Data, Environment and Society:
Lecture 21: Measuring Classifier Performance

Instructor: Duncan Callaway
GSI: Salma Elmallah

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Announcements

- Today: Elinor Benami guest lecture
- HW9 due today
- HW10 posts today, due in two weeks
- Thursday: Exam review
- Lab next week: Cancelled, prep for exam!
- Tuesday next week: Exam
    - Covers up to Lecture 19
    - Covers up to HW9

Objectives for today

- Methods for measuring performance of classifiers
- Reminders on parameter, hyperparameter tuning

Remidner: classification error rate

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The typical error, RSS $= \sum_{i=1}^{N}(y_i - \hat{y}_i)^2$ won't work.

Alternatives? Let's start by defining

$$p_{mk} = \text{ fraction of observations belonging to class } k \text{ in region } m.$$

Then a simple measure is:

Classification error rate = how many training observations don't fall into the assigned class.

Within-region this is simply:

$$E_m = 1 - \max_k(\hat{p}_{mk})$$

The confusion matrix

- A “confusion matrix” C is such that $C_{i,j}$ is equal to the number of observations known to be in group i but predicted to be in group j.
- Numbers on the diagonal of the matrix are correct predictions
- Rows sum to the number of actual observations in a category
- Columns sum to the number of predicted observations in a category

	Predict: True	Predict: False
Actual: True	True pos (TP)	False neg (FN)
Actual: False	False pos (FP)	True neg (TN)

Precision and recall

	Predict: True	Predict: False
Actual: True	True pos (TP)	False neg (FN)
Actual: False	False pos (FP)	True neg (TN)

- **Precision:** Correct number of positives divided by total number of positive predictions. (“True positive rate”)

$$= \frac{TP}{TP + FP}$$

- **Recall:** Correctly number of positives divided by total number of true positives. $(1-$ false positive rate)

$$= \frac{TP}{TP + FN}$$

Reminder / clarification for cross validation: Models have two different classes of parameters

1. Parameters that enter as decision variables for minimizing loss function:

$$\begin{aligned}
\text{Shrinkage: } \beta^* \quad &= \arg\min_{\beta} \sum_{i=1}^{N} (Y_i - X_i\beta)^2 + \lambda \cdot R(\beta) \\
\text{CART: } \{j^*, s^*\} \quad &= \arg \min_{j \in J, s \in X_j} \sum_{i: x_i \in R_1(j,s)} (y_i - \hat{y}_{R_1})^2 + \sum_{i: x_i \in R_2(j,s)} (y_i - \hat{y}_{R_2})^2
\end{aligned}$$

2. “Hyperparameters”: parameters that constrain how you solve the loss function. These generally prevent overfit.

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2. "Hyperparameters": parameters that constrain how you solve the loss function. These generally prevent overfit.
   - λ in shrinkage methods
   - How deep to grow a classification tree?
   - $\sum_{i=1}^{n} \epsilon_i$ in SVM

Ways to minimize the loss function

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- Closed form solution – e.g. normal equations
- Gradient search

In both cases, we're relying on the condition that the gradient of the loss function approaches zero as we approach the optimal solution

Ways to choose hyperparameters

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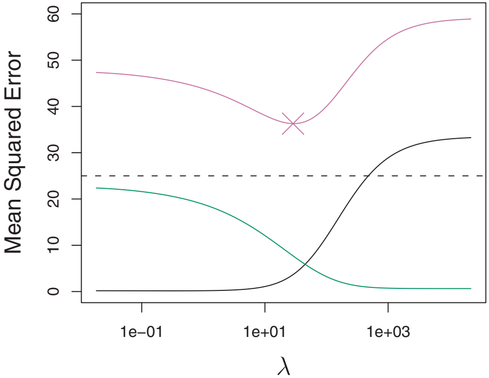
- **Grid search**: This is what we've done with shrinkage methods, when there is just one parameter to tune (λ)
- **Randomized parameter search**: This is what you're doing in HW10. Works well when you have lots of hyperparameters to tune.



In both cases, all we're doing is

- Creating a list of candidate hyperparameters (or sets of hyperparameters)
- Training the model (with the training data) for each hyperparameter in the list
- Choosing the hyperparameter with the best cross-validated error.