Homework 4

$f(x) = x\log x - x$

1). f is convex if $\nabla^2 f(x) \geq 0$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$\downarrow$

laplace operator

$$\frac{\partial f}{\partial x} = \log x + \frac{1}{\ln 10} - 1$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{x \ln 10}$$

$\boxed{\frac{1}{x\ln 10} \geq 0 \text{ for all } x > 0}$

Strongly convex if: $f(y) \geq f(x) + \nabla f(x)^T (y-x) + \frac{\mu}{2}\|y-x\|^2$

$$f(y) \geq f(x) + \left(\log x + \frac{1}{\ln 10} - 1\right)(y-x) + \|y-x\|^2$$

$f''(x)$ is close to 0 therefore $x \to \infty$ $f(x)$ looks linear

therefore it cannot be lower bounded by a quadratic and is not strongly convex

2) $\nabla f(x) = \begin{cases} 2x, & x < 0 \\ x, & x \geq 0 \end{cases}$

$x < 0$: $|f(x_1) - f(x_2)| \leq M|x_1 - x_2|$

$|2x_1 - 2x_2| \leq M|x_1 - x_2|$

$2|x_1 - x_2| \leq M|x_1 - x_2| \Rightarrow$ any $M > 2$ will satisfy lipschitz continuity for $x < 0$

$x > 0$: $|x_1 - x_2| \leq M|x_1 - x_2|$

$\Rightarrow$ any $M \geq 1$ will satisfy lipschitz continuity for $x > 0$

3) $|\theta_k - \theta_*| =$ something linear $(1.2 - \alpha)$

$\nabla f(\theta) = M(\theta - \theta_*)$

At each iteration:

$\underbrace{\theta = \theta - M(\theta - \theta_*)\alpha}_{\text{linear function}}$

$\theta_k > \alpha M(\theta - \theta_k)$

$\boxed{\alpha < \frac{\theta_k}{M(\theta - \theta_k)}}$

$\theta = \theta_k - \alpha \nabla f(\theta_k)$

$2M\alpha < 0$

$M\alpha < 0$

$\theta = \theta_k - \alpha M(\theta - \theta_k)$

$\theta = \theta_k - \alpha M\theta - \alpha M\theta_k$

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