

$$\textcircled{1} \text{ Let } K = (C_1 + C_2)(C_2 + C_3)(C_3 + C_4) - C_2^2(C_3 + C_4) - C_3^2(C_1 + C_2) \\ = C_1 C_2 C_3 + C_1 C_3 C_4 + C_1 C_2 C_4 + C_2 C_3 C_4$$

$$\text{Fixed-Free (8): } A^T C A = \begin{bmatrix} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 & -C_3 \\ 0 & -C_3 & C_3 \end{bmatrix}$$

$$|A^T C A| = (C_1 + C_2)((C_2 + C_3)C_3 - C_3^2) + C_2(-C_2 C_3) \\ = (C_1 + C_2)(C_2 C_3 + \cancel{C_3^2} - \cancel{C_3^2}) - C_2^2 C_3 = \cancel{C_1 C_2 C_3} + \cancel{C_2^2 C_3} - \cancel{C_2^2 C_3}$$

$$\text{Free-free (11): } A^T C A = \begin{bmatrix} C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 & -C_3 \\ 0 & -C_3 & C_3 \end{bmatrix}$$

$$|A^T C A| = C_2((C_2 + C_3)C_3 - C_3^2) + C_2(-C_2 C_3) = \cancel{C_2^2 C_2} - \cancel{C_2^2 C_3} = 0$$

$$\textcircled{4} A^T C A u = \begin{bmatrix} C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 & -C_3 \\ 0 & -C_3 & C_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad C = I$$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad u = (0, 1, 2) \leftarrow \text{all solutions!}$$