Homework 2

1). $p(x_1)p(x_2)\dots p(x_n)$

$\downarrow$

max $\Rightarrow$ max $\log p(x_1)p(x_2)\dots p(x_n)$

$= \max \sum_N \log(p(x_i)) = \max \sum_N \log(p(x_i))$

$\max \sum \log e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} = \frac{(-x_i-\mu)^2}{2\sigma^2}$

$\frac{\partial}{\partial x_i} \frac{(-x_i-\mu)^2}{2\sigma^2}$

2). $0\ 1\ 1\ 1\ 0 \longrightarrow (1-q)qqq(1-q)$

$= (1-q)^2 q^3 \longrightarrow$ maximize

$\Rightarrow \log(1-q)^2 q^3 = 2\log(1-q) + 3\log q$

$\frac{d}{dq} = -\frac{2}{1-q} + \frac{3}{q} = 0 \qquad \frac{3}{q} = \frac{2}{1-q}$

$3(1-q) = 2q \qquad q = \frac{3}{2}(1-q)$

$$\boxed{q = 3/5}$$

3). $p(x_1) = \int p(x,y)\,dy \qquad p(x,y) = \frac{1}{2\pi|\Sigma|^{1/2}} \exp(-1/2 (x,y)\Sigma^{-1}\begin{pmatrix} x \\ y \end{pmatrix})$

$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad |\Sigma| = 3 \qquad p(x_1) = \frac{1}{2\pi\sqrt{3}} \int e^{-(x^2-xy+y^2)}\,dy$

$\Sigma^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$(x\ y)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 2x-y \\ -x+2y \end{bmatrix}(x\ y) = \boxed{2x^2-2xy+2y^2}$

3). (cont.) $p(x_1) = \frac{e^{x^2}}{2\pi\sqrt{3}} \int e^{-(y^2 - xy)} dy$

$$p(x_1) = \frac{e^{x^2}}{2\pi\sqrt{3}} \int e^{y^2 - \frac{x^2}{4}y} = \int e^{y(y+\frac{x^2}{4})} \propto \int e^{y^2}$$

$$\boxed{p(x_1) = \frac{e^{x^2}}{2\pi\sqrt{3}} \int e^{y^2} dy}$$

4). $p(x_1 | x_2 = 1) = \frac{p(x_2 = 1 | x_1)}{p(x_2)}$ $\quad * x_2 = y$

$p(x_2 = 1 | x_1) = \frac{e^{(1)^2}}{2\pi\sqrt{3}} \int e^{y^2} dy$ do problem 3 again

$$p(x_2) = \frac{1}{2\pi\sqrt{3}} \int e^{-(x^2 - xy + y^2)} dx = \frac{e^{y^2}}{2\pi\sqrt{3}} \int e^{-(x^2 - xy)}$$

$$= \frac{e^{y^2}}{2\pi\sqrt{3}} \int e^{(x - \frac{y}{2})^2 - \frac{y}{4}x} dx \propto \frac{e^{y^2}}{2\pi\sqrt{3}} \int e^{x^2 - \frac{y^2}{4}x} dx$$

$$= \int e^{x(x - \frac{y^2}{4})} dx \propto \frac{e^{y^2}}{2\pi\sqrt{3}} \int e^{x^2} dx$$

$$\boxed{p(x_1 | x_2 = 1) = \frac{\frac{e}{2\pi\sqrt{3}} \int e^{y^2} dy}{\frac{e^{y^2}}{2\pi\sqrt{3}} \int e^{x^2} dx}}$$

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5). 2 steps: 1 do regular linear regression

$$\boxed{\begin{aligned} W_0 &= -36.9188 \\ W_1 &= 52.751 \end{aligned}}$$

My script estimated r^2 to be 2.5578

$$\sigma^2 = \frac{1}{N} \sum_N (t_n - X^T \hat{w})^2$$