EXERCISE SHEET 4

(E83) Let $a, b \in k^*$. For all $c \in k$, there exist $x, y \in k^*$ with $ax^2 + by^2 = c$.
(E84) Prove Lemma 38 of lectures for $\dim(V) = 2 = \mathrm{char}(k)$.
(E86) Prove Theorem 39 of lectures.
(E88) Let β_1 and β_2 be non-degenerate alternating bilinear forms defined on a $2r$-dimensional vector space V over a field k. Then $\mathrm{Isom}(\beta_1)$ and $\mathrm{Isom}(\beta_2)$ (resp. $\mathrm{Sim}(\beta_1)$ and $\mathrm{Sim}(\beta_2)$) are conjugate subgroups of $\mathrm{GL}_{2r}(k)$. Furthermore $\mathrm{SemiSim}(\beta_1)$ and $\mathrm{SemiSim}(\beta_2)$ are conjugate subgroups of $\Gamma\mathrm{L}_{2r}(k)$.
(E91) Let G act on a set Ω. Prove that the permutation rank is 2 if and only if G acts 2-transitively on Ω.
(E93) Prove that if $\beta(x, y) = 0$, then there exists z with $\beta(x, z), \beta(y, z) \neq 0$.
(E94) Prove that if $\beta(x, y) \neq 0$, then there exists z with $\beta(x, z) = \beta(y, z) = 0$.
(E96) Given a transvection t, there exists $f \in V^*$ and $a \in \ker(f)$ such that

$$vT = v + (vf)a \text{ for all } v \in V.$$

(E97) Prove that symplectic transvections in $\mathrm{Sp}_6(2)$ and $\mathrm{Sp}(4, 3)$ are commutators.