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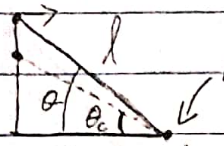
$F_{NW}$ = Normal force of the wall

1.1) 

Describes constrained motion

$$E_K(\theta) = \frac{1}{2} m v_{com}^2 + \frac{1}{2} I \dot\theta^2$$
$$= \frac{1}{2} m \frac{1}{4} l^2 \dot\theta^2 + \frac{1}{2} \cdot \frac{1}{3} m \frac{1}{4} l^2 \dot\theta^2$$
$$= \frac{1}{6} m l^2 \dot\theta^2$$

$$E_p(\theta) = mg \frac{1}{2} l \sin\theta$$

$$E_{Tot} = \frac{1}{6} m l^2 \dot\theta^2 + \frac{1}{2} mgl \sin\theta = mg \frac{l}{2} \sin\theta_0$$ , since energy is conserved

$$\frac{1}{6} l \dot\theta^2 + \frac{1}{2} g \sin\theta = \frac{1}{2} g \sin\theta_0$$
$$\frac{1}{3} l \dot\theta^2 + g \sin\theta = g \sin\theta_0$$

$$x_{com} = l\cos\theta - \frac{1}{2} l \cos\theta = \frac{1}{2} l \cos\theta$$
$$\dot x_{com} = -\frac{1}{2} l \sin\theta \dot\theta$$
$$\ddot x_{com} = -\frac{1}{2} l (\sin\theta \ddot\theta + \cos\theta \dot\theta^2)$$

$$F_{NW} = m \ddot x_{com} = -\frac{1}{2} m l (\sin\theta \ddot\theta + \cos\theta \dot\theta^2)$$ , we want the point where $F_{NW} = 0$ since that's when the wall no longer exerts a force on the ladder

$$0 = -\frac{1}{2} m l (\sin\theta \ddot\theta + \cos\theta \dot\theta^2)$$

$$\left.\begin{aligned} &\frac{1}{3} l \dot\theta^2 + g \sin\theta = g \sin\theta_0 \\ &\dot\theta^2 = 3 \frac{g}{l} (\sin\theta_0 - \sin\theta) \\ &2\dot\theta\ddot\theta = 3 \frac{g}{l} (-\cos\theta \dot\theta) \\ &\ddot\theta = -\frac{3}{2} \frac{g}{l} \cos\theta \end{aligned}\right\}$$ Finding $\dot\theta^2$ and $\ddot\theta$

$$0 = -\frac{1}{2} m l \left( \sin\theta \left(-\frac{3}{2} \frac{g}{l} \cos\theta\right) + \cos\theta \left(3 \frac{g}{l} (\sin\theta_0 - \sin\theta)\right)\right)$$
$$= \frac{3}{2} mg \left(\frac{1}{2} \sin\theta \cos\theta - \cos\theta \sin\theta_0 + \cos\theta \sin\theta\right)$$
$$= \frac{3}{2} mg \cos\theta \left(\frac{3}{2} \sin\theta - \sin\theta_0\right)$$
$$= \frac{9}{4} mg \cos\theta \left(\sin\theta - \frac{2}{3} \sin\theta_0\right)$$

$\therefore$ Normal force from wall is 0 when $\cos\theta_c = 0$ or $\sin\theta_c = \frac{2}{3} \sin\theta_0$. However, $\cos\theta_c = 0$ when $\theta_c = \pm 90°$, and $90°$ is not the case where the ladder is sliding down the wall, and $-90°$ is not possible as the ladder stops on the floor.

$\therefore$ $\sin\theta_c = \frac{2}{3} \sin\theta_0 \rightarrow \theta_c = \sin^{-1}\left(\frac{2}{3} \sin\theta_0\right)$

where $\theta_c$ is the angle where the ladder leaves the wall

$$\dot{x}_{com} = -\frac{1}{2} l \sin\theta \dot{\theta}$$
$$= -\frac{1}{2} l \sin\theta \sqrt{3\frac{g}{l}(\sin\theta_0 - \sin\theta)}$$
$$\dot{x}_{com}(\theta_c) = -\frac{1}{2} l \left(\frac{2}{3}\sin\theta_0\right)\sqrt{3\frac{g}{l}\left(\sin\theta_0 - \frac{2}{3}\sin\theta_0\right)}$$
$$= -\frac{1}{2} l \left(\frac{2}{3}\sin\theta_0\right)\sqrt{\frac{g}{l}\sin\theta_0}$$
$$= -\frac{1}{3}(gl)^{1/2}(\sin\theta_0)^{3/2}$$

$$x_{com} = \frac{1}{2} l \cos\theta$$
$$\dot{x}_{com} = -\frac{1}{2}\sin\theta\sqrt{3gl(\sin\theta_0 - \sin\theta)}$$
$$\ddot{x}_{com} = \frac{9}{4} g\cos\theta\left(\sin\theta - \frac{2}{3}\sin\theta_0\right)$$

1.2) $\mathcal{L} = T - V = E_k(\theta) - E_p(\theta)$

$= \frac{1}{6} m l^2 \dot{\theta}^2 - mg\frac{1}{2} l \sin\theta$

$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{1}{3} m l^2 \dot{\theta}$

$\frac{\partial \mathcal{L}}{\partial \theta} = -mg\frac{1}{2} l\cos\theta$

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{1}{3} m l^2 \ddot{\theta}$

$0 = -mg\frac{1}{2} l\cos\theta - \frac{1}{3} m l^2 \ddot{\theta}$

$\ddot{\theta} = \frac{-mg\frac{1}{2} l\cos\theta}{\frac{1}{3} m l^2} = -\frac{3}{2}\frac{g}{l}\cos\theta$

$x_{com} = \frac{1}{2} l\cos\theta$

$\dot{x}_{com} = -\frac{1}{2} l\sin\theta\,\dot{\theta}$

$\ddot{x}_{com} = -\frac{1}{2} l(\sin\theta\,\ddot{\theta} + \cos\theta\,\dot{\theta}^2)$

$\ddot{x}=0$ is the point where the ladder leaves the wall

$0 = -\frac{1}{2} l(\sin\theta\,\ddot{\theta} + \cos\theta\,\dot{\theta}^2)$

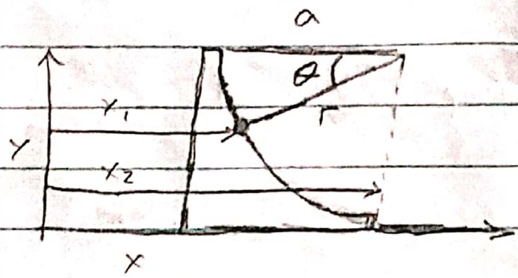
$= -\frac{1}{2} l(\sin\theta(-\frac{3}{2}\frac{g}{l}\cos\theta) + \cos\theta(3\frac{g}{l}(\sin\theta_0 - \sin\theta)))$

$= \frac{9}{4} g\cos\theta(\sin\theta - \frac{2}{3}\sin\theta_0)$

$\therefore$ ladder leaves wall at $\theta_c = \pm 90°$ (which we don't want) and $\sin\theta_c = \frac{2}{3}\sin\theta_0$

$\therefore \theta_c = \sin^{-1}(\frac{2}{3}\sin\theta_0)$

where $\theta_c$ is the angle where the ladder leaves the wall

2. 


$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2\dot{x}_2^2$ , $V = -mgr\sin\theta$

$x_1 = x_2 - r\cos\theta$

$\dot{x}_1 = \dot{x}_2 + r\sin\theta\dot{\theta} - \dot{r}\cos\theta$

$y_1 = -r\sin\theta$

$\dot{y}_1 = -r\cos\theta\dot{\theta} - \dot{r}\sin\theta$

$\dot{x}_1^2 = \dot{x}_2^2 + \dot{x}_2 r\sin\theta\dot{\theta} - \dot{x}_2\dot{r}\cos\theta$
$+ r\sin\theta\dot{\theta}\dot{x}_2 + r^2\sin^2\theta\dot{\theta}^2 - r\sin\theta\dot{\theta}\dot{r}\cos\theta$
$- r\cos\theta\dot{x}_2 - \dot{r}\cos\theta r\sin\theta\dot{\theta} + \dot{r}^2\cos^2\theta$
$= \dot{x}_2 + r^2\sin^2\theta\dot{\theta}^2 + \dot{r}^2\cos^2\theta + 2\dot{x}_2 r\sin\theta\dot{\theta} - 2r\sin\theta\dot{\theta}\dot{r}\cos\theta$
$- 2\dot{x}_2\dot{r}\cos\theta$

$\dot{y}_1^2 = r^2\cos^2\theta\dot{\theta}^2 + 2r\cos\theta\dot{\theta}\dot{r}\sin\theta + \dot{r}^2\sin^2\theta$

$T = \frac{1}{2}m_1(\dot{x}_2^2 + r^2\sin^2\theta\dot{\theta}^2 + \dot{r}^2\cos^2\theta + 2\dot{x}_2 r\sin\theta\dot{\theta} - 2r\sin\theta\dot{\theta}\dot{r}\cos\theta$
$- 2\dot{x}_2\dot{r}\cos\theta + r^2\cos^2\theta\dot{\theta}^2 + 2r\cos\theta\dot{\theta}\dot{r}\sin\theta + \dot{r}^2\sin^2\theta)$
$+ \frac{1}{2}m_2\dot{x}_2^2$

$T = \frac{1}{2}m_1(\dot{x}_2^2 + r^2\dot{\theta}^2 + \dot{r}^2 + 2\dot{x}_2 r\sin\theta\dot{\theta} - 2\dot{x}_2\dot{r}\cos\theta) + \frac{1}{2}m_2\dot{x}_2^2$

$\mathcal{L} = T - V$ , constraint $f = r - a = 0$

$\frac{\partial f}{\partial x_2} = 0$ , $\frac{\partial f}{\partial \theta} = 0$ , $\frac{\partial f}{\partial r} = 1$

$$\frac{\partial \mathcal{L}}{\partial x_2} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_2} - \lambda\frac{\partial f}{\partial x_2} = 0$$

$$0 = 0 - \frac{d}{dt}\left(\frac{1}{2}m_1(2\dot{x}_2 + 2r\sin\theta\dot{\theta} - 2\dot{r}\cos\theta) + m_2\dot{x}_2\right)$$

$$= -\frac{d}{dt}\left(m_1(\dot{x}_2 + r\sin\theta\dot{\theta} - \dot{r}\cos\theta) + m_2\dot{x}_2\right)$$

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$$0 = -(m_1(\ddot{x}_2 + r(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2) + \dot{r}\sin\theta\dot{\theta} + \dot{r}\sin\theta\dot{\theta} - \ddot{r}\cos\theta) + m_2\ddot{x}_2)$$

$r = a$, $\dot{r} = 0$, $\ddot{r} = 0$

$$0 = -(m_1(\ddot{x}_2 + a(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)) + m_2\ddot{x}_2)$$

$$= -(m_1 + m_2)\ddot{x}_2 - m_1 a(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)$$

$$\ddot{x}_2 = -\frac{m_1 a(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)}{m_1 + m_2}$$

$$0 = \frac{\partial \mathcal{L}}{\partial \theta} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}}$$

$$= \frac{1}{2}m_1(2\dot{x}_2 r\cos\theta\dot{\theta} + 2\dot{x}_2\dot{r}\sin\theta) + m_1 g r\cos\theta$$

$$- \frac{d}{dt}\left(\frac{1}{2}m_1(2r^2\dot{\theta} + 2\dot{x}_2 r\sin\theta)\right)$$

$$= m_1(\dot{x}_2 r\cos\theta\dot{\theta} + \dot{x}_2\dot{r}\sin\theta) + m_1 g r\cos\theta$$

$$- (m_1(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} + \dot{x}_2(r\cos\theta\dot{\theta} + \dot{r}\sin\theta) + \ddot{x}_2 r\sin\theta))$$

$r = a$, $\dot{r} = 0$, $\ddot{r} = 0$

$$0 = m_1\dot{x}_2 a\cos\theta\dot{\theta} + m_1 g a\cos\theta$$

$$- m_1(a^2\ddot{\theta} + \dot{x}_2 a\cos\theta\dot{\theta} + \ddot{x}_2 a\sin\theta)$$

$$= g a\cos\theta - a^2\ddot{\theta} - \ddot{x}_2 a\sin\theta$$

$$\ddot{\theta} = \frac{g\cos\theta - \ddot{x}_2\sin\theta}{a}$$

$$0 = \frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} + \lambda$$

$$= \frac{1}{2}m_1(2r\dot{\theta}^2 + 2\dot{x}_2\sin\theta\dot{\theta}) + mg\sin\theta$$

$$- \frac{d}{dt}\left(\frac{1}{2}m_1(2\dot{r} - 2\dot{x}_2\cos\theta)\right) + \lambda$$

$$= m_1(r\dot{\theta}^2 + \dot{x}_2\sin\theta\dot{\theta}) + m_1 g\sin\theta$$

$$- (m_1(\ddot{r} + \dot{x}_2\sin\theta\dot{\theta} - \ddot{x}_2\cos\theta)) + \lambda$$

$r = a$, $\dot{r} = 0$, $\ddot{r} = 0$

$$0 = a\dot{\theta}^2 + \dot{x}_2\sin\theta\dot{\theta} + g\sin\theta - \dot{x}_2\sin\theta\dot{\theta} + \ddot{x}_2\cos\theta + \frac{\lambda}{m_1}$$

$$= a\dot{\theta}^2 + g\sin\theta + \ddot{x}_2\cos\theta + \lambda/m_1$$

$$F_r = -m_1(a\dot{\theta}^2 + g\sin\theta + \ddot{x}_2\cos\theta)$$

$$\ddot{x}_2 = -\frac{m_1}{m_1+m_2} a\left(\sin\theta\left(\frac{g\cos\theta - \ddot{x}_2\sin\theta}{a}\right) + \cos\theta\,\dot{\theta}^2\right)$$

$$= -\frac{m_1}{m_1+m_2}\left(g\sin\theta\cos\theta - \ddot{x}_2\sin^2\theta + a\cos\theta\,\dot{\theta}^2\right)$$

$$\ddot{x}_2 - \frac{m_1}{m_1+m_2}\ddot{x}_2\sin^2\theta = -\frac{m_1}{m_1+m_2}\left(g\sin\theta\cos\theta + a\cos\theta\,\dot{\theta}^2\right)$$

$$\ddot{x}_2\left(1 - \frac{m_1\sin^2\theta}{m_1+m_2}\right) = \ldots$$

$$\ddot{x}_2\left(\frac{m_1+m_2-m_1\sin^2\theta}{m_1+m_2}\right) = \ldots$$

$$\ddot{x}_2\left(m_1(1-\sin^2\theta)+m_2\right) = -m_1\left(g\sin\theta\cos\theta + a\cos\theta\,\dot{\theta}^2\right)$$

$$\ddot{x}_2\left(m_1\cos^2\theta + m_2\right) = \ldots$$

$$\ddot{x}_2 = \frac{-m_1\left(g\sin\theta\cos\theta + a\cos\theta\,\dot{\theta}^2\right)}{m_1\cos^2\theta + m_2}$$

$$F_r = -m_1\left(a\dot{\theta}^2 + g\sin\theta + \ddot{x}_2\cos\theta\right)$$

$$= -m_1\left(a\dot{\theta}^2 + g\sin\theta - \frac{m_1\left(g\sin\theta\cos\theta + a\cos\theta\,\dot{\theta}^2\right)}{m_1\cos^2\theta + m_2}\cos\theta\right)$$

$$= \frac{-m_1\left(a\dot{\theta}^2 m_1\cos^2\theta + g\sin\theta\, m_1\cos^2\theta + a\dot{\theta}^2 m_2 + g\sin\theta\, m_2 - m_1 g\sin\theta\cos^2\theta - m a\cos^2\theta\,\dot{\theta}^2\right)}{m_1\cos^2\theta + m_2}$$

$$= -\frac{m_1}{m_1\cos^2\theta + m_2}\left(a\dot{\theta}^2 m_2 + g\sin\theta\, m_2\right)$$

$$F_r = -\frac{m_1 m_2\left(a\dot{\theta}^2 + g\sin\theta\right)}{m_1\cos^2\theta + m_2}$$

3. $x = a\cos(wt) + l\sin\theta$

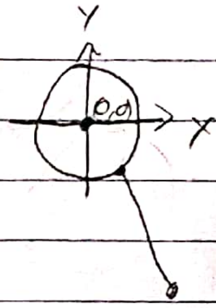
$\dot{x} = -aw\sin(wt) + l\cos\theta\dot{\theta}$

$\ddot{x} = -aw^2\cos(wt) + l(\cos\theta\ddot{\theta} - \sin\theta\dot{\theta}^2)$

$y = a\sin(wt) - l\cos\theta$

$\dot{y} = aw\cos(wt) + l\sin\theta\dot{\theta}$

$\ddot{y} = -aw^2\sin(wt) + l(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2)$




$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

$V = mgy$

$\dot{x}^2 = a^2w^2\sin^2(wt) - 2aw\sin(wt)\,l\cos\theta\dot{\theta} + l^2\cos^2\theta\dot{\theta}^2$

$\dot{y}^2 = a^2w^2\cos^2(wt) + 2aw\cos(wt)\,l\sin\theta\dot{\theta} + l^2\sin^2\theta\dot{\theta}^2$

$T = \frac{1}{2}m(a^2w^2 + l^2\dot{\theta}^2 + 2awl\dot{\theta}(\cos(wt)\sin\theta - \sin(wt)\cos\theta))$

$= \left(\frac{1}{2}[\sin(wt+\theta) - \sin(wt-\theta)] - \frac{1}{2}[\sin(wt+\theta) + \sin(wt-\theta)]\right)$

$= \frac{1}{2}[-2\sin(wt-\theta)] = -\sin(wt-\theta)$

$T = \frac{1}{2}m(a^2w^2 + l^2\dot{\theta}^2 - 2awl\dot{\theta}\sin(wt-\theta))$

$V = mg(a\sin(wt) - l\cos\theta)$

$\mathcal{L} = T - V$

$\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = ml^2\dot{\theta} - mawl\sin(wt-\theta)$

$\frac{\partial\mathcal{L}}{\partial\theta} = mawl\dot{\theta}\cos(wt-\theta) - mgl\sin\theta$

$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = ml^2\ddot{\theta} - mawl\cos(wt-\theta)(w-\dot{\theta})$

$0 = \frac{\partial\mathcal{L}}{\partial\theta} - \frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = mawl\dot{\theta}\cos(wt-\theta) - mgl\sin\theta$
$\quad - ml^2\ddot{\theta} + mawl\cos(wt-\theta)(w-\theta)$

$\ddot{\theta} = \frac{1}{l}aw\cos(wt-\theta)(1-w+\theta) - \frac{g}{l}\sin\theta$