

Problem set 2.5

$$\textcircled{1} u'' + 4u = 1, \quad s^2 u(s) + 4u(s) = \frac{1}{s}, \quad u(s) = \frac{1}{s(s^2+4)}$$

Laplace transform

$$\frac{B}{s} + \frac{A}{s^2+4} = \frac{1}{s(s^2+4)} \rightarrow \frac{(s^2+4)B}{s} + \frac{As}{s^2+4} = 1$$

$$As + B(s^2+4) = 1$$

$$s=0 \rightarrow B \cdot 4 = 1, \quad [B = \frac{1}{4}] \rightarrow \frac{s^2}{4} + \cancel{As} = \cancel{1}, \quad A + \frac{s}{4} = 0$$

$$[A = -\frac{1}{4}]$$

$$\mathcal{L}^{-1} \left[\frac{1}{4} \left(\frac{1}{s} \right) + \left(-\frac{1}{4} \right) \frac{s}{s^2+2^2} \right] = \left[\frac{1}{4} - \frac{1}{4} \cos(2t) \right] = u(t)$$

$$\textcircled{2} u'' + 4u' + 4u = 1, \quad s^2 u(s) + 4s u(s) + 4u(s) = 1/s$$

$$u(s)(s^2+4s+4) = 1/s \rightarrow u(s) = \frac{1}{s(s^2+4s+4)} \quad \text{transfer function}$$

$$\frac{1}{s(s^2+4s+4)} = \frac{1}{4s} - \frac{1}{4(s+2)} - \frac{1}{2(s+2)^2} = u(s)$$

$$\textcircled{3} \mathcal{L}^{-1} \left[\frac{1}{4s} - \frac{1}{4(s+2)} - \frac{1}{2(s+2)^2} \right] = \left[\frac{1}{4} - \frac{1}{4} e^{-2t} - \frac{1}{2} t e^{-2t} \right] = u(t)$$