10 Steps toward Empowering Students Mathematically

Marian Small October, 2023

Step 1

- Students are empowered by a focus on understanding the math and not just a focus on doing the math.

Step 1

- What's the difference?
- It's huge!

Step 1

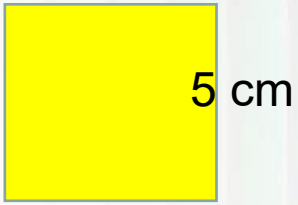
- Consider the differences in each of these examples.

K-2

- DOING: List the pairs of numbers that add to 10.
- UNDERSTANDING: If you add two numbers to make 10, can they both be less than 5?

G 3-5

- DOING: What is the perimeter of this square?



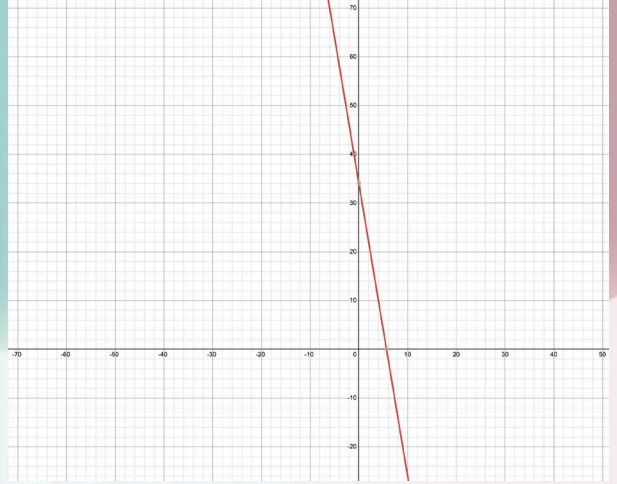
- UNDERSTANDING: I made a shape with a perimeter of 50 cm. Could one side be 25 cm?

G 6-8

- DOING: What is the sign if you divide a negative number by a positive one?

- UNDERSTANDING: Draw a picture that explains why (–12) ÷ 4 must be a negative.

G 9-12



- DOING: What is the equation of the line through (4,10) and (6,–2)?

- UNDERSTANDING: How do you know that there can only be one line that goes through (4, 10) and (6, –2)?

Why?

- Why is this empowering?
- If you only have rules, you have no power if you don’t remember the rules.
- If you have understanding, you can recreate ideas that you might have lost.
- This is about focusing on the “long haul” and not the “short haul”.

Step 2

- Students are empowered by less focus on answers and more focus on process, strategies and thinking.

Step 2

- Even if teachers use problem solving tasks, the focus is often on the answer and how various students got the answer.
- That is still a focus on answers.

Step 2

- It could involve activities like:
- WITHOUT MULTIPLYING, tell me something you know about 28 x 93.
- OR
- WITHOUT ADDING, tell me what you know about $\frac{2}{3} + \frac{1}{4}$.

Step 2

- OR
- WITHOUT ADDING, tell me something you know about 37 + 48.
- OR
- WITHOUT GRAPHING OR DOING ALGEBRA, tell me something you know about the common solution to $x + y = 9$ and $2x - y = 3$.

Step 2

- Let’s talk about what this looks like in terms of students working on a task.

Possible Task

Action Task

MATH UP

1. Ryan and Kyle served 249 meals at the community kitchen. Ryan served twice as many meals as Kyle. How many meals did each person serve? Show all your work.

Step 2

- Even before students get their answers, I could ask:
- How do you know that 100 and 200 are not the amounts?
- Will the numbers of meals each served be more or less than 100 and 200?
- Why might you divide?
- Why wouldn’t you just divide by 2?

Step 2

- Another task.

1. Describe a situation that fits each equation. Describe more than one situation for each equation, if possible.

a) $4x - 1 = 19$

b) $2x - 4 = 10$

c) $2x + 2y = 48$

d) $5x + 2y = 90$

2. Choose one of the situations above, and use a different equation to describe the same situation.

Step 2

- Even without all this being done, I can ask
- When you see a multiplication in an equation, how does that help you come up with a context?
- What about a subtraction?

Step 2

- Why can equations always describe more than one situation?
- Can situations usually be described by more than one equation?

Step 2

- This empowers students who are slower.

Step 2

- A strategy a teacher might use is to give students every answer and indicate that the students' only task is to figure out how you arrived at that solution.

Step 3

- Differentiation during instruction, and not only differentiation after instruction, empowers students.

Step 3

- This might be as simple as allowing students to choose values with which to work.
- Or allowing students to choose which of two or three tasks to pursue or which direction to take given an open-ended task.

Step 3

- For example, instead of a primary task where we ask students to represent the number 83 in three different ways, we could say:
- Choose a number greater than 10. Represent it in a bunch of ways.

Step 3

- For example, instead of a G5-7 task where we ask students to determine the sum of $\frac{2}{3} + \frac{4}{5}$ in whatever way they wish, we could say:
- Choose two fractions between $\frac{1}{2}$ and 1 so that the sum is closer to 1 than 1 $\frac{1}{2}$.

Step 3

- For example, instead of a higher grade task where we ask students to solve a particular quadratic equation, we could ask:
- Which of these do you think would be easiest for you to solve? Why?

  $(x + 4)^2 = 25$, $x^2 + 17x + 72 = 0$, or

  $x^2 + 5x + 6 = 0$.

Step 4

- Differentiation of assessment is another way to empower students.

Step 4

- A teacher might allow students these sorts of choices.
- Students might get to decide which of a group of items they choose to use to show their knowledge.
- They might get to decide whether they provide information in writing or orally.

Step 4

- This might be addressed by allowing students to choose which of a number of pieces of work they wish you to assess.

Step 5

- Less direction instruction and more learning through problem solving empowers students.

Step 5

- The main reason is that students would have their own personal connections to draw on when they hear what we want them to learn.
- A second reason is that you are showing students you actually trust them to figure things out.

Step 5

- A third reason is that DI is more likely in learning through problem solving than in direct instruction.

- Empowering students honours their differences. Direct instruction makes that more difficult.

Step 5

- Direct instruction is often quicker in the short term, but the question is whether the learning lasts.

Step 6

Some of the most successful math students notice a lot of things that not all students notice.

We empower students by helping them notice.

Step 6

We need to set tasks that make it possible for students to notice what is important learning.

Step 6

Consider this task:

57 – 37 =

58 – 37 =

59 – 37 =

60 – 37 =

Step 6

Students begin to notice,
It might be easier to do 59 – 37 than 60 – 37, so I'll do the first one and then add 1.

Step 6

For example, suppose I want students to notice that when you subtract a number from another one, the result could be more, less or the same as what you subtracted.

Step 6

I might ask students to complete these subtractions:

84 – 4 84 – 78 84 – 43

Then I'd ask: What do you notice about how the difference compares to what you subtracted?

Step 6

For example, suppose I want students to notice how the distributive principle works.

I might ask students what they notice here.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Step 6

Suppose I want students to notice that because an equation is a balance, you can also predict information about the solution before solving.

Step 6

For example, how could you predict that the solution to
$35x + 19 = 23x + 8$ could NOT be a positive value, but
$35x + 19 = 23x + 88$ could have a positive solution?

Step 7

- Conversation empowers students.

Step 7

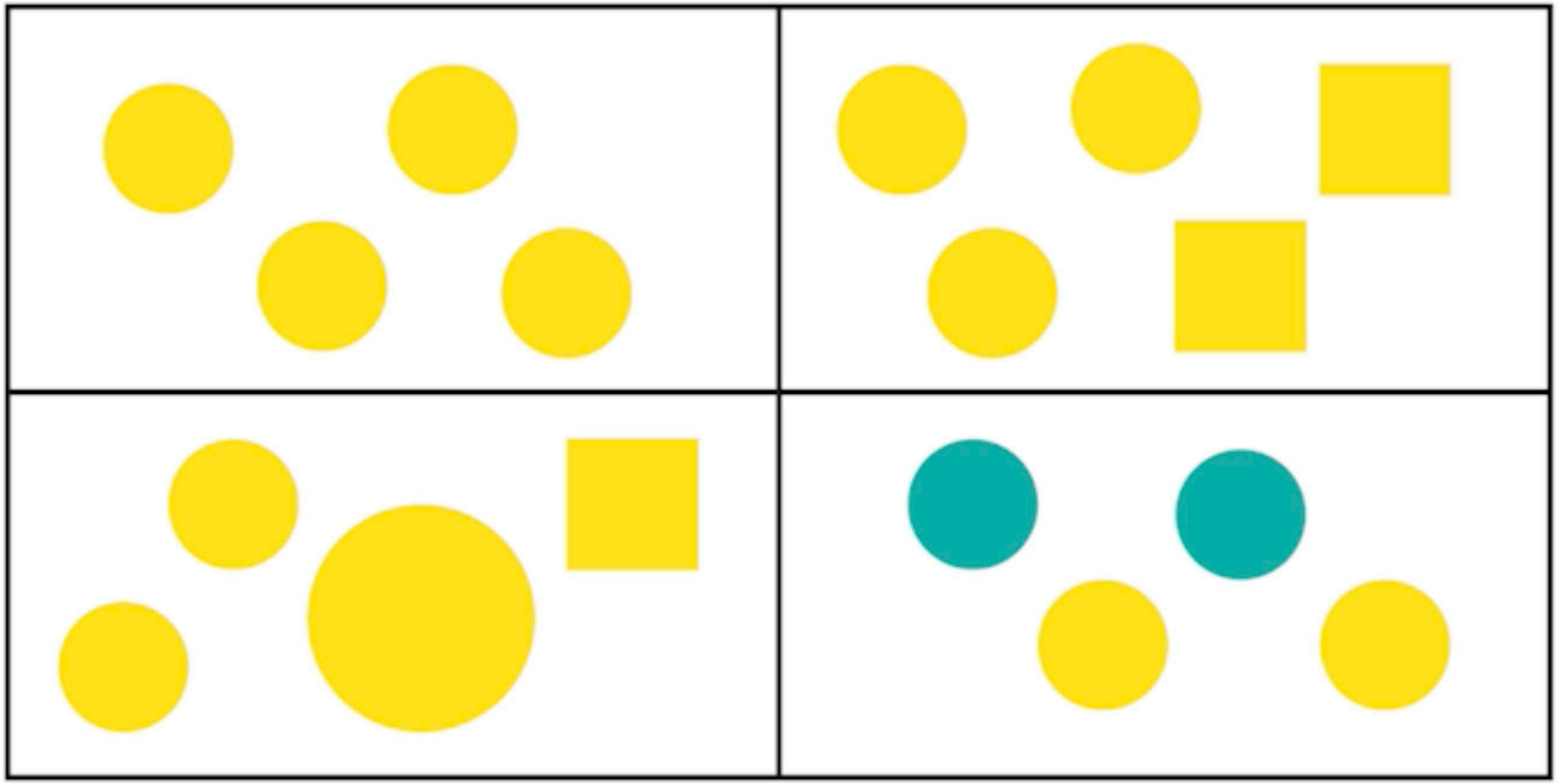
- We need to provide opportunities for student –student talk, student- teacher talk.

Step 7

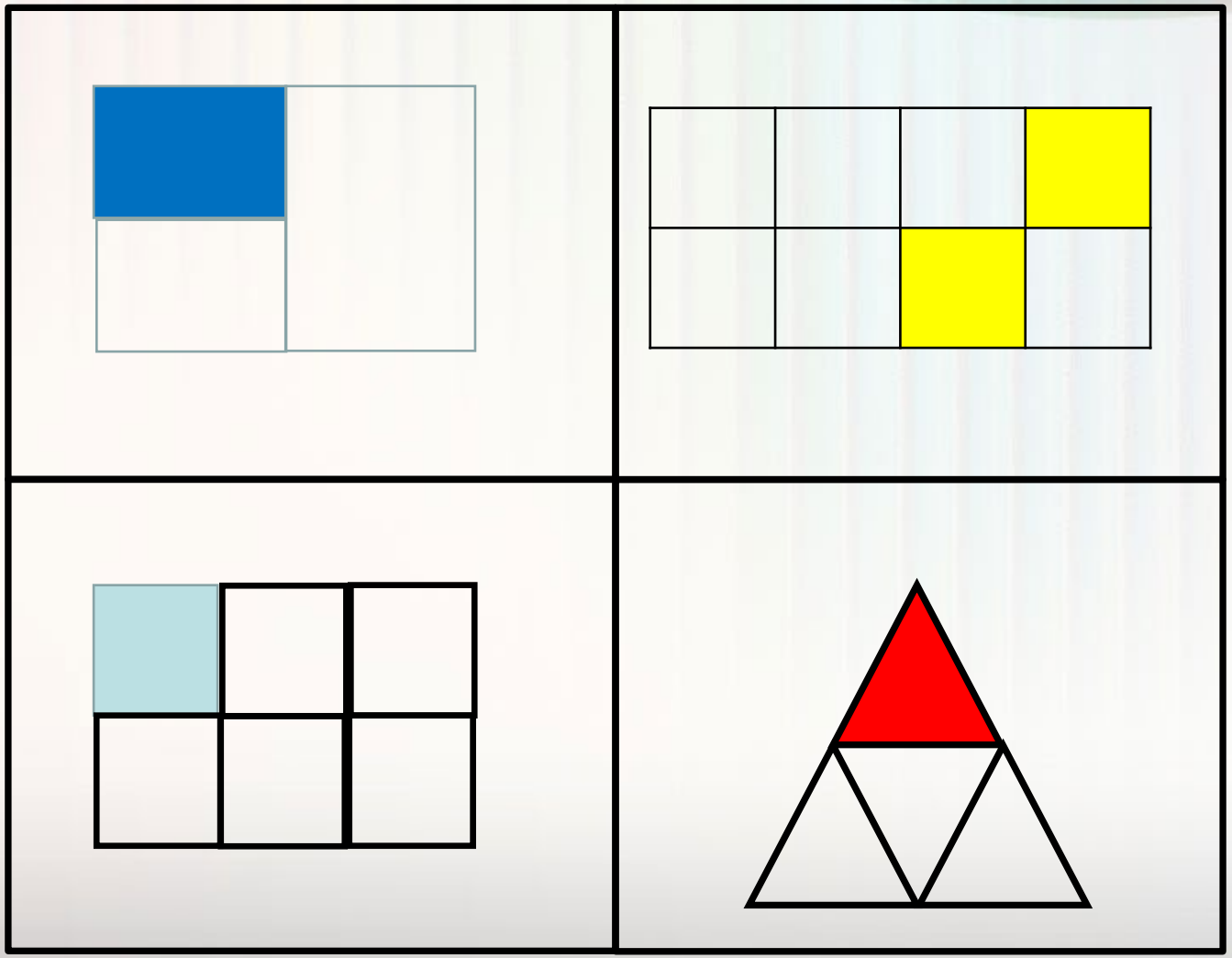
- Getting them to talk isn’t hard.
- Tell me what you notice and tell me what you wonder about.

Step 7

Step 7



Step 7



Step 7

- We need to show students we are listening to them in our responses to them.

Step 7

- Let's test that.
- I will ask a question and try to respond to what you say.
- You are adding two number and subtracting the same two numbers.
- The add answer is 10 more than the subtract answer.
- What could the numbers be?

Step 7

- Let's test that.
- I will ask a question and try to respond to what you say.
- You added two fractions and the sum has a denominator of 12.
- What do you think the denominators of the fractions you added were?

Step 7

- We might want to provide anonymous ways for students to ask questions.

Step 7

- We might want to find a way to turn a wrong answer into a correct one without misleading students.
- For example, I pose a problem:

Step 7

- I have a rectangle with whole number side lengths.
- The length is 4 times the width.
- I calculate the perimeter.
- How does the perimeter relate to the width?

Step 7

- The student says it is bigger than the width.
- What do I do?
- What made you say that?
- What if you used a really big width?
- Could you tell me the width if I tell you the perimeter is 20 units?

Step 7

- Another way to look at this is taking the position– ok, let's look at your problem instead of mine.

- For example, you ask students to solve the equation $2x + 6 = 84$ and they say 38.
- I could say give me another equation 38 solves (maybe $2x = 76$) and I keep going until I help them see the contradiction.

Step 7

- When students ask you what you mean, maybe you don’t always tell them, but then you respect the direction in which they go.

Step 8

- Students are empowered by learning to self-assess and not depend only on our confirmation.

Step 8

- We need to provide meaningful success criteria.
- We need to insist that students tell you if they are correct, not you tell them.
- We might ask, in response to a query about whether they are right, what do you think the best part of what you did was?

Step 9

- Students are empowered when they are free to ask for what they need and provided, without fanfare, the tools they need.

Step 9

- Students need to be provided with the tools that might empower them without feeling negatively judged for using them.
- We don’t’ say “You can use … IF YOU NEED THEM.”
- We don’t force them to get up in front of all their peers to get the materials they need.

Step 9

- Students need to have an opportunity to select tools and strategies.
- That means that we provide an assortment of tools on the table.
- That means they get to decide what an efficient strategy for them is.

10

- Empowered students need empowered teachers- teachers who reflect regularly on their practice and make professional judgments that are thought through.

Step 10

- So many teachers do not give themselves permission to make their own decisions.
- They talk about preparing kids for what the next teacher wants.
- They talk about what another teacher tells them they should do.

Step 10

- They choose resources to teach from where they just do what they are told.
- If a teacher does not feel empowered, it shows.

Step 10

- If it shows, the students see it and they are less comfortable believing they have choices.

In summary

- I am sure I could have stated the steps differently or I could have changed the order or the examples I used, but…
- These are the things that I have come to believe make the biggest difference.

Any questions?