1 Setup

The problem being solved is the rotated anisotropic diffusion equation in 2D,

$$-\left(\text{div } \boldsymbol{Q}\boldsymbol{A}\boldsymbol{Q}^T\right)\nabla^2\boldsymbol{u} = 0, \tag{1}$$

where $\boldsymbol{Q}$ is a rotation matrix encoding some rotation by θ, and $\boldsymbol{A}$ is a diagonal scaling matrix given by

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ 0 & \epsilon \end{bmatrix}. \tag{2}$$

A basic training set of 100 structured 15×15 grids was constructed using a finite-element discretization. Values of θ and ϵ were uniformly chosen within the ranges $[1, 5)$ and $[0, \pi)$, respectively. For rough testing, all figures and tables below are evaluted on a mesh with coefficients $\theta = \frac{\pi}{4}, \epsilon = 3$.

A TagConv network consisting of 5 convolutional layers was trained to predict optimal C/F partitioning. The unsupervised loss used consisted of a linear combination of the spectral radius of the multigrid error propagation operator and the 1 norm of the C/F partitioning that was output,

$$\ell\left(\boldsymbol{c}\right) := \rho\left(\boldsymbol{E}\left(\boldsymbol{c}\right)\right) + \alpha\left\|\boldsymbol{c}\right\|_1, \tag{3}$$

where $\boldsymbol{E}$ is defined as the error propagation for a two-level V-cycle multigrid method with using weighted Jacobi ($\omega = \frac{2}{3}$) relaxation,

$$\boldsymbol{E} := \left(\boldsymbol{I} - \omega\boldsymbol{D}^{-1}\boldsymbol{A}\right)\left(\boldsymbol{I} - \boldsymbol{P}\left(\boldsymbol{P}^T\boldsymbol{A}\boldsymbol{P}\right)^{-1}\boldsymbol{P}^T\boldsymbol{A}\right)\left(\boldsymbol{I} - \omega\boldsymbol{D}^{-1}\boldsymbol{A}\right), \tag{4}$$

and $\boldsymbol{P}$ is the operator obtained by Ruge-Stüben style interpolation on the C/F partitioning.

2 Results

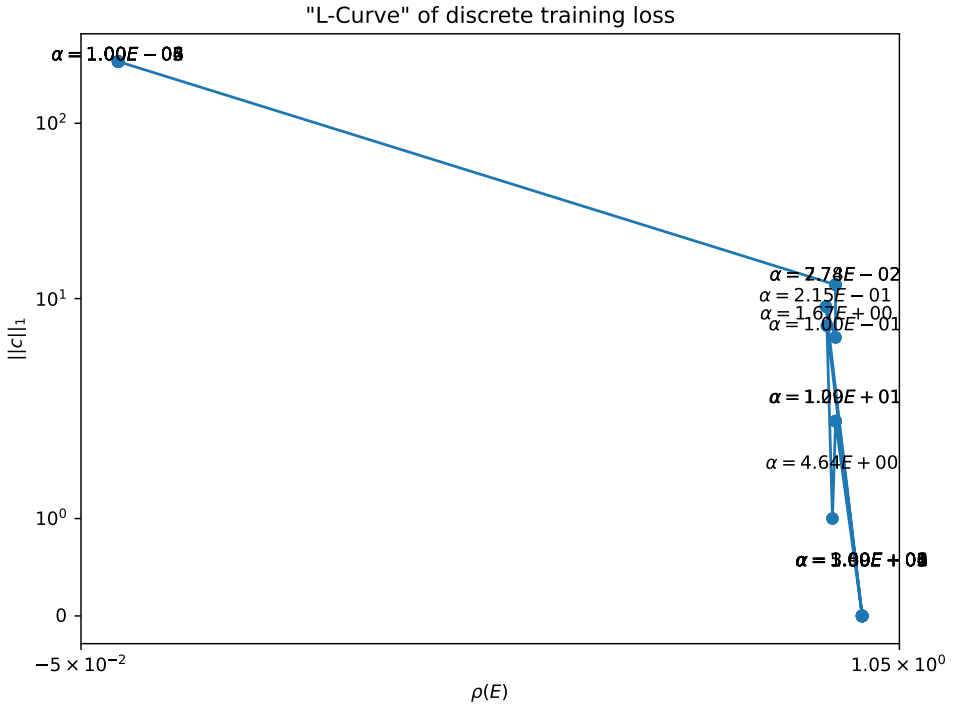
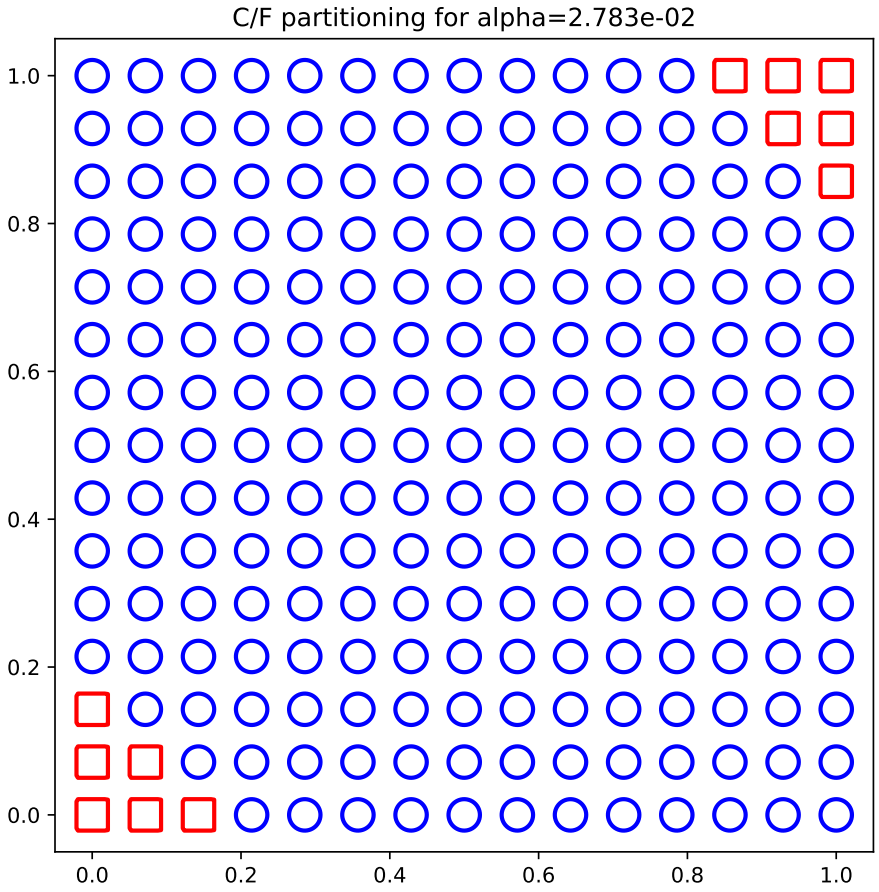
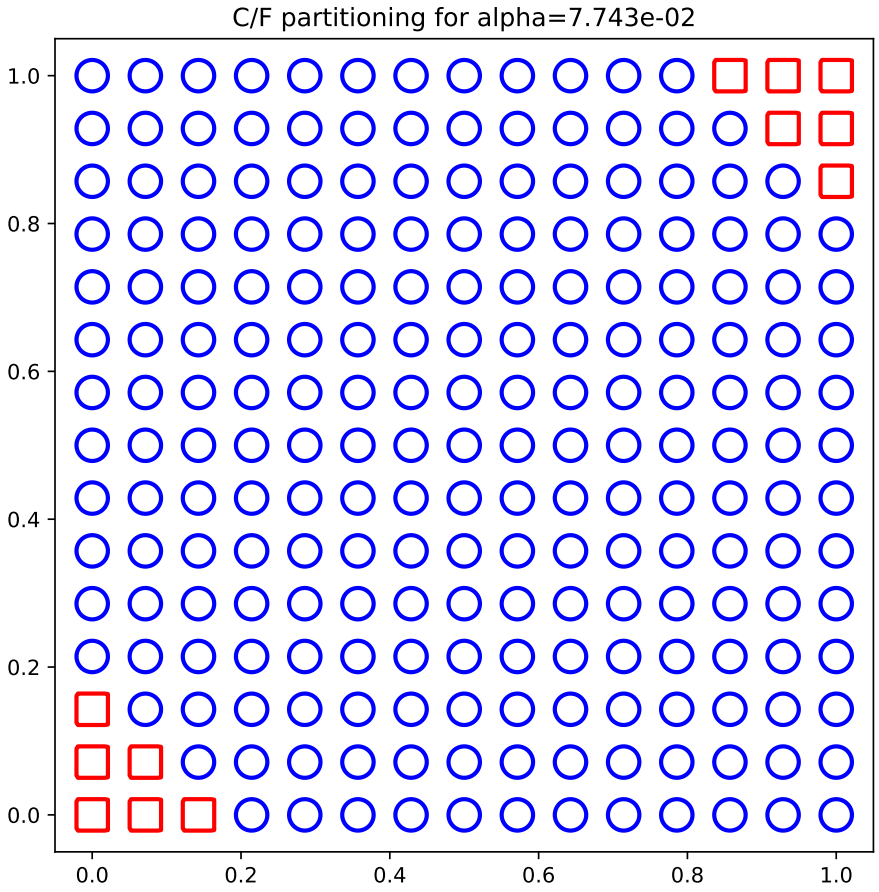


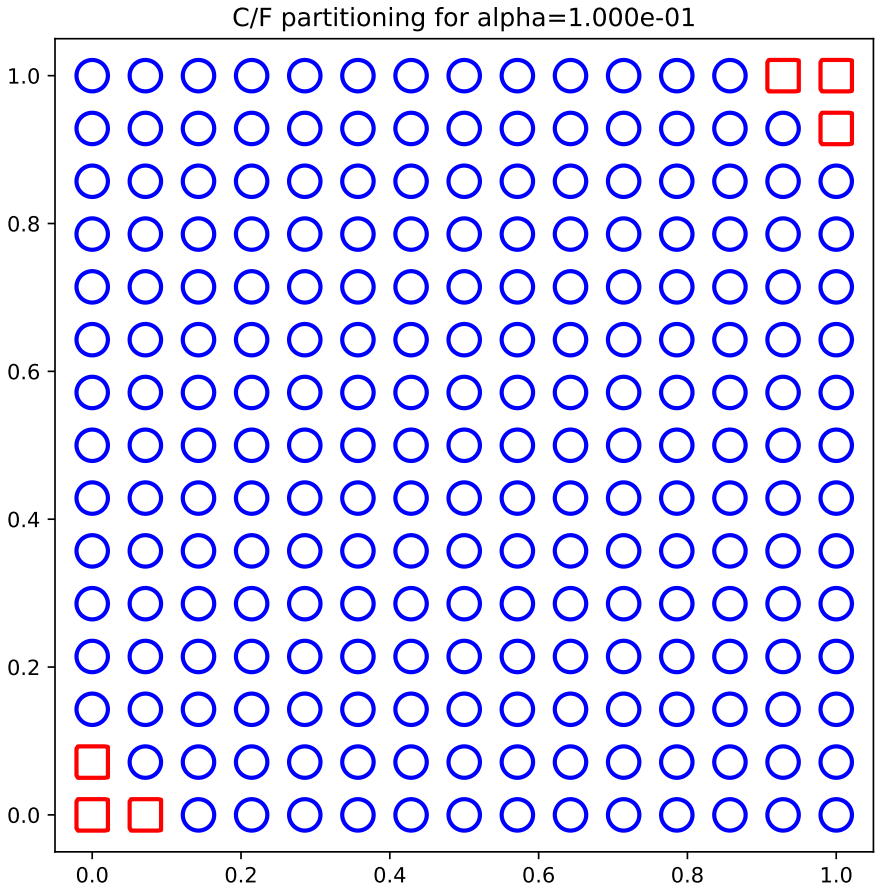
Figure 1: L-curve of $\rho(\boldsymbol{E})$ vs $\|\boldsymbol{c}\|_1$. This is using a swe ep of α values between 10^{-6} and 10^6.

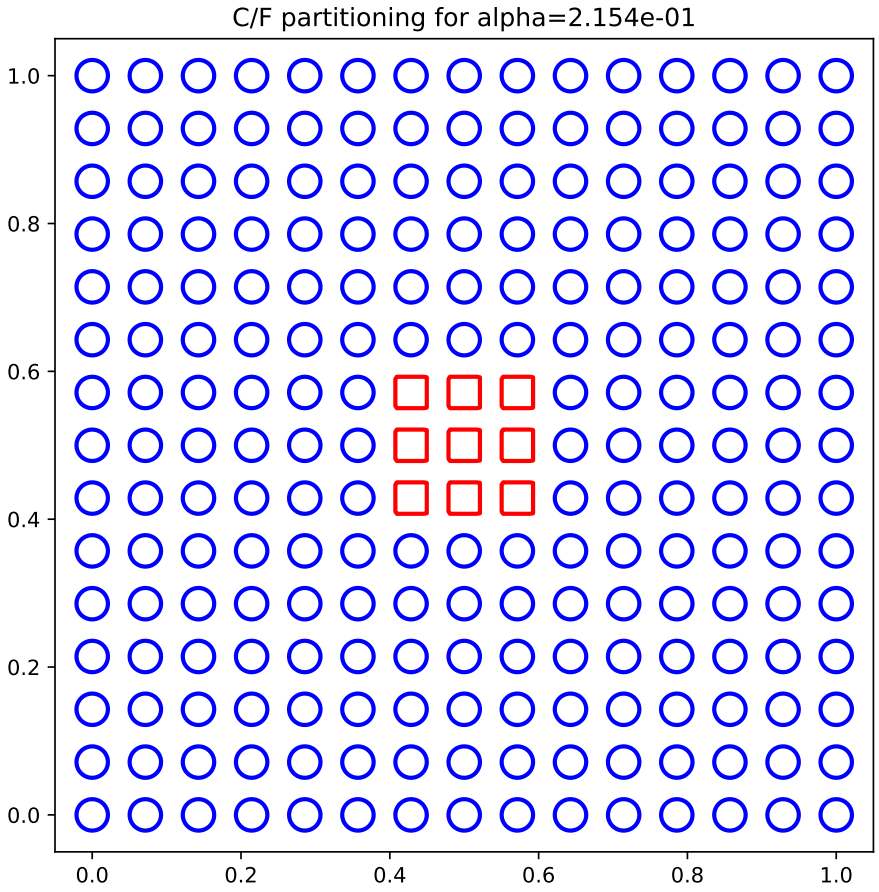
α	$\rho(\boldsymbol{E})$	$\|\boldsymbol{c}\|_1$	f-fraction
$1.000e-06$	$-3.223e-16$	225	0.000
$1.000e-05$	$-3.223e-16$	225	0.000
$1.000e-04$	$-3.223e-16$	225	0.000
$1.000e-03$	$-3.223e-16$	225	0.000
$1.000e-02$	$-3.223e-16$	225	0.000
$2.783e-02$	$9.641e-01$	12	0.947
$7.743e-02$	$9.641e-01$	12	0.947
$1.000e-01$	$9.642e-01$	6	0.973
$2.154e-01$	$9.512e-01$	9	0.960
$5.995e-01$		0	1.000
$1.000e+00$		0	1.000
$1.668e+00$	$9.525e-01$	7	0.969
$4.642e+00$	$9.600e-01$	1	0.996
$1.000e+01$	$9.642e-01$	2	0.991
$1.292e+01$	$9.642e-01$	2	0.991
$3.594e+01$		0	1.000
$1.000e+02$		0	1.000
$1.000e+03$		0	1.000
$1.000e+04$		0	1.000
$1.000e+05$		0	1.000
$1.000e+06$		0	1.000

Table 1: Spectral radius of error propagator, norm of C/F partitioning, and f-fraction for various values of α.


C/F partitioning for alpha=2.783e-02
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=7.743e-02
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=1.000e-01
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=2.154e-01
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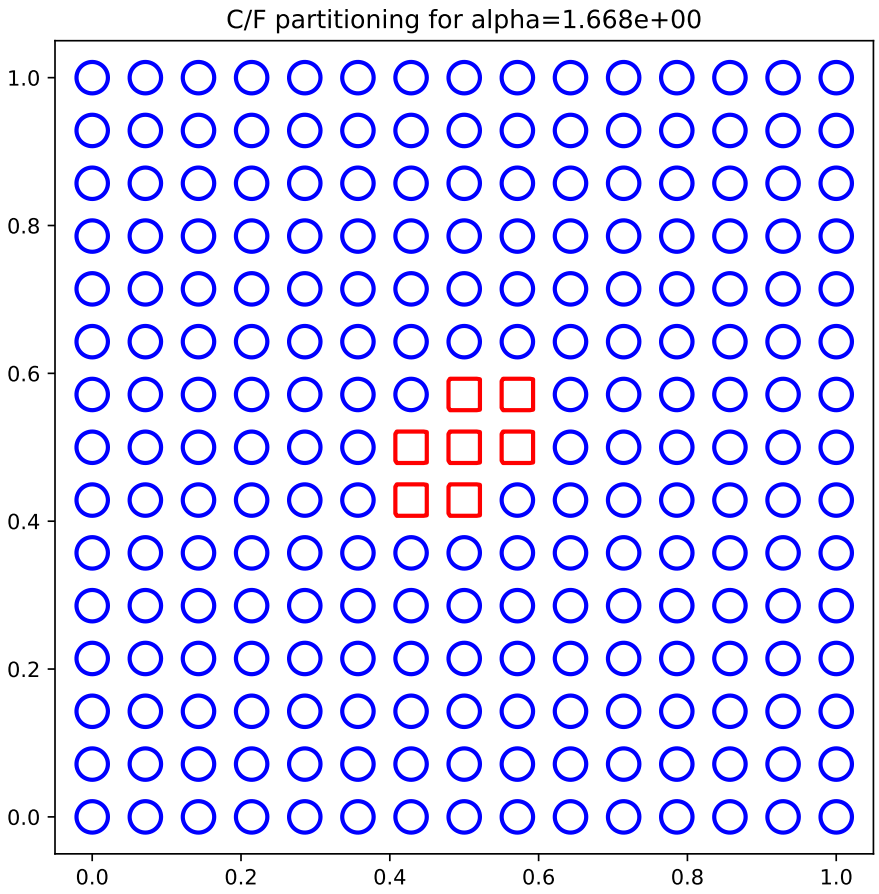
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=1.668e+00
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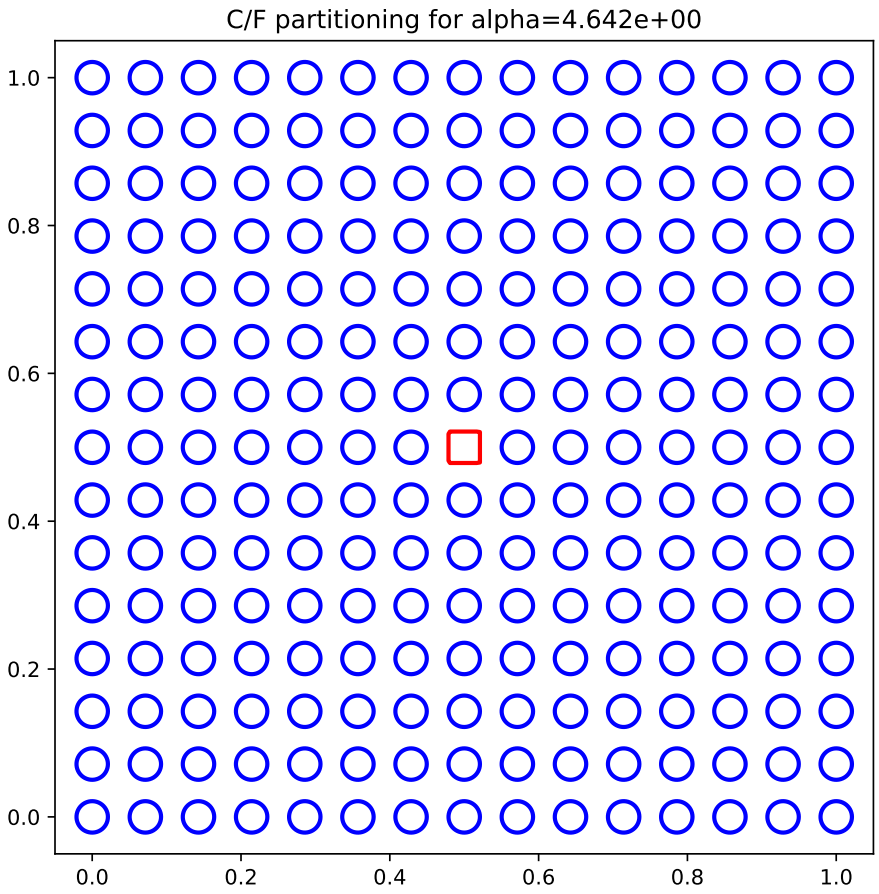
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=4.642e+00
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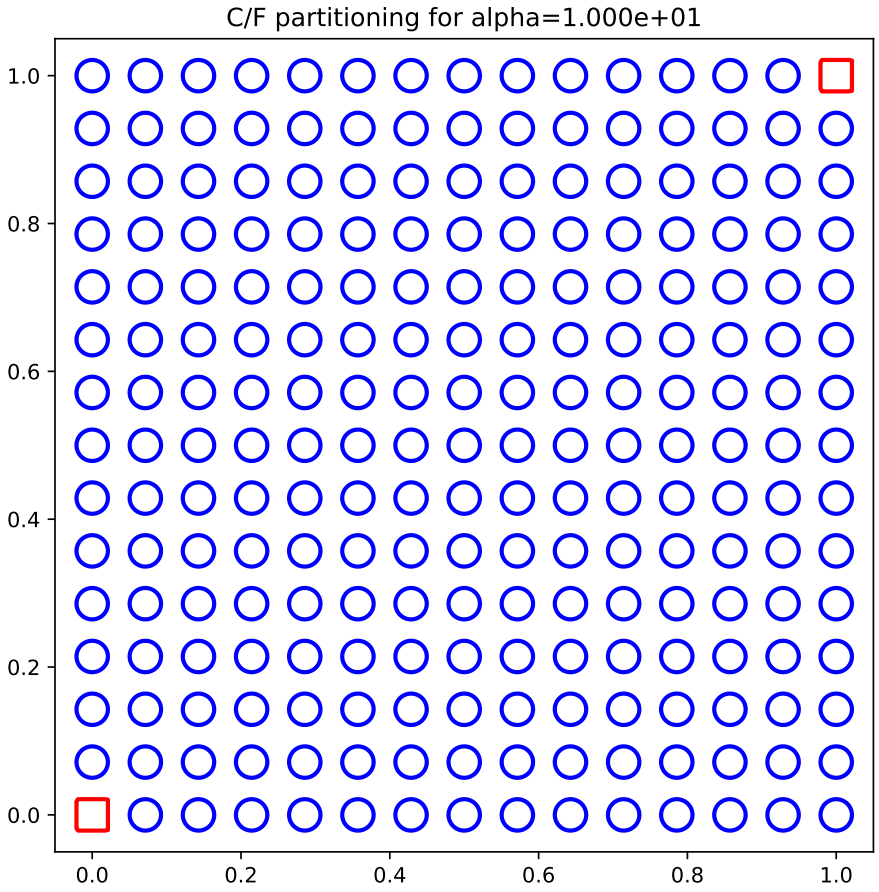
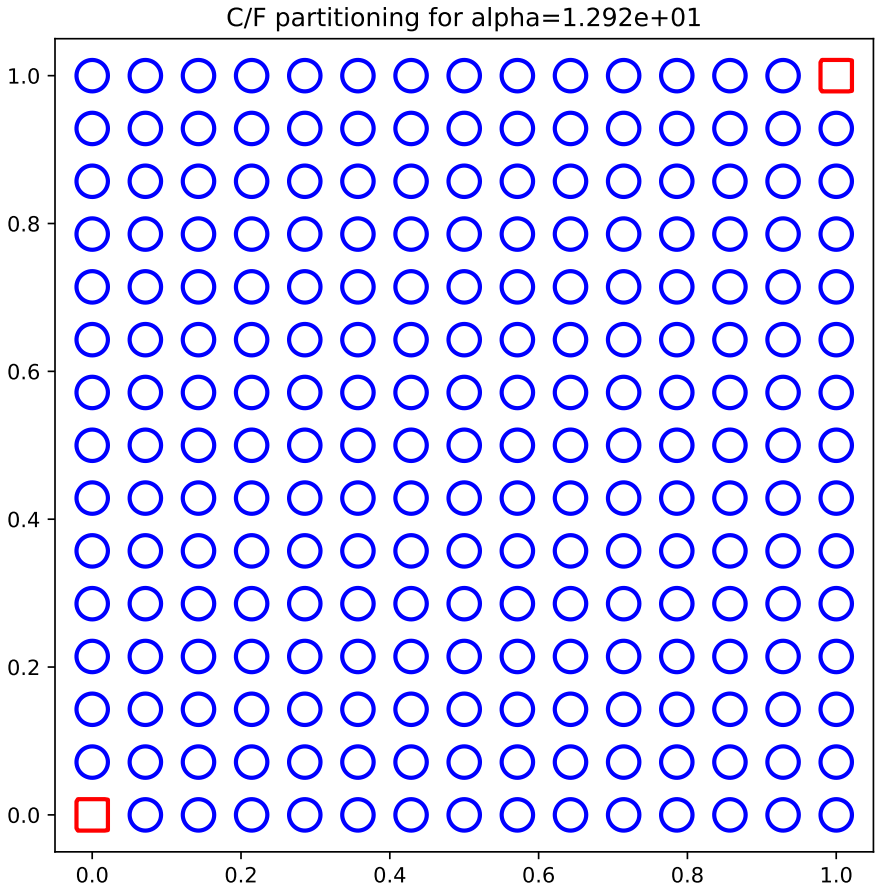
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=1.000e+01
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


C/F partitioning for alpha=1.292e+01
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0