Signal Filtering and Deconvolution

Filters

- Filters are mathematical functions that are multiplied into the frequency domain set to change the frequency distribution of the power spectrum.
- A filter is a magnitude vs. frequency function.

Ramp Filters

- A Ramp Filter emphasizes the high frequency components in our data sets.
- Therefore, a ramp filter enhances or emphasizes noise in our data sets.
- Since we use the ramp filter for filtered back projection reconstruction, we must somehow get rid of this additional noise in our image data sets. (smoothing filters)

3 Types of filters

- Low pass
  - Used to allow low frequencies through the filter and to reduce high frequencies (to attenuate)
- High pass
  - Used to allow high frequencies through the filter and to reduce or attenuate low frequencies.
  - Used to enhance edges or small objects
- Band pass
  - Used to allow a certain band or range of frequencies through the filter.

Filter Characteristics

- Characterized by 2 parameters
    1. Cutoff frequency
    2. Order

Note: Different facilities and manufacturers use different definitions and/or units to describe similar filters.

When using filters, make sure to know which set of definitions and units you are working with.

Commonly used filters

- Ramp filter
- Hann filter
- Hamming filter
- Shepp-Logan
- Parzen filter
- Buttersworth filter
- Combination filter
- Weiner Filter (for restoration)
- Metz Filter (for restoration)

Ramp Filters

- High pass filter
- Used for edge enhancement
- Drawback: Propagate high frequency noise in the images

$$\left|H(\omega)\right|^2 = m\omega,\ \text{ m is positive}$$

Hann and Hamming Filter

- Low pass filters
  - Used to remove high frequency noise
- The only parameter used to describe a Hann or Hamming filter is it's cutoff frequency.
- **Hamming =** $\left|H(\omega)\right|^2 = 0.5 + 0.5\cos\left(\dfrac{\omega}{n}\right)$
- **Hann =** $\left|H(\omega)\right|^2 = 0.54 + 0.46\cos\left(\dfrac{\omega}{n}\right)$
  - where n = *cut-off frequency (point where amplitude = 0)*

Hann vs. Hamming filters

- Functionally, the Hann and Hamming filters are very similar except that the Hamming filter goes to a non-zero value at the Nyquist Frequency

- These filters are used for studies where higher statistical accuracy is needed at the expense of a loss in spatial resolution.

Butterworth Filter

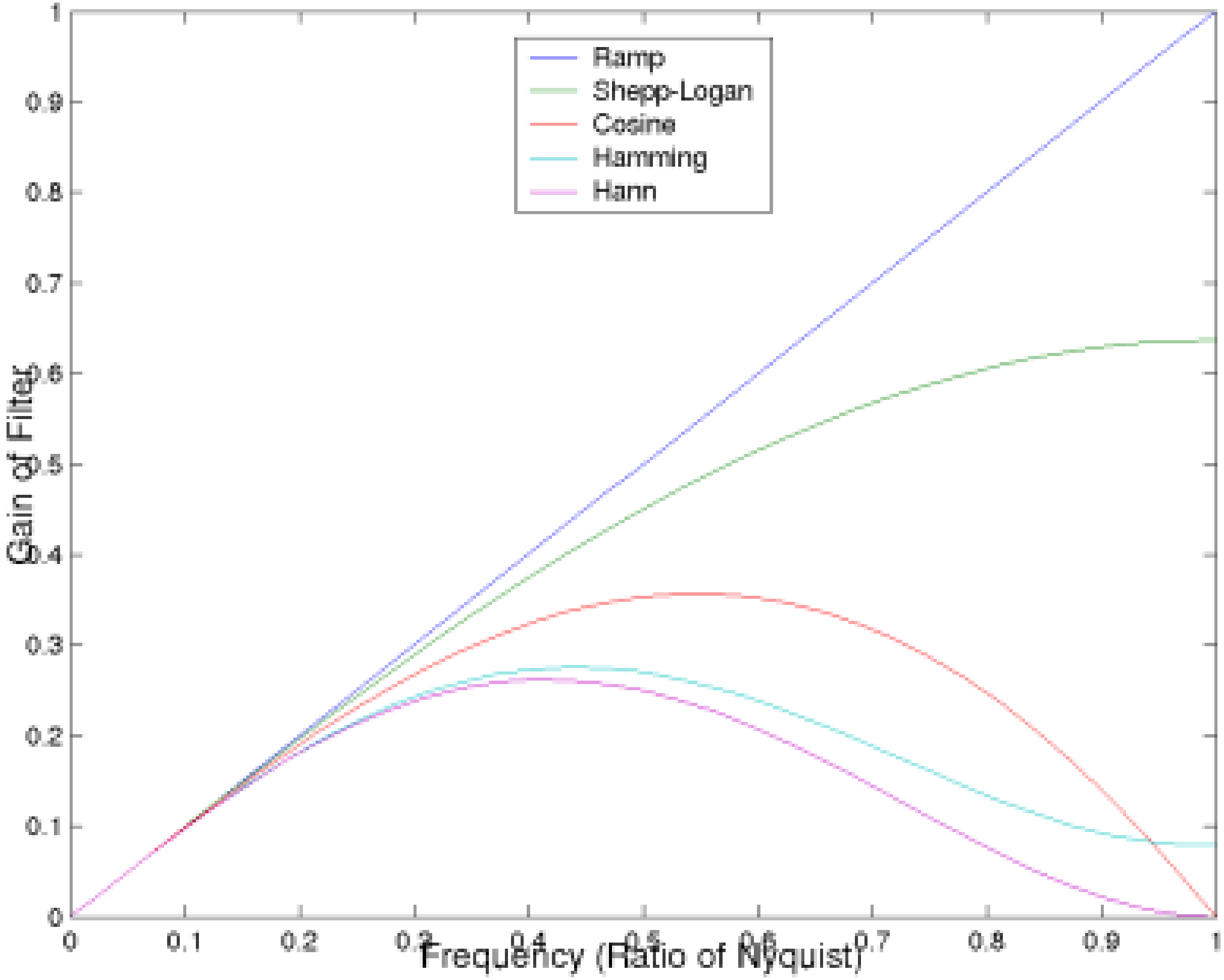
- Low pass filter
  - Better suited for studies where higher resolution needed to be preserved at the expense of higher statistical count fluctuations.
  - A BF needs 2 parameters to describe the filter.
    - Cutoff frequency
    - Order of the filter

$$\left|H(\omega)\right|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}$$

  - The order of the filter is related to how fast the filter is cutoff.

Cutoff Frequency & Order

- Cutoff
  - Allows us to retain information at higher frequencies while still eliminating noise.
  - Remember, frequency and size are related by the Fourier transform (larger objects are represented by lower frequencies of sine and cosines)
- Order
  - Determines how quickly the transition is made between frequencies that are kept and frequencies that are eliminated



Ramp
Shepp-Logan
Cosine
Hamming
Hann
Gain of Filter
Frequency (Ratio of Nyquist)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Combined Filters

- Filters designed to deconvolve (remove) both PSF and noise
  - **Wiener filter**
    - In frequency domain, Wiener filter *G(f) is given by:*

$$G(f) = \frac{H^*(f)S(f)}{|H(f)|^2 S(f) + N(f)}$$

    - G(f) and H(f) are the FT's of g and h (the PSF), respectively, S(f) is the mean power spectral density of the input signal, and N(t) is the mean power spectral density of the noise

The Metz Filter

- Modification to inverse filter.

$$L_M = \frac{1-\left[1-H^2(\omega)\right]^{\chi}}{H(\omega)}$$

Where H is the FT of the PSF

  - Supresses the high frequency noise instead of amplyfying it.
  - Selection of factor χ such that that mean-square error (MSE) between ideal and filtered spectrum is minimized.

- Alternative to H(w) as transform of PSF

$$H(\omega) = e^{\frac{-\omega^P}{S}}$$

Setting Filter Parameters

- The Cutoff Frequency is probably the most important parameter in filtering work.
  - The CF should be chosen based on the Frequency space distribution of the data and the associated noise level in the images.
  - One criteria is to set the CF to a value approximately equal to the NF
  - Match the point of the filter where it drops to zero.
- Note: Noise level in spectra or images will depend on the count density
  - i.e. how many counts per channel/pixel
- The higher the count density the lower the noise level in relation to the image's power spectrum
  - Think signal to noise ratio

Order or Roll off of the filter

- How quickly the transition is made between frequencies that are kept and frequencies that are eliminated.
- The only real “rule of thumb” is that if the order is set too high, then oscillations in signal intensity will be introduced.