Problem Set I.2

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1.1 # 1

$Ax = 0$ and $Ay = 0$

$B = \begin{bmatrix} x & y \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \end{bmatrix}$

$AB = C \implies A \begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

1.2 # 2

$a = a_1, a_2, \dots, a_m$ and $b = b_1, b_2, \dots, b_p$

$$ab^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \cdot \begin{bmatrix} b_1 & b_2 & \dots & b_p \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 & \dots & a_1 \cdot b_p \\ a_2 \cdot b_1 & a_2 \cdot b_2 & \dots & a_2 \cdot b_p \\ \vdots & \ddots & \dots & \vdots \\ a_m \cdot b_1 & a_m \cdot b_2 & \dots & a_m \cdot b_p \end{bmatrix}_{m \times p}$$

Row i, Col $j = a_i b_j$

$$aa^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \cdot \begin{bmatrix} a_1 & a_2 & \dots & a_m \end{bmatrix} =$$

$$\begin{bmatrix} a_1^2 & a_1 a_2 & \dots & a_1 a_m \\ a_2 a_1 & a_2^2 & \dots & a_2 a_m \\ \vdots & \dots & \ddots & \vdots \\ a_m a_1 & a_m a_2 & \dots & a_m^2 \end{bmatrix} = S$$

Symmetric matrix $S^T = S$

1.3 # 3

a) Sum of rank1 matrices

$$AB = \begin{bmatrix} | & & | \\ a_1 & \dots & a_n \\ | & & | \end{bmatrix} \cdot \begin{bmatrix} - & b_1^* & - \\ & \vdots & \\ - & b_n^* & - \end{bmatrix} = a_1 b_1^* + \dots + a_n b_n^* = C$$

b) $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

Example: $c_{45} = a_{41} b_{15} + a_{42} b_{25} + \dots + a_{4n} b_{n5} = \sum_{k=1}^{n} a_{4k} b_{k5}$

1.4 # 4

$$AB = \begin{bmatrix} | & & | \\ a_1 & \dots & a_n \\ | & & | \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + \dots + a_n b_n = C$$

$c_{ij} = \sum_{k=1}^{n} a_{ik} b_k$

1.5 # 5

$$A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}; B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix};$$

$(AB)C = A(BC)$

$(AB)C$:

$$AB = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} =$$

$$= \begin{bmatrix} b_1 + ab_3 & b_2 + ab_4 \\ b_3 & b_4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} b_1 + ab_3 & b_2 + ab_4 \\ b_3 & b_4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} b_1 + ab_3 + c(b_2 + ab_4) & b_2 + ab_4 \\ b_3 + cb_4 & b_4 \end{bmatrix}$$

$A(BC)$:

$$BC = \begin{bmatrix} b_1 + b_2 c & b_2 \\ b_3 + b_4 c & b_4 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b_1 + b_2 c & b_2 \\ b_3 + b_4 c & b_4 \end{bmatrix} =$$

$$= \begin{bmatrix} b_1 + b_2 c + a(b_3 + b_4 c) & b_2 + ab_4 \\ b_3 + b_4 c & b_4 \end{bmatrix}$$

$b_1 + ab_3 + c(b_2 + ab_4) = b_1 + b_2c + a(b_3 + b_4c)$

$b_1 + ab_3 + cb_2 + cab_4 = b_1 + cb_2 + ab_3 + acb_4$

So, $(AB)C = A(BC)$

1.6 # 6

$$AB = AI = \begin{bmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} | \\ a_3 \\ | \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} | & | & | \\ a_1 & 0 & 0 \\ | & | & | \end{bmatrix} + \begin{bmatrix} | & | & | \\ 0 & a_2 & 0 \\ | & | & | \end{bmatrix} + \begin{bmatrix} | & | & | \\ 0 & 0 & a_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{bmatrix} = A$$

$AI = A$

1.7 # 7

$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$,

$rank(A) = 2$;

$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$;

$AB = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$

$rank(AB) = 1$, $C(AB) < C(A)$

1.8 # 8

$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

$C = AB = (m \times n)(n \times p)$

Rows times cols

1) For $i = 1$ to m
2) For $j = 1$ to p
3) For $k = 1$ to n

$C(i, j) = C(i, j) + A(i, k) * B(k, j)$

Cols times rows

1) For $i = 1$ to m
2) For $j = 1$ to p
3) For $k = 1$ to n

$C(i, j) = C(i, j) + A(k, i) * B(j, k)$