

Pseudo-code for Prim's algorithm for Minimum Spanning Tree

Input: weighted digraph $G = (V, E, wt)$ assumed to be symmetric

Output: graph $T = (V, ET)$ that is symmetric and has the same vertices/edges and ET is a MST of G

$T :=$ new graph with vertices V (and no edges)

$r :=$ choose some element of V

$Q :=$ new min-priority queue

$link$: array indexed by V of vertices

$inQ :=$ array that indicates if a vertex is in the queue

$handles :=$ array that keeps track of our handles to our data

forall v :

$link[v] := 0$

$handles[v] := \text{null}$

$inQ := 1$

/* We write (v,d) for an object where the distance d is mutable.

* Comparison in Q based on d . */

$handles[0] := Q.enqueue((r,0))$ /* distance 0 for r */

for all v in V with $v \neq r$:

$handles[v] := Q.enqueue((v, \text{infinite}))$

while Q nonempty

$(v,d) := Q.dequeueMin()$

$handles[v] := \text{null}$

$inQ[v] := 0$

 for all u in $\text{successors}(G,v)$

 if $inQ[u]$ and $wt(u,v) < \text{distance of } u$ then

$link[u] := v$

 update distance of u to $wt(u,v)$

$Q.decreasedKey(handles[u])$ /*signal to the queue that

 we decreased out key, so that the Queue re-orders it*/

forall v :

 add to T the edges $(v, link[v])$ and $(link[v], v)$ //makes it symmetric

Invariants of main loop:

A: the edges in T , and vertices in $V - Q$, are part of a MST

B: the set $\{ (v, link[v]) \mid v \in V, v \neq r, v \text{ not in } Q \}$ is a tree and is part of an MST of G

C: for all (u,d) in Q ,

 if $link[u] \neq 0$ then $d < \text{infinity}$

 and d is the weight of a lightest edge connecting u to the rest of the current tree