) restrict to households with a single earner to abstract from joint household labor supply decisions. Robustness exercises are performed in Columns 2-9 and are detailed in the table notes. As can be seen, our estimate for  $\xi_{h^*}$  is imprecisely estimated. The baseline estimate in Column (1) is 0.16 with a 95% CI of [-0.15, 0.47]. The point-estimates are fairly stable but noisy across all specifications. Importantly however, across all specifications, the 95% confidence intervals can rule out very large elasticities, with the upper limits of these confidence intervals all being less than 1; moreover, in all columns except Column (6), upper limits are less than 0.5. Thus, in the next section, we will show how the size of adjustment frictions varies for values of  $\xi_{h^*} \in (0, 1]$ .<sup>14</sup>

Finally, while to the best of our knowledge no paper has attempted to estimate the ideal hours elasticity, several papers have estimated the earnings elasticity in absence of frictions (often referred to as the structural earnings elasticity), such as Kleven and Waseem (2013), Gelber et al. (2020), and Tazhitdinova (2020). To do so, these papers have utilized bunching masses

<sup>12</sup>When the initial tax schedule is linear (or piecewise linear) as in Gruber and Saez (2002) or Kopczuk (2005), then using the mechanical change in marginal tax rates as a simulated instrument or directly as a regressor is equivalent conditional on the assumption that individuals move smoothly in response to tax schedules (which is used to derive the Slutsky equation in the first place). If individuals stay within the same tax bracket then their mechanical change in marginal tax rates is equivalent to their actual change in marginal tax rates even after behavioral responses, meaning that theoretically the  $R^2$  from the first stage should be 1. When the tax schedule is non-linear, as in our setting, these are not theoretically equivalent due to a non-zero second derivative of the tax schedule.

<sup>13</sup>For comparison with prior studies, we also show regressions estimated using actual hours worked and income as the dependent variable: see Table 3 and Table 4 in Appendix C.2, respectively. While both are imprecisely estimated, the point estimates are consistent with those in the literature: the actual hours (income) elasticity is 0.08 (0.34) in our baseline specification.

<sup>14</sup> $\xi_{h^*} > 0$  if marginal utility of consumption is positive and second order conditions hold.

at notches and/or kinks in the income tax schedule along with functional form assumptions on the form of frictions. These papers all find that structural elasticities are larger than those estimated with frictions<sup>15</sup>; however, they also find fairly small structural earnings elasticities with values substantially lower than 1 (Chetty et al. (2011) also suggest this structural elasticity is less than 1). Given we expect compensated hours elasticities to be less than compensated earnings elasticities (given that earnings elasticities also capture additional, presumably positive, behavioral responses), this supports  $\xi_{h^*} < 1$ .

Table 1: Estimates of the Ideal Hours Worked Elasticity w.r.t. the Marginal Tax Rate

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Baseline	Cubic	Linear	Time Trends	<50th	Singles	All Hhs	2 Yr Diff	4 Yr Diff
$\xi_{h^*}$	0.16 ( 0.16)	0.10 ( 0.15)	0.14 ( 0.15)	0.15 ( 0.16)	0.13 ( 0.17)	0.38 ( 0.24)	0.13 ( 0.14)	0.04 ( 0.11)	0.13 ( 0.13)
$\eta_{h^*}$	0.06 ( 0.35)	-0.02 ( 0.33)	0.19 ( 0.32)	0.09 ( 0.36)	0.23 ( 0.50)	0.27 ( 0.52)	0.14 ( 0.31)	0.11 ( 0.24)	0.13 ( 0.32)
Obs.	2,830	2,845	2,845	2,830	1,690	1,599	3,940	4,067	1,952

*Notes:* Standard errors are clustered at the household level and are presented in parentheses. Each column presents estimates of  $\xi_{h^*}$  and  $\eta_{h^*}$  from regression (13) where the dependent variable is the change in log ideal hours worked for individual  $i$ ,  $\Delta \log h_{it}^*$ . Columns (1)-(7) use 3 year differences while Columns (8) and (9) use 2 and 4 year differences, respectively. All regressions are for years 1991-2001 inclusive (note, we do not have ideal hours for 1996), include year dummies, a married dummy, and restrict to individuals whose marital status did not change. All regressions are weighted using household survey weights. Column (1), (4), (5), (6), (7) include a spline in lagged log household income (i.e., lagged income decile dummies interacted with lagged log income); Column (4) also includes lagged log income interacted with a linear time trend. Instead of a spline, Column (2) includes a cubic polynomial in lagged log household income while Column (3) includes a linear polynomial in lagged log household income. All columns except (6) and (7) restrict to single-earner households. Column (6) restricts to single individuals only. Column (7) includes both single- and dual-earner households and the dependent variable for Column (7) is wage weighted ideal hours  $\Delta \log(w_i h_{it}^* + w_j h_{jt}^*)$  where  $j$  represents the second earner (if there is one) and the wages are equal to wages in time period  $t - 3$ . This specification thus estimates  $\xi_{z^*}$ , which is the relevant parameter for bounding the cost of frictions for multi-earner households in Proposition 5. Finally, Column (5) restricts to households with lagged household income below the 50th percentile.

### 3.3 Cost of Adjustment Frictions

Next, we put all of the pieces together: we use the values of  $\frac{h^F - h^*}{h^*}$  from the GSOEP and NSCW datasets combined with our range of estimates of the ideal hours elasticity from the previous section to estimate the cost of adjustment frictions from Proposition 1. As discussed in Section 2.2, some values of the ideal hours elasticity  $\xi_{h^*}$  may be inconsistent with a given individual's choices of  $h^F$  and  $h^*$ ; hence, if a certain value of  $\xi_{h^*}$  falls below the lower bound specified in Proposition 2 for a given individual, we use the lower bound of  $\xi_{h^*}$  for that individual to instead approximate the cost of frictions (equivalently, we bound the cost of frictions from above using the bounds for  $\frac{\delta(h^F)}{wh^*}$  from Proposition 2).<sup>16</sup> Importantly, our analysis only estimates frictions

<sup>15</sup>We too find that the ideal hours elasticity is higher than the actual hours elasticity (our baseline actual hours elasticity is 0.08 - see Table 3 Column 1 - whereas our baseline ideal hours elasticity is 0.16); however, we do not have precision to say that these elasticities are statistically different.

<sup>16</sup>The careful reader may note that when the bounds from Proposition 2 bind, this is effectively a rejection of the joint hypothesis that a given elasticity is consistent with their choices and that  $\frac{d^3 \delta}{dh^3} \approx 0$ . To explore



for employed individuals: all of our results require us to know the hourly wage  $w$  and this is not easily observable for unemployed individuals. To begin with, we restrict our analysis to households with a single earner; we will address joint labor supply decisions in Section 4.2.1. Because our estimates of the Hicksian elasticity of ideal hours w.r.t. the marginal tax rate are imprecisely estimated, Figure 3 first illustrates how the mean and median cost of adjustment frictions (as a fraction of ideal income  $wh^*$ ) vary with the elasticity of ideal hours w.r.t. the tax rate, showing the range of elasticities that are in  $[0.01, 1]$ .<sup>17</sup>

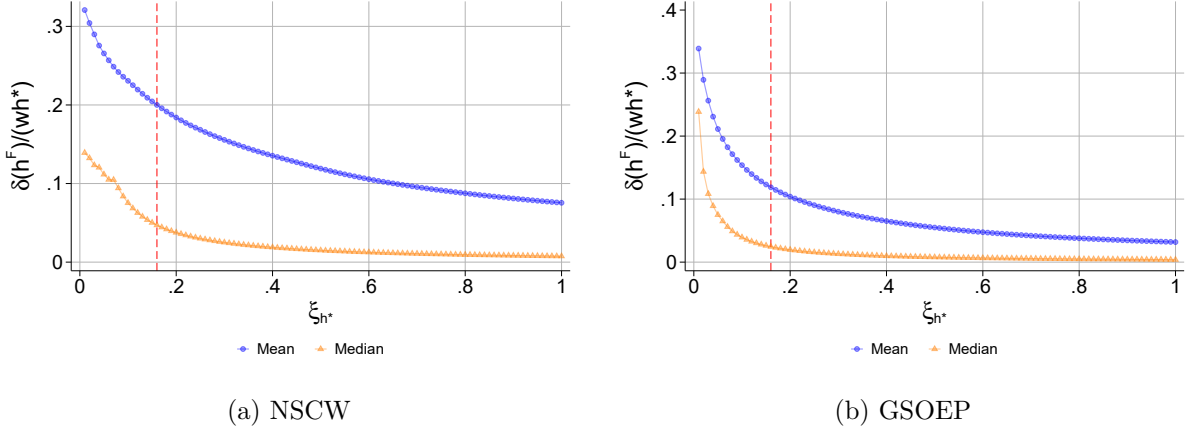


Figure 3: WTP to Remove Adjustment Frictions,  $\frac{\delta(h^F)}{wh^*}$ , vs. Ideal Hours Elasticity,  $\xi_{h^*}$   
*Notes:* This figure plots the average and median cost of adjustment frictions (as a fraction of ideal income) for single-earner households for different values of the ideal hours elasticity,  $\xi_{h^*}$ , using data on ideal hours,  $h^*$  and actual hours worked,  $h^F$ , from the NSCW (left) and the GSOEP (right). For all individuals, the cost of adjustment frictions are calculated via Proposition 1; if the assumed value of  $\xi_{h^*}$  violates the bounds from Proposition 2, then we use the lowest possible value of  $\xi_{h^*}$  for that person instead. The red vertical dashed line indicates our baseline estimate of  $\xi_{h^*} = 0.16$ .

Figure 3 illustrates that the cost of adjustment frictions is substantial: in the U.S., the average (median) WTP to remove adjustment frictions is 20% (4.3%) of income whereas in Germany the average (median) WTP to remove adjustment frictions is 12% (2.5%) of income assuming  $\xi_{h^*} = 0.16$ . Moreover, Figure 3 highlights the inverse relationship between the cost of frictions and the elasticity  $\xi_{h^*}$  because larger values of  $\xi_{h^*}$  imply there is less curvature in utility (recall Lemma 1). Note also that the mean cost of adjustment frictions is much larger than the median cost: this is because the cost of frictions cannot be less than 0 so the distribution of  $\frac{\delta(h^F)}{wh^*}$  is right skewed.<sup>18</sup> Figure 4 illustrates histograms of the cost of adjustment frictions  $\frac{\delta(h^F)}{wh^*}$  for

whether third order effects seem important, we illustrate in Figures 17 and 18 in Appendix C comparisons with exact calculation of frictions assuming quasi-linear isoelastic (QLIE) utility along with log consumption utility and power disutility of labor. Average frictions calculated using the second order approximation of Proposition 1 (applying the bounds on  $\xi_{h^*}$  from Proposition 2) are very similar to those calculated under both structural utility functions (our method appears to slightly *underestimate* frictions in both cases), suggesting that third order effects are not overly important.

<sup>17</sup>Figure 16 in Appendix C illustrates that the takeaways are almost entirely unchanged if we instead consider mean and median cost of adjustment frictions as a fraction of actual income  $wh$ .

<sup>18</sup>The mean being larger than the median is *not* driven by outliers: the cost of adjustment frictions is bounded at  $\approx 1$  by Proposition 2.



the NSCW and the GSOEP assuming the value of  $\xi_{h^*} = 0.16$  from our baseline specification: in both the NSCW and the GSOEP we see that there is a large fraction of people with no adjustment frictions (i.e., those with ideal hours equal to their actual hours) and the density is mostly monotonically decreasing. Figure 4 also shows how average frictions vary across the income distribution. In the U.S., we see that there is no clear relationship between the  $\frac{\delta(h^F)}{wh^*}$  and income: while the average low income person would prefer to work more and the average high income person would prefer to work less (see Figure 2), the cost of these frictions as a percentage of their income is not systematically different across the income distribution. In contrast, in Germany, we find a much clearer relationship between  $\frac{\delta(h^F)}{wh^*}$  and income: low income individuals face much larger frictions on average compared to middle and high income individuals.

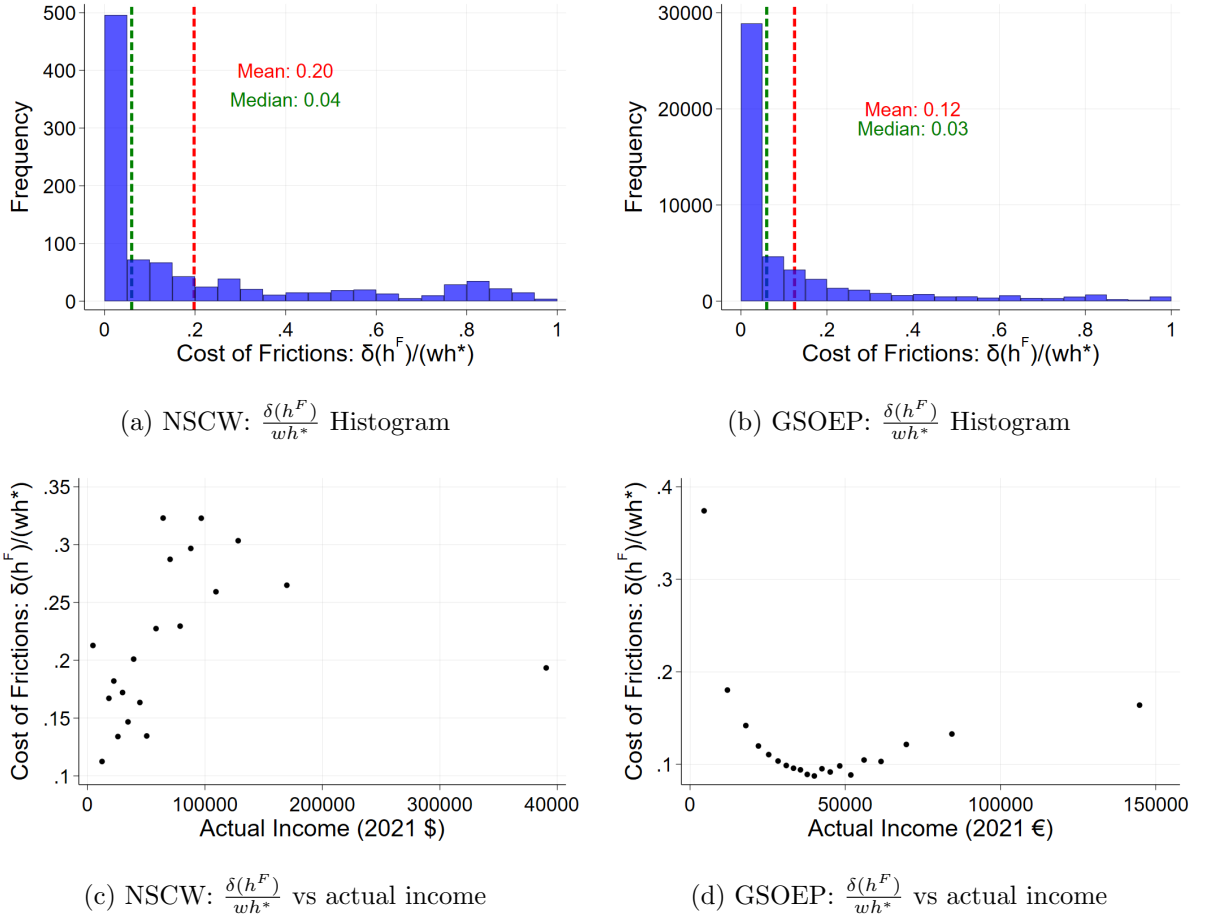


Figure 4: Cost of Adjustment Frictions  $\frac{\delta(h^F)}{wh^*}$

*Notes:* Panels a and b show the cost of adjustment frictions as a proportion of ideal income,  $\frac{\delta(h^F)}{wh^*}$ . For all individuals, the cost of adjustment frictions are calculated via Proposition 1 setting  $\xi_{h^*} = 0.16$ . However, if  $\xi_{h^*} = 0.16$  is lower than the revealed preference bounds from Proposition 2, we use the lower bound  $\xi_{h^*}$  for that individual. The red and green dashed vertical lines indicate the the mean and median values of  $\frac{\delta(h^F)}{wh^*}$ . Panels c and d show binned scatter plots of  $\frac{\delta(h^F)}{wh^*}$  vs. actual income (in 2021 currency).

## 4 Extensions

We now discuss how we can relax various implicit assumptions in Proposition 1 to understand the cost of frictions in more general environments.

### 4.1 Effort Per Hour Decisions (Endogenous Wages)

The model in Section 2.1 assumes that the only labor supply decision that individuals make is how many hours to work. We show how to recover the cost of frictions in a world with two labor supply decisions: hours worked and effort per hour.<sup>19</sup> Hence, in this model, the hourly wage rate is (partially) a choice variable. This second margin of adjustment can theoretically allow households to mitigate the cost of hours frictions by adjusting their effort per hour.

Consider the following labor supply model in which individuals have preferences over consumption,  $c$ , hours worked,  $h$ , and effort per hour,  $e$ . Individuals have a smooth utility function  $u(c, h, e)$  and face some non-linear tax schedule  $T(we)$  where  $w$  is their effort wage:

$$c = we - T(we) \quad (14)$$

We are now interested in understanding the cost of frictions,  $\delta(h^F, e^F)$ , which is the amount of money required to compensate an individual for frictions which require them to work  $(h^F, e^F)$  instead of their optimal choice  $(h^*, e^*)$ :

$$u(c(h^F, e^F) + \delta(h^F, e^F), h^F, e^F) = u(c(h^*, e^*), h^*, e^*) \quad (15)$$

Proposition 3 (proved in Appendix A.4) establishes that we can recover  $\delta(h^F, e^F)$  in terms of  $h^*$ ,  $h^F$ , and observable elasticities:

**Proposition 3.** *Suppose that there are no frictions in choosing effort per hour and let  $\chi_{z,h}$  represent the elasticity of observed income with respect to actual hours holding ideal hours fixed. Further, suppose the third derivatives of  $\delta(h^F, e^F)$  are  $\approx 0$ . Then:*

$$\frac{\delta(h^F, e^F)}{wh^*e^*} \approx (1 - T'(wh^*e^*)) \frac{\chi_{z,h}}{2\xi_{h^*}} \left( \frac{h^F - h^*}{h^*} \right)^2 \quad (16)$$

Note, the expression for  $\frac{\delta(h^F, e^F)}{wh^*e^*}$  from Proposition 3 is strikingly similar to the expression for  $\frac{\delta(h^F)}{wh^*}$  from Proposition 1: the only difference is that there is one extra term which captures the impacts of effort decisions on income. Note that  $\chi_{z,h}$  is equal to 1 if there is no effort decision so that Proposition 3 actually nests Proposition 1 as a special case. The core intuition is that, under the assumption that effort is chosen without frictions, we can infer how effort would change in absence of hours frictions via the parameter  $\chi_{z,h}$  which captures how effort

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<sup>19</sup>Examples of higher effort per hour (which lead to higher income) might be an Uber driver taking fewer breaks and thereby completing more trips per hour or a salaried worker increasing their effort to get a higher year-end bonus or promotion.

changes with exogenous changes in hours worked.<sup>20</sup>

#### 4.1.1 Calculating Cost of Frictions with Endogenous Wages

Using data from the NSCW and GSOEP, we explore how our estimates of the cost of frictions change if we incorporate endogenous wages as in Proposition 3. We repeat the exercise from Section 3.3 illustrating how the average and median cost of frictions varies with  $\xi_{h^*}$ . First, we need an estimate of  $\chi_{z,h} = \frac{\partial \log(whe(h))}{\partial \log(h)} \Big|_{h^*}$ . This parameter reflects how income varies if hours worked changes *exogenously* due to frictions. Fortunately, a number of papers have estimated this parameter. For example, Pencavel (2015) uses plausibly exogenous variation in hours worked for British munitions workers in the First World War to estimate the relationship between productivity and hours worked, estimating an average value of  $\chi_{z,h} = 0.8$  (see his Table 3); Collewet and Sauermann (2017) use exogenous variation in hours worked in call centers in the Netherlands to estimate a value of  $\chi_{z,h} \in [0.8, 0.9]$  (see their Table 3 which estimates  $1 - \chi_{z,h}$  to be between 0.11 and 0.18 depending on specification); and Aaronson and French (2004) use quasi-experimental variation generated by US social security rules to estimate  $\chi_{z,h} > 1$  ( $\chi_{z,h} \approx 1$ ) for men (women) nearing retirement age.  $\chi_{z,h} > 1$  will only increase our estimates for the WTP to remove adjustment frictions (see Equation 16); hence, we will set  $\chi_{z,h} = 0.8$  based on estimates from Pencavel (2015) and Collewet and Sauermann (2017).

Putting the pieces together, we form estimates for the cost of frictions in the NSCW and GSOEP from Proposition 3 setting  $\chi_{z,h} = 0.8$ . Analogously as in Proposition 2, with endogenous wages we apply a lower bound for the value of  $\xi_{h^*}$  consistent with each individual's choices of  $h^F$  and  $h^*$  based on revealed preference arguments (see Proposition 8 in Appendix B.3). Ultimately, accounting for effort decisions lowers mean frictions in the population by around 13% in the NSCW and 16% in the GSOEP (when  $\xi_{h^*} = 0.16$ ); see Figure 5.<sup>21</sup> The takeaway is that even if households can reduce the cost of hours frictions by substituting to supplying more effort per hour, the size of this effect empirically (measured by  $\chi_{z,h}$ ) is not large enough to drastically reduce our estimated cost of adjustment frictions.

## 4.2 Many Hours Worked Choices

The model in Section 2.1 assumed that there was only one hours worked decision. However, in reality, there are situations in which households make many hours decisions. For instance, one may consider (1) households where both spouses work, (2) households where a single individual works multiple jobs, or (3) households with a single individual that chooses how much to work throughout different portions of the day/week/year and has different disutility of work done

<sup>20</sup>Note that even when the effort per hour choice is frictionless,  $e^F$  will typically *not equal*  $e^*$  because optimal effort per hour is impacted by hours frictions.

<sup>21</sup>The value of  $\chi_{z,h}$  does not change the estimated cost of frictions exactly proportionally (as to be expected from Proposition 3) because the bounds in Proposition 8 are related in a non-linear way to the size of  $\chi_{z,h}$ .

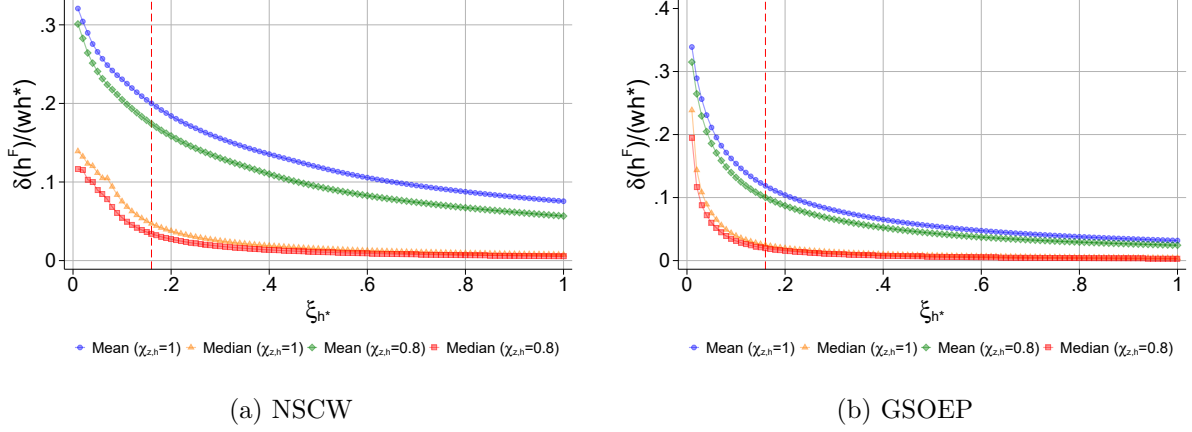


Figure 5: Cost of Adjustment Frictions with Endogenous Wages

*Notes:* This figure plots the average and median cost of adjustment frictions (as a fraction of ideal income) for single-earner households for different values of the ideal hours elasticity,  $\xi_{h^*}$ , using data on ideal hours,  $h^*$ , and actual hours worked,  $h^F$ , from the NSCW (left) and the GSOEP (right). For all individuals, the cost of adjustment frictions are calculated via Proposition 3 assuming a value of  $\chi_{z,h} = 1$  (see blue and orange lines; these are the same as Figure 3) and  $\chi_{z,h} = 0.8$  (see green and red lines); if the assumed value of  $\xi_{h^*}$  is lower than the revealed preference bounds from Proposition 8, we use the lower bound  $\xi_{h^*}$  for that individual. The red vertical dashed line indicates our preferred estimate of  $\xi_{h^*} = 0.16$ .

during different periods of time (Mas and Pallais, 2017). Appendix A.5 proves the following Proposition:

**Proposition 4.** *One can construct a second order approximation for the cost of frictions with many labor supply choices using the Slutsky matrix of own-price and cross-price compensated elasticities w.r.t. the after-tax wages and data on  $h_i^F$  and  $h_i^*$  for each hours choice  $i$ .*

In some situations it may be feasible to estimate the entire Slutsky matrix of ideal hours elasticities with respect to the (after-tax) wage rate of the given hours choice as well as with respect to the (after-tax) wage rate of all of the other hours choices. For instance, in a model of spousal labor supply choices, one would need exogenous variation in (after-tax) wage rates separately for each spouse; this might come from a tax reform in a country where couples are taxed individually rather than jointly. In other situations, however, recovering the full Slutsky matrix is almost certainly impossible. For instance, for an individual choosing how many hours to work on different days of the week, one would require separate variation in after-tax wage rates for each day of the week, which seems difficult to find quasi-experimentally.

This brings us to the following, arguably more useful result, which establishes that the logic of Proposition 1 can be used to provide a *lower bound* for the cost of frictions for households with multiple hours worked decisions using only a single elasticity of ideal income with respect to the tax rate. We denote the vector of labor supply decisions as  $\mathbf{h}$  and the cost of frictions as  $\delta(\mathbf{h}^F)$  and assume that taxes are a function solely of total income  $z = \sum_i w_i h_i$  where  $z^F \equiv \sum_i w_i h_i^F$  denotes observed income chosen with frictions and  $z^* \equiv \sum_i w_i h_i^*$  denotes ideal income:

**Proposition 5.** *Suppose that the third derivatives of  $\delta(\mathbf{h})$  are  $\approx 0$  and that utility is additively*

separable between  $c$  and  $\mathbf{h}$ . A lower bound for the cost of frictions expressed as a fraction of optimal income is given by:

$$\frac{\delta(\mathbf{h}^F)}{z^*} \geq (1 - T'(z^*)) \frac{\left(\frac{z^F - z^*}{z^*}\right)^2}{2\xi_{z^*}}$$

where  $\xi_{z^*}$  represents the Hicksian elasticity of  $z^*$  with respect to the marginal tax rate.

The intuition behind Proposition 5 (see the proof in Appendix A.6) is that the Hicksian elasticity of frictionless income  $z^*$  informs the curvature of utility with respect to ideal income assuming households optimize their labor supply decisions conditional on earning a given income (that is, assuming each  $h_i$  is chosen optimally subject to earning a total amount  $\sum_i w_i h_i$ ). Given that households who earn some amount  $z^F$  may *not* be optimizing each individual labor supply decision conditional on their earnings level, this forms a lower bound for the cost of frictions. In other words, Proposition 5 captures the cost of frictions which cause a household to deviate from frictionless income but does not capture the cost of frictions which lead to a sub-optimal allocation of hours worked given a total income level.

#### 4.2.1 Calculating Cost of Frictions for Dual-Earner Households

We now apply the results from the previous section to recover the cost of frictions for households with two earners (recall that our results in Section 3 were exclusively for households with a single earner). The NSCW only surveyed a single member of each household and therefore did not collect data on the ideal hours of work for both spouses; hence, all of the analysis in this section will only use the GSOEP. We implement Proposition 5 to recover a lower bound for the size of frictions for couples; Figure 6 shows how the lower bound estimate for the average and median size of frictions for two earner households varies with the parameter  $\xi_{z^*}$ . When  $\xi_{z^*} = 0.13$  (our point estimate of the ideal income elasticity from Column (7) of Table 1), the mean (median) WTP to remove adjustment frictions is 9% (3%) of pre-tax income.<sup>22</sup> Finally, implementing Proposition 4 to estimate the cost of frictions for dual-earners using the full Slutsky matrix is difficult in our setting because we do not have variation to identify the cross-price ideal hours elasticities (because all tax variation in Germany is at the household level). We nonetheless illustrate the cost of frictions using this method for plausible values of the relevant elasticities in Appendix B.4, finding that the cost of frictions is likely larger for dual-earner households than single earner households.

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<sup>22</sup>We still bound the values of  $\xi_{z^*}$  for each household using revealed preference arguments as Proposition 2 carries over immediately to bound  $\frac{\delta(z^F)}{z^*}$  defined in Appendix A.6.

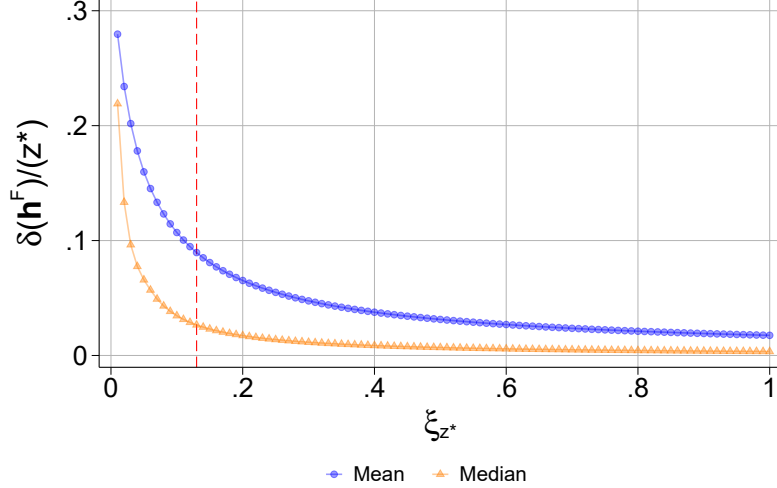


Figure 6: Lower Bound on  $\frac{\delta(h^F)}{z^*}$  for Dual-Earner Households in the GSOEP

*Notes:* This figure plots a lower bound for the average and median cost of adjustment frictions (as a fraction of ideal household income) for dual-earner households for different values of the ideal income elasticity,  $\xi_{z^*}$ , using data on ideal and actual hours from the GSOEP. For all households, the lower bound for the cost of adjustment frictions is calculated via Proposition 5; if the assumed value of  $\xi_{z^*}$  is lower than the revealed preference bounds from Proposition 2, we use the lower bound  $\xi_{z^*}$  for that household (note Proposition 2 carries over immediately to bound  $\frac{\delta(z^F)}{z^*}$  defined in Appendix A.6 and can therefore be used to bound the value of  $\xi_{z^*}$  for each household). The red vertical dashed line indicates our preferred estimate of  $\xi_{h^*} = 0.13$  from Column (7) in Table 1.

### 4.3 Dynamic Extension

In this section, we show how Proposition 1 extends to a dynamic setting. We consider a recursive formulation with two state variables: the wage in period  $t$ ,  $w_t$ , and the individual's wealth level in period  $t$ ,  $\kappa_t$ . There are two choice variables in each time period: how many hours to work  $h_t$  and how much to save  $\sigma_t$  subject to an interest rate  $r_t$ . Furthermore, we account for “career concerns” by allowing  $w_{t+1}$  to be a function of  $h_t$ . There may also be randomness in future wages or interest rates or preferences in the future. The individual's problem can be written as:

$$V(w_t, \kappa_t) = \max_{h_t} \left\{ \max_{\sigma_t} \{u(w_t h_t - T_t(w_t h_t) - \sigma_t, h_t) + \mathbb{E}[V(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t])]\} \right\} \quad (17)$$

Let us suppose that either there are no frictions in  $\sigma_t$  or that  $\sigma_t$  is at a corner solution and does not change with  $h_t$  (e.g., due to borrowing constraints). Let  $\sigma_t^*(h_t)$  denote the optimal level of savings for a given level of hours worked. Let us define  $\delta(h_t^F)$  as the WTP to remove frictions in hours worked in time period  $t$ :

$$\begin{aligned} & u(w_t h_t^F - T_t(w_t h_t^F) - \sigma_t^*(h_t^F) + \delta(h_t^F), h_t^F) + \mathbb{E}[V(w_{t+1}(h_t^F), r_t[\kappa_t + \sigma_t^*(h_t^F)])] \\ & = u(w_t h_t^* - T_t(w_t h_t^*) - \sigma_t^*(h_t^*), h_t^*) + \mathbb{E}[V(w_{t+1}(h_t^*), r_t[\kappa_t + \sigma_t^*(h_t^*)])] \end{aligned} \quad (18)$$

In Appendix A.7 we prove the following Proposition which establishes that with the properly defined elasticity, Proposition 1 still holds:

**Proposition 6.** Suppose that  $\frac{d^3 \delta}{dh_t^3} \approx 0$ . The WTP to remove frictions expressed as a fraction

of optimal income  $w_t h_t^*$  can be approximated as:

$$\frac{\delta(h_t^F)}{w_t h_t^*} \approx (1 - T_t'(w_t h_t^*)) \frac{\left(\frac{h_t^F - h_t^*}{h_t^*}\right)^2}{2\xi_{h_t^*}} \quad (19)$$

where  $\xi_{h_t^*}$  is the Hicksian elasticity of ideal hours in time period  $t$  w.r.t. the tax rate in time period  $t$  holding all future tax rates constant.<sup>23</sup>

There are a few things to note regarding Proposition 6. First, we do not assume anything about the continuation utility function  $V(w_{t+1}, \kappa_{t+1})$ . In particular, there may be frictions in future labor supply choices or future savings; hence, Proposition 6 recovers the cost of hours frictions in time period  $t$  *conditional* on future frictions. Second, the elasticity required,  $\xi_{h_t^*}$ , captures the compensated change in ideal hours in time period  $t$  for a change in marginal tax rates *in period  $t$* , holding constant *all future tax rates*. Hence, this elasticity is potentially difficult to estimate empirically not only because one requires data on ideal hours but also because one requires *temporary* variation in tax rates. Finally, let us discuss how Proposition 6 impacts the conclusions from Section 3. The relationships between the size of frictions and the elasticity in Figure 3 still hold (if we interpret the x-axis as now representing  $\xi_{h_t^*}$ ); however, to the extent that agents are not myopic, the elasticities estimated in Section 3.2 (which are estimated using permanent rather than temporary tax changes) are no longer the relevant elasticities for understanding the cost of frictions because these elasticities conflate hours changes today that result due to tax changes today with increases in hours worked today that result from tax changes in the future. The elasticities estimated in Section 3.2 are equal to  $\xi_{h_t^*}$  used in Proposition 6 if agents do not consider future tax changes when making labor supply decisions today.

## 5 Tax Misperceptions

Next, we explore how misperceptions impact our estimates of the cost of frictions. There are two key takeaways: (1) in the presence of misperceptions, the cost of adjustment friction estimates from Section 3 should be interpreted as the cost of adjustment frictions *conditional on misperceptions*, and (2) we can incorporate misperceptions of the tax schedule into a total cost of frictions inclusive of adjustment frictions. This exercise illustrates that the costs of adjustment frictions are much larger than tax misperceptions and that the cumulative cost of frictions is large *regardless* of the value of the Hicksian elasticity.

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<sup>23</sup>This is a Hicksian, rather than a Frisch, elasticity because the tax liability in period  $t$  (and future tax rates) are held constant.

## 5.1 Adjustment Frictions with Misperceptions

First, note that Proposition 1 does not require that the tax schedule in question is the true tax schedule. If the true tax schedule is given by  $T(wh)$  but agents misperceive the tax schedule as  $\hat{T}(wh)$ , then we can replace  $T$  with  $\hat{T}$  in Proposition 1 and interpret  $h^*$  as ideal hours conditional on misperceptions to recover the WTP to remove adjustment frictions conditional on misperceptions of the tax schedule.<sup>24</sup> Hence, if individuals misperceive the tax schedule when choosing how much to work and when answering the question about how much they would ideally like to work, our results from Section 3 recover how much individuals would be willing to pay to remove frictions conditional on existing misperceptions of the tax schedule.<sup>25</sup> The caveat is that when individuals misperceive the tax schedule, the relevant elasticity for Proposition 1 is now the Hicksian elasticity of ideal hours with respect to the *perceived* marginal tax rate holding the *perceived* tax level constant. Hence, unless individuals misperceive the existing tax schedule yet correctly perceive tax reforms, the elasticity estimates from Section 3.2 will no longer be relevant (but the relationships between the cost of frictions conditional on misperceptions and this unknown elasticity in Section 3 are otherwise still valid).

## 5.2 Cost of Adjustment Frictions and Misperceptions

But what if we want to estimate the cumulative, ex-post cost of both adjustment frictions and tax misperceptions? Going back to the baseline model of Proposition 1, suppose that an individual works  $h^F$  hours given their misperceptions of the tax schedule and their adjustment frictions. Let  $h^*$  denote ideal hours conditional on tax misperceptions (i.e., without any adjustment frictions). And let  $h^{**}$  denote ideal hours an individual would like to work if they faced neither adjustment frictions nor tax misperceptions. Thus, we now refer to  $h^*$  as ideal hours with misperceptions and  $h^{**}$  as ideal hours;  $h^F$  still denotes actual hours worked. Proposition 1 can be applied to recover the cumulative cost of frictions with the caveat that the relevant elasticity is now  $\xi_{h^{**}}$ , the Hicksian elasticity of  $h^{**}$  w.r.t. the marginal tax rate if the tax schedule was correctly perceived:

$$\frac{\delta(h^F)}{wh^{**}} \approx (1 - T'(wh^{**})) \frac{\left(\frac{h^F - h^{**}}{h^{**}}\right)^2}{2\xi_{h^{**}}} \quad (20)$$

Let us first discuss how we might recover  $h^{**}$  so as to gauge the cumulative cost of frictions. Let us assume that we have an estimate of  $h^*$ , which represents how many hours an individual would like to work in absence of adjustment frictions but conditional on tax misperceptions (this is presumably what one recovers from a survey such as the NSCW or GSOEP). So how can

<sup>24</sup>In the terminology of the behavioral economics literature, Proposition 1 can be used to recover the ex ante WTP to remove adjustment frictions under the decision utility function  $u(wh - \hat{T}(wh), h)$  rather than the ex post WTP to remove adjustment frictions under the experience utility function  $u(wh - T(wh), h)$ .

<sup>25</sup>Moreover, if individuals misperceive other elements of the environment, such as their utility function or their wage, the results from Section 3 capture the WTP to remove frictions conditional on these frictions as well.



we recover  $h^{**}$  from an estimate of  $h^*$ ? Using the discrete approximation to the Slutsky-esque Equation (12), the following characterizes (approximately) how  $\log(h^*) - \log(h^{**})$  is related to perceived tax rates:

$$\begin{aligned} \log(h^*) - \log(h^{**}) &\approx \xi_{h^{**}} [\log(1 - \hat{T}'(wh^{**})) - \log(1 - T'(wh^{**}))] \\ &+ \eta_{h^{**}} [\log(wh^{**} - \hat{T}(wh^{**})) - \log(wh^{**} - T(wh^{**}))] \end{aligned} \quad (21)$$

where  $\xi_{h^{**}}$  is the elasticity of ideal hours worked w.r.t. the marginal tax rate evaluated at  $h^{**}$  and  $\eta_{h^{**}}$  is the elasticity of ideal hours with respect to the after-tax income evaluated at  $h^{**}$ . Combining Equations (20) and (21) yields:

**Proposition 7.** *Suppose that  $\frac{d^3\delta}{dh^3} \approx 0$ . The WTP to remove frictions expressed as a fraction of optimal income  $wh^{**}$  can be approximated as:*

$$\frac{\delta(h^F)}{wh^{**}} \approx (1 - T'(wh^{**})) \frac{\left( \frac{h^F - h^* \left[ \frac{1-T'(wh^{**})}{1-\hat{T}'(wh^{**})} \right]^{\xi_{h^{**}}} \left[ \frac{wh^F - T(wh^{**})}{wh^F - \hat{T}(wh^{**})} \right]^{\eta_{h^{**}}} }{h^* \left[ \frac{1-T'(wh^{**})}{1-\hat{T}'(wh^{**})} \right]^{\xi_{h^{**}}} \left[ \frac{wh^F - T(wh^{**})}{wh^F - \hat{T}(wh^{**})} \right]^{\eta_{h^{**}}} } \right)^2}{2\xi_{h^{**}}} \quad (22)$$

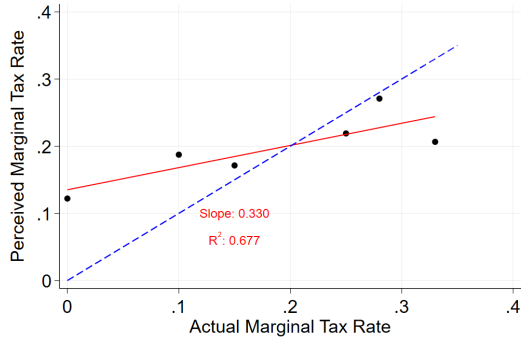
The key takeaway from Proposition 7 is that when there are both adjustment frictions and misperceptions, the relationship between the cost of frictions and  $\xi_{h^{**}}$  is now theoretically ambiguous. Intuitively, larger values of  $\xi_{h^{**}}$  still imply that there is less curvature in utility (so that the cumulative cost of frictions is smaller), but they now also imply that individuals have larger deviations from their optimal hours worked due to misperceptions (because people are more responsive to tax misperceptions). Hence, as  $\xi_{h^{**}}$  increases, the cost of adjustment frictions goes down but the cost of misperceptions may increase.

### 5.3 Data on Misperceptions

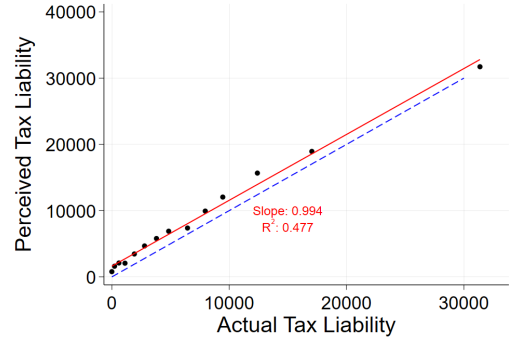
In order to gauge the impact of tax misperceptions via Proposition 7, we need data on both  $h^F$  and  $h^*$  along with the size of misperceptions in both the marginal tax rate and tax liability. Presumably, the measures of actual hours and ideal hours in the NSCW are both made conditional on any underlying misperceptions of the tax schedule; hence, we can interpret individuals' reported actual and ideal hours worked in the NSCW as being conditional on misperceptions. To gauge the magnitude of tax misperceptions, we use data from [Rees-Jones and Taubinsky \(2020\)](#) (RJT). RJT conducted an incentivized survey with a representative sample of 4,197 U.S. taxpayers to elicit perceptions of the U.S. income tax schedule. The survey asks respondents to assess the tax liability at their own income and at 15 other income levels.<sup>26</sup> Hence, the RJT survey provides data on individuals' perceptions of their own perceived tax liability  $\hat{T}$  and allows us to infer their perceptions about marginal tax rates from their misperceptions about how per-

<sup>26</sup> Respondents are asked to assess the tax liability for a fictional taxpayer "Fred" with income level  $z_F$ , but otherwise identical to themselves. Participants were informed they may receive a monetary compensation if their answers are within \$100 of the true tax liability.

ceived tax liability is changing with income. Precisely, we will use their perceived tax liability at the farthest income level that's in the same tax bracket to infer their perceived marginal tax rate (see Appendix B.5 for more discussion). Figure 7 plots binscatters of perceived marginal tax rates vs. actual marginal tax rates and perceived tax liability vs. actual tax liability. There are strong positive correlations between perceptions and actuality in both cases. Individuals with lower marginal rates tend to over-estimate their marginal rates while individuals with higher marginal rates tend to under-estimate their marginal rates. Most individuals seem to over-estimate their tax liability.



(a) Actual vs Perceived Mg Tax Rates



(b) Actual vs Perceived Tax Liability

Figure 7: Actual and Perceived Taxes

**Notes:** Each bin indicates the average perceived marginal tax rate (perceived tax liability) against the actual marginal tax rate (actual tax liability) in Panel a (b). The red line represents the best linear predictor of perceived marginal tax rates (perceived tax liability) given the actual marginal tax rate (actual tax liability). The dashed blue line represents the 45-degree line.

## 5.4 Estimating Cumulative Cost of Frictions

To gauge the cumulative cost of adjustment frictions and misperceptions, we need to “join” the NSCW data and the RJT data together. We now describe this data “join”. We start with the NSCW dataset from 2008 which has data on  $h^F$  and  $h^*$  (recall  $h^*$  represents ideal income without adjustment frictions but with misperceptions). For each individual, we calculate their rank in the income distribution from 1 to 100. Similarly, in the RJT data, which has information on  $\hat{T}'$  and  $\hat{T}$ , we calculate each individual’s rank in the income distribution from 1 to 100. Next, we perform a “join” of these two datasets to create a synthetic dataset of both adjustment frictions and tax misperceptions: we create a new dataset with all pairwise combinations of observations (one from each dataset) that have the same income percentile. There are two assumptions that we implicitly make when performing this exercise: (1) the distribution of tax misperceptions conditional on income rank has not changed from the 2008 NSCW to the RJT survey conducted in 2015, and (2) the size of tax misperceptions is independent of the size of adjustment frictions *conditional on income rank*. Under these two assumptions, the distribution of tax misperceptions faced by individuals at each income rank in the NSCW is

exactly the distribution of tax misperceptions in the RJT dataset; in this case, the joined dataset accurately captures the joint distribution of adjustment frictions and tax misperceptions.

With this joined dataset, we can apply Proposition 7 to gauge the cumulative cost of adjustment frictions and tax misperceptions as a function of  $\xi_{h^{**}}$ . We apply Proposition 2 (with  $h^*$  replaced by  $h^{**}$ ) when the assumed value of  $\xi_{h^{**}}$  violates the revealed preference bounds; in this case, we use the lowest possible value of  $\xi_{h^{**}}$  instead. Figure 8 displays how the mean and median size of frictions vary with the size of the unknown parameter  $\xi_{h^{**}}$ ; for comparison, Figure 8 also shows the cost of frictions assuming there are no misperceptions as in Figure 3a.<sup>27</sup>

There are two key takeaways from Figure 8. First, adjustment frictions appear to be much more costly for individuals than tax misperceptions: the cumulative cost of adjustment frictions and misperceptions is not drastically different from the cost of adjustment frictions on their own ignoring misperceptions, especially when  $\xi_{h^{**}}$  is small. To support this assertion more formally, we perform a decomposition exercise in Appendix B.6 which illustrates that the contribution of adjustment frictions to cumulative frictions is decreasing in  $\xi_{h^{**}}$  and that even when  $\xi_{h^{**}} = 1$ , adjustment frictions still contribute 81% of the cumulative cost of frictions. However, the cost of adjustment frictions and the cost of misperceptions move in opposite directions with  $\xi_{h^{**}}$ . Loosely, as  $\xi_{h^{**}}$  becomes larger, adjustment frictions get smaller (because there is less curvature in utility) but tax misperceptions lead to larger and larger deviations from ideal hours worked, creating larger costs of tax misperceptions. Hence, the second and most important takeaway from Figure 8 is that the cumulative cost of frictions appears large *no matter the value of  $\xi_{h^{**}}$* .<sup>29</sup>

<sup>27</sup>We assume that there are no income effects so that  $\eta = 0$ ; Figure 19 in Appendix C shows a corresponding plot when  $\eta = -0.5$ . When  $\eta \neq 0$ , the cost of frictions is slightly larger (regardless of whether  $\eta < 0$  or  $\eta > 0$ ).

<sup>28</sup>To simplify computations, we assume that individuals' misperceptions of their marginal tax rate at their ideal income level are equal to their misperceptions at their actual income level so that  $\log(1 - \hat{T}'(wh^{**})) - \log(1 - T'(wh^{**})) = \log(1 - \hat{T}'(wh^F)) - \log(1 - T'(wh^F))$ . In other words, we assume  $wh^{**}$  is in the same perceived tax bracket as  $wh^F$ .

<sup>29</sup>While we do not have individual level data on the size of tax misperceptions in Germany, Appendix B.7 explores the cumulative cost of frictions in the GSOEP under the assumption that all individuals misperceive their average tax rate as their marginal tax rate as suggested in De Bartolome (1995) or Rees-Jones and Taubinsky (2020). We find similar qualitative patterns as in the U.S: the mean cumulative cost of frictions is at least 9% of income regardless of the value of  $\xi_{h^{**}}$ .

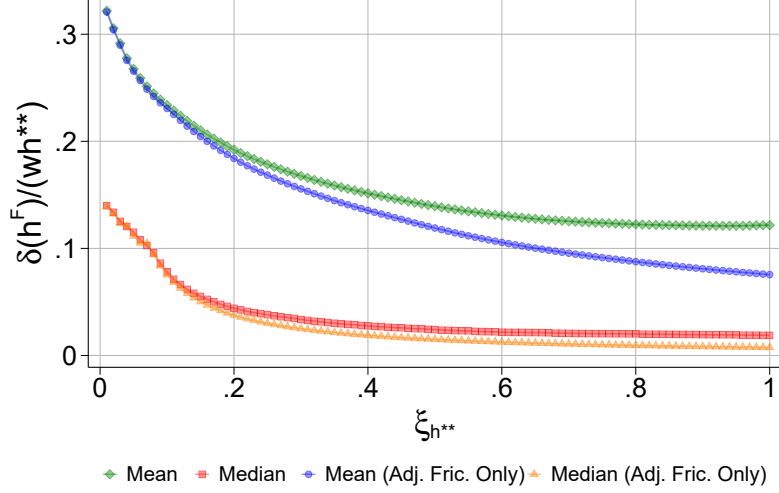


Figure 8: Cumulative Cost of Frictions

*Notes:* This figure plots the average and median cost of cumulative frictions (as a fraction of ideal income) for single-earner households for different values of the ideal hours elasticity,  $\xi_{h^{**}}$ , using data on ideal and actual hours worked from the NSCW and tax misperception data from [Rees-Jones and Taubinsky \(2020\)](#) (see green and red lines). Estimates of the cumulative cost of frictions are computed via Proposition 7; if the assumed value of  $\xi_{h^{**}}$  violates the bounds from Proposition 2, then we use the lowest possible value of  $\xi_{h^{**}}$  for that person instead. For comparison, we also display the cost of adjustment frictions assuming there are no misperceptions (or, equivalently, the cost of adjustment frictions conditional on misperceptions) from Figure 3a (shown in blue and orange).

## 6 Conclusion

This paper has developed a framework to estimate the cost of labor market frictions using data on ideal and actual hours of work along with the elasticity of ideal hours worked w.r.t. the tax rate. We have also shown how to extend this framework to account for endogenous wages, multiple labor supply decisions, dynamic decision making environments, and misperceptions of the tax schedule. Empirically, the core findings are that (1) the cost of adjustment frictions is large for any reasonable value of the Hicksian ideal hours elasticity, even when accounting for endogenous wages, multiple labor supply decisions, and dynamic decisions, (2) the cumulative cost of adjustment frictions and tax misperceptions is even larger: individuals would be willing to pay at least 10% of their income on average to remove these two frictions in hours worked, and (3) adjustment frictions appear to be much more costly than tax misperceptions. Moving forward, we believe our findings can offer guidance on (some of) the welfare impacts associated with policies that impact labor market frictions (e.g., policies that increase or decrease the availability of gig work, policies that reduce fixed costs of hiring employees, or government job guarantees). Our paper suggests that the welfare impacts from changing frictions could be substantial and also provides a framework to gauge the magnitude of these welfare impacts.

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## A Appendix: Proofs

### A.1 Proof of Equation (5)

*Proof.* Let us write the total derivative of Equation (2) (before substituting in  $h = h^*$  which yields Equation (3)):

$$\frac{d\delta}{dh}(h)u_c(c(h) + \delta(h), h) + u_c(c(h) + \delta(h), h)w(1 - T'(wh)) + u_h(c(h), h) = 0 \quad (23)$$

Totally differentiating Equation (23) and evaluating at  $h = h^*$ :

$$\frac{d^2\delta}{dh^2}\bigg|_{h^*} u_c(c(h^*), h^*) + \frac{du_c(c(h) + \delta(h), h)}{dh}\bigg|_{h^*} \frac{d\delta}{dh}\bigg|_{h^*} + \frac{d[u_c(c(h) + \delta(h), h)w(1 - T'(wh)) + u_h(c(h), h)]}{dh}\bigg|_{h^*} = 0$$

Recognizing that  $\frac{d\delta}{dh}\big|_{h^*} = 0$  and

$$\begin{aligned} & \frac{du_c(c(h), h)w(1 - T'(wh)) + u_h(c(h), h)}{dh}\bigg|_{h^*} \\ &= u_{cc}(c(h^*), h^*)w^2(1 - T'(wh^*))^2 - u_c(c(h^*), h^*)w^2T''(wh^*) \\ &+ 2u_{ch}(c(h^*), h^*)w(1 - T'(wh^*)) + u_{hh}(c(h^*), h^*) \end{aligned} \quad (24)$$

yields Equation (5) (omitting function arguments, noting everything is evaluated at  $h^*$ ).  $\square$

### A.2 Proof of Lemma 1

We first need to define the Hicksian elasticity when faced with a non-linear tax schedule as in [Jacquet et al. \(2013\)](#). First, consider how agents would respond if they faced no frictions to a non-linear tax change from  $T(wh)$  to  $T(wh) + \mu\tau(wh)$  (for some smooth perturbation direction  $\tau(wh)$ ). For an individual where  $T'(wh^*) + \mu\tau'(wh^*)$  exists at their optimal income level  $h^*$ , their FOC w.r.t.  $h$  under the perturbed tax schedule reads:

$$u_c(wh^* - T(wh^*) - \mu\tau(wh^*), h^*)w(1 - T'(wh^*) - \mu\tau'(wh^*)) + u_h(wh^* - T(wh^*) - \mu\tau(wh^*), h^*) = 0 \quad (25)$$

For individuals with a unique optimal  $h^*$  whose second order condition holds strictly, we can use the implicit function theorem to describe how their optimal income changes with  $\mu$  for any  $\tau(z)$ . Differentiating Equation (25) with respect to  $\mu$  and evaluating at  $\mu = 0$  yields:

$$\begin{aligned} \frac{\partial h^*}{\partial \mu} &= \frac{u_c w \tau'(z^*) + u_{cc} w (1 - T') \tau(z^*) + u_{ch} \tau(z^*)}{u_{cc} w^2 (1 - T')^2 - u_c w^2 T'' + 2u_{ch} w (1 - T') + u_{hh}} \\ &= \underbrace{\frac{u_c w}{d^2 u / dh^2} \tau'(z^*)}_{\text{Substitution Effect}} + \underbrace{\frac{u_{cc} w (1 - T') + u_{ch}}{d^2 u / dh^2} \tau(z^*)}_{\text{Income Effect}} \end{aligned} \quad (26)$$

where  $\frac{d^2 u}{dh^2} \equiv u_{cc} w^2 (1 - T')^2 - u_c w^2 T'' + 2u_{ch} w (1 - T') + u_{hh}$  represents the second (total) derivative of utility with respect to  $h$ . Equation (26) is essentially a non-linear version of the Slutsky equation and decomposes the behavioral impact of an arbitrary, non-linear tax change into the substitution effect of a change in the marginal tax rate (holding the tax level constant),



and the income effect of a change in the tax level (holding the marginal tax rate constant).

Dividing both sides by  $h^*$ , we can rewrite Equation (26) as:

$$\frac{\partial \log(h^*)}{\partial \mu} = \underbrace{\frac{u_c w}{-d^2 u / dh^2} \frac{1 - T'(wh^*)}{h^*}}_{\text{Substitution Elasticity, } \xi_{h^*}} \frac{-\tau'(wh^*)}{1 - T'(wh^*)} + \underbrace{\frac{wh^* - T(wh^*)}{h^*} \frac{u_{cc} w (1 - T') + u_{ch}}{-d^2 u / dh^2}}_{\text{Income Elasticity, } \eta_{h^*}} \frac{-\tau(wh^*)}{wh^* - T(wh^*)} \quad (27)$$

Equation (27) tells us that the behavioral impacts of small tax changes to a non-linear tax schedule can be decomposed as a substitution elasticity  $\xi_{h^*}$  multiplied by the percentage change in the keep rate (i.e., one minus the marginal tax rate) at optimal income  $wh^*$  as  $\frac{-\tau'(wh^*)}{1 - T'(wh^*)} = \frac{\partial \log(1 - T'(wh^*) - \mu \tau'(wh^*))}{\partial \mu} \big|_{\mu=0}$ , and an income elasticity parameter  $\eta_{h^*}$  multiplied by the percentage change in after tax income at optimal income  $wh^*$  as  $\frac{-\tau(z^*)}{z^* - T(z^*)} = \frac{\partial \log(wh^* - T(wh^*) - \mu \tau(wh^*))}{\partial \mu} \big|_{\mu=0}$ . Because  $\xi_{h^*}$  is the elasticity of frictionless hours worked with respect to the marginal tax rate holding the tax level constant, this is a Hicksian elasticity.<sup>30</sup>

Finally, using  $\frac{d^2 u}{dh^2} \equiv u_{cc} w^2 (1 - T')^2 - u_c w^2 T'' + 2u_{ch} w (1 - T') + u_{hh}$ , the definition of  $\xi_{h^*}$  from Equation (27), and Equation (5) yields the statement of the Lemma.

### A.3 Proof of Proposition 2

*Proof.* For individuals for whom  $h^* > h^F$ , we have that:

$$u(c(h^*), h^F) \geq u(c(h^*), h^*)$$

given that labor is presumed costly. Given that by definition:

$$u(c(h^F) + \delta(h^F), h^F) = u(c(h^*), h^*)$$

and

$$u(c(h^F), h^F) \leq u(c(h^*), h^*)$$

then the intermediate value theorem implies that  $\delta(h^F) \in [0, c(h^*) - c(h^F)] = [0, wh^* - T(wh^*) - (wh^F - T(wh^F))]$ .

For individuals with  $h^* < h^F$  we have that:

$$u(c(h^*), 0) \geq u(c(h^*), h^*)$$

given that labor is presumed costly. By definition:

$$u(c(0) + \delta(0), 0) = u(c(h^*), h^*)$$

and because  $u(c(0), 0) \leq u(c(h^*), h^*)$  by definition, the intermediate value theorem implies that  $\delta(0) \in [0, c(h^*) - c(0)] = [0, wh^* - T(wh^*) - (0 - T(0))]$ . By assumption  $\frac{d\delta}{dh}(h) \leq 0 \forall h < h^*$  so that  $\delta(0) \geq \delta(h^F)$  as  $h^F \geq 0$ .

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<sup>30</sup>Changing marginal tax rates while holding the level of taxes paid constant is equivalent to changing marginal tax rates while holding utility constant by the envelope theorem.

The final statement of the proof follows by noting that for sufficiently small  $\frac{d^3\delta}{dh^3}$  the cost of frictions from Proposition 1 must be smaller than the upper bounds given above:

$$\begin{aligned} \frac{wh^* - T(wh^*) - [wh^F - T(wh^F)]}{wh^*} &\geq \frac{(1 - T'(wh^*)) \left(\frac{h^F - h^*}{h^*}\right)^2}{2\xi_{h^*}} && \text{for } h^* > h^F \\ \frac{wh^* - T(wh^*) + T(0)}{wh^*} &\geq \frac{(1 - T'(wh^*)) \left(\frac{h^F - h^*}{h^*}\right)^2}{2\xi_{h^*}} && \text{for } h^* < h^F \end{aligned}$$

Rearranging the above expressions finishes the proof.  $\square$

#### A.4 Proof of Proposition 3

Towards showing how to (approximately) measure  $\delta(h^F, e^F)$  with the difference between  $h^F$  and  $h^*$  and a set of observable elasticities, we will first establish the following Lemma:

**Lemma 2.** *As long as all of the third derivatives of  $\delta$  w.r.t.  $h$  and  $e$  are small then the WTP to remove frictions expressed as a fraction of optimal income  $z^* = wh^*e^*$  can be approximated as:*

$$\frac{\delta(h^F, e^F)}{z^*} \approx -\frac{U_{hh}}{z^*u_c} \frac{(h^*)^2}{2} \left(\frac{h^F - h^*}{h^*}\right)^2 - \frac{U_{ee}}{z^*u_c} \frac{(e^*)^2}{2} \left(\frac{e^F - e^*}{e^*}\right)^2 - \frac{U_{eh}}{z^*u_c} e^* h^* \frac{e^F - e^*}{e^*} \frac{h^F - h^*}{h^*} \quad (28)$$

where  $U_{hh}$ ,  $U_{he}$ , and  $U_{ee}$  are second derivatives of the indirect utility function evaluated at the optimum  $(h^*, e^*)$ :

$$\begin{aligned} U_{hh} &= u_{cc}[(1 - T')we^*]^2 - u_c(we^*)^2 T'' + 2u_{ch}(1 - T')we^* + u_{hh} \\ U_{he} &= u_{cc}(1 - T')^2 w^2 e^* h^* - u_c w^2 e^* h^* T'' + u_{ce}(1 - T')we^* + u_{ch}(1 - T')wh^* + u_c(1 - T')w + u_{he} \\ U_{ee} &= u_{cc}[(1 - T')wh^*]^2 - u_c(wh^*)^2 T'' + 2u_{ce}(1 - T')wh^* + u_{ee} \end{aligned}$$

*Proof.* As in Section 2, let us first show how  $\delta$  changes as  $h$  and  $e$  deviate from  $h^*$  and  $e^*$  by applying the implicit function theorem to Equation (15) to calculate:

$$\left. \frac{\partial \delta}{\partial h} \right|_{h^*, e^*} = -\frac{u_c(c(h^*, e^*), h^*, e^*)we^*(1 - T') + u_h(c(h^*, e^*), h^*, e^*)}{u_c(c(h^*, e^*), h^*, e^*)} = 0 \quad (29)$$

$$\left. \frac{\partial \delta}{\partial e} \right|_{h^*, e^*} = -\frac{u_c(c(h^*, e^*), h^*, e^*)wh^*(1 - T') + u_e(c(h^*, e^*), h^*, e^*)}{u_c(c(h^*, e^*), h^*, e^*)} = 0 \quad (30)$$

where both expressions in Equations (29) and (30) equal 0 by the FOC w.r.t.  $h$  and  $e$  respectively evaluated at  $h^*$  and  $e^*$ . Noting that the numerators of the RHS of Equations (29) and (30) are the first derivatives of the indirect utility function w.r.t.  $h$  and  $e$ , respectively, we can rewrite as:

$$u_c \frac{\partial \delta}{\partial h} + U_h = 0 \quad (31)$$

$$u_c \frac{\partial \delta}{\partial e} + U_e = 0 \quad (32)$$

Differentiating Equations (31) and (32) and using the fact that  $\left. \frac{\partial \delta}{\partial h} \right|_{h^*, e^*} = \left. \frac{\partial \delta}{\partial e} \right|_{h^*, e^*} = 0$ , we get

that:

$$\frac{\partial^2 \delta}{\partial h^2} \Big|_{h^*, e^*} = -\frac{U_{hh}}{u_c} \quad (33)$$

$$\frac{\partial^2 \delta}{\partial e^2} \Big|_{h^*, e^*} = -\frac{U_{ee}}{u_c} \quad (34)$$

$$\frac{\partial^2 \delta}{\partial h \partial e} \Big|_{h^*, e^*} = -\frac{U_{eh}}{u_c} \quad (35)$$

As a result of Equations (29) and (30), we can use the fundamental theorem of calculus twice to deduce:

$$\begin{aligned} \delta(h^F, e^F) &= \int_{h^*}^{h^F} \frac{\partial \delta}{\partial h}(s, e^*) ds + \int_{e^*}^{e^F} \frac{\partial \delta}{\partial e}(h^F, s) ds \\ &= \int_{h^*}^{h^F} \int_{h^*}^s \frac{\partial^2 \delta}{\partial h^2}(t, e^*) dt ds + \int_{e^*}^{e^F} \left( \int_{e^*}^s \frac{\partial^2 \delta}{\partial e^2}(h^*, t) dt + \int_{h^*}^{h^F} \frac{\partial^2 \delta}{\partial h \partial e}(t, s) dt \right) ds \end{aligned} \quad (36)$$

If third derivatives of  $\delta$  are small then the second derivatives of  $\delta$  are  $\approx$  constant so that:

$$\begin{aligned} \delta(h^F, e^F) &\approx \frac{\partial^2 \delta}{\partial h^2} \Big|_{h^*, e^*} \int_{h^*}^{h^F} \int_{h^*}^s dt ds + \frac{\partial^2 \delta}{\partial e^2} \Big|_{h^*, e^*} \int_{e^*}^{e^F} \int_{e^*}^s dt ds + \frac{\partial^2 \delta}{\partial h \partial e} \Big|_{h^*, e^*} \int_{e^*}^{e^F} \int_{h^*}^{h^F} dt ds \\ &= -\frac{U_{hh}}{2u_c} (h^F - h^*)^2 - \frac{U_{ee}}{2u_c} (e^F - e^*)^2 - \frac{U_{eh}}{u_c} (e^F - e^*)(h^F - h^*) \end{aligned} \quad (37)$$

Dividing Equation (37) by  $z^*$  and multiplying the three terms by  $\frac{(h^*)^2}{(h^*)^2}$ ,  $\frac{(e^*)^2}{(e^*)^2}$ , and  $\frac{h^* e^*}{h^* e^*}$  (respectively) yields the statement of the Lemma.  $\square$

Next, using Lemma 2, we can prove Proposition 3:

*Proof.* The first step is to find expressions for the ideal hours elasticity w.r.t. the marginal tax rate,  $\xi_{h^*}$ , in terms of second derivatives of the utility function. Let us first consider utility as a function of an arbitrary non-linear tax schedule perturbation  $\tau(wh^*e^*)$ :

$$u(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*)$$

First, at the optimal bundle we know that the following two FOCs are satisfied (assuming the tax schedule is differentiable):

$$\begin{aligned} &u_c(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*)[1 - T'(wh^*e^*) - \epsilon\tau'(wh^*e^*)]we^* \\ &+ u_h(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*) = 0 \\ &u_c(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*)[1 - T'(wh^*e^*) - \epsilon\tau'(wh^*e^*)]wh^* \\ &+ u_e(wh^*e^* - T(wh^*e^*) - \epsilon\tau(wh^*e^*), h^*, e^*) = 0 \end{aligned} \quad (38)$$

Next, let us apply the implicit function theorem to the FOCs in (38) to calculate how optimal hours and optimal effort vary with an arbitrary tax reform in the direction of  $\tau(z)$ :  $\frac{\partial h^*}{\partial \epsilon}$  and  $\frac{\partial e^*}{\partial \epsilon}$ . Differentiating both equations w.r.t.  $\epsilon$  and evaluating at  $\epsilon = 0$  We have (omitting function

arguments for brevity, noting that everything is evaluated at  $(h^*, e^*)$ :

$$\begin{aligned} -u_c w e^* \tau' - u_{cc}(1 - T') w e^* \tau - u_{ch} \tau + U_{hh} \frac{\partial h^*}{\partial \epsilon} + U_{he} \frac{\partial e^*}{\partial \epsilon} &= 0 \\ -u_c w h^* \tau' - u_{cc}(1 - T') w h^* \tau - u_{ce} \tau + U_{eh} \frac{\partial h^*}{\partial \epsilon} + U_{ee} \frac{\partial e^*}{\partial \epsilon} &= 0 \end{aligned} \quad (39)$$

In the Proposition we assumed that we can observe the elasticity of  $h^*$  with respect to the marginal net of tax rate holding the tax level constant. For an individual with optimal income  $z^*$  this corresponds to the tax perturbation  $\tau(z)$  that decreases marginal rates at  $z^*$  but leaves the tax level unchanged:  $\tau(z) = (z^* - z)$  so that  $\tau'(z^*) = -1$  and  $\tau(z^*) = 0$ . So let us consider this perturbation, denoting:

$$\begin{aligned} \xi_{h^*} &\equiv \left. \frac{\partial h^*}{\partial \epsilon} \frac{1 - T'}{h^*} \right|_{\tau(z) = (z^* - z)} \\ \xi_{e^*} &\equiv \left. \frac{\partial e^*}{\partial \epsilon} \frac{1 - T'}{e^*} \right|_{\tau(z) = (z^* - z)} \end{aligned}$$

Hence, for the perturbation direction  $\tau(z) = (z^* - z)$  at  $z = z^*$ , we have:

$$\begin{aligned} u_c w e^* + U_{hh} \frac{h^*}{1 - T'} \xi_{h^*} + U_{he} \frac{e^*}{1 - T'} \xi_{e^*} &= 0 \\ u_c w h^* + U_{eh} \frac{h^*}{1 - T'} \xi_{h^*} + U_{ee} \frac{e^*}{1 - T'} \xi_{e^*} &= 0 \end{aligned} \quad (40)$$

From here, multiply the first equation in (40) by  $h^*$  and the second equation by  $e^*$  then divide both equations by  $z^* u_c$  to yield:

$$\begin{aligned} 1 + \frac{U_{hh}}{z^* u_c} \frac{(h^*)^2}{1 - T'} \xi_{h^*} + \frac{U_{he}}{z^* u_c} \frac{h^* e^*}{1 - T'} \xi_{e^*} &= 0 \\ 1 + \frac{U_{he}}{z^* u_c} \frac{h^* e^*}{1 - T'} \xi_{h^*} + \frac{U_{ee}}{z^* u_c} \frac{(e^*)^2}{1 - T'} \xi_{e^*} &= 0 \end{aligned} \quad (41)$$

Solving Equation (41), we have the following expression for  $\xi_{h^*}$ :

$$\xi_{h^*} = \frac{\frac{U_{he}}{z^* u_c} \frac{h^* e^*}{1 - T'} - \frac{U_{ee}}{z^* u_c} \frac{(e^*)^2}{1 - T'}}{\frac{U_{hh}}{z^* u_c} \frac{(h^*)^2}{1 - T'} - \left[ \frac{U_{he}}{z^* u_c} \frac{h^* e^*}{1 - T'} \right]^2} \quad (42)$$

Next, we will utilize the assumption that effort is chosen without any frictions. Specifically, we consider how the effort decision changes as hours change exogenously. First, the effort decision  $e$  always satisfies the following FOC for any given exogenously determined  $h$  because effort is chosen without any frictions:

$$u_c(whe - T(whe), h, e)(1 - T')wh + u_e(whe - T(whe), h, e) = 0$$

Applying the implicit function to the above FOC we have that:

$$\frac{de}{dh} = -\frac{U_{he}}{U_{ee}}$$

or in terms of elasticities:

$$\frac{d \log(e)}{d \log(h)} = \frac{de}{dh} \frac{h}{e} = -\frac{U_{he} h}{U_{ee} e} \quad (43)$$

Thus, let us consider how observed income changes with an exogenous shift in actual hours  $h$

as we move away from  $h^*$ , holding  $w$  and  $T(\cdot)$  (and therefore ideal hours  $h^*$ ) constant:

$$\chi_{z,h} \equiv \frac{d \log(z)}{d \log(h)} \Big|_{h^*} = \frac{d \log(w e(h) h)}{d \log(h)} \Big|_{h^*} = 1 + \frac{d \log(e)}{d \log(h)} \Big|_{h^*} = 1 - \frac{U_{he}/(z^* u_c) h^* e^*}{U_{ee}/(z^* u_c) (e^*)^2} \quad (44)$$

Next, we can take a Taylor series of  $\log(e(h))$  around  $\log(h^*)$  using Equation (43) where  $e(h)$  represents optimal  $e$  for any exogenously determined  $h$ :

$$\log(e) = \log(e^*) - \frac{U_{he}/(z^* u_c) h^* e^*}{U_{ee}/(z^* u_c) (e^*)^2} [\log(h) - \log(h^*)] \quad (45)$$

We can use Equation (45) to infer (a first order approximation for)  $\frac{e^F - e^*}{e^*}$  from  $\frac{h^F - h^*}{h^*}$ :

$$\begin{aligned} \frac{e^F - e^*}{e^*} &\approx \log(e(h^F)) - \log(e^*) = -\frac{U_{he}/(z^* u_c) h^* e^*}{U_{ee}/(z^* u_c) (e^*)^2} [\log(h^F) - \log(h^*)] \\ &\approx -\frac{U_{he}/(z^* u_c) h^* e^*}{U_{ee}/(z^* u_c) (e^*)^2} \frac{h^F - h^*}{h^*} = (\chi_{z,h} - 1) \frac{h^F - h^*}{h^*} \end{aligned} \quad (46)$$

Next, let us rearrange our expression for  $\frac{\delta(h^F, e^F)}{z^*}$  from Equation (28) slightly:

$$\begin{aligned} & -\frac{U_{hh}(h^*)^2}{2z^* u_c} \left( \frac{h^F - h^*}{h^*} \right)^2 - \frac{U_{ee}(e^*)^2}{2z^* u_c} \left( \frac{e^F - e^*}{e^*} \right)^2 - \frac{U_{eh} h^* e^*}{z^* u_c} \frac{e^F - e^*}{e^*} \frac{h^F - h^*}{h^*} \\ &= \left\{ -\frac{U_{hh}(h^*)^2}{2z^* u_c} - \frac{U_{ee}(e^*)^2}{2z^* u_c} (\chi_{z,h} - 1)^2 - \frac{U_{eh} h^* e^*}{z^* u_c} (\chi_{z,h} - 1) \right\} \left( \frac{h^F - h^*}{h^*} \right)^2 \\ &= \left\{ -\frac{U_{hh}(h^*)^2}{2z^* u_c} + \frac{U_{eh} h^* e^*}{2z^* u_c} (\chi_{z,h} - 1) - \frac{U_{eh} h^* e^*}{z^* u_c} (\chi_{z,h} - 1) \right\} \left( \frac{h^F - h^*}{h^*} \right)^2 \\ &= \left\{ -\frac{U_{hh}(h^*)^2}{2z^* u_c} - \frac{U_{eh} h^* e^*}{2z^* u_c} (\chi_{z,h} - 1) \right\} \left( \frac{h^F - h^*}{h^*} \right)^2 \end{aligned} \quad (47)$$

where the first equality substitutes in the Equation for  $\frac{e^F - e^*}{e^*}$  from Equation (46), the second equality uses the fact that  $(\chi_{z,h} - 1) = -\frac{U_{he}/(z^* u_c) h^* e^*}{U_{ee}/(z^* u_c) (e^*)^2}$ , and the third equality just adds terms together.

The final step is to use a bit of algebra to rearrange our expression for the ideal hours elasticity from Equation (42):

$$\begin{aligned} (1 - T') \frac{1}{\xi_{h^*}} &= \frac{\frac{U_{hh}(h^*)^2}{z^* u_c} \frac{U_{ee}(e^*)^2}{z^* u_c} - \left[ \frac{U_{he} h^* e^*}{z^* u_c} \right]^2}{\frac{U_{he} h^* e^*}{z^* u_c} - \frac{U_{ee}(e^*)^2}{z^* u_c}} \\ &= \frac{-\frac{U_{hh}(h^*)^2}{z^* u_c} - \frac{U_{he} h^* e^*}{z^* u_c} \left[ -\frac{U_{he} h^* e^*}{z^* u_c} / \frac{U_{ee}(e^*)^2}{z^* u_c} \right]}{\left[ -\frac{U_{he} h^* e^*}{z^* u_c} / \frac{U_{ee}(e^*)^2}{z^* u_c} \right] + 1} \\ &= \frac{-\frac{U_{hh}(h^*)^2}{z^* u_c} - \frac{U_{he} h^* e^*}{z^* u_c} (\chi_{z,h} - 1)}{\chi_{z,h}} \end{aligned} \quad (48)$$

Combining Equations (47) and (48), we get that:

$$\frac{\delta(h^F, e^F)}{z^*} \approx \left\{ -\frac{U_{hh}(h^*)^2}{2z^* u_c} - \frac{U_{eh} h^* e^*}{2z^* u_c} (\chi_{z,h} - 1) \right\} \left( \frac{h^F - h^*}{h^*} \right)^2 = (1 - T') \frac{\chi_{z,h}}{2\xi_{h^*}} \left( \frac{h^F - h^*}{h^*} \right)^2 \quad (49)$$

□

## A.5 Proof of Proposition 4

We show that Proposition 1 can be generalized to settings when individuals make multiple choices of hours worked (e.g., spousal labor supply) and consumption choices. Suppose that individuals have  $m$  hours choices  $h_1, h_2, \dots, h_m$  each with wage  $w_1, w_2, \dots, w_m$ . We assume that there is a linear tax rate  $T'_i$  that applies to earnings from source  $i$  and that individuals have some unearned income  $R$ .<sup>31</sup> Suppose individuals have a smooth utility function. The budget constraint for the individual is given by:

$$c = [(1 - T'_1)w_1h_1 + (1 - T'_2)w_2h_2 + \dots + (1 - T'_m)w_mh_m] + R$$

To condense notation a bit, let us denote the after-tax price of hours as:  $p_i \equiv w_i(1 - T'_i)$ . Defining  $c$  as a function of  $(h_1, h_2, \dots, h_m) \equiv \mathbf{h}$  via  $c(\mathbf{h}) = [p_1h_1 + p_2h_2 + \dots + p_mh_m] + R$  we are interested in the cost of frictions  $\delta(\mathbf{h}^F)$  defined as:

$$u(c(\mathbf{h}^F) + \delta(\mathbf{h}^F), h_1^F, \dots, h_m^F) = u(c^*(\mathbf{h}^*), h_1^*, \dots, h_m^*)$$

where  $\mathbf{h}^F = (h_1^F, h_2^F, \dots, h_m^F)$  denotes hours chosen under frictions. Identical fundamental theorem of calculus arguments as in the proof to Proposition 1 can be used to show that a second order approximation for  $\delta(\mathbf{h}^F)$  is given by:<sup>32</sup>

$$\delta(\mathbf{h}^F) \approx \frac{1}{2}(\mathbf{h}^F - \mathbf{h}^*)^T \frac{-\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)}(\mathbf{h}^F - \mathbf{h}^*) \quad (50)$$

where  $\mathbf{H}(\mathbf{h}^*)$  is the Hessian matrix of  $u(c(\mathbf{h}), h_1, \dots, h_m)$  with respect to  $\mathbf{h} = (h_1, h_2, \dots, h_m)$  evaluated at  $\mathbf{h}^* = (h_1^*, h_2^*, \dots, h_m^*)$ . Next, we establish that we can recover  $\frac{-\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)}$  from the matrix of (observable) compensated elasticities.

The system of individual first order conditions expressed in matrix notation is (recognizing that  $c = [p_1h_1 + p_2h_2 + \dots + p_mh_m] + R$ ):

$$\begin{aligned} U_{h_1} &\equiv u_c([p_1h_1^* + \dots + p_mh_m^*] + R, h_1^*, \dots, h_m^*)p_1 + u_{h_1}([p_1h_1^* + \dots + p_mh_m^*] + R, h_1^*, \dots, h_m^*) = 0 \\ U_{h_2} &\equiv u_c([p_1h_1^* + \dots + p_mh_m^*] + R, h_1^*, \dots, h_m^*)p_2 + u_{h_2}([p_1h_1^* + \dots + p_mh_m^*] + R, h_1^*, \dots, h_m^*) = 0 \\ &\vdots \\ U_{h_m} &\equiv u_c([p_1h_1^* + \dots + p_mh_m^*] + R, h_1^*, \dots, h_m^*)p_m + u_{h_m}([p_1h_1^* + \dots + p_mh_m^*] + R, h_1^*, \dots, h_m^*) = 0 \end{aligned} \quad (51)$$

Let us apply the implicit function theorem to System (51) to compute the (uncompensated)

<sup>31</sup>We assume that tax rates are constant solely to make the subsequent matrix expressions simpler but we can allow for a non-linear tax schedule exactly as in Proposition 1.

<sup>32</sup>Essentially, parametrize the line segment between  $\mathbf{h}^F$  and  $\mathbf{h}^*$ , apply Proposition 1 for this single parameter and then apply the multivariable chain rule; the proof is nearly identical to the proof of the the multivariable version of Taylor's Theorem.

Slutsky matrix:

$$\begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} & \frac{\partial h_1^*}{\partial p_2} & \cdots \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m^*}{\partial p_1} & \cdots & \frac{\partial h_m^*}{\partial p_m} \end{bmatrix} = - \begin{bmatrix} U_{h_1 p_1} & U_{h_1 p_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m p_1} & \cdots & U_{h_m p_m} \end{bmatrix} \quad (52)$$

where  $U_{h_i p_j}$  corresponds to the partial derivative of  $U_{h_i}$  with respect to  $p_j$ . Note that  $\mathbf{H}(\mathbf{h}^*)$  is given by:

$$\mathbf{H}(\mathbf{h}^*) = \begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix}$$

To compute the income effect parameter vector, we also apply the implicit function theorem, where  $U_{h_i R}$  is the partial derivative of  $U_{h_i}$  with respect to unearned income  $R$ :

$$\begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix} \begin{bmatrix} \frac{\partial h_1^*}{\partial R} \\ \vdots \\ \frac{\partial h_m^*}{\partial R} \end{bmatrix} = - \begin{bmatrix} U_{h_1 R} \\ \vdots \\ U_{h_m R} \end{bmatrix} \quad (53)$$

Next, we note that by direct computation one has the relationships:

$$\begin{aligned} U_{h_i p_i} &= u_c + U_{h_i R} h_i^* \\ U_{h_i p_j} &= U_{h_i R} h_j^* \\ U_{h_i R} &= u_{cc} p_i + u_{ch_i} \end{aligned}$$

Thus, we get the usual Slutsky equations (in vector form) that state, for example:

$$\begin{aligned} \frac{1}{h_1^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} | C \\ \frac{\partial h_2^*}{\partial p_1} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1} | C \end{bmatrix} &\equiv \frac{1}{h_1^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} \\ \frac{\partial h_2^*}{\partial p_1} \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1} \end{bmatrix} - \begin{bmatrix} \frac{\partial h_1^*}{\partial R} \\ \frac{\partial h_2^*}{\partial R} \\ \vdots \\ \frac{\partial h_m^*}{\partial R} \end{bmatrix} = - \begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix}^{-1} \begin{bmatrix} \frac{U_{h_1 p_1}}{h_1} - U_{h_1 R} \\ \vdots \\ \frac{U_{h_m p_1}}{h_1} - U_{h_m R} \end{bmatrix} \\ &= - \begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix}^{-1} \begin{bmatrix} \frac{u_c}{h_1^*} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{aligned} \quad (54)$$

The left hand side is just the vector of compensated (Hicksian) price effects. For clarity, the corresponding equation for the compensated price effects with respect to  $p_2$  equals:

$$\frac{1}{h_2^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_2} | C \\ \frac{\partial h_2^*}{\partial p_2} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2} | C \end{bmatrix} \equiv \frac{1}{h_2^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_2} \\ \frac{\partial h_2^*}{\partial p_2} \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2} \end{bmatrix} - \begin{bmatrix} \frac{\partial h_1^*}{\partial R} \\ \frac{\partial h_2^*}{\partial R} \\ \vdots \\ \frac{\partial h_m^*}{\partial R} \end{bmatrix} = - \begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{u_c}{h_2^*} \\ \vdots \\ 0 \end{bmatrix} \quad (55)$$

Next, consider the following matrix of compensated semi-elasticities (which are easily recover-

able from the matrix of elasticities):

$$\begin{aligned}
A &\equiv \begin{bmatrix} \frac{1}{h_1^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} | C \\ \frac{\partial h_2^*}{\partial p_1} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1} | C \end{bmatrix} & \frac{1}{h_2^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_2} | C \\ \frac{\partial h_2^*}{\partial p_2} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2} | C \end{bmatrix} & \cdots & \frac{1}{h_m^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_m} | C \\ \frac{\partial h_2^*}{\partial p_m} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_m} | C \end{bmatrix} \end{bmatrix} \\
&= - \begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix}^{-1} \begin{bmatrix} \frac{u_c}{h_1^*} & 0 & \cdots & 0 \\ 0 & \frac{u_c}{h_2^*} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{u_c}{h_m^*} \end{bmatrix}
\end{aligned} \tag{56}$$

Inverting both sides of the above equation:

$$A^{-1} = - \begin{bmatrix} \frac{h_1^*}{u_c} & 0 & \cdots & 0 \\ 0 & \frac{h_2^*}{u_c} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{h_m^*}{u_c} \end{bmatrix} \begin{bmatrix} U_{h_1 h_1} & U_{h_1 h_2} & \cdots \\ \vdots & \ddots & \vdots \\ U_{h_m h_1} & \cdots & U_{h_m h_m} \end{bmatrix} = \begin{bmatrix} h_1^* & 0 & \cdots & 0 \\ 0 & h_2^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_m^* \end{bmatrix} \frac{-\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)} \tag{57}$$

Yielding:

$$-\frac{\mathbf{H}(\mathbf{h}^*)}{u_c(\mathbf{h}^*)} = \begin{bmatrix} h_1^* & 0 & \cdots & 0 \\ 0 & h_2^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_m^* \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_1} | C \\ \frac{\partial h_2^*}{\partial p_1} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_1} | C \end{bmatrix} \frac{1}{h_1^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_2} | C \\ \frac{\partial h_2^*}{\partial p_2} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_2} | C \end{bmatrix} \frac{1}{h_2^*} \cdots \frac{1}{h_m^*} \begin{bmatrix} \frac{\partial h_1^*}{\partial p_m} | C \\ \frac{\partial h_2^*}{\partial p_m} | C \\ \vdots \\ \frac{\partial h_m^*}{\partial p_m} | C \end{bmatrix}^{-1} \tag{58}$$

Proposition 4 shows that we can recover the utility costs of frictions in substantially more general settings as long as we can observe the matrix of compensated price effects. While this is useful in situations where we can feasibly estimate the Slutsky matrix of all labor supply decisions, the chief limitation of Proposition 4 is that it requires estimation of a large set of statistics which may be difficult or impossible to estimate.

## A.6 Proof of Proposition 5

This section will show how we can construct a lower bound for the value of frictions when households have many labor supply decisions using “aggregate” elasticities. Consider the following labor supply model where household makes multiple labor supply decisions  $(h_1, \dots, h_j) = \mathbf{h}$  which are converted into labor income via a production function  $z(\mathbf{h}) = \sum_i w_i h_i$ . Individuals



are taxed according to a schedule  $T(z)$ . The household problem is thus:<sup>33</sup>

$$\begin{aligned} \max_{c, \mathbf{h}} \quad & \tilde{u}(c, \mathbf{h}) \\ \text{s.t.} \quad & c \leq z(\mathbf{h}) - T(z(\mathbf{h})) \end{aligned} \quad (59)$$

Towards constructing a lower bound for the cost of frictions, let us consider the following two-stage maximization problem:

$$\begin{aligned} \max_{c, z} \quad & u(c, z) \\ \text{s.t.} \quad & c \leq z - T(z) \end{aligned} \quad (60)$$

with

$$\begin{aligned} u(c, z) = \max_{c, \mathbf{h}} \quad & \tilde{u}(c, \mathbf{h}) \\ \text{s.t.} \quad & c \leq z - T(z) \text{ and } z(\mathbf{h}) = z \end{aligned} \quad (61)$$

Next, let us define the cost of frictions  $\delta(z^F)$  that lead households to earn income  $z^F$  rather than  $z^*$  assuming that hours are chosen optimally via sub-problem (61):

$$u(c(z^F) + \delta(z^F), z^F) = u(c(z^*), z^*)$$

We can immediately apply Proposition 1 (simply replacing  $h^F$  by  $z^F$  everywhere) to recover an approximation for  $\delta(z^F)$  as long as  $\frac{d^3\delta}{dz^3} \approx 0$  (which is implied by  $\frac{d^3\delta}{d\mathbf{h}^3} \approx 0$  because  $\frac{d^3\delta}{dz^3}$  is just a third directional derivative in the direction of a particular vector  $\mathbf{h}$ ):

$$\frac{\delta(z^F)}{z^*} \approx (1 - T'(z^*)) \frac{\left(\frac{z^F - z^*}{z^*}\right)^2}{2\xi_{z^*}} \quad (62)$$

where  $\xi_{z^*}$  is the elasticity of  $z^*$  with respect to the marginal tax rate, defined analogously as in Equation (27).

In actuality, individuals earning an income  $z^F$  due to frictions may also be constrained so that their hours choices conditional on an earnings level  $z^F$  may be inferior to their conditional optimum given by sub-problem (61). Intuitively, households may ideally prefer to earn the same income  $z^F$  with a different mix of labor supply. Hence, for any observed values of  $\mathbf{h}^F$  satisfying  $z(\mathbf{h}^F) = z^F$  (and therefore  $c(\mathbf{h}^F) = c(z^F)$ ) we will show that

$$\delta(\mathbf{h}^F) > \delta(z^F)$$

where

$$\tilde{u}(c(\mathbf{h}^F) + \delta(\mathbf{h}^F), \mathbf{h}^F) = \tilde{u}(c(\mathbf{h}^*), \mathbf{h}^*) = u(c(z^*), z^*)$$

as long as utility is additively separable with  $\tilde{u}(c, \mathbf{h}) = \tilde{u}_{(1)}(c) - \tilde{u}_{(2)}(\mathbf{h})$ . Under additive separability:

$$\delta(\mathbf{h}^F) = \tilde{u}_{(1)}^{-1} [u(c(z^*), z^*) + \tilde{u}_{(2)}(\mathbf{h}^F)] - c(\mathbf{h}^F)$$

---

<sup>33</sup>Note, if we can observe (and have exogenous variation in) the wages  $w_i$ , then we could apply the multidimensional procedure from Section 4.2 using the full set of (compensated) elasticities of all choice variables  $\mathbf{h}$  with respect to the full set of wages.

$$\delta(z^F) = \tilde{u}_{(1)}^{-1} [u(c(z^*), z^*) + \tilde{u}_{(2)}(\mathbf{h}(z^F))] - c(z^F)$$

where  $\mathbf{h}(z^F)$  represents  $\mathbf{h}$  chosen according to sub-problem (61). Hence, disutility of labor is greater under  $\mathbf{h}^F$  than  $\mathbf{h}(z^F)$  so  $\tilde{u}_{(2)}(\mathbf{h}(z^F)) < \tilde{u}_{(2)}(\mathbf{h}^F)$ . Recognizing that  $c(\mathbf{h}^F) = c(z^F)$  and  $\tilde{u}_{(1)}^{-1}$  is strictly increasing (as utility of consumption  $\tilde{u}_{(1)}$  is strictly increasing) we have that  $\delta(\mathbf{h}^F) > \delta(z^F)$ . Thus, we have the following lower bound for  $\delta(\mathbf{h}^F)$ :

$$\delta(\mathbf{h}^F) > (1 - T'(z^*)) \frac{\left(\frac{z^F - z^*}{z^*}\right)^2}{2\xi_{z^*}} \quad (63)$$

## A.7 Proof of Proposition 6

*Proof.* First, let us rewrite the maximization problem (17) with a perturbed tax schedule in period  $t$  of  $T_t(w_t h_t) + \epsilon \tau_t(w_t h_t)$ :

$$\max_{h_t} \left\{ \max_{\sigma_t} u(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t, h_t) + \mathbb{E}[V(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t])] \right\} \quad (64)$$

Next, let us solve the inner problem. Suppose for the time being that the solution  $\sigma_t^*(h_t)$  is interior and let  $u_1$  and  $V_2$  represent derivatives of  $u$  and  $V$  w.r.t. the first and second arguments:

$$-u_1(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V_2(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t^*(h_t)])] r_t = 0 \quad (65)$$

Hence, we can rewrite Equation (64) as follows:

$$\max_{h_t} u(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t^*(h_t)])] \quad (66)$$

Next, we have the first order condition for the outer problem which is satisfied when  $h_t = h_t^*$ :

$$\begin{aligned} & u_1(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) w_t (1 - T'_t - \epsilon \tau'_t) \\ & + u_2(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E} \left[ V_1(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t^*(h_t)]) \frac{dw_{t+1}}{dh_t} \right] \\ & + [-u_1(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V_2(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t^*(h_t)])]] r_t \frac{\partial \sigma_t^*}{\partial h_t} = 0 \end{aligned} \quad (67)$$

Next, let us consider how  $h_t$  changes when we perturb the tax schedule in the direction of the function  $\tau_t(w_t h_t) = (w_t h_t^* - w_t h_t)$  which decreases marginal tax rates at  $w_t h_t^*$  but does not change the tax level at  $w_t h_t^*$  (i.e., a compensated tax change). Next, we apply the implicit function theorem. Differentiating Equation (67) w.r.t.  $\epsilon$  with  $\tau_t(w_t h_t) = (w_t h_t^* - w_t h_t)$  and evaluating at  $\epsilon = 0$  and  $h_t = h_t^*$ , noting that  $\tau'_t(w_t h_t^*) = -1$  and  $\tau_t(w_t h_t^*) = 0$ , we have that:

$$u_1(w_t h_t^* - T_t(w_t h_t^*) - \sigma_t^*(h_t^*), h_t^*) w_t + U_{h_t h_t}(h_t^*) \frac{\partial h_t^*}{\partial \epsilon} = 0 \quad (68)$$

where  $U_{h_t h_t}(h_t^*)$  represents the second derivative  $u(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + \mathbb{E}[V(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t^*(h_t)])]$  w.r.t  $h_t$  evaluated at  $h_t^*$ . Note, we have used the fact that by the

FOC for  $\sigma_t$ :

$$[-u_1(w_t h_t - T_t(w_t h_t) - \epsilon \tau_t(w_t h_t) - \sigma_t^*(h_t), h_t) + r_t \mathbb{E}[V_2(w_{t+1}(h_t), r_t[\kappa_t + \sigma_t^*(h_t)])]] = 0$$

so that we can ignore calculating  $\frac{\partial}{\partial \epsilon} \left( \frac{\partial \sigma_t^*}{\partial h_t} \right)$  when differentiating Equation (67) w.r.t.  $\epsilon$ . From Equation (68) we have that:

$$\frac{-U_{h_t h_t}(h_t^*)}{u_1(h_t^*)} = w_t \left( \frac{\partial h_t^*}{\partial \epsilon} \right)^{-1} \equiv w_t \left( \xi_{h_t^*} \frac{h_t^*}{1 - T'} \right)^{-1} = \frac{w_t(1 - T')}{\xi_{h_t^*} h_t^*} \quad (69)$$

But what if  $\sigma_t^*(h_t)$  is not interior? In this case, we have that  $\frac{\partial \sigma_t^*}{\partial h_t} = 0$  as we assume either that  $\sigma_t^*(h_t)$  is interior or  $\sigma_t^*$  does not change with small changes in  $h_t$  so that Equation (69) still holds.

From here, it should be clear that because we define  $\delta(h_t^F)$  as:

$$\begin{aligned} & u(w_t h_t^F - T_t(w_t h_t^F) - \sigma_t^*(h_t^F) + \delta(h_t^F), h_t^F) + \mathbb{E}[V(w_{t+1}(h_t^F), r_t[\kappa_t + \sigma_t^*(h_t^F)])] \\ & = u(w_t h_t^* - T_t(w_t h_t^*) - \sigma_t^*(h_t^*), h_t^*) + \mathbb{E}[V(w_{t+1}(h_t^*), r_t[\kappa_t + \sigma_t^*(h_t^*)])] \end{aligned}$$

then the analogue of Equation (5) is that (this follows exactly by the logic of Appendix A.1):

$$\left. \frac{d^2 \delta}{dh^2} \right|_{h_t^*} = \frac{-U_{h_t h_t}(h_t^*)}{u_1(h_t^*)} \quad (70)$$

Putting Equations (69) and (70) together and applying the fundamental theorem of calculus exactly as in Proposition 1 we get Proposition 6.  $\square$

## B Appendix: Additional Results and Discussion

### B.1 German Tax Reforms

We will explore the impacts of tax changes on ideal hours worked in the context of a large German tax reform in 1996 that changed marginal tax rates for most taxpayers. Figure 9 shows the German marginal tax rate schedule in three different time periods. Before 1996, incomes below 5,626 Deutschemark (DM) were taxed at 0%, with a discontinuous jump in the marginal tax rate to 19% for incomes between 5,626 to 8,153 DM.<sup>34</sup> The marginal tax rate then increased linearly with income up to 120,041 DM before leveling out at a constant 53% marginal tax rate. In 1996, Germany underwent its only significant reform to the marginal rate schedule of the decade, nearly doubling the income threshold for 0% marginal taxes to 12,095 DM and increasing the marginal rate of the first non-zero bracket from 19% to  $\approx 26\%$ .<sup>35</sup>

Germany underwent another tax reform in 2001, illustrated by the green line in Figure 9. Prior to 2001, the tax rate increased linearly from 48.5% to 53% between incomes of 107,567 DM and 120,041 DM; in 2001 the tax schedule changed so that the top marginal tax rate remained

<sup>34</sup>Germany used the Deutschemark before 2002. The conversion factor in 2002 was 1.96 DM/Euro.

<sup>35</sup>There were very small adjustments to the linearly-increasing portion of the schedule in 1998 and 1999, and an additional (small) linearly-increasing tax bracket was also created in 1999.

constant at 48.5% starting at an income of 107,567 DM. There were also some small adjustments at the bottom of the income distribution, which are shown in the figure. When estimating the elasticity of  $h^*$  with respect to the marginal tax rate, we will leverage the variation in tax rates from 1995-2001 (and use pre-reform data from 1991-1995). Note, when estimating the elasticity, we only use data from 1991-2001 (inclusive) due to German reunification in 1990, and the introduction of the euro in 2002.

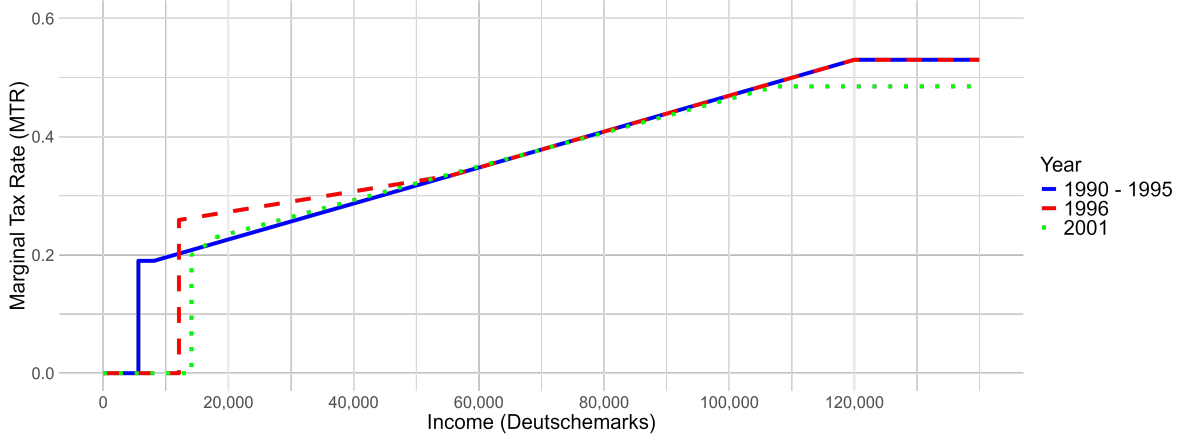


Figure 9: Marginal Tax Rate Schedule in Germany, 1991-2001

**Notes:** This figure displays the variation in German marginal tax rates over 1991-2001. The tax schedule was the same from 1991 to 1995, then was reformed in 1996. The tax schedule then underwent small reforms each year between 1998, 1999, and 2000. There was another reform in 2001 which was larger than the reforms in 1998, 1999, and 2000 but smaller than the reform in 1996.

## B.2 Deriving Equation (12)

We begin with Equation (27), which describes behavioral responses to arbitrary non-linear tax perturbations, reproduced below:

$$\frac{\partial \log(h^*)}{\partial \mu} = \underbrace{\frac{u_c w}{-d^2 u} \frac{1 - T'(wh^*)}{h^*}}_{\text{Substitution Elasticity, } \xi_{h^*}} \frac{-\tau'(wh^*)}{1 - T'(wh^*)} + \underbrace{\frac{wh^* - T(wh^*)}{h^*} \frac{u_{cc} w (1 - T') + u_{ch}}{-d^2 u}}_{\text{Income Elasticity, } \eta_{h^*}} \frac{-\tau(wh^*)}{wh^* - T(wh^*)}$$

Recall that for purposes of determining the willingness-to-pay to remove frictions, the relevant parameter is  $\xi_{h^*}$  which captures how  $h^*$  changes with the marginal tax rate, holding the tax level constant. Equation (27) implies that the impact of a small tax change to a non-linear tax schedule on  $h^*$  can be decomposed into a substitution elasticity  $\xi_{h^*}$  multiplied by the percentage change in the keep rate at optimal income  $wh^*$ ,  $\frac{-\tau'(wh^*)}{1 - T'(wh^*)} = \frac{\partial \log(1 - T'(wh^*) - \mu \tau'(wh^*))}{\partial \mu} \Big|_{\mu=0}$ , and an income elasticity parameter  $\eta_{h^*}$  multiplied by the percentage change in after tax income at the optimal income  $wh^*$ ,  $\frac{-\tau(z^*)}{z^* - T(z^*)} = \frac{\partial \log(wh^* - T(wh^*) - \mu \tau(wh^*))}{\partial \mu} \Big|_{\mu=0}$ . Hence, we have:

$$\frac{\partial \log(h^*)}{\partial \mu} = \xi_{h^*} \frac{\partial \log(1 - T'(wh^*) - \mu \tau'(wh^*))}{\partial \mu} + \eta_{h^*} \frac{\partial \log(wh^* - T(wh^*) - \mu \tau(wh^*))}{\partial \mu} \quad (71)$$

Approximating the derivatives in Equation (71) yields Equation (12). Importantly, note that the terms multiplying the substitution elasticity and the income elasticity are  $\frac{\partial \log(1 - T'(wh^*) - \mu \tau'(wh^*))}{\partial \mu}$

and  $\frac{\partial \log(wh^* - T(wh^*) - \mu\tau(wh^*))}{\partial \mu}$  which capture the *mechanical* changes to the keep rate and after-tax income that result from the tax perturbation, *holding decisions fixed*.

### B.3 Cost of Friction Bounds with Effort Decision

We can also bound the cost of frictions in a model with effort decisions, letting  $z^F \equiv wh^F e^F$ :

**Proposition 8.** *Suppose that there are no frictions in choosing effort per hour and that  $e(h)$  represents optimal effort for any given hours:*

$$\frac{\delta(h^F, e(h^F))}{z^*} \leq \frac{z^* - T(z^*) - [z^F - T(z^F)]}{z^*} \quad \text{for } h^* > h^F \quad (72)$$

$$\frac{\delta(h^F, e(h^F))}{z^*} \leq \frac{z^* - T(z^*) + T(0)}{z^*} \quad \text{for } h^* < h^F \quad (73)$$

where  $z^* \approx z^F \frac{h^*}{h^F} \frac{1}{1 + (\chi_{z,h} - 1) \frac{h^F - h^*}{h^*}}$  as long as:

- $\frac{du(c, h, e(h))}{dh} \leq 0 \forall c, h$  so that working more hours is costly, even accounting for changes in effort that occur concurrently (i.e., hours and effort per hour aren't strong substitutes).
- $\frac{d\delta(h, e(h))}{dh} \big|_h \leq 0 \forall h < h^*$  so that WTP to remove frictions is increasing as  $h$  gets farther away from  $h^*$ .

Finally, we have the following bounds on  $\xi_{h^*}$  assuming the third derivatives of  $\delta(h^F, e^F)$  are small:

$$\xi_{h^*} \leq \frac{\chi_{z,h}}{2} \frac{(1 - T'(wh^*e^*)) \left(\frac{h^F - h^*}{h^*}\right)^2}{\frac{z^* - T(z^*) - [z^F - T(z^F)]}{z^*}} \quad \text{for } h^* > h^F$$

$$\xi_{h^*} \leq \frac{\chi_{z,h}}{2} \frac{(1 - T'(wh^*e^*)) \left(\frac{h^F - h^*}{h^*}\right)^2}{\frac{z^* - T(z^*) + T(0)}{z^*}} \quad \text{for } h^* < h^F$$

*Proof.* For individuals for whom  $h^* > h^F$ , we have that:

$$u(c(h^*, e^*), h^F, e(h^F)) \geq u(c(h^*, e^*), h^*, e^*)$$

given that our first assumption ensures working more hours is costly (and  $e^* = e(h^*)$  because effort is chosen frictionlessly). Given that by definition:

$$u(c(h^F, e(h^F)) + \delta(h^F, e(h^F)), h^F, e(h^F)) = u(c(h^*, e^*), h^*, e^*)$$

and

$$u(c(h^F, e(h^F)), h^F, e(h^F)) \leq u(c(h^*, e^*), h^*, e^*)$$

then the intermediate value theorem implies that  $\delta(h^F, e(h^F)) \in [0, c(h^*, e^*) - c(h^F, e(h^F))] = [0, z^* - T(z^*) - (z^F - T(z^F))]$  where  $z^* = wh^*e^*$  and  $z^F = wh^Fe(h^F)$ .

For individuals with  $h^* < h^F$  we have that:

$$u(c(h^*, e^*), 0, 0) \geq u(c(h^*, e^*), h^*, e^*)$$

given that  $h$  and  $e$  are presumed costly. Given that by definition:

$$u(c(0, 0) + \delta(0, 0), 0, 0) = u(c(h^*, e^*), h^*, e^*)$$

and  $u(c(0, 0), 0, 0) \leq u(c(h^*, e^*), h^*, e^*)$  by definition, then the intermediate value theorem implies that  $\delta(0, 0) \in [0, c(h^*, e^*) - c(0, 0)] = (0, z^* - T(z^*) - (0 - T(0))]$ . By assumption  $\frac{d\delta(h, e(h))}{dh}|_h \leq 0 \forall h < h^*$  so that  $\delta(0, 0) \geq \delta(h^F, e(h^F))$  as  $h^F > 0$  and  $e(0) = 0$ .

Finally,  $z^* \approx z^F \frac{h^*}{h^F} \frac{1}{1 + (\chi_{z,h} - 1) \frac{h^F - h^*}{h^*}}$  follows from the fact that  $z^F = wh^F e^F$  and  $z^* = wh^* e^* = z^F \frac{h^*}{h^F} \frac{e^*}{e^F}$  and the approximation for  $\frac{e^*}{e^F}$  from Equation (46) derived in the proof to Proposition 3.

To conclude, we note that for sufficiently small third derivatives of  $\delta(h^F, e^F)$ , the cost of frictions from Proposition 3 must be smaller than the upper bounds given above:

$$\begin{aligned} (1 - T'(wh^* e^*)) \frac{\chi_{z,h}}{2\xi_{h^*}} \left( \frac{h^F - h^*}{h^*} \right)^2 &\leq \frac{z^* - T(z^*) - [z^F - T(z^F)]}{z^*} && \text{for } h^* > h^F \\ (1 - T'(wh^* e^*)) \frac{\chi_{z,h}}{2\xi_{h^*}} \left( \frac{h^F - h^*}{h^*} \right)^2 &\leq \frac{z^* - T(z^*) + T(0)}{z^*} && \text{for } h^* < h^F \end{aligned}$$

Rearranging the above completes the proof. □

The intuition is essentially the same as Proposition 2: for individuals who would ideally like to work more hours than they currently do, the WTP to remove frictions cannot be larger than the increase in your after tax income because labor supply is costly; for individuals who would ideally like to work fewer hours than they currently do, the WTP to remove frictions is bounded above by the WTP of moving from unemployment to her optimum, which is bounded by the consumption gain that she gets from working at her optimal hours relative to unemployment.

#### B.4 Implementing Proposition 4 Empirically

Implementing Proposition 4 to estimate the cost of frictions for dual-earners using the full Slutsky matrix is difficult in our setting because we do not have variation to identify the cross-price ideal hours elasticities (because all tax variation in Germany is at the household level). Furthermore, while numerous papers have estimated own and cross wage labor supply elasticities, all of these estimates (to the best of our knowledge) are for actual hours worked rather than ideal hours worked. We nonetheless illustrate the cost of frictions using this method for various values of the relevant elasticities, finding that the cost of frictions is likely larger for dual-earner households than single earner households. Figure 10 shows a heat map of how WTP to remove adjustment frictions varies jointly with the ideal hours elasticity of the primary earner and the

ideal hours elasticity of the secondary earner assuming the cross wage elasticity is 0.<sup>36</sup> If both of these elasticities are 0.2, then the average WTP to remove adjustment frictions is 22% of household income and the median WTP is 4%; even if both ideal hours elasticities are 1, the average WTP to remove adjustment frictions for couples is around 4% of household income (and the median is 1%). The takeaway is that frictions appear larger for couples than for single earner households as a percentage of household income unless the ideal hours elasticity for singles is much smaller than the elasticities for couples.

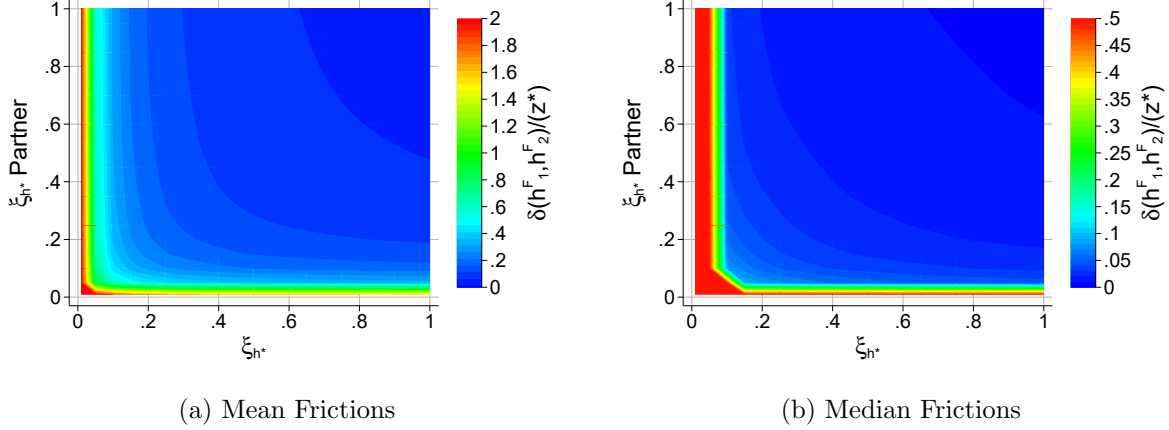


Figure 10:  $\delta(h_1^F, h_2^F)/(z^*)$  vs. Elasticities for Dual-Earner Households

*Notes:* This figure plots the average and median cost of adjustment frictions (as a fraction of ideal household income) for dual-earner households over different values of the ideal hours elasticities for both partners using data on ideal hours,  $h^*$ , and actual hours worked,  $h^F$ , from the GSOEP. The partner on the x-axis is the higher earning partner. For all households, the cost of adjustment frictions is calculated via Proposition 4; if the cost of frictions exceeds the sum of individual WTP bounds from Proposition 2, the bounds are applied.

## B.5 Inferring Perceived Tax Rates from Rees-Jones and Taubinsky (2020)

The survey data from Rees-Jones and Taubinsky (2020) asks individuals about their perceived tax liability but not their perceived marginal tax rates. To infer their perceptions about marginal tax rates, we use the fact that individuals were also asked about their perceptions of the tax liability at 15 different income levels other than their own. Hence, we can get an estimate of the perceived marginal tax rate between two income levels  $z_1$  and  $z_2$  with the approximation:

$$\hat{T}' = \frac{\hat{T}(z_2) - \hat{T}(z_1)}{z_2 - z_1}.$$

Letting  $z_1$  represent the individual's income, one may be inclined to use the  $z_2$  which is closest to  $z_1$  to infer the individual's perception of their marginal tax rate. However, this turns out to be problematic in this setting. If individuals make errors,  $\sigma_i$  in their assessment of  $\hat{T}(z_i)$ , then when  $|z_2 - z_1| \approx 0$ , then  $\hat{T}' = \frac{\hat{T}(z_2) - \hat{T}(z_1) + \sigma_2 - \sigma_1}{z_2 - z_1}$  can be significantly biased if  $\sigma_i \perp z_i$  (as the

<sup>36</sup>It turns out that both the mean and median WTP to remove frictions are mostly unimpacted by the size of the cross wage elasticity essentially because the correlation between  $(h_1^F - h_1^*)$  and  $(h_2^F - h_2^*)$  within a couple is quite small (0.15) and the impact of the cross price elasticity on the cost of frictions is directly related to the size of this correlation (this can be seen by expanding the second order approximation for the cost of adjustment frictions from the proof to Proposition 4 in Appendix A.5).



denominator gets very small but the errors do not become correspondingly small). In particular, if individuals are unsure about the tax level at  $z_1, z_2$  they may just guess  $T(z_1) = T(z_2)$  if  $z_1 \approx z_2$  heuristically; however, this is probably not a good indication of their perception of marginal tax rates and is rather a reflection of their uncertainty about tax liabilities more generally. Alternatively, individuals may “hedge” their answers by answering that  $T(z_2) = T(z_1) + 1000$  if  $z_1 \approx z_2$ , but this will lead to non-sensical tax rates above 100% if  $|z_2 - z_1| < 1000$ . Both of these biases seem to be quite common in the survey data of [Rees-Jones and Taubinsky \(2020\)](#): when using the closest  $z_2$  to each  $z_1$  to infer marginal tax rates, 20.25% of individuals have  $\hat{T}' < 0$  and 18% of individuals have  $\hat{T}' > 1$ .

Hence, there is a significant concern that using values of  $z_2$  that are very close to  $z_1$  will yield incorrect estimates of individual’s perceived marginal tax rates. In contrast, using values of  $z_2$  that are very far from  $z_1$  will conflate their perceptions about marginal tax rates and their perceptions about average tax rates. As a middle ground approach, we use the value of  $z_2$  that is farthest away from  $z_1$  yet still in the same tax bracket to infer their perceptions about marginal tax rates; this is what is shown in Figure 7 in the text. In contrast to using the closest  $z_2$  to each  $z_1$  to infer marginal tax rates, 9.6% of individuals have  $\hat{T}' < 0$  and 3.1% of individuals have  $\hat{T}' > 1$  when using the farthest  $z_2$  that is still in the same tax bracket as  $z_1$ .

## B.6 Decomposition: Adjustment Frictions vs. Misperceptions

From Equation (20), we can decompose the cost of frictions as follows by using the fact that  $h^F - h^{**} = (h^F - h^*) + (h^* - h^{**})$ :

$$\begin{aligned} \frac{\delta(h^F)}{wh^{**}} &\approx (1 - T') \frac{\left(\frac{h^F - h^{**}}{h^{**}}\right)^2}{2\xi_{h^{**}}} \\ &= \left[ \underbrace{\frac{1 - T'}{2\xi_{h^{**}}} \left(\frac{h^F - h^*}{h^{**}}\right)^2}_{\text{Adjustment Frictions}} + \underbrace{\frac{1 - T'}{2\xi_{h^{**}}} \left(\frac{h^* - h^{**}}{h^{**}}\right)^2}_{\text{Misperceptions}} + 2 \underbrace{\frac{1 - T'}{2\xi_{h^{**}}} \frac{h^F - h^*}{h^{**}} \frac{h^* - h^{**}}{h^{**}}}_{\text{Covariance}} \right] \end{aligned} \quad (74)$$

If our estimate of  $\frac{\delta(h^F)}{wh^{**}}$  comes from the bounds in Proposition 2, then we can decompose the impact of adjustment frictions vs. misperceptions using Equation (75) if  $h^{**} > h^F$  or Equation (76) if  $h^{**} < h^F$ :

$$\begin{aligned} \frac{\delta(h^F)}{wh^{**}} &= \frac{wh^{**} - T(wh^{**}) - [wh^F - T(wh^F)]}{wh^{**}} \\ &= \left[ \underbrace{\frac{wh^* - T(wh^*) - [wh^F - T(wh^F)]}{wh^{**}}}_{\text{Adjustment Frictions}} + \underbrace{\frac{wh^{**} - T(wh^{**}) - [wh^* - T(wh^*)]}{wh^{**}}}_{\text{Misperceptions}} \right] \end{aligned} \quad (75)$$

$$\begin{aligned}
\frac{\delta(h^F)}{wh^{**}} &= \frac{wh^{**} - T(wh^{**}) + T(0)}{wh^{**}} \\
&= \left[ \underbrace{\frac{wh^* - T(wh^*) + T(0)}{wh^{**}}}_{\text{Adjustment Frictions}} + \underbrace{\frac{wh^{**} - T(wh^{**}) - (wh^* - T(wh^*))}{wh^{**}}}_{\text{Misperceptions}} \right] \tag{76}
\end{aligned}$$

Using Equations (74), (75), and (76) we can decompose the cumulative cost of frictions into the contributions of the adjustment frictions, misperceptions, and their covariance. Figure 11 presents the results from this exercise setting  $\eta = 0$ . Overall, we see that at small values of  $\xi_{h^{**}}$ , misperceptions are essentially not contributing to the cumulative cost of frictions. Even at  $\xi_{h^{**}} = 1$ , adjustment frictions generate 80% of the cumulative cost of frictions while misperceptions generate the remaining 20%. The covariance has essentially no contribution to the cumulative cost of frictions because the correlation between  $\frac{h^F - h^*}{h^{**}}$  and  $\frac{h^* - h^{**}}{h^{**}}$  turns out to only be 0.01.<sup>37</sup>

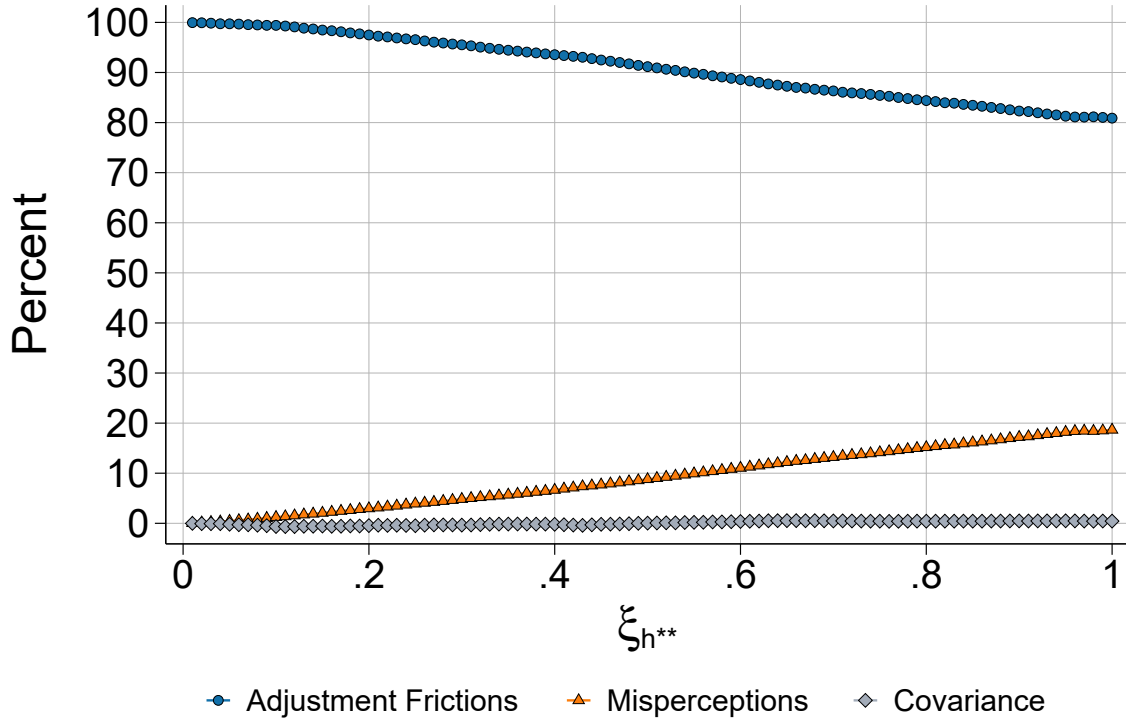


Figure 11: Decomposition of Cumulative Cost of Frictions

**Notes:** This figure illustrates our decomposition of the cumulative cost of frictions for single-earner households in the NSCW based on Equations (74), (75), and (76). Estimates of the cumulative cost of frictions are computed via Propositions 7 and 2.

<sup>37</sup>For comparison, if we set  $\eta = -0.5$ , then misperceptions contribute about 10% to the cumulative cost when  $\xi_{h^{**}} = 0.01$  (and the covariance contributes about -7%) and misperceptions contribute about 19% when  $\xi_{h^{**}} = 1$  (and the covariance contributes about -1%).

## B.7 Adjustment Frictions vs. Misperceptions in the GSOEP

We do not have data on the size of misperceptions in Germany. However, we nonetheless explore what the impact of misperceptions might look like in the context of Germany by exploring the possibility that all individuals misperceive their average tax rate as their marginal tax rate (both [De Bartolome \(1995\)](#) and [Rees-Jones and Taubinsky \(2020\)](#) suggest that  $\approx 50\%$  of taxpayers use this heuristic). Figure 12 shows the cumulative size of frictions if all individuals misperceive their average tax rate as their marginal tax rate in the GSOEP (and assuming they correctly perceive their total tax burden). Qualitatively, the takeaways from this exercise are the same as in the NSCW in Section 5.4: (1) adjustment frictions are larger than misperceptions (even when  $\xi_{h^*} = 1$ , adjustment frictions account for 60% of the cumulative cost) and (2) even though adjustment frictions become smaller with  $\xi_{h^*}$ , the cost of misperceptions goes up with  $\xi_{h^*}$  so that the cumulative cost of frictions is large no matter the value of  $\xi_{h^*}$ .

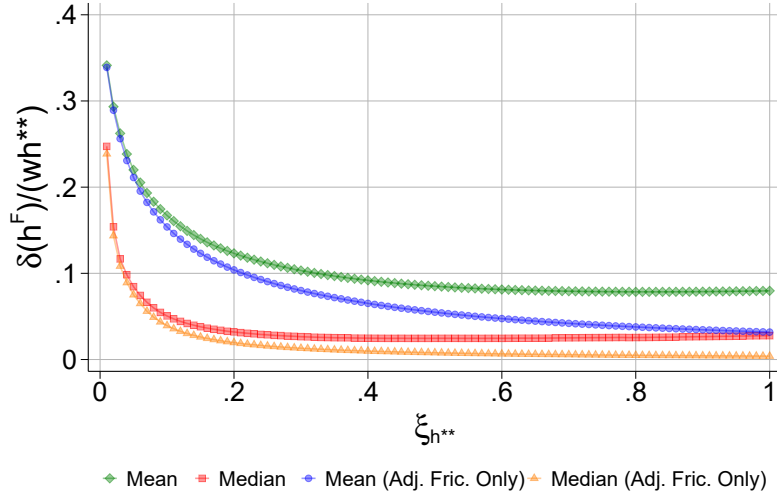


Figure 12: Cumulative Size of Frictions in the GSOEP

*Notes:* This figure illustrates the cumulative cost of frictions for single-earner households as a fraction of ideal income in the GSOEP accounting for both adjustment frictions and misperceptions (see green and red lines). We assume that all individuals perceive their marginal tax rate as their average tax rate, consistent with [De Bartolome \(1995\)](#) and [Rees-Jones and Taubinsky \(2020\)](#). Estimates of the cumulative cost of frictions are computed via Proposition 7; if the assumed elasticity violates the bounds in Proposition 2, then we use the lowest possible value of  $\xi_{h^{**}}$  for that person instead. For comparison, we also display the cost of adjustment frictions without misperceptions (or, equivalently, the cost of adjustment frictions conditional on misperceptions) from Figure 3b (shown in blue and orange).

## C Appendix: Figures and Tables

### C.1 Figures

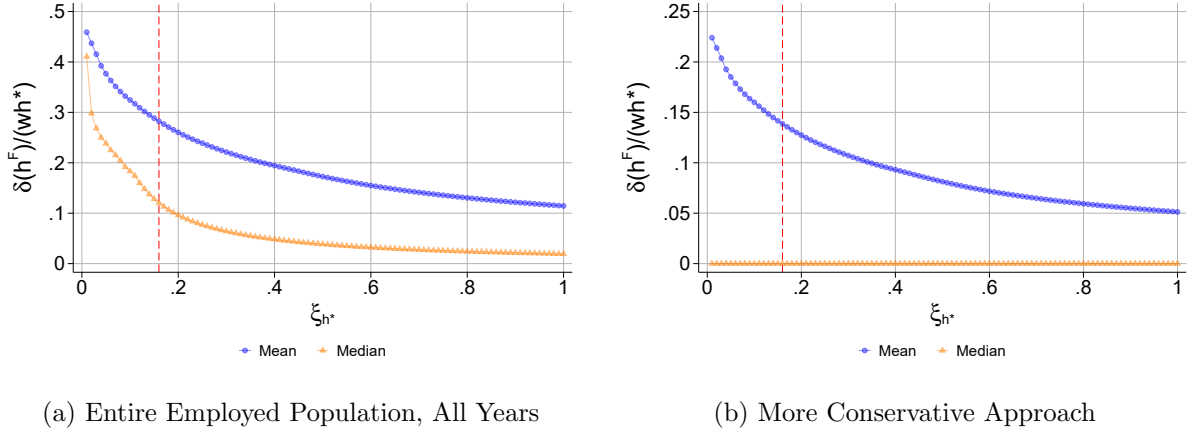


Figure 13: Figure 3a Under Different Sample Selections/Assumptions

*Notes:* This figure plots the average and median cost of adjustment frictions (as a fraction of ideal income) for single-earner households for different values of the ideal hours elasticity,  $\xi_{h^*}$ , using data on ideal hours,  $h^*$  and actual hours worked,  $h^F$  from the NSCW under two different sample selections and assumptions. Panel **a** uses the entire employed population from all four survey waves of the NSCW and assumes that all respondents interpret the question on ideal hours of work as being about hours worked under zero adjustment frictions. Panel **b** provides a much more conservative estimate of adjustment frictions by using only the 2008 NSCW and setting  $h^* = h^F$  for all individuals that do not interpret the question on ideal hours worked as being about the removal of adjustment frictions (these individuals are excluded from our sample in the main text). The cost of adjustment frictions are calculated via Proposition 1; if the assumed value of  $\xi_{h^*}$  violates the bounds from Proposition 2, then we use the lowest possible value of  $\xi_{h^*}$  for that person instead. The red vertical dashed line indicates our baseline estimate of  $\xi_{h^*} = 0.16$ .

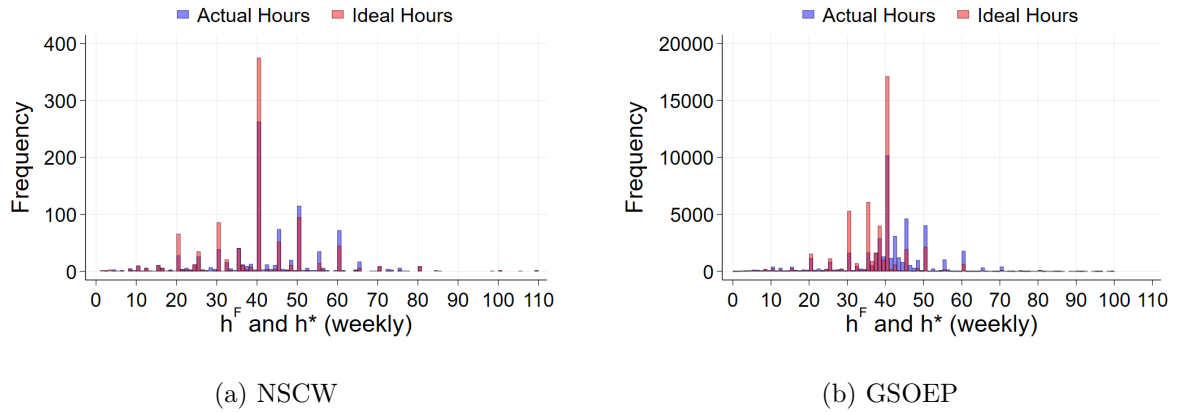
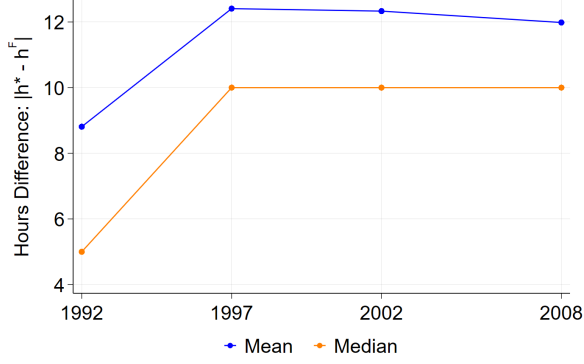
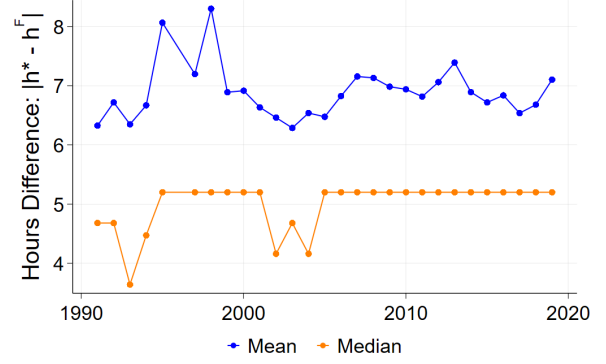


Figure 14:  $h$  vs.  $h^*$  Distributions

**Notes:** These histograms show the distributions of  $h^F$  and  $h^*$  for the 2008 NSCW included sample and the 1990-2019 GSOEP.



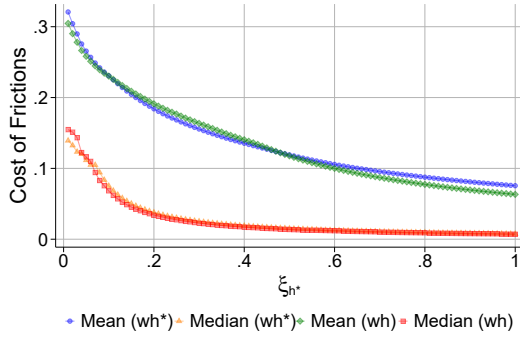
(a) NSCW



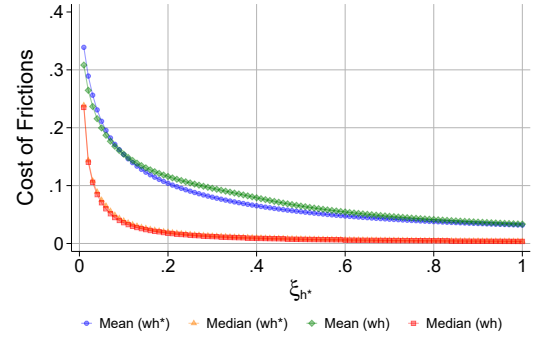
(b) GSOEP

Figure 15: Mean and Median  $|h^F - h^*|$  by Year

*Notes:* This figure shows the mean and median weekly hours difference between ideal and actual hours,  $|h^F - h^*|$ , by year for the entire employed population of the NSCW and the GSOEP.



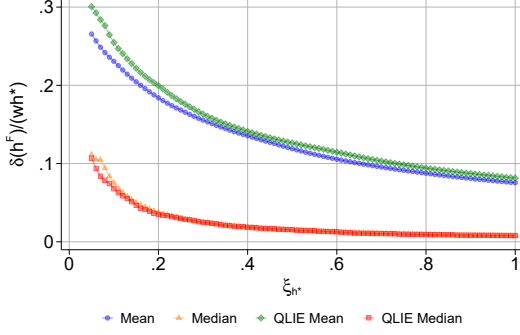
(a) NSCW



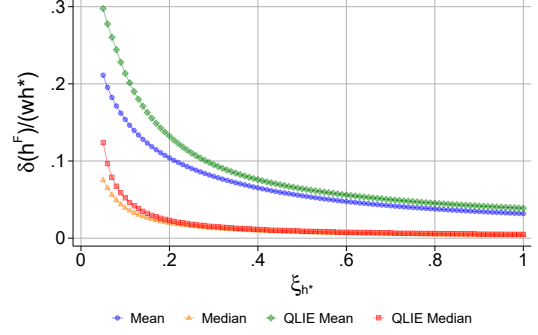
(b) GSOEP

Figure 16:  $\delta(h^F)/(wh)$  vs.  $\xi_{h^*}$

**Notes:** This figure plots the mean and median cost of adjustment frictions (as a fraction of actual income as well as ideal income) for single-earner households for different values of the ideal hours elasticity,  $\xi_{h^*}$ , using data on ideal hours,  $h^*$  and actual hours worked,  $h^F$ , from the NSCW (left) and the GSOEP (right). The legend entries indicate whether each line corresponds to adjustment frictions as a fraction of ideal income ( $wh^*$ ) or actual income ( $wh$ ).



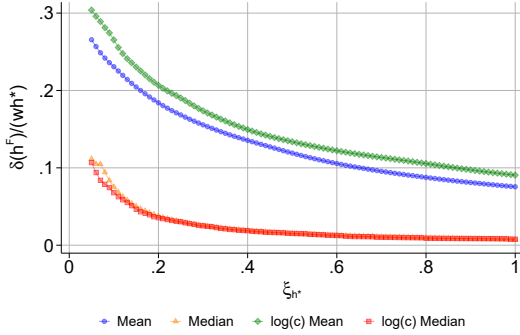
(a) NSCW



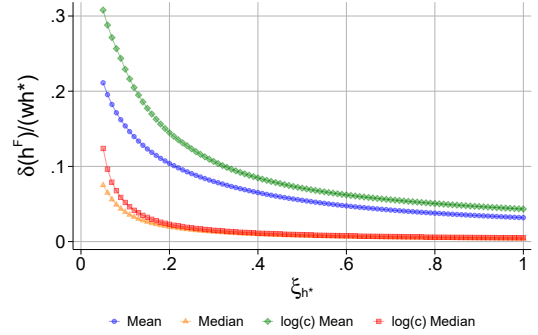
(b) GSOEP

Figure 17:  $\frac{\delta(h^F)}{wh^*}$  vs.  $\xi_{h^*}$  with Quasi-Linear Isoelastic Utility

*Notes:* This figure displays a comparison between the cost of frictions for single-earner households calculated using our second order approximation in Proposition 1 and the cost of frictions calculated assuming that utility is quasi-linear iso-elastic (QLIE) with elasticity  $\xi_{h^*}$ :  $u(c, h) = \alpha c - \frac{h^{1+1/\xi_{h^*}}}{1+1/\xi_{h^*}}$ . For each value of  $\xi_{h^*}$ , we calculate the WTP to remove frictions from Equation (2) where  $\alpha$  is pinned down from each individuals FOC evaluated at  $h^*$ . Note, we still apply Proposition 2 to bound  $\xi_{h^*}$  even when we assume QLIE utility because for some individuals the choice to work much more than their ideal hours rather than to not work may not be rationalizable under the assumed elasticity.



(a) NSCW



(b) GSOEP

Figure 18:  $\frac{\delta(h^F)}{wh^*}$  vs.  $\xi_{h^*}$  when  $u(c, h) = \alpha \log(c) - \frac{h^{1+1/k}}{1+1/k}$

*Notes:* This figure displays a comparison between the cost of frictions for single-earner households calculated using our second order approximation in Proposition 1 and the cost of frictions calculated assuming that utility is given by:  $u(c, h) = \alpha \log(c) - \frac{h^{1+1/k}}{1+1/k}$  where  $k$  is chosen so that the compensated elasticity equals  $\xi_{h^*}$ . For each value of  $\xi_{h^*}$ , we calculate the WTP to remove frictions from Equation (2) where  $\alpha$  is pinned down from each individuals FOC evaluated at  $h^*$ . Note, we still apply Proposition 2 to bound  $\xi_{h^*}$  even when we assume log consumption utility because for some individuals the choice to work much more than their ideal hours rather than to not work may not be rationalizable under the assumed elasticity.

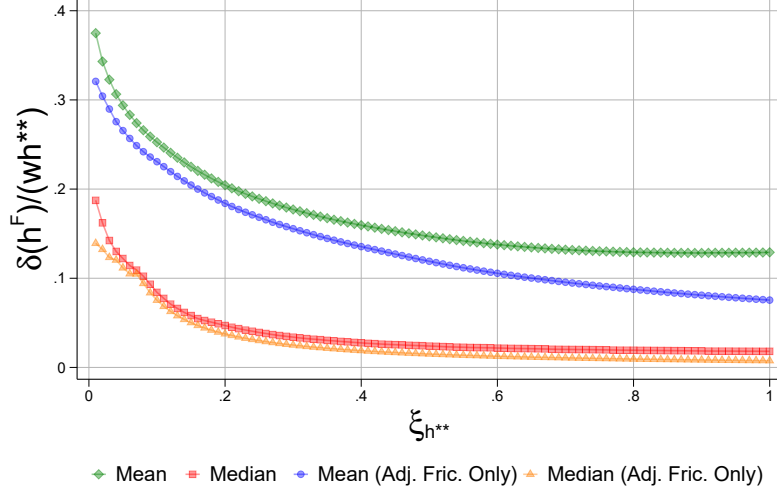


Figure 19: Cumulative Cost of Frictions,  $\eta = -0.5$

*Notes:* This figure plots the average and median cost of cumulative frictions (as a fraction of ideal income) for single-earner households for different values of the ideal hours elasticity,  $\xi_{h^{**}}$ , using data on ideal and actual hours worked from the NSCW and tax misperception data from [Rees-Jones and Taubinsky \(2020\)](#) (see green and red lines). Estimates of the cumulative cost of frictions are computed via Proposition 7 setting  $\eta = -0.5$  (this means that each exogenous 1% change in after-tax income leads to a 0.5% decrease in ideal hours (see Equation (12)); if the assumed value of  $\xi_{h^{**}}$  violates the bounds from Proposition 2, then we use the lowest possible value of  $\xi_{h^{**}}$  for that person instead. For comparison, we also display the cost of adjustment frictions without misperceptions (or, equivalently, the cost of adjustment frictions conditional on misperceptions) from Figure 3a (shown in blue and orange).

## C.2 Tables

Table 2: Why Individuals Do Not Work Their Ideal Hours in the NSCW

Reasons for not working fewer hours	Percent	Reasons for not working more hours	Percent
<b>I would lose my job</b>	18.15%	<b>Can't find job with more hours</b>	33.87%
I would not be as successful as I want	4.67%	<b>Can't find job that fits schedule</b>	9.22%
<b>Pressure from boss to work more</b>	1.21%	<b>Can't find flexibility in another job</b>	4.84%
<b>Pressure from coworkers to work more</b>	0.29%	Transportation issues	0%
Could not achieve own standards	7.2%	Childcare too expensive	0.69%
<b>Must work long hours to finish work</b>	14.63%	Children are too young for childcare	4.15%
I need the money	48.62%	Elder care responsibilities	1.15%
<b>Other</b>	15.26%	Other family responsibilities	4.38%
		Partner doesn't want me to work more	0%
		Personal health problems	4.15%
		<b>Other</b>	46.08%
<hr/>			
Individuals where $h = h^*$	814		
Individuals where $h < h^*$	438		
Individuals where $h > h^*$	1755		

*Notes:* This table displays responses to the question “Why don’t you work more hours per week?” or “Why don’t you work fewer hours per week?” (depending on whether ideal hours is more or less than actual hours) for individuals in the 2008 NSCW. Responses in bold text correspond to the individuals we include in our baseline analysis. Responses in non-bold text are those that we believe plausibly indicate that the individual either did not account for the fact that their income would change when determining their optimal hours or interpreted the question as also allowing them to change other circumstances in their life (e.g., their childcare options or their health status).



Table 3: Estimates of the Actual Hours Worked Elasticity w.r.t. the Marginal Tax Rate

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Baseline	Cubic	Linear	Time Trends	<50th	Singles	All Hhs	2 Yr Diff	4 Yr Diff
$\xi_h^*$	0.08 ( 0.09)	0.08 ( 0.08)	0.11 ( 0.09)	0.08 ( 0.09)	0.04 ( 0.10)	0.29 ( 0.15)	0.07 ( 0.08)	-0.01 ( 0.09)	0.15 ( 0.12)
$\eta_h^*$	0.21 ( 0.26)	0.18 ( 0.22)	0.50 ( 0.22)	0.25 ( 0.25)	-0.06 ( 0.34)	0.40 ( 0.40)	0.29 ( 0.23)	0.09 ( 0.25)	0.12 ( 0.27)
Obs.	4,328	4,344	4,344	4,328	2,392	2,496	6,229	5,935	3,082

*Notes:* Standard errors are clustered at the household level and are presented in parentheses. Each column presents estimates from regression (13) except the dependent variable is now the change in log hours worked for individual  $i$ ,  $\Delta \log h_{F,it}$ . Columns (1)-(7) use 3 year differences while Columns (8) and (9) use 2 and 4 year differences, respectively. All regressions are for years 1991-2001 inclusive, include year dummies, a married dummy, and restrict to individuals whose marital status did not change. All regressions are weighted using household survey weights. Column (1), (4), (5), (6), (7) include a spline in lagged log household income (i.e., lagged income decile dummies interacted with lagged income); Column (4) also includes lagged income interacted with a linear time trend. Instead of a spline, Column (2) includes a cubic polynomial in lagged log household income while Column (3) includes a linear polynomial in lagged log household income. All columns except (6) and (7) restrict to single-earner households. Column (6) restricts to single households only. Column (7) includes both single- and dual-earner households and the dependent variable for Column (7) is now wage weighted actual hours  $\Delta \log(w_i h_{it}^F + w_j h_{jt}^F)$  where  $j$  represents the second earner (if there is one) and the wages are equal to wages in time period  $t - 3$ . Finally, Column (5) restricts to households with lagged household income below the 50th percentile.

Table 4: Estimates of the Taxable Income Elasticity w.r.t. the Marginal Tax Rate

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Baseline	Cubic	Linear	Time Trends	<50th	Singles	All Hhs	2 Yr Diff	4 Yr Diff
$\xi_h^*$	0.34 ( 0.19)	0.36 ( 0.18)	0.54 ( 0.18)	0.27 ( 0.20)	0.33 ( 0.20)	0.67 ( 0.31)	0.19 ( 0.15)	0.62 ( 0.16)	0.50 ( 0.21)
$\eta_h^*$	-0.67 ( 0.38)	-0.38 ( 0.35)	0.93 ( 0.35)	-0.77 ( 0.39)	-0.55 ( 0.59)	-0.72 ( 0.61)	-0.48 ( 0.29)	0.16 ( 0.36)	0.21 ( 0.43)
Obs.	4,610	4,627	4,627	4,610	2,564	2,673	7,669	6,343	3,255

*Notes:* Standard errors are clustered at the household level and are presented in parentheses. Each column presents estimates from regression (13) except the dependent variable is now the change in log taxable income for individual  $i$ ,  $\Delta \log w_{it} h_{F,it}$ . Columns (1)-(7) use 3 year differences while Columns (8) and (9) use 2 and 4 year differences, respectively. All regressions are for years 1991-2001 inclusive, include year dummies, a married dummy, and restrict to individuals whose marital status did not change. All regressions are weighted using household survey weights. Column (1), (4), (5), (6), (7) include a spline in lagged log household income (i.e., lagged income decile dummies interacted with lagged income); Column (4) also includes lagged income interacted with a linear time trend. Instead of a spline, Column (2) includes a cubic polynomial in lagged log household income while Column (3) includes a linear polynomial in lagged log household income. All columns except (6) and (7) restrict to single-earner households. Column (6) restricts to single households only. Column (7) includes both single- and dual-earner households. Finally, Column (5) restricts to households with lagged household income below the 50th percentile.