

Homework 7

17.

(1,1)	(1.5,2)	(3,4)	(5,7)	(3.5,5)	(4.5,5)	(3.5,4.5)
0	1	1	1	1	1	1

updated centroids: (1,1)
(3.375, 4)

$$E[f(x)] \geq f(E(x))$$

2). $\frac{1}{n} \sum x_i = E[x_i]$ for uniform distribution

$$f(x) = x^3$$

$$E[x^3] \geq (E[x])^3$$

$$\boxed{\frac{1}{n} \sum x_i^3 \geq \left(\frac{1}{n} \sum x_i\right)^3} \quad \checkmark$$

3). Weighted likelihoods after one iteration: $q_{n1} = (1, 1, 1, 1, 0, 0)$ $q_{n2} = (0, 0, 0, 0, 2, 1, 1)$

$$M_i = \frac{\sum q_{ni} x_n}{\sum q_{ni}} = (1.43, 1.8) = M_1$$

$$M_2 = (4.77, 4.71)$$

$$\Sigma_i = \frac{\sum q_{ni} (x_n M_i) (x_n M_i)^T}{\sum q_{ni}} = \Sigma_1 = \begin{pmatrix} 0.36 & 0.31 \\ 0.31 & 0.5 \end{pmatrix}$$

$$\Sigma_2 = \begin{pmatrix} 1.47 & 0.52 \\ 0.52 & 0.21 \end{pmatrix}$$