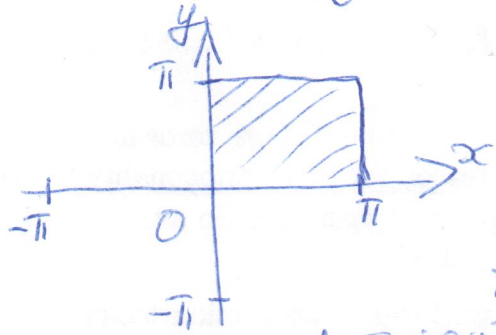# Problem Set 4.2

(1) **a)** $F = $ Quarter square $= \begin{cases} 1 & \text{for } 0 \le x \le \pi, \, 0 \le y \le \pi \\ 0 & \text{if } -\pi < x < 0 \text{ or } -\pi < y < 0 \end{cases}$
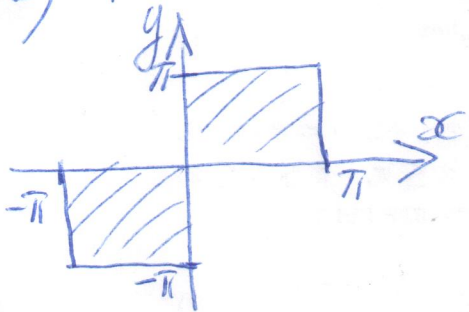

$$C_{ke} = \left(\frac{1}{2\pi}\right)^2 \int_0^\pi \int_0^\pi e^{-ikx} e^{-iey} \, dx \, dy$$

$$\int_0^\pi e^{-ikx} \, dx = -\frac{1}{ik}\left[e^{ikx}\right]_0^\pi = -\frac{1}{ik}\left(e^{ik\pi}-1\right)$$

$$\int_0^\pi e^{-iey} \, dy = -\frac{1}{ie}\left[e^{iey}\right]_0^\pi = -\frac{1}{ie}\left(e^{ie\pi}-1\right)$$

$$\boxed{C_{ke} = \left(\frac{1}{2\pi}\right)^2 \frac{1}{(i)^2 ke}\left(e^{ik\pi}-1\right)\left(e^{ie\pi}-1\right)}$$

b) $F = $ Checkerboard $= \begin{cases} 1 & \text{if } xy \ge 0 \quad -\pi < x \le \pi \\ 0 & \text{if } xy < 0 \quad -\pi < y \le \pi \end{cases}$

$$C_{ke} = \left(\frac{1}{2\pi}\right)^2 \int_0^\pi \int_{-\pi}^\pi 1 \cdot e^{-ikx} e^{-iex} \, dx \, dy$$

$$\int_{-\pi}^\pi e^{-ikx} \, dx = -\frac{1}{ik}\left[e^{-ikx}\right]_{-\pi}^\pi = -\frac{1}{ik}\left(e^{-ik\pi}-e^{ik\pi}\right)$$
$$= \frac{2}{k}\sin(k\pi)$$

$$e^{-ik\pi} - e^{ik\pi} = \cos k\pi - i\sin k\pi - \cos k\pi - i\sin k\pi = -2i\sin k\pi$$

$$\boxed{C_{ke} = \left(\frac{1}{2\pi}\right)^2 \left(-\frac{2}{iek}\right)\sin(k\pi)\left(e^{ie\pi}-1\right)}$$

(3) $F(x,y) = \displaystyle\sum_{n=1}^\infty \sum_{m=1}^\infty b_{nm} \sin(nx)\sin(my)$

$$b_{nm} = \frac{1}{\pi^2}\int_{-\pi}^\pi \int_{-\pi}^\pi F(x,y)\sin(nx)\sin(ny)\,dx\,dy$$