Homeworks Heaps 1

Plasencia Palacios Milton Nicolas

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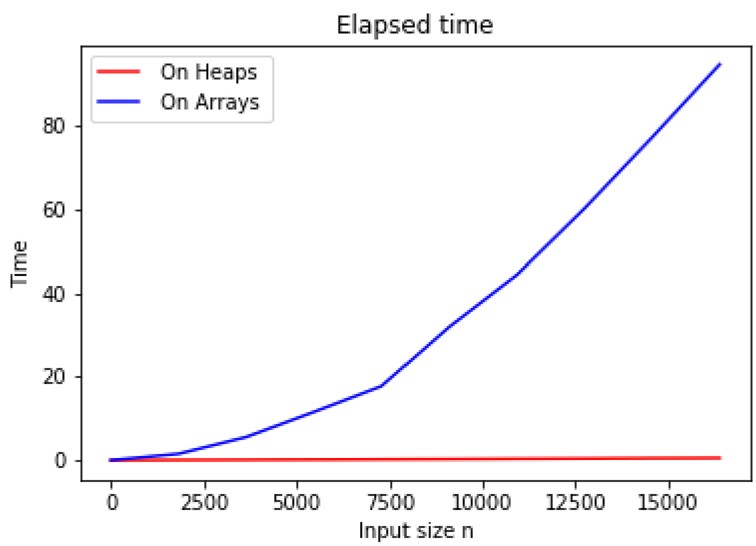
Exercises 1. and 2. are implemented in the code "binheap.c" in the "src" directory.

3. I've used the file "test_delete_min.c" to test the time differences in removing the minimum in a heap or in an array, those are the results (in seconds):



As we can see the heap implementation is faster than an array one.

4. To show that the leave nodes are indexed using $\lfloor\frac{n}{2}\rfloor + 1, \lfloor\frac{n}{2}\rfloor + 2 \ldots, n$ we must prove that those nodes does not have a left or right child, so, given an index i such that $i \in [\lfloor\frac{n}{2}\rfloor + 1,\ \lfloor\frac{n}{2}\rfloor + 2 \ldots, n]$, by taking $i = \lfloor\frac{n}{2}\rfloor + 1$, we can write its left child as $2i = 2\lfloor\frac{n}{2}\rfloor + 2 \geq n$, but it cannot be because our heap is made up by at most n nodes. So the other values in the interval, which are greater than $\lfloor\frac{n}{2}\rfloor + 1$, cannot have a left child (and also a right child), then those are leave nodes.

5. Considering a max-heap in which the elements $i \in \{1, 2, \dots 100\}$, we have the heap built such that $i = 1$ is the root, so the values of the other nodes are $\preceq$ $(\geq)$ with respect to the root. We may apply heapify at every level of the heap in this case (worst case), which will have a complexity of $\Omega(\log_2 n)$ with $\log_2 n$ the height of the heap.

6. Given a n nodes Binary-Heap we can show that the leave nodes are $\lceil \frac{n}{2^{h+1}} \rceil$.
We can prove this by induction.
In the base case we have $h = 0$ so the leave nodes are $\lceil \frac{n}{2} \rceil$.
In the inductive step we assume that in the $h - 1 - th$level we have $\lceil \frac{n}{2^n} \rceil$ nodes. Knowing that we are considering a binary heap, so every node can have at most 2 children, we can write the the upper bound for the number of nodes at height h in the following way:

$$h - 1 \text{ level has } \left\lceil \frac{n}{2^h} \right\rceil \text{ nodes } \rightarrow h \text{ level } \left\lceil \frac{n}{2 * 2^h} \right\rceil = \left\lceil \frac{n}{2^{h+1}} \right\rceil$$

in this way thesis has been proven.