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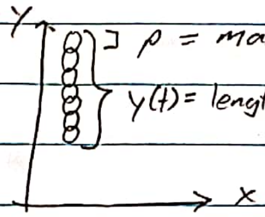

ρ = mass per unit length

$y(t)$ = length hanging at time t

1. $F_{ext} = m\dot{v} + v\dot{m}$

$mg = m\dot{v} + v\dot{m}$

$\rho y g = \rho y \ddot{y} + \dot{y}\rho\dot{y}$

$\ddot{y} = \frac{d\dot{y}}{dt} = \frac{d\dot{y}}{dy} \cdot \frac{dy}{dt} = \frac{1}{2}\frac{d\dot{y}^2}{dy}$

$\rho y g = \rho y \left(\frac{1}{2}\frac{d\dot{y}^2}{dy}\right) + \rho\dot{y}^2$

$yg = \frac{1}{2} y(\dot{y}^2)' + \dot{y}^2$

$(\dot{y}^2)' + \frac{2}{y}\dot{y}^2 = 2g \rightarrow$ (First-Order, Non-linear ODE)

Integrating factor: $r(t) = e^{\int \frac{2}{y} dy} = e^{2\ln y} = y^2$

$y^2(\dot{y}^2)' + 2y\dot{y}^2 = 2gy^2$

$\frac{d}{dy}(y^2\dot{y}^2) = 2gy^2$

$y^2\dot{y}^2 = \int_b^a 2gy^2\, dy = \left[\frac{2}{3}gy^3\right]_b^a = \frac{2}{3}g(a^3-b^3)$

$\dot{y}^2 = \frac{2g(a^3-b^3)}{3y(t)^2} = v(t)^2$

$v(t) = \left(\frac{2g(a^3-b^3)}{3y(t)^2}\right)^{1/2}$

At time t_f, $y(t_f) = a$

$v(t_f) = \left(\frac{2g(a^3-b^3)}{3y(t_f)^2}\right)^{1/2}$

$v_f = \left(\frac{2g(a^3-b^3)}{3a^2}\right)^{1/2}$

2. Before the drop each person carries half the weight

$F_{o(1)} = \frac{1}{2} mg$

$\tau_1 = F \cdot d = mg \cdot \frac{1}{2} l = I \dot{\omega}$, $I_{rod} = \frac{1}{3} m l^2$

$\frac{1}{2} mgl = \frac{1}{3} m l^2 \dot{\omega}$

$\dot{\omega} = \frac{3}{2} g/l$

$F_{F(1)} = mg - ma_{com} = mg - m \frac{l}{2} \dot{\omega} = mg - \frac{3}{4} mg$

$F_{F(1)} = \frac{1}{4} mg$ down

On free end,

$a = \dot{\omega} \cdot l = \frac{3}{2} g/l \cdot l = \frac{3}{2} g$