1 Background

Learning aggregation for 3D anisotropic diffusion on a structured grid

$$-\nabla \cdot (\boldsymbol{D} \nabla u) = 0, \tag{1}$$

$$\boldsymbol{D} := \boldsymbol{R}^T \begin{bmatrix} \varepsilon_x & & \\ & \varepsilon_y & \\ & & 1 \end{bmatrix} \boldsymbol{R}, \tag{2}$$

$$\boldsymbol{R} := \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix}. \tag{3}$$

For now, this is on the specific problem for $N_x = N_y = N_z = 12$ (number of elements in each dimension), $\theta_z = 1.11\pi$, $\theta_y = 0.58\pi$, $\varepsilon_x = 0.001$, $\varepsilon_y = 1000.0$.

This was discretized in Firedrake [2] using piecewise linear tetrahedral finite elements with homogeneous Dirichlet boundary conditions. Afterwards, the degrees-of-freedom corresponding to the Dirichlet boundary were removed from the system. Overall, there are $11 \times 11 \times 11 = 1331$ degrees of freedom total.

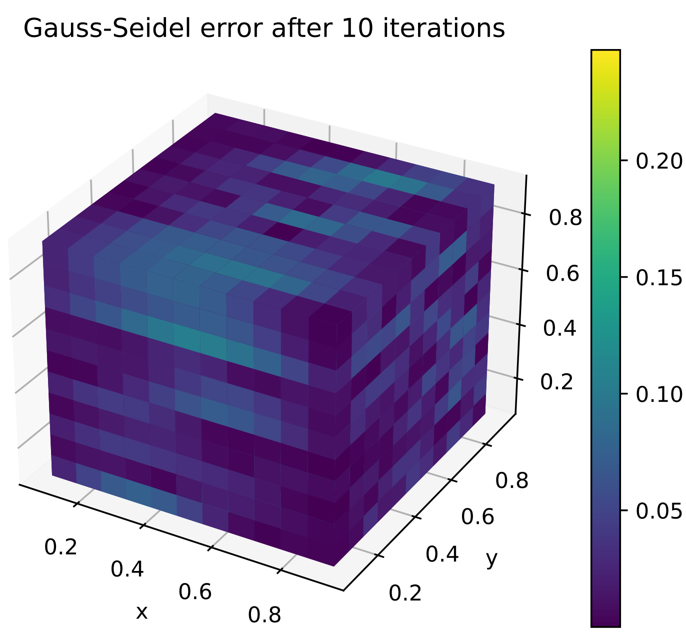

Figure 1: Error behavior on the problem with a random initial guess, solving $\boldsymbol{Ax} = \mathbf{0}$ with Gauss-Seidel iterations. The direction and strength of anisotropy are visible by looking at in which areas the error is geometrically smooth.

References

[1] L. N. Olson and J. B. Schroder, *Pyamg: Algebraic multigrid solvers in python v4.0*. `https://github.com/pyamg/pyamg`, 2018. Release 4.0.

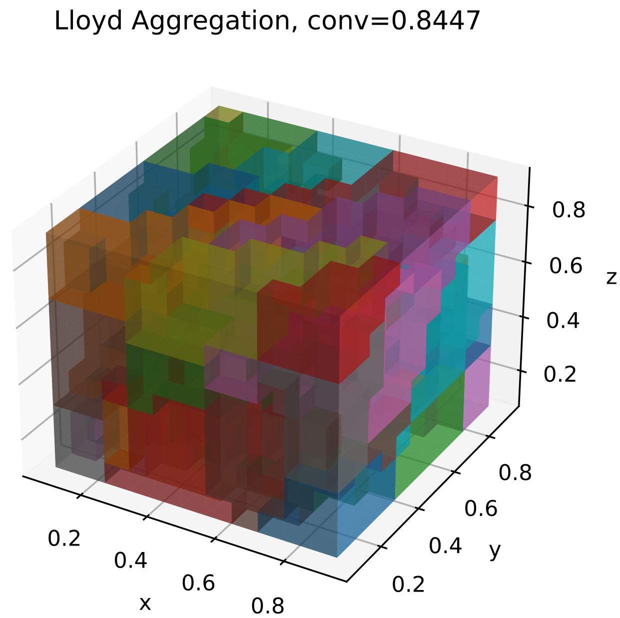

Figure 2: Aggregate selection when using PyAMG's (regular) Lloyd aggregation [1].

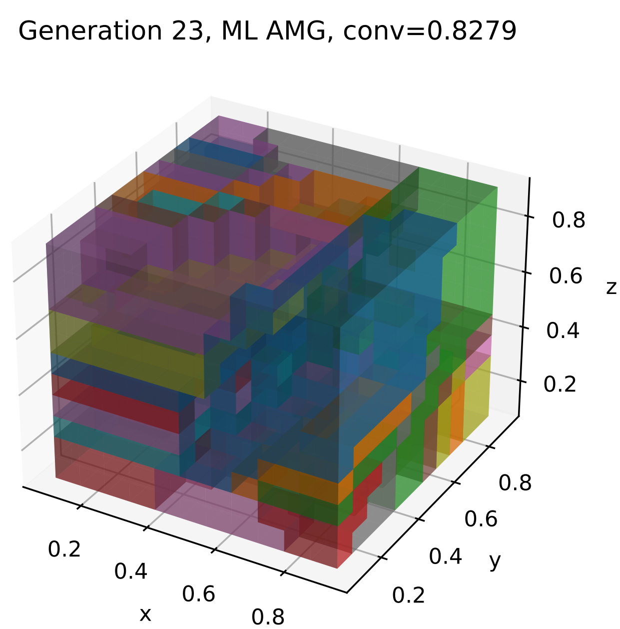

Figure 3: Aggregate selection from the ML method. This is trained on the specific problem being shown.

[2] F. Rathgeber, D. A. Ham, L. Mitchell, M. Lange, F. Luporini, A. T. T. McRae, G. Bercea, G. R. Markall, and P. H. J. Kelly, *Firedrake: automating the finite element method by composing abstractions*, CoRR, abs/1501.01809 (2015).