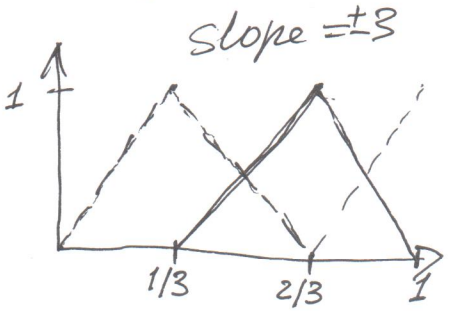
(11) $-u'' = x$ with $u(0)=u(1)=0$

$u = -\frac{x^3}{6} + Ax + C$; $u(0)=0 \to C=0$; $u(1)=0 \to -\frac{1}{6}+A=0$

$A=\frac{1}{6}$

$\left[u_p(x) = -\frac{x^3}{6} + \frac{x}{6}\right]$ $u(\frac{1}{3})=\frac{4}{81}$, $u(\frac{2}{3})=\frac{5}{81}$



with 2 hat functions $h=\frac{1}{3}$

$F_1 = \int v_1 \cdot 1\,dx = \frac{1}{3}$, $F_2 = \int v_2 \cdot 2\,dx = \frac{2}{3}$

$K_{ij} = \int_0^1 c(x) \frac{dv_i}{dx} \frac{d\varphi_j}{dx} dx$

$K_{11} = \int_0^1 \underbrace{\left(\frac{dv_1}{dx}\right)^2}_{3\cdot 3 = 9} dx = \int_0^{1/3} 9\,dx + \int_{1/3}^{2/3} 9\,dx = 3+6+3 = 12$

$K_{12} = \int_0^1 \underbrace{\frac{dv_1}{dx}}_{3} \underbrace{\frac{d\varphi_2}{dx}}_{-3} = \int_{1/3}^{2/3} (-9)\,dx = -\frac{18}{3} + \frac{9}{3} = -6+3 = -3$

$K_{21} = -3$, $K_{22} = 12$

$K = \begin{bmatrix} 12 & -3 \\ -3 & 12 \end{bmatrix}$, $Ku = F \to \begin{bmatrix} 12 & -3 \\ -3 & 12 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 4/90 \\ 5/75 \end{bmatrix}$

u_2 has the biggest error