

# Problem Set 4.4

$$(1) (12)(15) = 180; \underbrace{(\dots, 1, 2, \dots)}_3 \times \underbrace{(\dots, 1, 5, \dots)}_6 = \underbrace{(\dots, 1, 7, 10, \dots)}_{18}$$

$$(2) F(C \otimes D) = (F_C) \cdot (F_D) \text{ for } N=2$$

$$(F_C) \cdot (F_D) = \begin{bmatrix} c_0 + c_1 \\ c_0 - c_1 \end{bmatrix} \begin{bmatrix} d_0 + d_1 \\ d_0 - d_1 \end{bmatrix} = \begin{bmatrix} (c_0 + c_1)(d_0 + d_1) \\ (c_0 - c_1)(d_0 - d_1) \end{bmatrix}$$

$$F(C \otimes D) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} c_0 d_0 + c_1 d_1 \\ c_0 d_1 + c_1 d_0 \end{bmatrix} = \begin{bmatrix} c_0 d_0 + c_1 d_1 + c_0 d_1 + c_1 d_0 \\ c_0 d_0 + c_1 d_1 - c_0 d_1 - c_1 d_0 \end{bmatrix}$$

$$= \begin{bmatrix} (c_0 + c_1)(d_0 + d_1) \\ (c_0 - c_1)(d_0 - d_1) \end{bmatrix} \checkmark$$

$$(3) F^{-1} \text{diag}(F_C) F = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} c_0 + c_1 & 0 \\ 0 & c_0 - c_1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} c_0 + c_1 & c_0 + c_1 \\ c_0 - c_1 & c_1 - c_0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} c_0 + c_1 + c_0 - c_1 & c_0 + c_1 + c_1 - c_0 \\ c_0 + c_1 - c_0 + c_1 & c_0 + c_1 - c_1 + c_0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2c_0 & 2c_1 \\ 2c_1 & 2c_0 \end{bmatrix} = \begin{bmatrix} c_0 & c_1 \\ c_1 & c_0 \end{bmatrix} \checkmark$$

$$(6) \delta_N \otimes D = D \rightarrow \delta_N = (1, 0, \dots, 0) \text{ with } N-1 \text{ zeroes}$$

$$(8) a) f = (0, 0, 0, 1, 0, 0), N=6 \quad f \otimes f = (f_0 \omega^0 + f_1 \omega^1 + \dots + f_5 \omega^5)(f_0 \omega^0 + \dots + f_5 \omega^5) = (\omega^3)(\omega^3) = \omega^6$$

$$f \otimes f = (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0), N=6 \Rightarrow \omega^6 = 1$$

$$f \otimes f = (1, 0, 0, 0, 0, 0)$$

$$b) F_6^{-1} = \frac{1}{6} \begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = C$$

$$c) f \otimes f = N F_v(C * C)$$

$$C * C = \frac{1}{36} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad 6 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \dots & \dots & \omega^5 \\ 1 & & \ddots & & \\ 1 & & & \ddots & \\ 1 & & & & \omega^5 \\ 1 & \omega^5 & \dots & \dots & \omega^{25} \end{bmatrix} \frac{1}{36} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$