Problem Set I.1

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1.1 #1

Example: $Ax = 0$ in R^4

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & 2 & 5 \\ 1 & 2 & 5 \\ 1 & 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Dimensions: $(4 \times 3) \cdot (3 \times 1) = (4 \times 1)$

$u + v - (u + v) = 0$

So, any u, v:

$$\begin{bmatrix} u & v & u+v \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = 0$$

In the Example:

$$\begin{bmatrix} u & v & u+2v \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = u + 2v - (u + 2v) = 0$$

1.2 #2

Example of $Ax = Ay$

$$A = \begin{bmatrix} u & v & u+v \end{bmatrix}$$

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = A \cdot \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \implies u + v - (u + v) = -u - v + (u + v) = 0$$

$Az = 0$

$$\begin{bmatrix} u & v \end{bmatrix} \cdot z = 0 \implies z_1 \cdot u + z_2 \cdot v = 0$$

So: $z = 0$

Or: $z = \begin{bmatrix} -v \\ u \end{bmatrix}$

Or: $z = \begin{bmatrix} v \\ -u \end{bmatrix}$

1.3 #2

The vectors $a_1, a_2, \ldots, a_n$ in R^m and $c_1 \cdot a_1 + \cdots + c_n \cdot a_n = 0$

Matrix notation

$$\begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = 0$$

Or

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = c_1 \cdot \begin{bmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1n} \end{bmatrix} + \cdots + c_n \cdot \begin{bmatrix} a_{m1} \\ a_{m2} \\ \vdots \\ a_{mn} \end{bmatrix} = 0$$

Sigma notation:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} * c_j = 0$$

1.4 #4