Experiment #2
LCR Resonance Circuit *

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1 Pre lab exercise (due at 2 PM of the day of your lab)

- Consider the circuit in Fig. 1. What are γ and ω_0 in Eqns 1, 4 and 5 in units of inverse seconds? **(1 mark)**
- Assume V_o and V_{in} in Eqns 1 and 5 are both +1V. Use the above values of γ and ω_0 to make plots of Eqns. 1 and 5 with units on the x and y axes. Modify the Python script Plotfunction.py for this purpose. **(2 marks)**
- In section 4.1 estimate the period of the square wave you need to observe 10 oscillations in $V_r(t)$. **(1 mark)**

2 Objective

The properties of a series LCR circuit will be examined in the time and frequency domain. First, the voltage drop across the resistor (the output voltage) will be measured as a function of time after a sudden change in the otherwise constant (DC) input voltage driving the circuit. Secondly, the circuit will be driven with a sinusoidal (AC) input voltage. The amplitude and the phase shift of the output voltage will be measured as a function of the frequency of the input voltage. The data will be fit using Python and the fitted parameters will be compared with each other and with theory.

3 Introduction/Background

A mass suspended on the end of a spring acts like a simple harmonic oscillator with a sinusoidal displacement around equilibrium. If the model is refined by adding a damping force proportional to the velocity of the mass, the resulting

*This lab writeup is based on a similar lab in ENPH 259 by David Jones motion is a damped. As we will see in the tutorial an equation of a similar form can be used to model the current in a series LCR circuit (shown in Fig. 1).

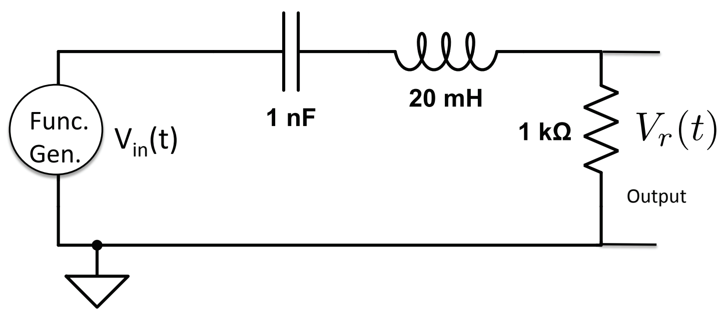

Figure 1: Resonant series LCR circuit. The properties of the circuit can be analyzed by (1) making a sudden change to an otherwise constant input voltage V_{in}^0 or by (2) driving the circuit with a sinusoidal input voltage $V_{in}(t)$. The voltage across the resistor is (output voltage from the circuit) is a measure of the current in the circuit.

As we will show in the tutorial the transient time response of the voltage across the resistor, $V_r(t)$, after the input voltage instantaneously switches from a constant (DC) value V_o to zero, is approximately given by:

$$V_r(t) = \frac{2V_o}{\omega_o \tau} \exp\left(-t/\tau\right) \cos(\omega_o t + \phi_o) \tag{1}$$

where $1/\tau = R/2L$ is the damping rate, $\omega_o = 1/\sqrt{LC}$ is the resonant *angular frequency* and $\phi_o = \pi/2$ is the phase shift relative to a cosine. The approximation is valid in the so called under-damped limit where $\omega_o \tau >> 1$. Note the resonant frequency $f_o = 1/(2\pi\sqrt{LC})$.
Alternately, if the input voltage is sinusoidal,

$$V_{in}(t) = V_{in}^0 \cos\left[\omega t\right] \tag{2}$$

the output voltage across the resistor is an undamped sinusoidal but with an amplitude and phase which depend on the frequency of the input voltage:

$$V_r(\omega, t) = V_r^0(\omega) \cos\left[\omega t + \phi(\omega)\right] \tag{3}$$

One can derive explicit expressions for the amplitude $V_r^0(\omega)$ and phase $\phi(\omega)$ versus the angular frequency ω of the input voltage. The output amplitude:

$$V_r^0(\omega) = \frac{V_{in}^0}{\sqrt{1 + \left(\omega \frac{L}{R} - \frac{1}{\omega CR}\right)^2}} = \frac{V_{in}^0}{\sqrt{1 + \left(\frac{1}{\gamma\omega}\right)^2 (\omega^2 - \omega_o^2)^2}} \tag{4}$$

where $\omega_o = 1/\sqrt{LC}$ is the resonant angular frequency and $\gamma = R/L$ is a parameter which determines the width of the resonance. One can rewrite this as a function of frequency $f = \omega/2\pi$:

$$V_r^0(f) = \frac{V_{in}^0}{\sqrt{1 + \left(\frac{2\pi}{\gamma f}\right)^2 (f^2 - f_o^2)^2}} \tag{5}$$

where $f_o = 1/(2\pi\sqrt{LC})$ is the resonant frequency.

The predicted phase shift may be obtained from the tutorial notes on the LCR circuit. However note that the phase shift in the notes is for $q(t)$ on the capacitor whereas the current in the circuit is proportional to $\frac{dq}{dt}$ which introduces a shifted by $\pi/2$. Why is this?

4 Experiment

4.1 Transient Response

Assemble the circuit shown in Fig. 1. Make sure you configure the inductor bank to give you the correct inductance. To investigate the transient response the input must be switched from a constant DC value to zero. As in the RC lab you can conveniently accomplish this by applying a low frequency square wave from the function generator so that the voltage switches between a positive value and zero. Discuss with your partner what low frequency means in this context and explain your reasoning. Use the oscilloscope to observe the output voltage across the resistor $V_r(t)$. Split the OUTPUT signal from the generator and use one of the signals to trigger the oscilloscope and the other as $V_{in}(t)$ to drive the circuit. Make sure the input to the scope is DC coupled. Describe what you observe in $V_r(t)$ and compare with Eqn 1. How can you adjust the vertical and horizontal scales to make the most accurate measurement? Discuss this with your partner and one other group and explain your reasoning. Remember your goal is to determine both the oscillation frequency and damping term and also to test the theoretical expression given by Eqn 1. Create a .csv file of the data from the scope on a USB stick. Transfer the data to your labtop and use Python script Curvefitexpcos.py to pack and fit the data to Eqn 1. You will need to determine the random uncertainty (point to point variation) in $V_r(t)$ (ysigma) using the histogram of residuals. Recall ysigma is one of the input parameters for the program. Report the fitted parameters and their uncertainties. Discuss the quality of the fit. Try changing the range of the fit to see how this affects the fitted parameters. What does this say about the model and how well you can determine the parameters of interest? Make sure you can fit the data before leaving for the day.

Before leaving for the day place the resistor(s) and capacitor(s)in an envelope with your names on it and put it in a drawer under the bench so you can use the same resistor(s) and capacitor(s) for the next part of the experiment. Don't put away the inductor but make sure you label it so you can use the same one for the next section. This will help you make a better comparison of resonant frequency and damping rate measured in the time domain with that you will measure in the frequency domain the following week.

4.2 Steady State AC Response

Use the same resistor, capacitor and inductor as you used in the time domain measurements described in the previous section.

To investigate the AC frequency response of the circuit, switch the function generator to a sinusoidal output and observe both the input voltage $V_{in}(t)$ to the circuit and the output voltage $V_r(t)$ on two channels of the oscilloscope. Measure the amplitude of V_r with a fixed V_{in}^0 over a frequency range wide enough to clearly show the resonant behaviour and test the validity of Eqn 5 at the extreme values (between 10 Hz and 150 kHz). Record the measurements and uncertainties in a spread sheet which you can save to a .csv file. The first column should be the frequency $f = \omega/2\pi$, the second column the amplitude of $V_r(t)$ and the third column the uncertainty in the amplitude of $V_r(t)$. Make a rough measurement first and then fill in any regions of interest. Pay particular attention to the frequencies in the vicinity of the resonance peak. There are 3 points of particular interest:the resonant frequency (where V_r reaches its maximum value) and the two points on either side of the resonance (where $V_r/V_{in} = 1/\sqrt{2}$. The frequency difference between these last two points is known as the *bandwidth*. It is also important to investigate the behaviour on both sides of the resonance to see how well the theory does well away from the resonance. Explain why you think this might be important? Think in terms of the assumptions being made to arrive at the theory.

Use the Python script Curvefitlcrres.py to read the .csv file and fit the data to Eqn 5. Look at the residuals to decide if this is a good fit. Where does it do well and where does it have problems? Decide how to modify the fitting function to improve the fit? Discuss this with your partner. Modify the script accordingly and see how it affects the Chi^2 and the fitted parameters.

4.3 Comparison between the Transient and Steady State Response

From the transient and steady state measurements you now have two independently determined values of the resonance frequency of your circuit. What is the T-score and discuss what it means. In addition, you found a second parameter in each case: the damping term in the transient response and the bandwidth in the steady state response. Is there underlying agreement in the ratio R/L? Discuss.

4.4 Influence of R on the LCR Resonance

Replace the 1 kOhm resistor with a 100 Ohm resistor and remeasure the transient response. Adjust the frequency of the square wave appropriately. Save the data to a .csv file and refit using the Curvefitexpcos.py. Compare the fitted parameters with what you expect based on the ratio between the two resistors measured with a DMM. Discuss any significant differences.

4.5 Phase shift of the Output Signal

Measure the phase shift of the $V_r(t)$ relative to $V_{in}(t)$ on resonance, well above resonance, and well below resonance. Do this by downloading the two signals from the scope. Fit the data to Curvefitsin.py. Comment on what you find. Compare your observations with the theory for the LCR circuit in the tutorial notes.

5 Conclusion

In your lab writeup make sure you answer all questions and include all circuit designs with explanations. In addition you should compare your two measured values for the resonant frequency and bandwidth/damping term along with the expected values based on what you know about R, L and C. As always include an uncertainty for each measurement. Discuss how the measurement agree or not and how they compare with expected values based on measured or nominal values of R, L and C. Discuss how you might improve the measurement or the theory. Finally reflect on what the lab has taught you about experimental physics.