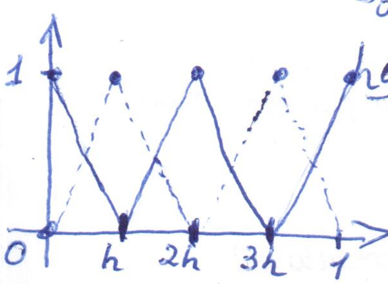
(10) $h=\frac{1}{4}$, to solve $-u''=2$ with $u(0)=u(1)=0$

(1) Weak form $\int_0^1 c(x)\frac{du}{dx}\frac{dv}{dx}dx=\int_0^1 f(x)v(x)dx$



hat functions, $v_i=\varphi_i$ and $\left.\begin{matrix} c(x)=1 \\ f(x)=2 \end{matrix}\right]$

$\underline{\text{Slope}=\pm 4}$

$F_1=F_2=F_3=2\int_0^1 v_i\,dx=\frac{1}{2} \leftarrow F_i=\int_0^1 f(x)v_i(x)dx$

$\underline{K_{ij}=\int_0^1 c(x)\frac{dv_i}{dx}\frac{d\varphi_i}{dx}dx}$

$K_{11}=\int_0^1 \underbrace{\frac{dv_1}{dx}\frac{d\varphi_1}{dx}}_{\left(\frac{dv_1}{dx}\right)^2=(4)^2=16}dx=\int_0^{1/4}16\,dx+\int_{1/4}^{1/2}16\,dx=4+4=8$

$K_{12}=\int_0^1 \underbrace{\frac{dv_1}{dx}\frac{d\varphi_2}{dx}}_{4\cdot(-4)=-16}dx=\int_{1/4}^{1/2}1\cdot(-16)dx=-16\,\frac{1}{4}=-4$; $K_{13}=\int_0^1 \underbrace{\frac{dv_1}{dx}\cdot\frac{d\varphi_3}{dx}}_{4\cdot 0=0}dx=0$ (no intersections)

$K_{21}=K_{23}=-4$, $K_{22}=8$ and $K_{31}=0$, $K_{32}=-4$, $K_{33}=8$

$$KU=F \rightarrow \begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}=\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}=\frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}\cdot\frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\frac{1}{32}\begin{bmatrix} 6 \\ 8 \\ 6 \end{bmatrix}=\frac{1}{16}\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}=\begin{bmatrix} .1875 \\ .25 \\ .1875 \end{bmatrix}$$

$u(x)=x-x^2 \rightarrow x=\begin{bmatrix} 1/4 \\ 1/2 \\ 3/4 \end{bmatrix}$, $u(x)=\begin{bmatrix} .1875 \\ .25 \\ .1875 \end{bmatrix}$, Checked!

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(18) $u'(1)=0 \rightarrow$ (a) $\frac{1}{h}K=\frac{1}{h}\begin{bmatrix} \cdots & & \\ \cdots & & \\ \cdots & -1 & 1 \end{bmatrix}$

$\underline{-u_n+u_{n+1}=f_{n+1}}$