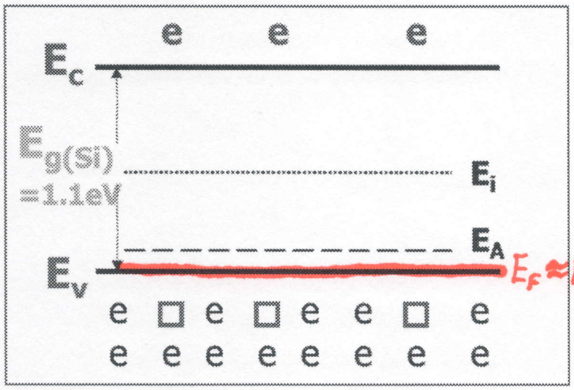
Problem 1: Energy Bands and Charge Carriers

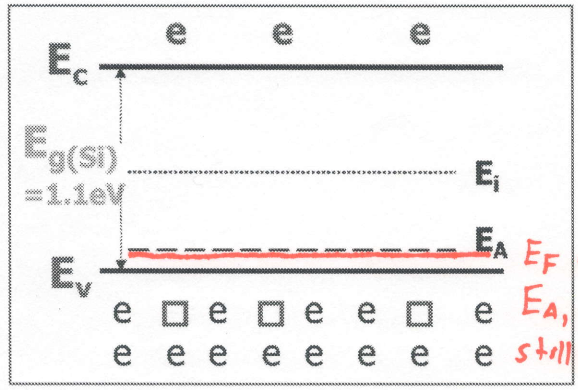
For a p-type Si semiconductor with an acceptor concentration $N_A = 10^{14}$ cm^{-3}, an accepter level 15 meV away from the valence band edge, and an intrinsic carrier concentration $n_i(300K) \sim 1.5 \times 10^{10}$ cm^{-3} and $n_i(500K) \sim 5 \times 10^{14}$ cm^{-3}:

Draw the Fermi level for the 4 different temperature regions a) 0K, b) 50K, c) 300K, and d) 500K:

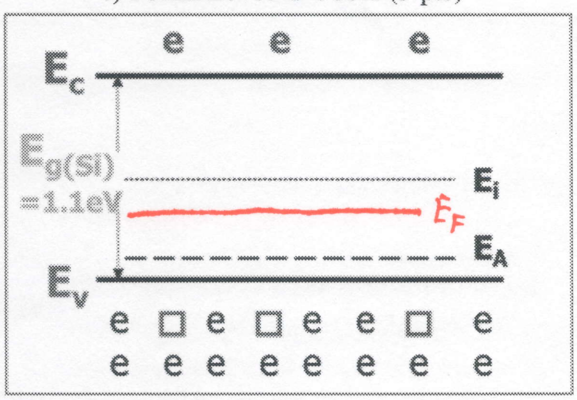
a) Fermi Level @ 0K (5 pts)



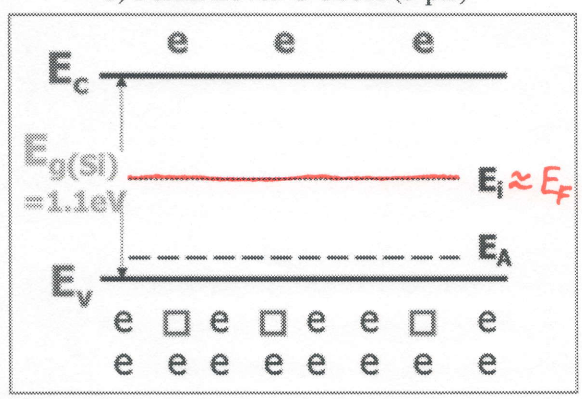
b) Fermi Level @ 50K (5 pts)



c) Fermi Level @ 300K (5 pts)



d) Fermi Level @ 500K (5 pts)



T = 0 K: E_F represents the top of the "electron sea" meaning that it should be placed at exactly the uppermost filled electron state, which is E_v.

T = 50 K: The temperature is still very low, though it is finite so some of the acceptors have ionized, meaning electrons now exist at E_A and holes in E_v. Since the Fermi function changes very rapidly around E_F, we know it must be close to E_A. It cannot be above EA since that would imply a significant number of acceptors are ionized, which is not the case.

T = 300 K: All acceptors ionized and $p_0 \gg n_i$ so E_F is in the middle (or close to) $E_i - E_v$

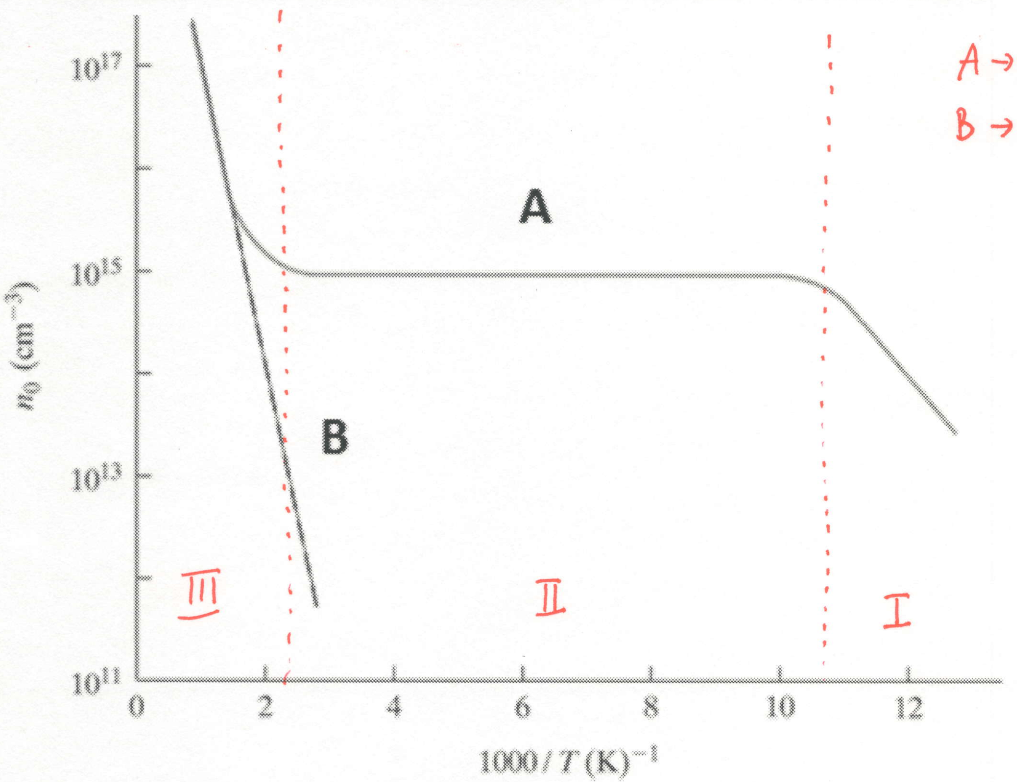
T = 500 K: $n_i > N_A^-$ so $E_F - E_i \ll \frac{E_g}{2}$, or $E_F \approx E_i$

Problem 2: Carrier Concentration and Space Charge Neutrality

a) What is the probability of an energy state at the Fermi level to be occupied by an electron at temperature T=0K? How about at 300K? (2 pts)

The Fermi level is defined as the energy level where the probability of electron occupancy is $\frac{1}{2}$ ($f(E_F) = \frac{1}{2}$). This is true for all temperatures.

b) The figure below shows the temperature dependence of the carrier centration for two silicon samples, "A" and "B." One of the samples is intrinsic, and the other sample is doped. Please identify which sample is intrinsic and which one is doped. (3 pts) Explain the behavior of sample "A." (5 pts)



A → doped

B → intrinsic

Region I (low temperatures):

At low T, donor states are only partially ionized, leading to a slow increase in n_0 with increasing T. Very little thermal generation is present.

Region II (medium temperatures):

All of the donor states are fully ionized, contributing to an electron concentration of $n_0 = N_D$. Since $N_D \gg n_i$ in this temperature range, the electron concentration behaves temperature-independent.

Region III (high temperatures):

Thermally generated carriers dominate over the donated electrons ($n_i > N_D$) and the sample resembles an intrinsic semiconductor.

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$N_D = 3\times10^{13}$ $N_A = 1\times10^{13}$

A silicon wafer is doped with $3\text{x}10^{13}$ cm^{-3} phosphorus and $1\text{x}10^{13}$ cm^{-3} boron. Assume the doping is uniform in the wafer and all the impurities are ionized.

1 fewer valence electron → p-type dopant

1 extra valence electron → n-type dopant

IIB	IIIA	IVA	VA	VIA
	5 B	6 C	7 N	8 O
	13 Al	14 Si	15 P	16 S
30 Zn	31 Ga	32 Ge	33 As	34 Se
48 Cd	49 In	50 Sn	51 Sb	52 Te

c) At 300K, the intrinsic carrier concentration of silicon is approximately $1\text{x}10^{10}$ cm^{-3}. Is this sample n type or p type? What is the majority carrier concentration? Show your work. (4 pts)

$N_D > N_A \rightarrow$ $\boxed{\text{n-type}}$

$N_D - N_A = 2\times10^{13} \gg n_i$

$n \simeq N_D - N_A = \boxed{2\times10^{13}\ cm^{-3}}$

d) At 410K, the intrinsic carrier concentration of silicon is approximately $1\text{x}10^{13}$ cm^{-3}. Is this sample n type or p type? Is the majority carrier concentration the same, larger, or smaller than the value calculated in c) above? Explain. (6 pts)

Since n_i is now on the same order of magnitude as N_D and N_A, we need to solve the quadratic equation:

$n + N_A^- = p + N_D^+$ ← charge neutrality

$p = \frac{n_i^2}{n}$ ← mass action law

$n + N_A^- = \frac{n_i^2}{n} + N_D^+$

$n^2 + (N_A^- - N_D^+)\, n - n_i^2 = 0$

$$n = \frac{-(N_A^- - N_D^+) \pm \sqrt{(N_A^- - N_D^+)^2 + 4 n_i^2}}{2}$$

$$= \frac{2\times10^{13} \pm \sqrt{(2\times10^{13})^2 + 4(10^{13})^2}}{2}$$ ← negative root leads to negative (nonphysical) carrier concentration

$= (1+\sqrt{2})\times10^{13}\ cm^{-3}$

Since $N_D > N_A$, the sample is still $\boxed{\text{n-type}}$.

Since there are more EHPs generated at higher T, both the majority & minority carrier concentrations have to be $\boxed{\text{larger}}$

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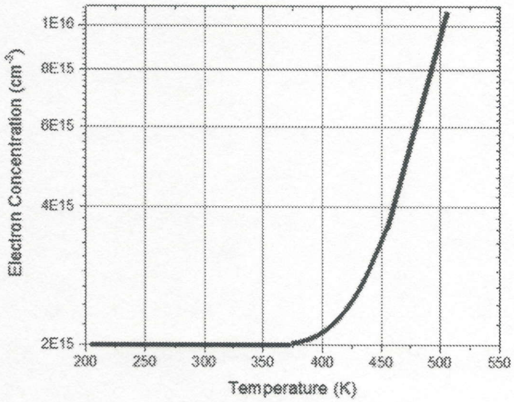
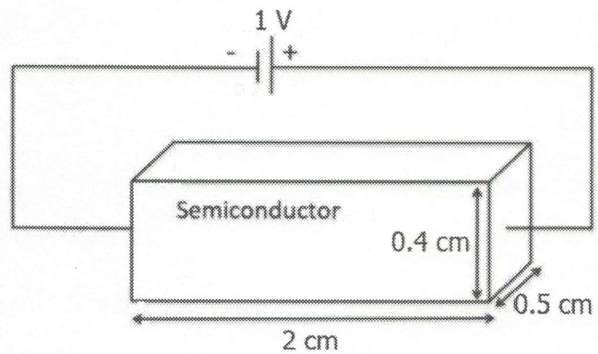
Problem 3: Drift and Resistance

The temperature dependence of the electron concentration in a semiconductor that is doped only with donors is shown below. A bias voltage is applied to a bar of this material as shown below. The electron and hole mobilities at 300K are 5000 and 3000 $cm^2/V\text{-sec}$, respectively. The electron and hole mobilities at 500K are 3000 and 2500 $cm^2/V\text{-sec}$, respectively. At 300K, assume complete donor ionization and an intrinsic carrier concentration of $10^{10}\ /cm^3$.





a) Why do both the electron and hole mobility decrease at high temperatures (~500K) relative to room temperature (~300K)? Explain briefly. (4 pts)

The reason the mobility decreases for both types of carriers is based on the notion of an increase in $\boxed{\text{scattering}}$ events. As temperature increases, the $\boxed{\text{lattice vibrations}}$ (atomic vibrations) increase, thereby increasing the likelihood of collisions between charged carriers, scattering off, and further affecting lattice vibrations. As scattering events increase, the mobility reduces for a free carrier.

b) Find the current flowing through the sample at 300K. (6 pts)

$$J = \sigma \vec{E} = q(n \cdot \mu_n + p \cdot \mu_p)\left(\frac{\text{Volts}}{\text{length}}\right)$$

$$= 1.6 \cdot 10^{-19}\left(2 \cdot 10^{15}_{cm^{-3}} \cdot 5000_{\frac{cm^2}{V \cdot s}} + \frac{(1.5 \cdot 10^{10})^2}{2 \cdot 10^{15}}_{cm^{-3}} \cdot 3000_{\frac{cm^2}{V \cdot s}}\right)\left(\frac{1\ \text{Volt}}{2\,cm}\right)$$

since $n >> p$, we can neglect the $p \cdot \mu_p$ contribution. We also must make sure we match length units.

$$= 1.6 \cdot 10^{-19}\,C\left(1 \cdot 10^{19}\ cm^{-1}V^{-1}s^{-1}\right)\left(.5\ V \cdot cm^{-1}\right) = (1.6)(.5)\,C \cdot s^{-1}cm^{-2}$$

$$I = J \cdot \text{Area} = \left(0.8\ \frac{\text{Coulombs}}{s \cdot cm^2}\right)\left(0.4\,cm \cdot 0.5\,cm\right)$$

$$\boxed{= 0.16\ \text{Amps}}$$

c) At 500K does the ratio of hole current to electron current increase, decrease, or stay the same? Explain. (6 pts)

At room temperature (300 K), we see that the electron contribution is much larger than holes, yet as we raise temperature to 500K this changes. At lower temperatures, the electron carrier concentration dominates due to the n-type donor doping; at elevated temperatures, the thermalization increases n_i and thus increases more electron-hole pairs (EHP's) so much so that thermalized carriers dominate doping. Since thermalization creates both electrons and holes equally, the minority carriers (holes) will feel a larger increase as $p \to n$, so the ratio increases. To confirm that mobility doesn't offset this, we check: $\frac{J_p}{J_n} \propto \frac{p\mu_p}{n\mu_n}$. Between 300K and 500K, the relative change in mobility ratios changes but concentration changes more and dominates.

d) What will happen to the value of the electron current when we lower the temperature from 300K to 250K? Circle one and explain briefly. (4 pts)

INCREASE DECREASE STAY THE SAME NOT ENOUGH INFORMATION

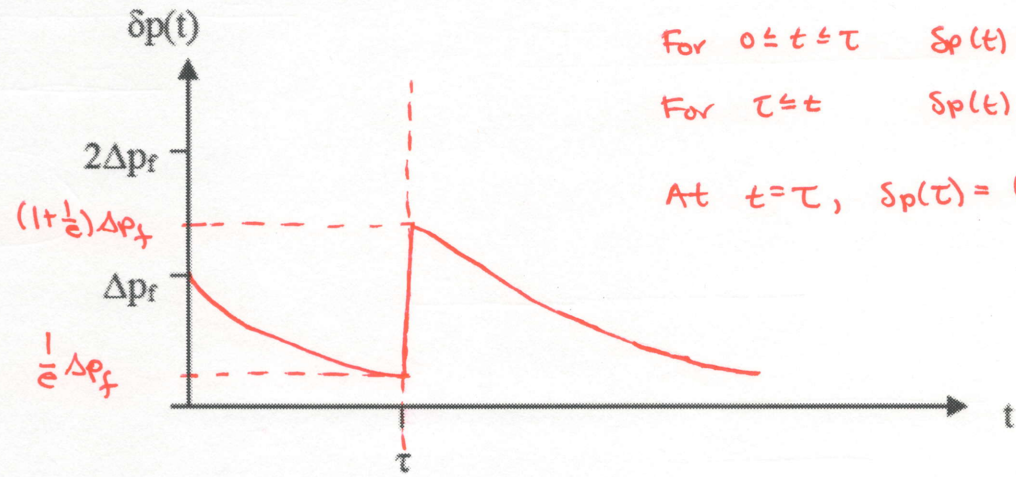
Based on the graph, we can expect n to stay relatively constant between 250K and 300K, yet since $I_n \propto n\mu_n$ we must also consider changes in mobility for electrons. Around the temperature change $300K \to 250K$, we would expect μ_n to increase as you drop in temperature due to a reduction in scattering events.

However, it can also be argued that since the material used is not given, it is not known for sure where the crossover from increasing μ_n to decreasing μ_n per temperature is, so not enough information is known to determine whether this change increases or decreases current. This is also acceptable if explained fully.

Problem 4: Excess Carriers with Generation and Recombination

An n-type Si sample with $N_D = 10^{15}$ cm^{-3} and maintained at room temperature is illuminated by **two flashes of light** from a stroboscope. Each flash creates $\Delta p_f = 10^{12}$ cm^{-3} excess holes **uniformly** throughout the Si sample. The first flash occurs at $t = 0$ and the second occurs at $t = \tau$, where τ is the minority carrier lifetime in the sample. The flashes of light themselves are of infinitesimally short duration compared to τ. For this sample answer the following questions. You may use: $\ln 10^5 = 11.5$, $\ln 100 = 4.61$, $\ln 1.5 = 0.405$, $e = 2.72$, $e^{-1} = 0.368$, $n_i = 1.5 \times 10^{10}$ cm^{-3}, E_g for Si = 1.12 eV

a) Sketch on the linear scale shown below (based on qualitative reasoning), the expected general variation of $\delta p(t)$ as a function of time, t. Explain. (8 pts)



For $0 \le t \le \tau$ $\quad \delta p(t) = \Delta p_f e^{-t/\tau}$

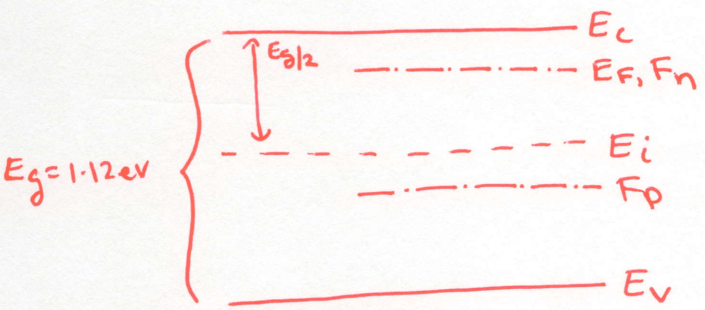
For $\tau \le t$ $\quad \delta p(t) = \Delta p_f e^{-t/\tau} + \Delta p_f e^{-(t-\tau)/\tau}$

At $t = \tau$, $\delta p(\tau) = (1+\frac{1}{e})\Delta p_f$

b) Do low-level injection conditions prevail at all times inside the Si sample? Explain. (5 pts)

- $\delta p(t)|_{max} = \delta p(t=\tau) = (1+\frac{1}{e})\Delta p_f = (1+\frac{1}{e}) \times 10^{12}$
- Since $N_D >> n_i$, $n_0 \simeq N_D = 10^{15}$
- Since $\delta p(t)_{max} << n_0$ $\forall t$, Low-level injection prevails at all times

c) Draw the energy band diagram that is applicable to this sample immediately after the first flash, at $t=0^+$. Show, **qualitatively**, the values of F_n and F_p. Also **qualitatively** show E_c, E_i, E_F for equilibrium, and E_v on the diagram. (7 pts)



$n = n_0 + \Delta n \simeq n_0 \Rightarrow \boxed{E_F \simeq F_N}$

$p = p_0 + \Delta p \simeq \Delta p = n_i e^{(E_i - F_p)/kT}$

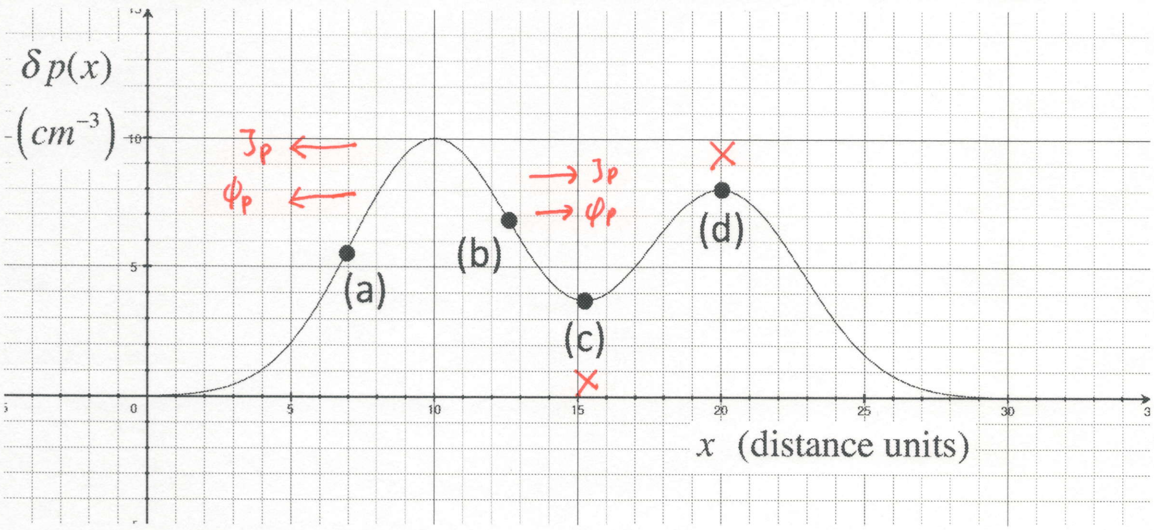
$E_i - F_p = kT \ln \frac{10^{12}}{1.5E10} \simeq \underline{0.108 eV}$

$E_F - E_i = kT \ln \frac{10^{15}}{1.5E10} \simeq \underline{0.290 eV}$

(*) Numerical answers not required for full credit.

Problem 5: Diffusion

Following a perturbing event, a semiconductor bar has the following excess carrier distribution:



Please answer the following questions regarding the diffusion of the excess holes $\delta p(x)$:

a) At points (a), (b), (c), and (d) draw arrows to indicate the direction of hole flow $\Phi_p(x)$ and current density $J_p(x)$, or an "X" if the magnitude is 0. Clearly label the arrows to differentiate direction of flow from direction of current. (8 pts)

b) The distribution above is a sum of Gaussians and can be described by the equation:

$$\delta p(x) = Ae^{\left(-\frac{(x-a)^2}{c}\right)} + (0.8 \times A)e^{\left(-\frac{(x-b)^2}{c}\right)}$$

Write the expression for the current density $J_p(x)$ given no applied electric field. (12 pts)

$$J_p(x) = -qD_p \frac{dp(x)}{dx}$$

$$= -qD_p \frac{d}{dx}\left(Ae^{-\frac{(x-a)^2}{c}} + 0.8A\,e^{-\frac{(x-b)^2}{c}}\right)$$

$$= -qD_pA\left(-\frac{2(x-a)}{c}e^{-\frac{(x-a)^2}{c}} + 0.8\left(-\frac{2(x-b)}{c}\right)e^{-\frac{(x-b)^2}{c}}\right)$$

$$= \frac{2qD_pA}{c}\left((x-a)e^{-\frac{(x-a)^2}{c}} + 0.8(x-b)e^{-\frac{(x-b)^2}{c}}\right)$$

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