* Code and graphs in GitHub Repo

Homework 1

1) * The normal equation will be the gradient of $\mathcal{L}$ with respect to w_0 and w_1

$$\frac{\partial \mathcal{L}}{w_0} = \sum_{i=1}^{N} g_i \left| y_i - (w_0 + w_1 x_i) \right| \cdot (-1)$$

$$= -\sum_{i=1}^{N} g_i \left| y_i - (w_0 + w_1 x_i) \right|$$

$$\frac{\partial \mathcal{L}}{w_1} = -\sum_{i=1}^{N} g_i \left| y_i - (w_0 + w_1 x_i) \right| x_i$$

2). $\hat{w}_0 = \bar{t} + w_1 \bar{x}$ ($\bar{t}$ = y, $\bar{x}$ = x)

$\hat{w}_1 = \frac{\overline{xt} - \bar{x}\bar{t}}{\overline{x^2} - (\bar{x})^2}$ → avg. of x times avg. of t

avg. of $x \cdot t$

$t = w_0 + w_1 x$

My regression achieved an average error of 0.1856825

3)

3	-4
1	-1
0	1
-1	2
-3	3

$\bar{t} = 1/5 \quad \bar{x} = 0$

$\overline{xt} = -24/5 \quad \bar{x}\bar{t} = 0 \quad w_1 = \frac{-24/5}{4} = \frac{-24}{20} = \frac{-12}{10} = \frac{-6}{5}$

$\overline{x^2} = 4 \quad (\bar{x})^2 = 0$

$t = 1/5 - 6/5 x$

Linear : $w_0 + w_1 x$

Quadratic: $w_0 + w_1 x + w_2 x^2$

Cubic: $w_0 + w_1 x + w_2 x^2 + w_3 x^3$

→ I calculated the coefficients in python

$$\begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & -3 & 9 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{bmatrix} \quad \begin{bmatrix} 1 & -3 & 9 & -27 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \\ -4 \end{bmatrix}$$

$\hat{w} = (x^T x)^{-1} x^T t$

4) $\ln(h(x)) = \ln(w_0 e^{w_1 x}) = \ln(w_0) + w_1 x = b + w_1 x$ where $b = \ln(w_0)$

now we perform linear regression (using my script) and arrive at $b = 39.005$

$39.005 = \ln(w_0)$ $\quad w_0 = e^b = 8.703 \times 10^{16}$

$\ln(h(x)) = 8.703 \times 10^{16} e^{-0.015x}$

$e^{\ln(h(x))} = e^{8.703 \times 10^{16} e^{-0.015x}}$

$h(x) =$

$\ln(h(x)) = \ln(w_0) + \ln(e^{w_1 x})$

$\ln(h(x)) = \ln(8.703 \times 10^{16}) + \ln(e^{-0.015x})$

$h(x) = e^{\ln(8.703 \times 10^{16}) + -0.015x}$

$h(x) = e^{\ln(8.7 \times 10^{16})} \cdot e^{-0.015x}$

$\boxed{h(x) = 8.703 \times 10^{16} e^{-0.015x}}$

5) $\hat{w} = (x^T x)^{-1} x^T t$

5) On GitHub