

Data-Driven Models for Discrete Hedging Problem

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Practitioner Black-Scholes (BS) Delta Hedging



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- ▶ BS model:

$$\frac{dS}{S} = \mu dt + \sigma dZ$$

σ : Constant

- ▶ Implied Volatility

$$\sigma_{imp} = V_{BS}^{-1}(V_{mkt}, \cdot)$$

V_{mkt} : market option price
 V_{BS}^{-1} : inverse of BS pricing function

- ▶ BS Delta:

$$\delta_{BS} = \frac{\partial V_{BS}}{\partial S}$$

Data-driven Approach



The data-driven approach is

$$\min_f \left[\frac{1}{N} \sum_{i=1}^N (\Delta V_i - \Delta S_i f(X_i))^2 \right]$$

- ▶ $f(X_i)$: option hedging position function.
- ▶ Data-driven models outperform other delta hedging strategies¹.
- ▶ Incorporating sequential Information further improves the performance of data-driven models ².

¹Ke, Nian, Thomas F. Coleman, and Yuying Li. "Learning minimum variance discrete hedging directly from the market." Quantitative Finance (2018): 1-14.

²Ke, Nian, Thomas F. Coleman, and Yuying Li. "Learning Sequential Option Hedging Models from Market Data. " To be submitted.

Call Option Daily Hedging



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Introduction

Delta	MV (%)	SABR(%)	LVF(%)	Data-Driven Model			
				DKL _{SPL} (%)		DRNN (%)	
				Traded	All	Traded	All
0.1	42.1	39.4	42.6	47.1	48.6	32.3	33.8
0.2	35.8	33.4	36.2	37.8	40.0	33.7	36.4
0.3	31.1	29.4	30.3	34.1	35.1	34.1	35.5
0.4	28.5	26.3	26.7	32.3	32.0	33.7	34.2
0.5	27.1	24.9	25.5	29.3	29.4	35.1	33.0
0.6	25.7	25.2	25.2	29.9	28.4	35.6	32.1
0.7	25.4	24.7	25.8	29.0	26.8	31.8	29.7
0.8	24.1	23.5	25.4	25.9	24.7	28.6	26.5
0.9	16.6	17.0	16.9	17.7	13.9	19.3	18.9
Overall	25.7	24.6	25.5	31.3	26.0	32.9	28.7

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Call Option Weekly Hedging and Monthly Hedging



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Introduction

Delta	Data-Driven Model			
	DKL _{SPL} (%)		DRNN(%)	
	Traded	All	Traded	All
0.1	38.9	38.3	47.8	45.6
0.2	29.0	26.9	48.5	46.0
0.3	23.5	25.3	48.5	46.6
0.4	20.8	24.3	45.9	45.4
0.5	19.9	22.8	46.6	45.0
0.6	17.3	19.5	44.8	43.1
0.7	16.8	17.7	43.9	42.4
0.8	12.5	12.3	37.7	39.0
0.9	6.2	5.1	16.4	29.1
Overall	20.2	17.1	43.7	40.5

Delta	Data-Driven Model			
	DKL _{SPL} (%)		DRNN (%)	
	Traded	All	Traded	All
0.1	22.7	24.8	53.9	39.4
0.2	23.5	25.5	51.7	48.3
0.3	24.0	24.6	50.2	49.1
0.4	21.0	20.7	47.8	48.3
0.5	13.5	12.7	44.5	47.6
0.6	14.3	13.5	44.6	47.4
0.7	6.1	7.0	35.3	42.9
0.8	5.3	4.1	24.8	34.1
0.9	4.1	2.3	10.5	19.9
Overall	16.3	12.5	44.5	42.3

Table: Weekly(Left) and Monthly(Right)

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