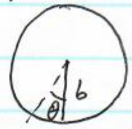
Nicholas Pun
27872167

Phys 216 Homework 3

1.



$$E_f = mgh_{max} = \frac{1}{2}mv_y^2 + mg(b - b\cos\theta_c) = E_0$$

$$h_{max} = \frac{v_y^2}{2g} + b(1-\cos\theta_c)$$

$$h_{max} = \frac{1}{2g}v_0^2\sin^2\theta_c + b(1-\cos\theta_c)$$

$$h = \frac{1}{2g}v_0^2\sin^2\theta + b(1-\cos\theta)$$ and a function of θ

h is a $^{(-)}$ parabolic function, so h_{max} is when $\frac{d}{d\theta}h = 0$

$$\frac{d}{d\theta}h = \frac{1}{g}v_0^2\sin\theta_c\cos\theta_c + b\sin\theta_c = 0$$

$$\frac{1}{g}v_0^2\sin\theta_c\cos\theta_c = -b\sin\theta_c$$

$$\frac{1}{g}v_0^2\cos\theta_c = -b$$

$$\cos\theta_c = \frac{-gb}{v_0^2}, \quad \theta_c = \cos^{-1}\left(\frac{-gb}{v_0^2}\right),$$ where θ_c is the angle at which the mud leaves wheel

$$h_{max} = \frac{1}{2g}v_0^2(1-\cos^2\theta_c) + b(1-\cos\theta_c)$$

$$= \frac{1}{2g}v_0^2\left(1-\left(\frac{-gb}{v_0^2}\right)^2\right) + b\left(1-\left(\frac{-gb}{v_0^2}\right)\right)$$

$$= \frac{v_0^2}{2g} - \frac{gb^2}{2v_0^2} + b + \frac{gb^2}{v_0^2}$$

$$h_{max} = b + \frac{v_0^2}{2g} + \frac{gb^2}{2v_0^2}$$ //

2.a) $$m\vec{a}' = F - m\dot{\vec{\omega}}\times\vec{r} - 2m\vec{\omega}\times\vec{v}' - m\vec{\omega}\times(\vec{\omega}\times\vec{r}') - mA$$

$$\dot{\omega} = 0, \quad A = 0$$

$$m\vec{a}' = F - 2m\vec{\omega}\times\vec{v}' - m\vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$F = m\vec{a}' + 2m\vec{\omega}\times\vec{v}' + m\vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$-F_f = \mu_s mg = m\vec{a}' + 2m\vec{\omega}\times\vec{v}' + m\vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$-\mu_s g = \vec{a}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$ ← point of slip so $a = 0$

$$\vec{a}' = -\frac{(v')^2}{b}\hat{e}_r, \quad \vec{\omega} = \omega\hat{e}_z, \quad \vec{r}' = b\hat{e}_r, \quad \vec{v}' = v'\hat{e}_\theta$$

$$\mu_s g = -\vec{a}' - 2\vec{\omega}\times\vec{v}' - \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$= \frac{(v')^2}{b} + 2\omega v' + \omega^2 b$$

$$0 = (v')^2 + 2b\omega v' + \omega^2 b^2 - \mu_s gb$$ ← quadratic

$$v' = \frac{-2b\omega \pm \sqrt{4b^2\omega^2 - 4(\omega^2b^2 - \mu_s gb)}}{2}$$

$$= -b\omega \pm \sqrt{b^2\omega^2 - \omega^2b^2 + \mu_s gb}$$

$$v'_{max} = -b\omega + \sqrt{\mu_s gb}$$ // ← want (+) sqr root

b) $\vec{v}' = -v' \hat{e}_\theta$

$$\mu_s g = \frac{(v')^2}{b} - 2wv' + w^2 b$$

$$0 = (v')^2 - 2wbv' + w^2 b^2 - \mu_s g b$$

$$v'_{max} = bw + \sqrt{\mu_s g b} \quad //$$

x_final = -123.313229 m, y_final = 182.431474 m

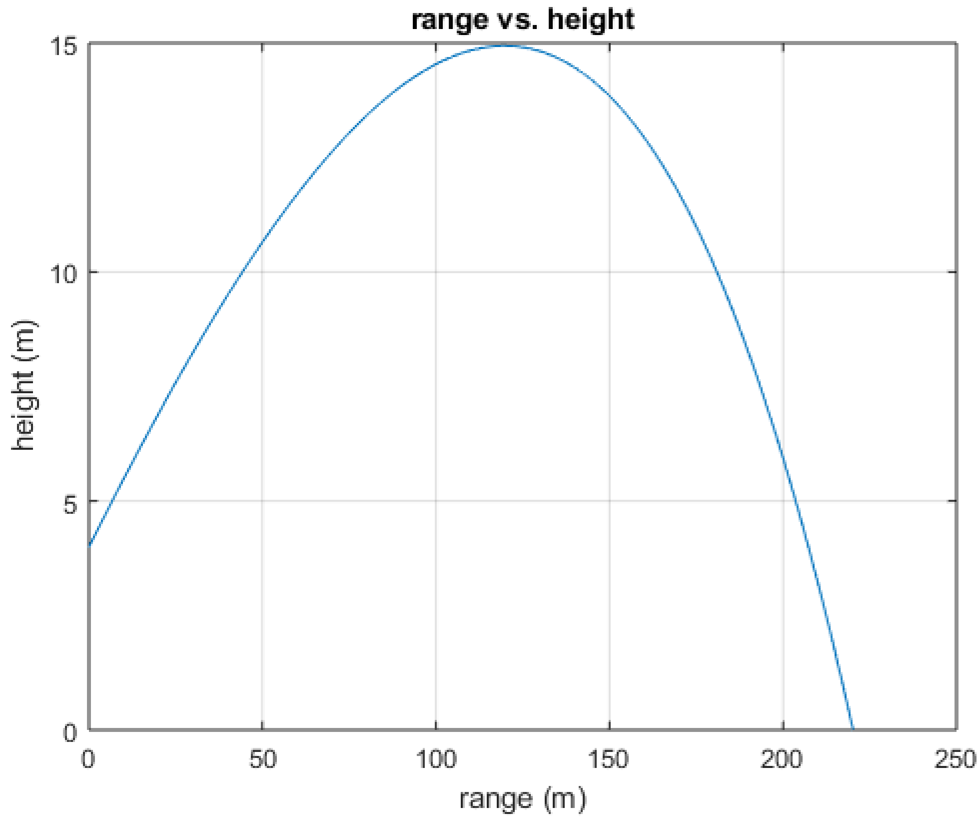
I calulated these numbers by first using a very small timestep

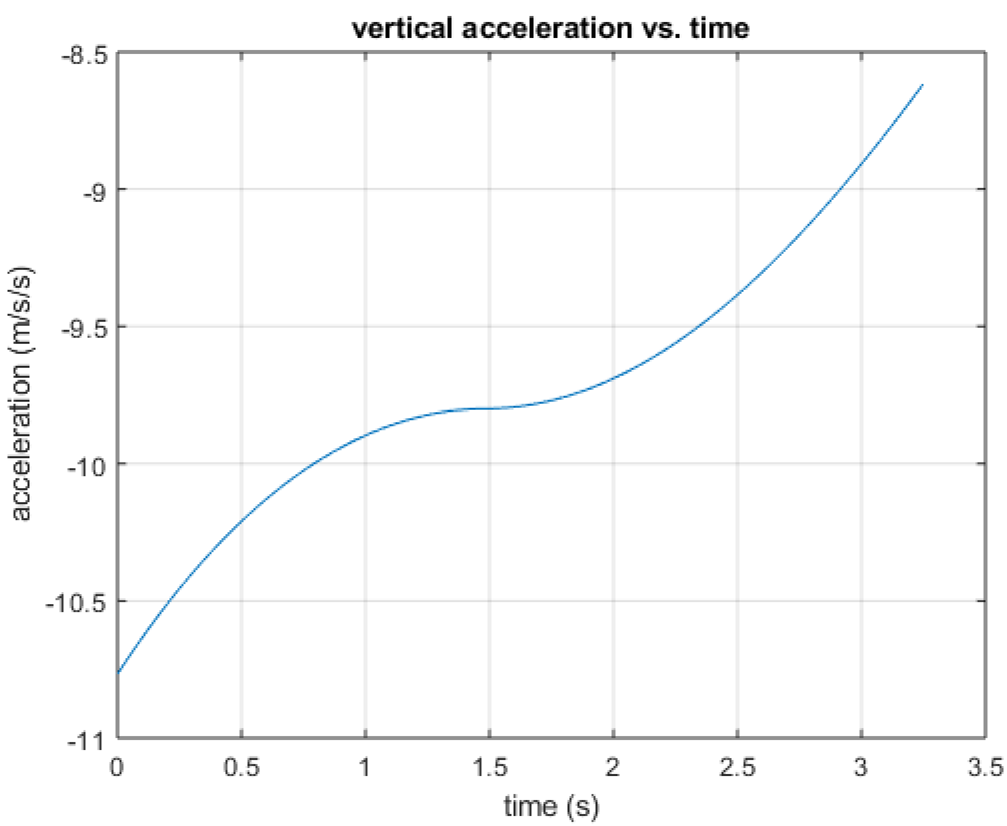
to produce an accurate result, then increased the timestep until is was no longer within 1cm error

altitude = 19.000000 degrees, azimuth = 100.000000 degrees

flight time = 3.247130 s

velocity vector final = 51.583635 m/s


range vs. height
height (m)
range (m)
0
5
10
15
0
50
100
150
200
250


vertical acceleration vs. time
acceleration (m/s/s)
time (s)
-8.5
-9
-9.5
-10
-10.5
-11
0
0.5
1
1.5
2
2.5
3
3.5