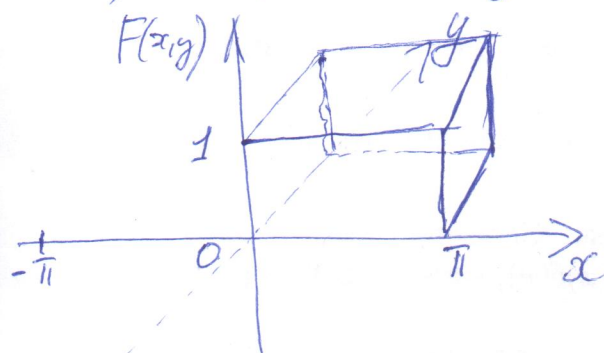


Problem Set 4.2

① a) $F = \text{quarter square} = \begin{cases} 1, & 0 \leq x \leq \pi, 0 \leq y \leq \pi \\ 0, & -\pi < x < 0 \text{ or } -\pi < y < 0 \end{cases}$



$$C_{mn} = \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} F(x,y) e^{-imx} e^{-iny} dx dy$$

$$= \left(\frac{1}{2\pi}\right)^2 \int_0^{\pi} \int_0^{\pi} e^{-imx} e^{-iny} dx dy$$

$$\int_0^{\pi} e^{-iny} dy = \left[\frac{e^{-iny}}{-in} \right]_0^{\pi} = \left[\frac{e^{-iny}}{in} \right]_{\pi}^0 = \frac{1}{in} - \frac{e^{-in\pi}}{in} = \frac{1}{in} (1 - e^{-in\pi})$$

$$\int_0^{\pi} e^{-imx} dx = \frac{1}{im} (1 - e^{-im\pi})$$

$$C_{mn} = \left(\frac{1}{2\pi}\right)^2 \left(\frac{1}{in}\right) \left(\frac{1}{im}\right) (1 - e^{-im\pi}) (1 - e^{-in\pi}) = \frac{1}{4\pi^2} \left(\frac{-1}{nm}\right) (1 - \cos(\pi m)) (1 - \cos(\pi n))$$

$$(1 - e^{-im\pi}) (1 - e^{-in\pi}) = (1 - \cos(\pi m) + i \sin(\pi m)) (1 - \cos(\pi n) + i \sin(\pi n))$$

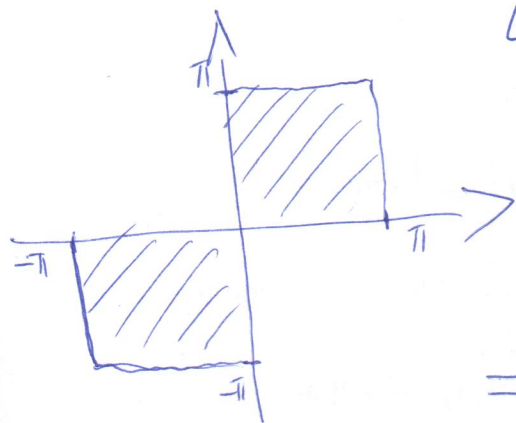
For $m, n \neq 0$

$$C_{mn} = \begin{cases} -\frac{1}{4\pi^2 nm} & \text{for } m \text{ and } n \text{ odd} \\ 0 & \text{for } m \text{ and } n \text{ even} \end{cases}$$

$$C_{0n} = \left(\frac{1}{2\pi}\right)^2 \int_0^{\pi} \int_0^{\pi} e^{-iny} dx dy = \left(\frac{1}{2\pi}\right)^2 \left(\frac{\pi}{in}\right) (1 - e^{-in\pi}) = \begin{cases} \frac{1}{2\pi in}, & \text{for } n \text{ odd} \\ 0, & \text{for } n \text{ even} \end{cases}$$

$$C_{m0} = \begin{cases} \frac{1}{2\pi im}, & \text{for } m \text{ odd} \\ 0, & \text{for } m \text{ even} \end{cases}$$

b) $F = \text{checkerboard} = \begin{cases} 1 & \text{if } xy > 0 \\ 0 & \text{if } xy < 0 \end{cases} \quad \begin{matrix} -\pi < x \leq \pi \\ -\pi < y \leq \pi \end{matrix}$



$$\begin{aligned} C_{mn} &= \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{-imx} e^{-iny} dx dy \\ &\quad + \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^0 \int_0^{\pi} e^{-imx} e^{-iny} dx dy \\ &= \left(\frac{1}{2\pi}\right)^2 \left(\frac{1}{im}\right) \left(\frac{1}{in}\right) \left[(1 - e^{-im\pi})(1 - e^{-in\pi}) + (-1 + e^{im\pi})(-1 + e^{in\pi}) \right] \end{aligned}$$

For $m, n \neq 0$

$$C_{mn} = \begin{cases} -\frac{2}{\pi^2 mn} & \text{for } m, n \text{ odd} \\ 0 & \text{if } m \text{ even or } n \text{ even} \end{cases}$$

③ $F(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} b_{nm} \sinh(nx) \sinh(my)$

$$b_{nm} = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} F(x, y) \sinh(nx) \sinh(my) dx dy$$