

$$(10) \frac{df}{dx} = -a f + \delta(x) \rightarrow -a \hat{f}(k) + 1 = \frac{-a}{a+ik} + 1 = \frac{ik}{a+ik}$$

which agrees with $ik \hat{f}(k)$ ✓

$$(11) u' + au = \delta(x-d), \quad ik \hat{u}(k) + a \hat{u}(k) = e^{-ikd}$$

$$\hat{u}(k) = \frac{e^{-ikd}}{ik+a}$$

If $v(x) = u(x+d)$ then $\hat{v}(k) = e^{ikd} \hat{u}(k) = \frac{1}{a+ik}$

$$v(x) = \begin{cases} e^{-ax}, & x > 0 \\ 0, & x < 0 \end{cases} \quad \text{and } u(x) = \begin{cases} e^{-a(x-d)}, & x > d \\ 0, & x < d \end{cases}$$

$$(12) (\text{integral of } u(x)) - (\text{derivative of } u(x)) = \delta(x)$$

$$\frac{\hat{u}(k)}{ik} - ik \hat{u}(k) = 1, \quad \hat{u}(k) = \frac{1}{\frac{1}{ik} - ik} = \frac{1+k^2}{ik} \quad \text{transfer function}$$

$$(23) \delta * \delta = \delta$$