Data, Environment and Society: Lecture 15: Classification

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October 11, 2018

Announcements

Today

- Introduction to classification
  - Corresponding reading: ISLR Ch 4.1 through 4.3.1, and also Ch 2.2.3.
- Classification in practice: water quality violations in California

Next week

- Lab: you'll collaboratively write study guides for each other.
- Lecture on Tuesday: I'll go over the review slides, and you can ask questions.
- Exam on Thursday!

Remember qualitative variables? This is how you do it

$$x_1 = \begin{cases} 1, & \text{Likes split pea.} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_2 = \begin{cases} 1, & \text{Likes minestrone.} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_3 = \begin{cases} 1, & \text{Doesn't like soup.} \\ 0, & \text{otherwise.} \end{cases}$$

Question: What about the “other” category?

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Question: What about the “other” category?
Answer: The answers are mutually exclusive, so if x_1, x_2, x_3 are all zero, then the answer must be “other”.

Cooked-up example: Predicting age with qualitative variables

$$\text{AGE}_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \beta_3 x_{i,3} + \beta_4 x_{i,4} + \epsilon_i$$

where x_1, x_2, x_3 are defined on previous slide and x_4 is how spicy the respondent likes their food.

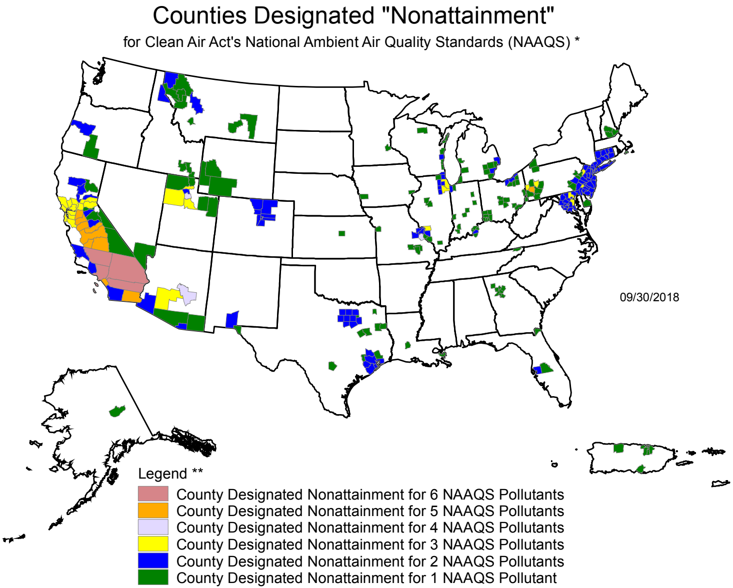
With these variables the qualitative predictors are just modifying the intercept.

- When $x_1 = x_2 = x_3 = 0$ (i.e. the answer is “other”) then the intercept is β_0.
- Otherwise the intercept is $\beta_0 + \beta_i$ where i is the x variable that is nonzero.

What it you want to *predict* a categorical variable?

Examples:

- Clean air act attainment status for a region (attainment or non-attainment)
- Is an area going to be a candidate for a new refinery in the next 10 years
- Disease presented in emergency dept of a hospital



Why not linear regression?

For example: EPA Criteria Pollutants. For each region i indicate nonattainment as follows:

- $y_i = 1 \rightarrow$ Ozone
- $y_i = 2 \rightarrow$ PM2.5
- $y_i = 3 \rightarrow$ PM10
- $y_i = 4 \rightarrow$ Sulfur Dioxide
- $y_i = 5 \rightarrow$ Lead
- $y_i = 6 \rightarrow$ Carbon Monoxide
- $y_i = 7 \rightarrow$ Nitrogen Dioxide

Then,

$$y_i = \beta \mathbf{x}_i + \epsilon$$

where $\mathbf{x}_i$ is a vector of observed independent variables for each location i

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The problem: a different ordering of variables would imply a different relationship between statuses, different models, and different predictions.

Predicting categorical variables the right way: Similar coding to using them as predictors

For example: EPA Criteria Pollutants:

- $y_{i,1} \rightarrow$ Ozone status
- $y_{i,2} \rightarrow$ PM2.5 status
- $y_{i,3} \rightarrow$ PM10 status
- $y_{i,4} \rightarrow$ Sulfur Dioxide status
- $y_{i,5} \rightarrow$ Lead status
- $y_{i,6} \rightarrow$ Carbon Monoxide status
- $y_{i,7} \rightarrow$ Nitrogen Dioxide status

where *i* indexes the region of interest, i.e. observations

Then, for each,

$$y_{i,j} = \begin{cases} 1, & \text{Nonattainment status} \\ 0, & \text{Attainment status} \end{cases}$$

where *j* indexes criteria pollutants

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First, build your model so that its output estimates a *probability* that a given outcome happens.

For example, a model might give:

p(PM-2.5 nonattainment) $= 0.734$ and p(PM-2.5 attainment) $= 0.266$

(the probabilities add to one).

How do you get a model to output a $\{0, 1\}$ result? (ctd)

Second, we use something called the *Bayes Classifier*.

Bayes classifier:

For a given observation x_i, choose j as the value for which

$$\Pr(Y = j | X = x_i)$$

is largest.

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is largest.

More formally,

$$\hat{y}_i = \arg\max_{j \in \mathcal{J}} \Pr(Y = j | X = x_i)$$

where $\mathcal{J}$ is the set of possible (mutually exclusive) outcomes.

Classification error rate

If the "true model" for $\Pr(Y = j|X = x_i)$ is known, then using the Bayes classifier will provide the lowest possible error rate.

The *error rate* quantifies how frequently a model mis-classifies a categorical variable.

Let $I(\cdot)$ denote the *indicator function*:

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Let $I(\cdot)$ denote the *indicator function*:

$$I(y_i \neq \hat{y}_i) = \begin{cases} 1, & \text{when } y_i \neq \hat{y}_i. \\ 0, & \text{otherwise.} \end{cases}$$

$$\Rightarrow \text{error rate} = \frac{1}{n} \sum_{i=1}^{n} I(y_i \neq \hat{y}_i)$$

What’s the error rate?

Location	**Actual status**	**Predicted (hypothetical)**	$I(y_i \neq \hat{y}_i)$
Allegheny County, PA	Non-attainment	Non-attainment	
Cleveland, OH	Non-attainment	Non-attainment	
Delaware County, PA	Non-attainment	Non-attainment	
Imperial County, CA	Non-attainment	Attainment	
Lebanon County, PA	Non-attainment	Attainment	
South Coast Air Basin, CA	Non-attainment	Non-attainment	
Plumas County, CA	Non-attainment	Non-attainment	
Sacramento County, CA	Attainment	Non-attainment	
San Francisco, CA	Attainment	Non-attainment	
San Joaquin Valley, CA	Non-attainment	Non-attainment	
West Silver Valley, ID	Non-attainment	Non-attainment	
Luzerne County, PA	Attainment	Attainment	
Lancaster County, PA	Attainment	Attainment	

What's the error rate?

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Imperial County, CA	Non-attainment	Attainment	
Lebanon County, PA	Non-attainment	Attainment	
South Coast Air Basin, CA	Non-attainment	Non-attainment	
Plumas County, CA	Non-attainment	Non-attainment	
Sacramento County, CA	Attainment	Non-attainment	
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Lebanon County, PA	Non-attainment	Attainment	1
South Coast Air Basin, CA	Non-attainment	Non-attainment	0
Plumas County, CA	Non-attainment	Non-attainment	0
Sacramento County, CA	Attainment	Non-attainment	
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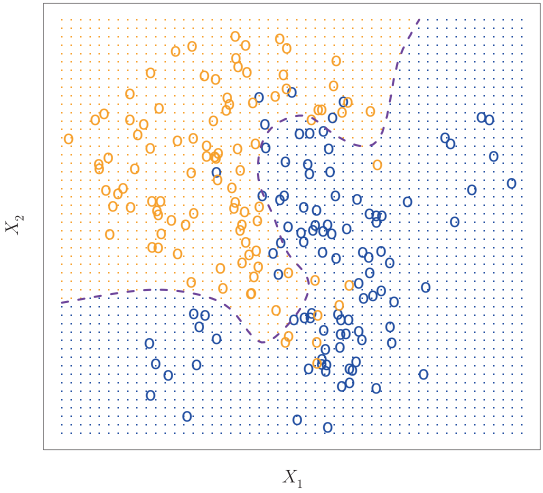
San Joaquin Valley, CA	Non-attainment	Non-attainment	0
West Silver Valley, ID	Non-attainment	Non-attainment	0
Luzerne County, PA	Attainment	Attainment	0
Lancaster County, PA	Attainment	Attainment	0

$\rightarrow$ Overall error rate is $\frac{1}{13} \cdot 4 = .31$

Finally, the decision boundary

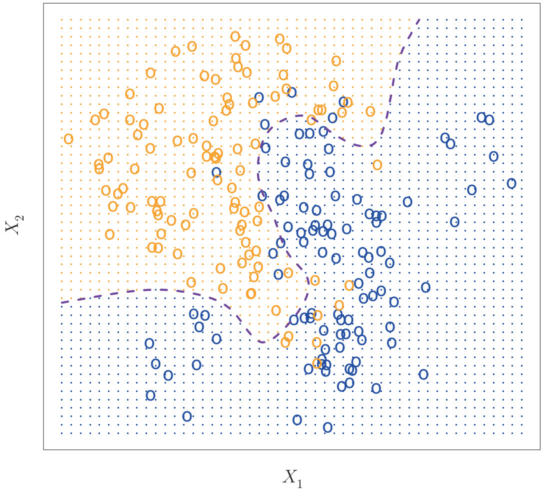


ISLR Fig 2.13

Along the boundary the probability of one outcome equals the probability of the other.

If the true model $\Pr(Y = j|X = x_i)$ is known, we call this the...

Finally, the decision boundary



ISLR Fig 2.13

Along the boundary the probability of one outcome equals the probability of the other.

If the true model $\Pr(Y = j | X = x_i)$ is known, we call this the...*Bayes decision boundary.*

While not all methods directly estimate $\Pr(Y = j | X = x_i)$, they *all do* try to *estimate* the Bayes decision boundary.

KNN for categorical variables

It's pretty simple! Estimate the probabilities as

$K =$ number of neighboring ***training*** points to consider
$\mathcal{N}_0 =$ set of K ***training*** points closest to observation x_0.

...Then apply the Bayes decision rule to choose the outcome variable.

KNN for categorical variables

It's pretty simple! Estimate the probabilities as

$$\Pr(Y = j | X = x_0) = \frac{1}{K} \sum_{i \in \mathcal{N}_0} I(y_i = j)$$

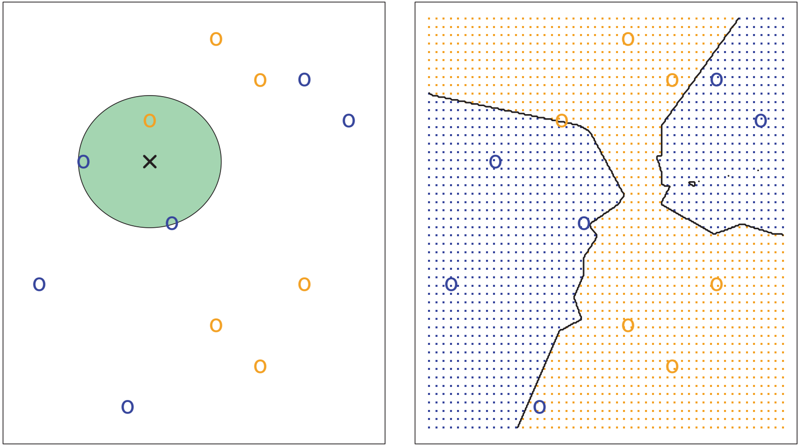
K = number of neighboring ***training*** points to consider
$\mathcal{N}_0$ = set of K ***training*** points closest to observation x_0.

...Then apply the Bayes decision rule to choose the outcome variable.

$$\hat{y}_i = \arg\max_{j \in \mathcal{J}} \Pr(Y = j | X = x_i)$$

where x_i is any feasible point in the space of independent variables.

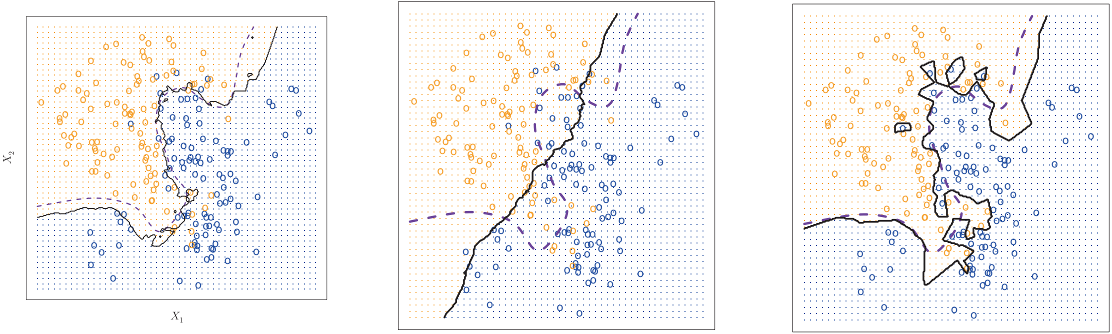
Simple KNN Example, $K = 3$



ISLR Fig 2.14

Which has the highest K? Which has the lowest?

Dashed = Bayes decision boundary
Solid = KNN estimate of Bayes decision boundary



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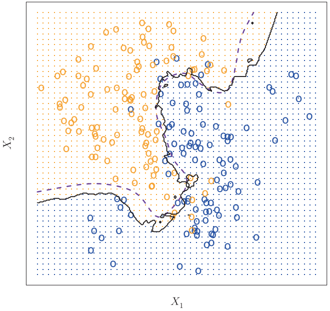
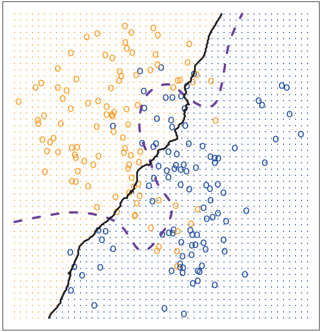
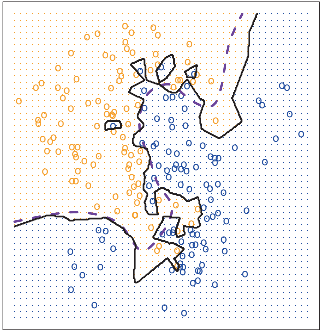


K = 10

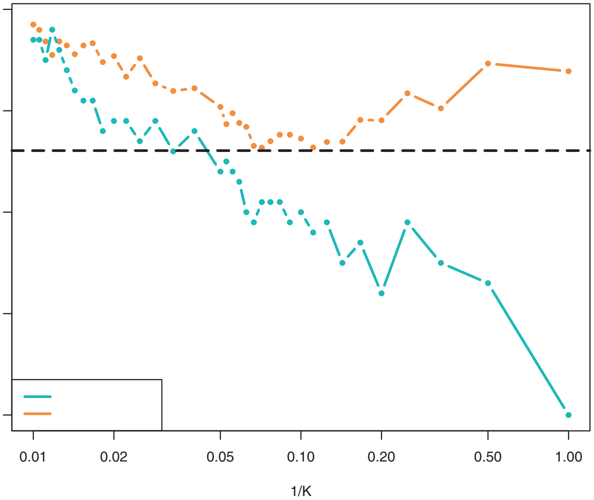


K = 100



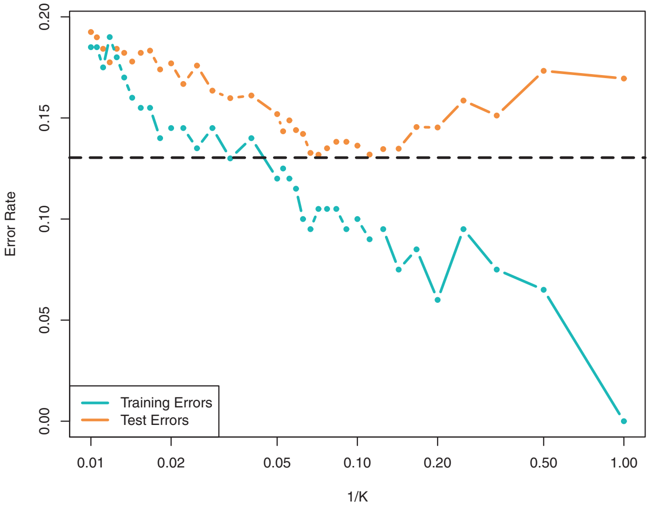
K = 1

KNN test and training error



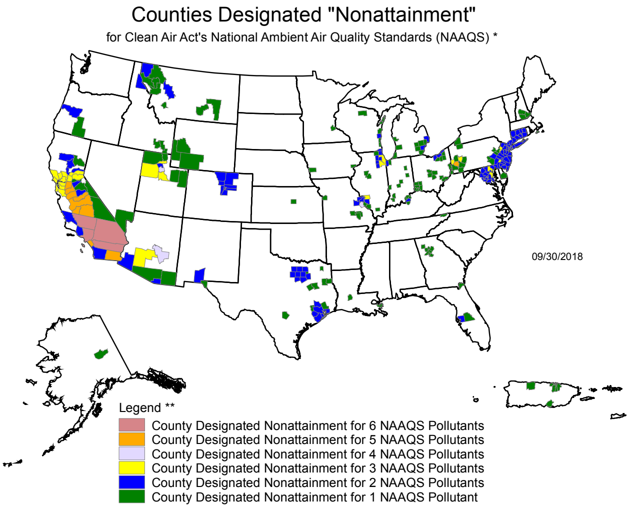
ISLR 2.17

KNN test and training error



ISLR 2.17

How would KNN perform with nonattainment areas?



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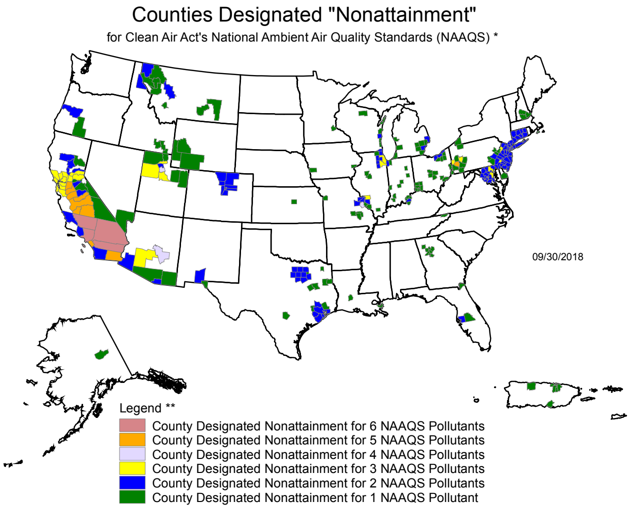

- Challenge: If we only use location as independent variables, the Bayes decision boundary is very complex!
- But if we use other independent variables (a simple one would be local emissions) we might do ok.