Math 452 Homework 2

Due September 8, 2023

1. Let $x_1, x_2, \cdots, x_n \in \mathbb{R}$ be a set of i.i.d. random variables sampled from Gaussian distribution. Find the MLE of the Gaussian mean and variance.

2. Assume that a data set of binary values, $0, 1, 1, 1, 0$, are sampled from a Bernoulli distribution. Compute the MLE of the Bernoulli parameter q.

3. Let $\boldsymbol{x} \in \mathbb{R}^2$ be a 2 dimensional Gaussian random variable with zero mean and covariance

$$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

Compute the marginal $p(x_1)$.

4. Following the previous problem, compute $p(x_1|x_2 = 1)$.

5. (Computer project) The mean height and weight of boys ages 2-11 were collected in the U.S. National Health and Nutrition Examination Survey by the Centers for Disease Control (CDC) in 2002, resulting in the following table. Apply MLE to determine the model $y = w_0 + w_1 x + w_2 x^2 + \epsilon$ with x being the height, y being the weight and $\epsilon \sim \mathcal{N}(0, \sigma^2)$. Find an estimate of σ. Attach you code.

age (yrs.)	height (m)	weight (kg)
2	0.9120	13.7
3	0.9860	15.9
4	1.0600	18.5
5	1.1300	21.3
6	1.1900	23.5
7	1.2600	27.2
8	1.3200	32.7
9	1.3800	36.0
10	1.4100	38.6
11	1.4900	43.7